Accelerated quark and holography for confining gauge theory

Kazuo Ghoroku,[1,*] Masafumi Ishihara,[2,†] Kouki Kubo,[3,‡] and Tomoki Taminato[3,§]

[1]*Fukuoka Institute of Technology, Wajiro, Higashi-ku Fukuoka 811-0295, Japan*

[2]*Department of Electrophysics, National Chiao-Tung University, Hsinchu, Taiwan, Republic of China*

[3]*Department of Physics, Kyushu University, Hakozaki, Higashi-ku Fukuoka 812-8581, Japan*

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We show a constantly accelerated quark as a string solution of the Nambu-Goto action, which is embedded in the bulk background dual to the $\mathcal{N} = 2$ supersymmetric confining Yang-Mills theory. The induced metric of the world sheet for this string solution has an event horizon specified by the fifth coordinate. By an extended Rindler transformation proposed by Xiao, we move to the comoving frame of the accelerated quark string. Then we find that this horizon is transferred to the event horizon of the bulk and the causal part of the accelerated quark is transformed to a static free quark in the Rindler coordinate. As a result, the confinement of the Minkowski vacuum is lost in the Rindler vacuum. This point is assured also by studying the potential between the quark and antiquark. However, the remnants of the original confining force are seen in various thermal quantities. We also discuss the consistency of our results and the claim that the Green's functions will not be changed by the Rindler transformation.

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I. INTRODUCTION

An observer, who is accelerated with a constant acceleration, a, in the Minkowski space-time would observe a thermal bath of particles with the temperature $a/2\pi$ (the Rindler temperature) [1–3]. This phenomenon is known as the Unruh effect (see the review for example [4]). Recently, a similar situation has been studied for the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in the context of the holography [5–8]. In the approach of [6,7], an accelerated quark has been introduced as string solutions of the Nambu-Goto action which are embedded in the AdS_5 (anti-de Sitter) background dual to the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. The solution in this background has been found by Xiao [6], and one finds an event horizon in the induced metric (in its world sheet) of this string configuration. The position of this horizon is specified by the fifth coordinate of the bulk.

Xiao has proposed an extended form of Rindler transformation (ERT) to move to a comoving frame of the accelerated quark. Performing this ERT, the event horizon appears in the bulk. Thus the theory dual to the geometry after ERT is considered as a Yang-Mills theory with finite temperature. The temperature is given by the Rindler temperature $a/2\pi(= T_R)$. At the same time, the position of the bulk horizon can be put at the same fifth-coordinate point with the one of the world sheet horizon of the accelerated string. As a result, in the new coordinate, one finds a static string which connects the boundary and the event horizon of the bulk.

This is nothing but a free quark-string configuration in the Rindler vacuum. Since the theory dual to the AdS_5 is in the deconfinement phase, there is also a free quark in the Minkowski vacuum at zero temperature. However we should notice that the free quark in the one vacuum is not the same as the one in the other vacuum, because the static free quark in the Minkowski vacuum cannot be transformed to the one of the Rindler vacuum by ERT, and vice versa. Anyway, in both theories dual to AdS_5 and to the one transformed by ERT, the quarks are not confined and chiral symmetry is not broken. So the confinement-deconfinement transition or chiral symmetry restoration has not been discussed as the thermal effect in the Rindler vacuum. Thus, it remains an important point to perform this analysis for the gauge theory in the confinement phase and also for the theory with the chiral symmetry broken in the Minkowski vacuum. Up to now, there has been no such attempt.

Here we consider a confining Yang-Mills theory in the Minkowski vacuum in order to examine properties of its Rindler vacuum, which is obtained by performing ERT. As a concrete model, we consider a supersymmetric background solution of type IIB theory. This background is dual to the $\mathcal{N} = 2$ supersymmetric Yang-Mills theory, and the quark is confined in this theory [9–11]. In other words, it is impossible to find a free quark-string solution for the Nambu-Goto action embedded in this background.

Then we solve the equation of motion for the Nambu-Goto action to find a constantly accelerated quark-string solution, which has a similar functional form to Xiao's. Actually, we could find such a solution. Then the original coordinate with the Minkowski vacuum is transformed to the comoving coordinate of the accelerated string solution by ERT given by Xiao. After performing this transformation, we could find the free quark-string configuration in the Rindler vacuum as in the case of AdS_5. This implies

*gouroku@dontaku.fit.ac.jp
†masafumi.ishihara@gmail.com
‡kkubo@higgs.phys.kyushu-u.ac.jp
§taminato@higgs.phys.kyushu-u.ac.jp

that this Rindler vacuum is in the quark deconfinement phase. However, we should again notice that the quark in the Rindler vacuum is not the one in the Minkowski vacuum. Then this phase change between the Minkowski and Rindler vacuum cannot be interpreted as the phase transition, which is seen in the usual finite temperature theory. In the latter case, the quark is considered to be common in both phases.

In the new coordinate vacuum, the dual theory can be regarded as the thermal Yang-Mills theory with the Rindler temperature. Its thermal properties are then examined furthermore, and we could assure that the confinement has been lost at any value of the finite Rindler temperature. So, there is no critical temperature in this case. On the other hand, some remnants of the confining force are seen in various quantities. The situation is similar to the case of the finite temperature theory dual to the AdS_5-Schwarzschild background.

One may consider that our result seems to be inconsistent with the statement of [2]. There is a claim that the quark deconfinement is not expected in the Rindler vacuum. This is based on the equivalence of the vacuum expectation value of Green's functions. However, it can be understood that our calculations and the results derived from them are not contradicting with the claim given in [2] based on the Green's functions since our results are derived from the Wilson-loop calculation in each vacuum for the corresponding quarks, which cannot be related by the coordinate transformation as mentioned above. This point is explained more in the Sec. V.

In Sec. II, we give the setting of our model for the supersymmetric confining Yang-Mills theory. In Sec. III, the accelerated string solutions for the supersymmetric theory are given, and then the effect of the confining force is examined by comparing with the solution given for AdS_5 background. In Sec. IV, new coordinates are given by ERT, and we find the same Rindler temperature with the one given for AdS_5. The thermal effects are studied to see that the confinement is lost in the Rindler vacuum. However, the remnant of the confining force has been observed in the Wilson-loop calculation and the drag force. In Sec. V, a brief comment related to the work of four-dimensional (4D) field theory is given. The summary and discussions are given in Sec. VI.

II. D3 MODEL FOR CONFINING YANG-MILLS THEORY

We consider the 10D supergravity action based on the type IIB superstring theory retaining the dilaton Φ, axion χ and self-dual five form field strength $F_{(5)}$. Under the Freund-Rubin ansatz for $F_{(5)}$, $F_{\mu_1\cdots\mu_5} = -\sqrt{\Lambda}/2\epsilon_{\mu_1\cdots\mu_5}$ [9,10], and for the 10D metric as $M_5 \times S^5$ or $ds^2 = g_{MN}dx^M dx^N + g_{ij}dx^i dx^j$, we find the solution. The five dimensional M_5 part of the solution is obtained by solving the following reduced five-dimensional (5D) action,

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R + 3\Lambda - \frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}e^{2\Phi}(\partial\chi)^2\right), \tag{1}$$

which is written in the string frame and taking $\alpha' = g_s = 1$.

The solution is obtained under the ansatz,

$$\chi = -e^{-\Phi} + \chi_0, \tag{2}$$

which is necessary to obtain supersymmetric solutions. And the solution is expressed as

$$ds^2_{10} = G_{MN}dX^M dX^N = e^{\Phi/2}\left\{\frac{r^2}{R^2}A^2(r)(-dt^2 + (dx^i)^2) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2\right\}. \tag{3}$$

Then, the supersymmetric solution is obtained as

$$e^{\Phi} = 1 + \frac{q}{r^4}, \qquad A(r) = 1, \tag{4}$$

where $M, N = 0 \sim 9$ and $R = \sqrt{\Lambda}/2 = (4\pi N_c)^{1/4}$. And q represents the vacuum expectation value (VEV) of gauge fields condensate [10,11]. In this configuration, the four dimensional boundary represents the $\mathcal{N} = 2$ supersymmetric Yang-Mills theory. In this model, we find quark confinement in the sense that we find a linear rising potential between quark and antiquark with the tension $\sqrt{q}/R^2$ [9,11]. However, chiral symmetry is preserved in the sense that the vacuum expectation value of the order parameter is zero. In other words, the dynamical mass generation of massless quarks does not occur.

In this case, we notice that the space-time is regular at any point. In the ultraviolet limit, $r \to \infty$, the dilaton part e^{ϕ} approaches to one and the metric (1) is reduced to $AdS_5 \times S^5$. On the other hand, the dilaton part e^{ϕ} diverges in the infrared limit $r \to 0$, so that one may expect singularity at $r = 0$. However there is no such singularity. It is assured by rewriting the metric (1) in terms of a new coordinate z, where $z = R^2/r$. Then we obtain

$$ds^2_{10} = e^{\Phi/2}\frac{R^2}{z^2}(-dt^2 + (dx^i)^2 + dz^2 + z^2 d\Omega_5^2). \tag{5}$$

In the infrared limit $z \to \infty$, we have

$$e^{\Phi/2}\frac{R^2}{z^2} = R^2\sqrt{\frac{q}{R^8} + \frac{1}{z^4}} \sim \frac{\sqrt{q}}{R^2}. \tag{6}$$

Therefore, we find 10D flat space-time in this limit and no singular point [9,10].

III. ACCELERATING STRING SOLUTION

Here we restrict to the supersymmetric case. The metric for the supersymmetric case (4) is given as,

$$ds_{10}^2 = e^{\Phi/2}R^2\left\{u^2(-dt^2 + (dx^i)^2) + \frac{1}{u^2}du^2 + d\Omega_5^2\right\}. \quad (7)$$

where $u = r/R^2$. The world sheet coordinates of the string are taken as (t, u), and it is assumed to be stretching in the direction $x_1 \equiv x = x(t, u)$. Then the induced metric on the world sheet of the string is given as

$$g_{tt} = -e^{\Phi/2}R^2u^2(1 - \dot{x}^2) \quad (8)$$

$$g_{uu} = e^{\Phi/2}R^2\left(\frac{1}{u^2} + u^2x'^2\right) \quad (9)$$

$$g_{ut} = g_{tu} = e^{\Phi/2}R^2u^2\dot{x}x' \quad (10)$$

Then the Nambu-Goto action for a string stretching in the x direction is given as follows,

$$S = -\frac{R^2}{2\pi\alpha'}\int dtdu\, e^{\Phi/2}\sqrt{1 - \dot{x}^2 + u^4x'^2} \quad (11)$$

for the world sheet of the string $(\tau, \sigma) = (t, u)$, where $r = R^2u$, $x' = \partial_u x$, $\dot{x} = \partial_t x$ and

$$e^{\Phi} = 1 + \frac{\tilde{q}}{u^4}, \qquad \tilde{q} = \frac{q}{R^8}. \quad (12)$$

The equation of motion for $x(t, u)$ is obtained as

$$\frac{1}{2}\Phi'\frac{u^4x'}{\tilde{g}^{1/2}} + \left(\frac{u^4x'}{\tilde{g}^{1/2}}\right)' - \partial_t\left(\frac{\dot{x}}{\tilde{g}^{1/2}}\right) = 0, \quad (13)$$

$$\tilde{g} = 1 - \dot{x}^2 + u^4x'^2, \quad (14)$$

where dash and dot denote the derivative with respect to u and t respectively as given above.

$q = 0$ *case.*—Before solving (13), we briefly review Xiao's analytic solution given for $q = 0$ or $\Phi = 0$, namely, in AdS_5 background. In this case, the solution is obtained as

$$x(t, u) = \sqrt{t^2 + f_0(u)}, \qquad f_0(u) = 1/a^2 - 1/u^2 \quad (15)$$

This solution represents the accelerated quark in the supersymmetric Yang-Mills theory in the deconfinement phase. The value a denotes the acceleration of the quark sitting at the boundary. The speed of the string to x direction depends on the position u, and it exceeds light velocity in the region $u \leq a$.

We notice here the following point. For this solution, the induced metric g_{tt} of the string world sheet is given as

$$g_{tt} = -u^2\frac{f_0(u)}{t^2 + f_0(u)}. \quad (16)$$

This implies that g_{tt} has a zero point at $u = a$ and this point could be regarded as the horizon on the world sheet. So we could see the Hawking radiation of fields on the world sheet. However, there is no horizon in the 5D bulk background, so this situation is interpreted as the gauge field radiation of the accelerated color charged particle, namely, the quark.

In the present case, since the world sheet metric of the accelerated string is time dependent and nondiagonal, then we should go to the other coordinate which would provide a clear vacuum of the theory. This is performed by the Rindler coordinate transformation given by (26)–(28) in the next section. In this case, we find the following world sheet metric

$$g_{\tau\tau} = -R^2(s^2 - a^2) \quad (17)$$

$$g_{ss} = R^2\frac{1}{s^2 - a^2} \quad (18)$$

$$g_{s\tau} = g_{\tau s} = 0 \quad (19)$$

These represent the bulk metric at the same time, so we find the horizon of the world sheet and the one of the bulk are common. As a result, we find the thermal bath of the radiation in the new coordinate. This is known as the Unruh effect.

Here we give some comment on the string. In the new coordinate, the string world sheet is transformed to (τ, s), and it stretches in the direction $\beta = \beta(\tau, s)$. Then the string solution given above is transformed as

$$\beta(s, \tau) = 0. \quad (20)$$

The new horizon point is equivalent to the point of $f(u) = 0$, which is the horizon point of (16). The string represented by (20) is the straight line from the horizon $s = a$ to the boundary $s = \infty$. Then the part $u < a$ of the string in the Minkowski coordinate has been hidden in the thermal bath (inside the horizon) in the new vacuum. We can see the similar situation for the accelerated solutions given in a different form of background.

$q > 0$ *case.*—Next, we solve Eq. (13) for the case of nontrivial dilaton, namely, for $q > 0$. Also in this case, we solve (13) by assuming the following functional form,

$$x(t, u) = \sqrt{t^2 + f(u)}. \quad (21)$$

We would find $f(u) \simeq f_0(u)$ at large u where the effect of q could be neglected. However the function $f(u)$ deviates from $f_0(u)$ when u decreases. Then it may be written as

$$f(u) = f_0(u) + \tilde{f}(u), \quad (22)$$

where $\tilde{f}(u)$ is expected to be proportional to q. These points are seen as follows.

Equation (13) is rewritten as the one of $f(u)$ as follows,

$$\sqrt{f/f'^2 + u^4/4}\left(\frac{u^4}{2\sqrt{f/f'^2 + u^4/4}}\right)' = \frac{1}{f'} + q\frac{u^3}{q + u^4} \quad (23)$$

where prime denotes the derivative with respect to u. First, we consider the asymptotic behavior near the boundary

($u \to \infty$). Substituting $f(u) = f_0(u) + \tilde{f}(u)$ into Eq. (23), we find

$$\tilde{f}(u) = \frac{k}{u^3} + O(1/u^5) \tag{24}$$

at large u. Here k is an arbitrary constant as a in f_0. The solution has two arbitrary constants since the equation is the second order differential equation. Then we can set the asymptotic value of $f(u)$ as $f(\infty) = 1/a^2$ for any q as the boundary condition used here. However, this condition restricts the boundary condition in the infrared region of small u.

For the boundary condition, $f(\infty) = 1/a^2$, the solution with positive q deviates definitely from $f_0(u)$ at small u. In general the zero point of $f(u)$ increases with q, namely

$$u_1 > a, \quad \text{for } f(u_1) = 0. \tag{25}$$

This is obtained when we fix the point at $u = \infty$ as $f(\infty) = 1/a^2$ for any q. On the other hand, the value of $f(\infty)$ moves to the large value with increasing q when we fix u_1. The situation depends on the boundary condition in solving the equation of motion of $f(u)$. The understandable situation would be to fix the acceleration of the quark as a by $f(\infty) = 1/a^2$. In this case, the "horizon" u_1 on the string moves to a larger value and the string configuration $f(u)$ is modified from $f_0(u)$ with increasing q. However the Rindler temperature, which is obtained after a coordinate transformation where the string is seen to be static, does not depend on q and another parameter coming from the coordinate transformation. On the other hand, Rindler Temperature changes with the parameter of ERT, namely a. We show this point in the next section.

For $q > 0$, we give here the numerical solutions. The solutions for $q = 0.5, 3.0, 10$ are shown in Fig. 1 and it is compared with the case of $q = 0$. We can see the zero point of $f(u)$ moves to the larger value of u as stated above.

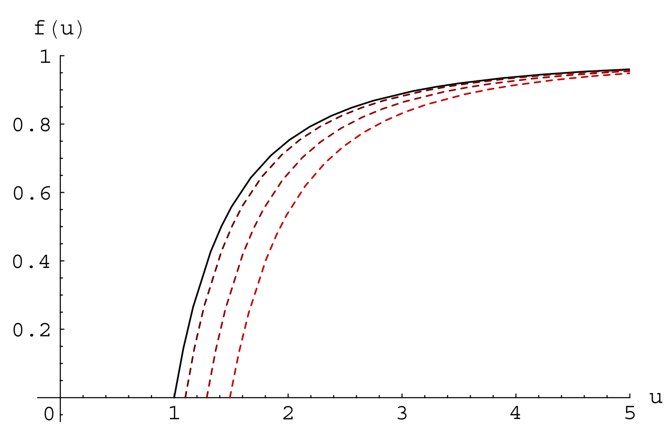

FIG. 1 (color online). The numerical results of $f(u)$ for $a = 1$ ($f(\infty) = 1/a^2 = 1$) and $q = 0$, 0.5, 3.0 and 10 are shown from left to right. The zero point of the solution moves to the right with increasing q.

IV. EXTENDED RINDLER TRANSFORMATION

Next we move to the comoving coordinate of the accelerated quark. Usually this is performed by the transformation among two coordinates in 4D space-time, time and accelerated direction, as seen in [2,5]. However, here we use another form of Rindler transformation proposed by Xiao [6]. This transformation is performed among the three coordinates, time, the accelerated direction and the fifth coordinates of the original AdS_5 space-time, namely, for (t, x, u). We call this the extended Rindler transformation (ERT). After the transformation, in the new coordinate, the accelerated quarks given above in the original coordinate are seen as static and they are in a thermal bath with a finite temperature. Especially, we notice that these quarks are moving as free particles. In this sense, the quark is not confined in the dual gauge theory for the Rindler coordinate given by the extended Rindler transformation. We study the details of this point in the following cases.

For the case of $q = 0$, the comoving coordinates, (s, τ, β), of this accelerated quark are obtained from the original coordinates, (u, t, x), by the following extended coordinate transformation [6],

$$x = \sqrt{\frac{1}{a^2} - s^{-2}}\, e^{a\beta} \cosh(a\tau), \tag{26}$$

$$t = \sqrt{\frac{1}{a^2} - s^{-2}}\, e^{a\beta} \sinh(a\tau), \tag{27}$$

$$u = s e^{-a\beta} \tag{28}$$

Then, the new metric is given as

$$ds_{10}^2 = R^2\left(\frac{ds^2}{s^2 - a^2} - (s^2 - a^2)d\tau^2 + s^2(d\beta^2 + e^{-2a\beta}[dx_2^2 + dx_3^2]) + d\Omega_5^2\right) \tag{29}$$

and the above string configuration (15) is given by

$$\beta = 0 \tag{30}$$

which is static since it is independent of the new time coordinate τ. This string represents a free[1] quark string which connects a d-brane put at some $s > a$ and the event horizon $s = a$.

We should notice here that we can replace the parameter a by another value, for example, by $\tilde{a}$, in the above coordinate transformation (26)–(28). In this case, we find the transformed accelerated string at

$$\beta = \frac{1}{\tilde{a}} \log(\tilde{a}/a) \tag{31}$$

[1]Here, "free" means that the quark is not bounded with an antiquark as a meson.

where we imposed the condition that the position of the quark on the boundary $(x, t, u) = (1/a, 0, \infty)$ is transformed to $(\beta, \tau = 0, s = \infty)$. In this new coordinate with $\tilde{a}$, the Rindler temperature is obtained as $\tilde{a}/2\pi$. We, however, set here as $\tilde{a} = a$ for simplicity. In this case, the Rindler temperature is directly related to the acceleration of the particles as in particle physics.

For the case of $q > 0$, we consider the following similar transformation,

$$x = g(s)e^{a\beta}\cosh(a\tau), \tag{32}$$

$$t = g(s)e^{a\beta}\sinh(a\tau), \tag{33}$$

$$u = h(s)e^{-a\beta} \tag{34}$$

where a denotes the acceleration of the quark at the boundary. Then we find the new metric,

$$ds_{10}^2 = e^{\Phi/2}\left\{R^2h^2\left(\left(\frac{h'^2}{h^4} + g'^2\right)ds^2 - g^2a^2d\tau^2 + a^2\left(g^2 + \frac{1}{h^2}\right)d\beta^2 + e^{-2a\beta}[dx_2^2 + dx_3^2]\right) + R^2d\Omega_5^2\right\} \tag{35}$$

where prime represents the derivative with respect to s, and we set as

$$(h^2)' = h^4(g^2)' \tag{36}$$

to eliminate the $g_{s\beta}$. Then g is related to h by solving (36) as

$$g^2 = c_0^2 - \frac{1}{h^2} \tag{37}$$

where c_0 is an arbitrary constant. Then we find again the Rindler coordinate for $c_0 = 1/a$ and $h = s$, except for the prefactor $e^{\Phi/2}$. In the present case, we change the radial coordinate from s to $h(s)$, then we have

$$ds_{10}^2 = e^{\Phi/2}R^2\left\{\frac{dh^2}{h^2 - a^2} - (h^2 - a^2)d\tau^2 + h^2(d\beta^2 + e^{-2a\beta}[dx_2^2 + dx_3^2]) + d\Omega_5^2\right\} \tag{38}$$

The accelerated strings therefore can be seen in the same Einstein frame coordinate also in the confining phase (for $q > 0$).

Transformed String Configurations.—We consider how the string configuration of the accelerated strings is seen in the comoving coordinate. The configuration is different from the $q = 0$ case. The new configuration is shown in the β-h plane by using the relation,

$$\tilde{f}(h\,e^{-a\beta}) = \frac{1}{a^2}(e^{2a\beta} - 1) \tag{39}$$

which is obtained from the form of the solution, $x^2 - t^2 = f(u) = f_0(u) + \tilde{f}(u)$. Then, using the solution, $\tilde{f}(u)$, we find the string configurations in the Rindler coordinate in terms of Eq. (39).

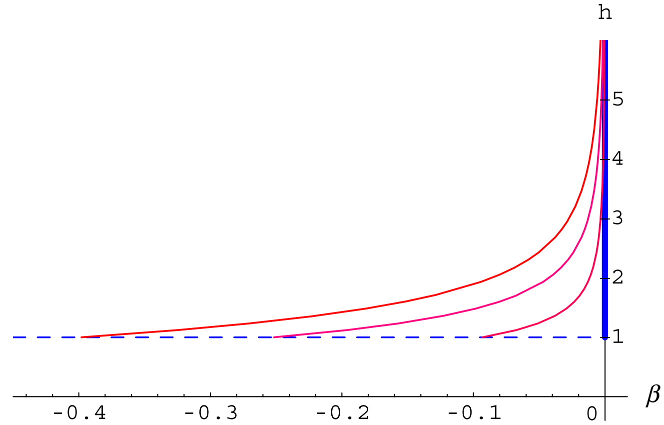

FIG. 2 (color online). Examples of the string solution $\beta(h)$ for $q = 0$ (straight line) and $q = 0.5$, 3, 10 (from right to left) with $a = 1$. The dotted line, $h = a(= 1)$, represents the Rindler horizon. Solutions for finite q are bent due to the Yang-Mills force expressed by the dilaton. Then the larger q becomes, the larger the deformation of the solution grows.

The boundary condition should be taken as $\tilde{f}(h = \infty) = 0$ at fixed β since a in $f_0 = 1/a^2 - 1/u^2$ represents the acceleration of the quark at the boundary. This implies $\beta(h = \infty) = 0$ from (39). In Fig. 2, some examples of the solution for $q = 10, 3, 0.5, 0$ with $a = 1$ are shown. When we set $q = 0$, we obtain $\tilde{f}(u) = 0$, which is the straight line of $\beta = 0$, namely, the h-axis. The larger q becomes, the larger the deviation of the string configurations from the straight line grows. (i) However, in any case, each string configuration shows the free-quark state in a heat bath of Rindler temperature $a/2\pi$ and (ii) the horizon is given by the point, $h = a$ (or $u = ae^{-a\beta} > a$), where the velocity of the string to x direction in the original coordinate arrives at the speed of light.

Temperature and asymmetry of three space.—Here we consider the temperature from two viewpoints. First, it could be given from the condition to evade a conical singularity in the (τ, h) plane for the Euclidean metric $(\tau \to i\tau)$.

It is seen near the horizon. By setting $h = a(1 + \epsilon^2/2)$, the metric (38) is rewritten for small ϵ as follows

$$ds_{10}^2 = e^{\Phi_a/2}R^2\{d\epsilon^2 + a^2\epsilon^2d\tau^2 + \cdots\}, \tag{40}$$

where

$$e^{\Phi_a} = 1 + \frac{\tilde{q}e^{4a\beta}}{a^4}. \tag{41}$$

From the above, the temperature is given by

$$T_R = \frac{a}{2\pi} \tag{42}$$

for fixed β. Here we must notice that the prefactor $e^{\Phi_a/2}$ depends on the new coordinate β, then the temperature is well defined only for fixed β in this definition. However, we notice that the temperature given by (42) is independent of β. In other words, the thermal equilibrium with the temperature T_R would be seen at any point of β. Then this temperature is well defined in the three dimensional space, $ds^2_{(3)} = d\beta^2 + e^{-2a\beta}[dx_2^2 + dx_3^2]$.

The other way to define the temperature is given by considering the timelike Killing vector $\xi^\mu = \delta^{\mu 0}$, which satisfies

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0. \tag{43}$$

In this case, the surface curvature at the horizon is given as

$$k_H^2 = -\frac{1}{2}(\nabla^\mu \xi^\nu)(\nabla_\mu \xi_\nu) = a^2, \tag{44}$$

then we obtain the same result with (42),

$$T_H = \frac{k_H}{2\pi|\xi^\mu|} = \frac{a}{2\pi} = T_R. \tag{45}$$

So the temperature could be well defined in the Rindler coordinate, but the three space, $ds^2_{(3)} = d\beta^2 + e^{-2a\beta}[dx_2^2 + dx_3^2]$, is asymmetric. It is separated to the longitudinal (β) and the transverse (x_2-x_3) directions. This asymmetric behavior is also reflected to the color force, and it can be seen through the dilaton which affects the force between the quark and the antiquark as being found through the Wilson-Loop. Its analyses are given in the next section.

Actually, the gauge coupling constant is defined by $g^2_{\rm eff} = e^{\Phi_a}$ and it depends on β and h as follows,

$$g^2_{\rm eff} = 1 + \frac{\tilde{q}e^{4a\beta}}{h^4}. \tag{46}$$

This implies that the Yang-Mills force depends on the energy scale h and also on the coordinate β in the real three space. In the present case, the temperature is finite and the Yang-Mills force between a quark and antiquark is completely screened when they are separated by the distance (L) larger than a critical value (L_*). Namely the quark is independent of the antiquark which is separated by the distance $L > L_*$, but we know that the quark can feel the force from the antiquark in the region of $L < L_*$ and this force is nearly equivalent to the one given at zero temperature.

In the present case, we find the linear rising force in the region of $L_0 < L < L_*$, where $L < L_0$ defines the ultraviolet region of the conformal symmetric limit. And we find the tension parameter [11]

$$\tau_{\rm eff} = \frac{\sqrt{q}}{2\pi\alpha' R^2} \tag{47}$$

at zero temperature in the present model. Then we expect the tension parameter in the Rindler coordinate would be given by

$$\tau_{\rm R} = \frac{\sqrt{\tilde{q}}R^2 e^{2a\beta}}{2\pi\alpha'}, \tag{48}$$

which is however coordinate dependent. We can assure this point through the Wilson-Loop calculation given below. Therefore we study the dynamical properties in this vacuum by separating into two cases, the longitudinal and transverse directions in three dimensional space in the new coordinate.

A. Wilson loop and the force between quarks

From the gauge coupling given above (46), we can say that the force between the quark and antiquark would depend on a and also on β. In our original metric, the quarks are confined due to the strong infrared gauge coupling constant. Namely, it diverges for $u \to 0$. In the new coordinate (Rindler coordinate), the infrared strong force would be screened by the fluctuations of the thermal matter with the temperature $T_R = a/(2\pi)$. The situation would be parallel to the case of the AdS-Schwarzschild background which is dual to the high temperature gauge field theory. Because of this screening, we would find the deconfinement phase in the Rindler vacuum. This is the Unruh effect in the confinement theory. In order to assure this point, we study the force between the quark and the antiquark, which are represented by the static strings in the Rindler vacuum (38).

1. Strings stretched to the longitudinal (β) direction

Consider the string which is extending to the β direction. Taking its world sheet as $(\tau, h, \beta(h))$, then the Nambu-Goto action of this string is given as

$$S = -\frac{1}{2\pi\alpha'}\int d\tau dh R^2 e^{\Phi/2}\sqrt{1 + h^2(h^2 - a^2)\beta'^2} \tag{49}$$

where $\beta' = \partial_h \beta$. Namely the prime denotes the derivative with respect to h. The equation of motion for β is given as

$$\partial_h\left(\frac{e^{\Phi/2}h^2(h^2 - a^2)\beta'}{\sqrt{1 + h^2(h^2 - a^2)\beta'^2}}\right) = \sqrt{1 + h^2(h^2 - a^2)\beta'^2}\,\partial_\beta e^{\Phi/2} \tag{50}$$

We notice here that the dilaton Φ depends on β and h, and we find

$$\partial_\beta e^{\Phi/2} = 2ae^{-\Phi/2}\frac{\tilde{q}e^{4a\beta}}{h^4}. \tag{51}$$

Then the right hand side of (50) is not zero, so we cannot obtain the symmetric U shaped string configuration. We therefore solve the equation of motion by using a parameter s along the string.[2]

[2]The parameter s introduced here has nothing to do with the one given in (26)–(28) above.

Before studying the Wilson loop, we would like to show that the Rindler coordinate is dual to the deconfinement phase of the gauge theory. We consider the energy E of the string, which is given by changing the variable from h, $\beta(h)$ to β, $h(\beta)$ as

$$E = \frac{R^2}{2\pi\alpha'} \int d\beta n \sqrt{1 + \frac{h'^2}{h(h^2 - a^2)}} \tag{52}$$

$$n = e^{\Phi/2} h \sqrt{(h^2 - a^2)} \tag{53}$$

If quarks are confined, we would find

$$E = \tau_{\rm eff} L, \qquad \tau_{\rm eff} = \frac{R^2}{2\pi\alpha'} n(h^*) \tag{54}$$

where $\tau_{\rm eff}$ denotes the tension of the linear rising potential between the quark and the antiquark, which are separated by L, and h^* is the minimum point of $n(h)$. In the present case, $h^* = a$ and $n(a) = 0$, so we cannot find linear potential with a definite tension for large L. In other words, the quarks are not confined. The main reason of this deconfinement might be the screening of the color force at finite distance due to the thermal effect of the thermal bath with the Rindler temperature given above.

Reparametrization invariant formulation.—These points can be seen more explicitly by calculating the Wilson loop in the Rindler vacuum. As mentioned above, we rewrite the action (49) by using the parameter s along the string as follows

$$S = -\int d\tau U, \tag{55}$$

$$U = \frac{R^2}{2\pi\alpha'} \int ds \mathcal{L} = \frac{R^2}{2\pi\alpha'} \int ds e^{\Phi/2} \sqrt{\dot{h}^2 + h^2(h^2 - a^2)\dot{\beta}^2}, \tag{56}$$

where the dot denotes the derivative with respect to s. We notice that the above Lagrangian U is reparametrization invariant with respect to s. Then we can give the Hamiltonian $H = H(h, p_h, \beta, p_\beta)$ where

$$p_h = \frac{\partial \mathcal{L}}{\partial \dot{h}}, \qquad p_\beta = \frac{\partial \mathcal{L}}{\partial \dot{\beta}}, \tag{57}$$

and we obtain

$$H = \frac{\tilde{H}}{\Delta}, \qquad \Delta^{-1} = 2e^{-\Phi/2} \sqrt{\dot{h}^2 + h^2(h^2 - a^2)\dot{\beta}^2}, \tag{58}$$

$$\tilde{H} = \frac{1}{2}\left\{ p_h^2 + \frac{p_\beta^2}{h^2(h^2 - a^2)} - e^{\Phi} \right\}. \tag{59}$$

Notice that we can use the Hamiltonian $\tilde{H}$ instead of H. This is allowed since the theory we are solving is written in the reparametrization invariant form. In this case, we impose the Hamiltonian constraint as in the gravitational theory,

$$H = 0 \quad \text{then } \tilde{H} = 0. \tag{60}$$

Under this constraint, we find that the difference of the solutions of the Hamilton's equations written by H and $\tilde{H}$ is in the parametrization of the solution. For example we write the solution as $h(s)$ or $h(f(s))$, where

$$\frac{\partial f(s)}{\partial s} = \Delta \tag{61}$$

So in both cases of H and $\tilde{H}$, we will find the same string configuration.

Then we can derive the equations of motion of the string by the following Hamilton's equations of $\tilde{H}$,

$$\dot{h} = p_h, \qquad \dot{\beta} = \frac{p_\beta}{h^2(h^2 - a^2)}, \tag{62}$$

$$\dot{p}_h = \frac{2h^2 - a^2}{h^3(h^2 - a^2)^2} p_\beta^2 - \frac{2\tilde{q}}{h^5} e^{4a\beta}, \qquad \dot{p}_\beta = \frac{2a\tilde{q}}{h^4} e^{4a\beta}. \tag{63}$$

Boundary condition and numerical solutions.—Solving the above equations (62) and (63) numerically, the solutions are obtained as the functions of s. We solve them with the boundary conditions,

$$h_{\rm max} > h(0) = h_0 > a, \qquad \beta(0) = \beta_0, \qquad p_h(0) = 0 \tag{64}$$

and $p_\beta(0)$ is given from the constraint $\tilde{H} = 0$ at $s = 0$ as,

$$p_\beta(0) = h_0 \sqrt{(h_0^2 - a^2)\left(1 + \frac{\tilde{q} e^{4a\beta_0}}{h_0^4}\right)}. \tag{65}$$

Here $h_{\rm max}$ denotes the end point of the string, and it is fixed so that this point is interpreted as the flavor brane's position or a UV cutoff near the boundary. And h_0 denotes the bottom of the string solution. This point is varied to obtain the string solutions with various different energy E, which is obtained by substituting the solution into the following equation,

$$E = \frac{R^2}{2\pi\alpha'} \int_{s_{\rm dw}}^{s_{\rm up}} ds \mathcal{L}. \tag{66}$$

Here the values of $s_{\rm dw}$ and $(s_{\rm dw} <) s_{\rm up}$ are obtained from the solution $h(s)$ by solving the following equations,

$$h(s_{\rm dw}) = h_{\rm max} = h(s_{\rm up}). \tag{67}$$

They indicate the two end points of the string solution.

As for β_0, it denotes the coordinate β at the bottom of the string solution and it controls the end point values $\beta(s_{\rm dw})$ and $\beta(s_{\rm up})$. Then, we adjust β_0 such that $\beta(s_{\rm dw}) = 0$ and $\beta(s_{\rm up}) \neq 0$. In this case, the distance (L) between the quark and antiquark for these solutions is given as

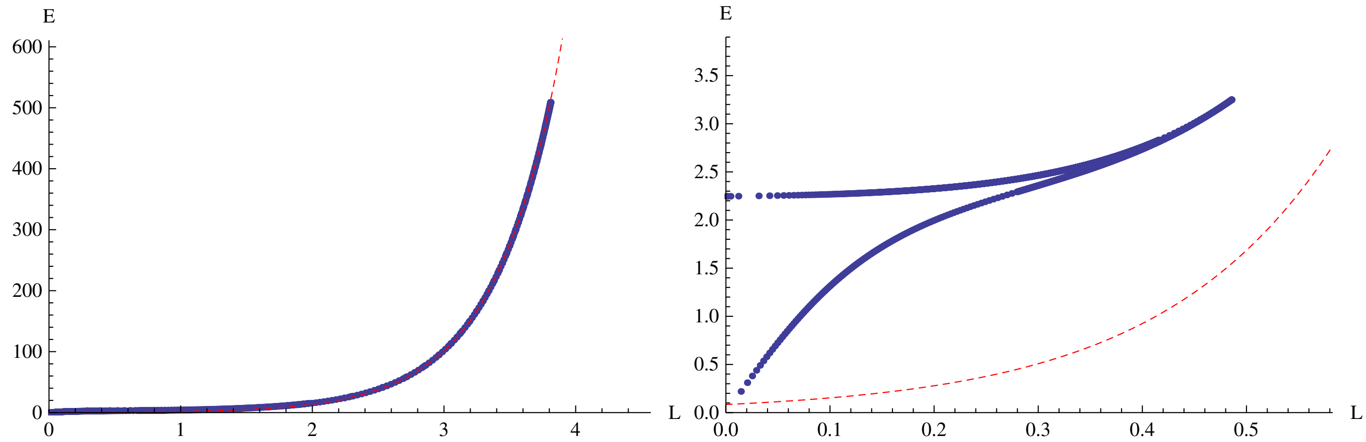

FIG. 3 (color online). *E-L* relations for quark and antiquark in the longitudinally extended case for $h_{\max} = 10$, $R = 1$, $\tilde{q} = 10$ and $a = 1$ (left), $a = 3$ (right). The (red) dashed curves represent Eq. (69), which is expected as the effect of the color force existing in the confinement phase.

$$L = |\beta(s_{\mathrm{up}}) - \beta(s_{\mathrm{dw}})| = |\beta(s_{\mathrm{up}})|. \tag{68}$$

In our present calculation, h is cut at $h_{\max}(= 10)$ and the quarks are supposed to be at this point. This regularization does not affect the large L behavior of the Wilson loop estimations.

We show the results inFig. 3 for the region $\beta(s_{\mathrm{up}}) > 0$, where the parameters are set as $R = 1$, $q = 10$, $a = 1.0$ (left) and $a = 3.0$ (right).

For sufficiently large q compared to the temperature $a/2\pi$, as mentioned above, we can see the color force with a definite tension before the screening effects become dominant. In this region, the string energy is approximated as $E = \tau_{\mathrm{R}} L$ in terms of the tension τ_{R} and the length L of the string. In the present case the tension is given by (48). However L is measured in the β direction and the tension τ_{R} depends on β. Therefore the energy could be estimated as follows,

$$E \simeq \int_0^L \tau_{\mathrm{R}} d\beta \simeq \frac{\sqrt{\tilde{q}} R^2 e^{2aL}}{4a\pi\alpha'}. \tag{69}$$

Actually, we can see a good fit with this curve for low temperature case $a = 1.0$, and the numerical result as shown in the left figure of Fig. 3.

On the other hand, at high temperature, for the case of $a = 3.0$, we can find that the potential is screened before it meets the dashed curve as shown in the right one of Fig. 3. As for the screening, we can see it through the existence of the maximum point ($L_{\max}$) of L which we discussed in the previous subsection. This point moves smaller L with the increasing temperature as expected from the usual high temperature theories in the confinement phase.

The same analysis has been performed also for $\beta(s_{\mathrm{up}}) < 0$. In this case, the force becomes weak even if a is small, so the screening becomes dominant at rather small L and it becomes difficult to see the remnant of the confinement force. We abbreviated here to show the numerical results.

2. Strings stretched to the transverse direction

Next, we consider the string stretched to the transverse direction, for example, to $x_2 \equiv x$ direction which is transverse to the accelerated direction β. In this case, the action is given as

$$S = -\frac{R^2}{2\pi\alpha'} \int d\tau dh e^{\Phi/2} \times \sqrt{1 + h^2(h^2 - a^2)\beta'^2 + x'^2 e^{-2a\beta} h^2(h^2 - a^2)} \tag{70}$$

where $x' = \partial x/\partial h$. Here $\beta'(= \partial\beta/\partial h)$ is also retained since we cannot fix β as a constant value as can be seen from the equations of motion for β.

In order to obtain the relation of E and L as given above, we must obtain the string solutions. It is convenient to solve the equations after rewriting the action in the reparametrization invariant form as above,

$$U_x = \frac{R^2}{2\pi\alpha'} \int ds \mathcal{L}_x = \frac{R^2}{2\pi\alpha'} \int ds e^{\Phi/2} \times \sqrt{\dot{h}^2 + h^2(h^2 - a^2)\dot{\beta}^2 + \dot{x}^2 e^{-2a\beta} h^2(h^2 - a^2)} \tag{71}$$

where dot denotes the derivative with respect to the parameter s. Then we have the canonical momentum,

$$p_h = \frac{\partial \mathcal{L}_x}{\partial \dot{h}}, \qquad p_\beta = \frac{\partial \mathcal{L}_x}{\partial \dot{\beta}}, \qquad p_x = \frac{\partial \mathcal{L}_x}{\partial \dot{x}}, \tag{72}$$

and the following Hamiltonian

$$\tilde{H}_x = \frac{1}{2}\left\{p_h^2 + \frac{p_\beta^2}{h^2(h^2 - a^2)} + \frac{p_x^2 e^{2a\beta}}{h^2(h^2 - a^2)} - e^{\Phi}\right\} \tag{73}$$

Then we can solve the equations of motion by the following Hamilton equations,

$$\dot{h} = p_h, \qquad \dot{\beta} = \frac{p_\beta}{h^2(h^2-a^2)}, \qquad \dot{x} = \frac{p_x e^{2a\beta}}{h^2(h^2-a^2)}, \tag{74}$$

$$\dot{p}_h = \frac{2h^2-a^2}{h^3(h^2-a^2)^2}(p_\beta^2 + p_x^2 e^{2a\beta}) - \frac{2\tilde{q}}{h^5}e^{4a\beta}, \tag{75}$$

$$\dot{p}_\beta = -\frac{a p_x^2 e^{2a\beta}}{h^2(h^2-a^2)} + \frac{2a\tilde{q}}{h^4}e^{4a\beta}, \qquad \dot{p}_x = 0. \tag{76}$$

Boundary condition and numerical solutions.—Solving the above equations numerically, we imposed the following boundary conditions,

$$\beta(0) = \beta_0, \qquad h_{\rm max} > h(0) = h_0 > a, \qquad x(0) = 0, \tag{77}$$

$$p_\beta(0) = 0, \qquad p_h(0) = 0, \tag{78}$$

and $p_x(0)$ is given from the constraint $\tilde{H}_x = 0$ at $s = 0$ as

$$p_x(0) = h_0 e^{a\beta_0}\sqrt{(h_0^2-a^2)\left(1+\frac{\tilde{q}e^{4a\beta_0}}{h_0^4}\right)}. \tag{79}$$

In this case, the distance between the quark and the antiquark is measured in the direction of x by setting the coordinate β as the same value at the two end points. Namely, we suppose at $h = h_{\rm max}$,

$$x_+ \equiv x(s_{\rm up}), \qquad x_- \equiv x(s_{\rm dw}), \qquad \beta(s_{\rm up}) = \beta(s_{\rm dw}) \equiv \beta_{\rm end}, \tag{80}$$

where $s_{\rm up}$ and $s_{\rm dw}$ are defined as (67) and $x_+(s_{\rm up})$ $(x_-(s_{\rm dx}))$ denotes the position of the quark (antiquark). Then the distance between quark and antiquark is obtained as,

$$L = x_+ - x_-. \tag{81}$$

We must be careful about the following fact that, as shown above, the color force is in the present case depending on β and h. Now, we like to see the force in the x direction through the solution of (74)–(76). So we should solve these equations by imposing the condition that the end point coordinate $\beta = \beta_{\rm end}$ is kept as a fixed value. So here we must tune the boundary values, β_0 and h_0 in order to realize the same $\beta_{\rm end}$ for each solution. Here, we obtain the solution for

$$\beta_{\rm end} = 0,$$

and a typical string solution in the present case is shown in Fig. 4.

Defining the energy of the quark and antiquark system as (66) in the previous case,

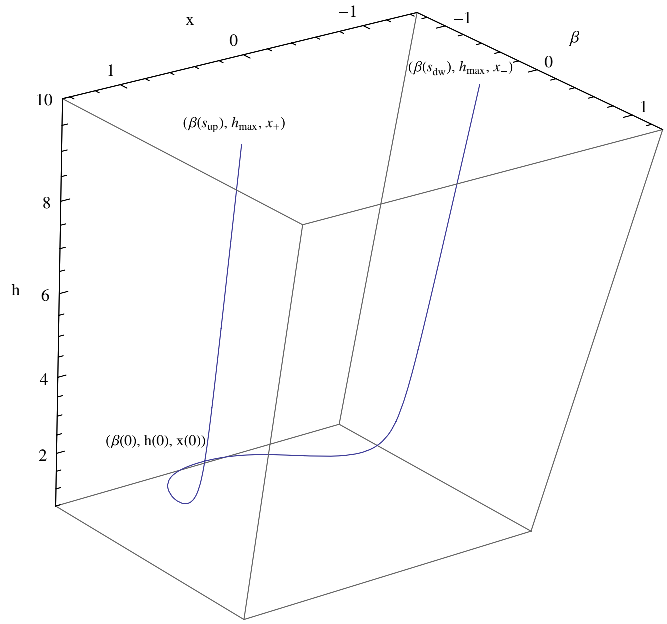

FIG. 4 (color online). 3D string configuration stretched in the y direction. As y decreases the bottom of the string is stretched to β direction.

$$E = \frac{R^2}{2\pi\alpha'}\int_{s_{\rm dw}}^{s_{\rm up}} ds\,\mathcal{L}_x,$$

we can see the relation of E and L as above. Typical results are shown in Fig. 5. In this case we can see the linear rising part before the screening. This rising part is fitted by the formula

$$E = \frac{\sqrt{\tilde{q}}e^{2a\bar{\beta}}R^2}{2\pi\alpha'}L, \tag{82}$$

where $\bar{\beta}$ would be approximately given by the β_x on the horizon $(h = a)$ as,

$$\bar{\beta} \approx \frac{\beta_0|_{h_0=a}}{3}. \tag{83}$$

As for the upper part of the E-L relation, the curve increases with decreasing L. This point is understood as follows. The upper part is obtained by pulling the bottom point of the string to near the horizon. Then the lower part of the string grows to the direction of β and the energy of the string becomes large as shown in Fig. 4. This kind of behavior cannot be seen in the case of the AdS-Schwarzschild background.

Screening length and temperature.—Next, we turn to the temperature dependence of the screening length, which is defined by the maximum value allowed L for a given temperature. It is denoted by $L_{\rm max}$, and it usually decreases with temperature as observed in the theory dual to the AdS-Schwarzschild background [12]. We find finite $L_{\rm max}$ for any finite a or temperature $a/2\pi$ as the reflection

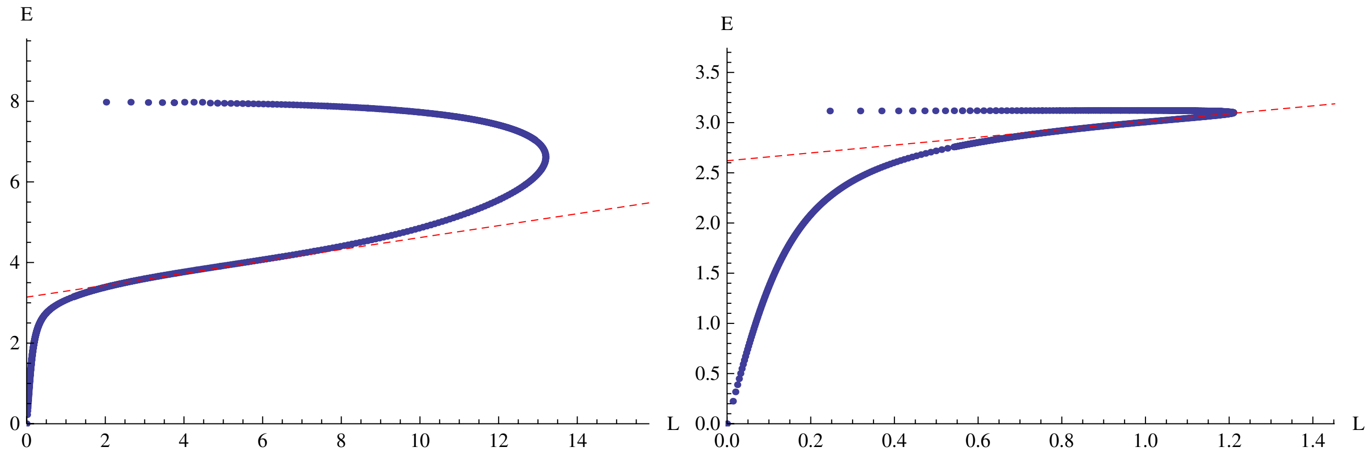

FIG. 5 (color online). Typical E-L relations for quark and antiquark for $a = 0.1$ (left) and $a = 1.0$ (right), $h_{\max} = 10$, $R = 1$ and $\tilde{q} = 10$. Each (red) dashed line represents the tension of the potential of linear rising part, and it is given by (82) and (83), which is expected as the effect of the color force existing in the confinement phase.

of the screening of the color force. Our result of the relation between $L_{\max}$ and a is given in Fig. 6.

The dotted (solid) curve represents the results from the Wilson loop stretched in the transverse (x) (longitudinal (β)) direction. The behavior of two curves clearly differs from each other in the small a region. This implies that the color force enhancement in the longitudinal direction at low temperature is greater than the force in the transverse direction. In the case of the transverse direction, the behavior is similar to the case of the AdS-Schwarzschild background, where $aL_{\max}$ is almost constant or varies slowly with the temperature. On the other hand, at high temperature, the screening becomes dominant and the asymmetric behavior disappears. As a result, the two curves coincident at large a (at high temperature) region. This is also assured from the fact that the potential in two cases approaches the similar form.

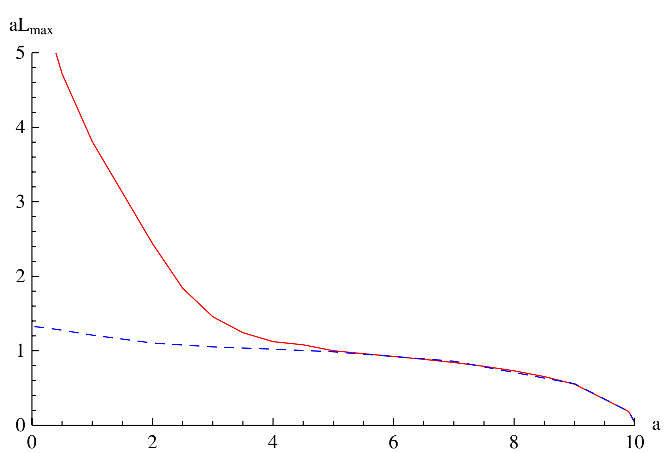

FIG. 6 (color online). a-$aL_{\max}$ relations for $\tilde{q} = 10$, $R = 1$ and $h_{\max} = 10(\geq a)$. The (red) solid line is obtained from the string stretched to the longitudinal direction. The (blue) dashed line is obtained from the string stretched to the transverse direction.

B. Trailing string and drag force

Next we examine the drag force working on the quark moving with a constant velocity in the hot gluons. This is done by studying the trailing solution, which was discussed in [13–15] in the AdS-Schwarzschild background. In the present case, it is performed in the Rindler background (38) given here. According to the work [13–15], we consider a heavy quark moving with a fixed velocity v in the thermal medium. This running quark is expressed through a string with the velocity v, and its end point is on the boundary.

When we choose the coordinate β as the moving direction, the string solution is supposed as

$$\beta(\tau, h) = v\tau + \tilde{\xi}(h). \tag{84}$$

For the coordinate (38), however, it is easily found that there is no such a form of solution. The reason is that the dilaton depends on both time β and $\tilde{\xi}(h)$ as $e^{\Phi} = 1 + \tilde{q}e^{4a\beta}/h^4$. Then the equation of motion for $\tilde{\xi}$ contains time explicitly through the dilaton as,

$$e^{\Phi} = 1 + \frac{\tilde{q}e^{4a(v\tau+\tilde{\xi}(h))}}{h^4}. \tag{85}$$

As a result, $\tilde{\xi}$ should depend on h and also on τ to satisfy the equation of motion.

Therefore, the moving direction should be chosen as x_2 or x_3 in order to obtain a string solution with a constant velocity as given in the right hand side of (84) and to see the conserved momentum flow along the string from the boundary to the horizon. So we embed the string with its world sheet (ξ_1, ξ_2) into the space $(\tau(=\xi_1), h(=\xi_2), \beta, x_2, x_3)$ through the ansatz that $x_3 =$ constant and

$$\beta = \beta(h), \qquad x_2 \equiv y(\tau, h) = v\tau + \xi(h). \tag{86}$$

We notice that we should also keep h dependence of $\beta(h)$ in this case due to the nontrivial dilaton. Then, the induced metric of the string is given as

$$g_{\tau\tau} = -e^{\Phi/2}R^2((h^2 - a^2) - h^2 e^{-2a\beta}v^2) \tag{87}$$

$$g_{hh} = e^{\Phi/2}R^2\left(\frac{1}{h^2 - a^2} + h^2 e^{-2a\beta}\xi'^2 + h^2\beta'^2\right) \tag{88}$$

$$g_{h\tau} = g_{\tau h} = e^{\Phi/2}R^2 h^2 e^{-2a\beta} v\xi' \tag{89}$$

and the Nambu-Goto action of this string is written as

$$S = \int d\tau dh \mathcal{L}_{\rm tr} = -\frac{R^2}{2\pi\alpha'}\int d\tau dh e^{(\Phi/2)}e^{-a\beta}\sqrt{\left(e^{2a\beta} - \frac{h^2v^2}{h^2 - a^2}\right)(1 + h^2(h^2 - a^2)\beta'^2) + h^2(h^2 - a^2)\xi'^2}. \tag{90}$$

From this action, we find the equation of motion for ξ, and its conserved conjugate momentum π_ξ ($\pi'_\xi = 0$) is obtained as

$$\pi_\xi = \frac{\partial\mathcal{L}_{\rm tr}}{\partial\xi'} = -\frac{R^2}{2\pi\alpha'}e^{(\Phi/2)}e^{-a\beta}\frac{h^2(h^2 - a^2)\xi'}{\sqrt{(e^{2a\beta} - \frac{h^2v^2}{h^2 - a^2})(1 + h^2(h^2 - a^2)\beta'^2) + h^2(h^2 - a^2)\xi'^2}}. \tag{91}$$

Here we notice that we also find $\pi_\beta = \frac{\partial\mathcal{L}_{\rm tr}}{\partial\beta'}$. However there is no momentum in the β direction now, so we do not need to consider the drag force in this direction. However, we must solve both $\beta(h)$ and $\xi(h)$ in order to obtain π_ξ as seen from Eq. (91). As a result, the string configuration moving with a constant velocity is deformed also to β direction. The profile of such string is shown in Fig. 7.

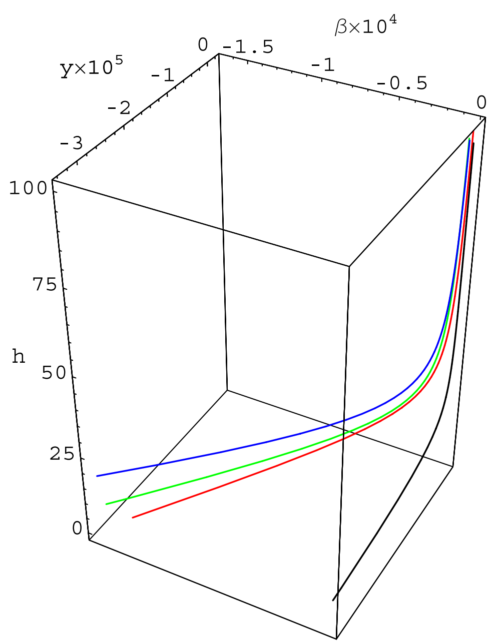

FIG. 7 (color online). Strings trailing along with y axis for $v = 0.1$, $a = 1$, $R = 1$ and $q = 0$ (on $\beta = 0$ plane), $q = 1$ (front), $q = 2$ (middle), $q = 5$ (back). These strings that have finite q are curved to β axis due to the existence of dilaton and the curve is sharper with the larger value of q.

Here we give the following comment instead of showing the numerical behavior of the drag force. The drag force on the quark can be determined by the momentum flow π_ξ which is lost as the flow from the string to the horizon [15]. We find the horizon on the string world sheet at $h = h_*$ from (87),

$$h_* = \frac{a}{\sqrt{1 - v^2 e^{-2a\beta}}}, \tag{92}$$

which depends on v and β. And $h_* > a$, namely, it is larger than the bulk horizon. Another point to be noticed is that the velocity is constrained in the present case as

$$v \leq e^{a\beta} \tag{93}$$

for fixed β.

In any case, thus, we can estimate the drag force at the horizon $h = h_*$, where ξ' and β' dependence disappears, as

$$F_{\rm drag} = \pi_\xi = -\frac{1}{2\pi\alpha'}\sqrt{(h_* e^{-a\beta(h_*)})^4 R^4 + \frac{q}{R^4}}v. \tag{94}$$

Here we notice the following two points. First, we can see $\pi_\beta(h_*) = 0$, then the force observed at the horizon $h = h_*$ is only the drag force $F_{\rm drag}$ for dragging the string to the y direction. While the value of h_* is determined by solving $\beta(h)$ for the first time, it is not performed here. Second, the force is modified by the color force due to the dilaton from the form which is proportional to the temperature, $F_{\rm drag} \propto T^2$.

Finally as for the friction constant η, one can define it as

$$F_{\rm drag} = \frac{dp}{dt} = -\eta p, \qquad p = m_q \frac{v}{\sqrt{1-v^2}} \tag{95}$$

where m_q denotes the quark mass, and η is written as,

$$\eta = \frac{\sqrt{1-v^2}}{2\pi\alpha' m_q}\sqrt{(h_* e^{-a\beta(h_*)})^4 R^4 + \frac{q}{R^4}} \tag{96}$$

For small $v \simeq 0$, we find $h_* \simeq a$ and $h_* e^{-a\beta(h_*)} \simeq u_1$, then we get

$$\eta \simeq \frac{1}{2\pi\alpha' m_q}\sqrt{u_1^4 R^4 + \frac{q}{R^4}}. \tag{97}$$

Then the friction constant is related to the radiation power given in AdS_5 background [6].

In the case of the AdS_5-Schwarzschild background, the friction constant is given as follows [14],

$$\eta_{AdS} = \frac{\pi}{2\alpha' m_q} R^2 T^2. \tag{98}$$

On the other hand, in the limit of $q = 0$, (97) leads to

$$\eta \simeq \frac{2\pi}{\alpha' m_q} R^2 T^2, \tag{99}$$

since $u_1 = a$ in this limit. Thus we recognize the difference of factor four between (99) and (98). This point should be explained from some physical insight, but it is remained as an open problem here.

V. RELATION TO THE 4D FIELD THEORY

Here we give a comment on the statement for the Unruh effect given in [2], where the analysis is performed within the 4D field theory. The main result is that the vacuum expectation values (VEVs) of any Green's functions in the vacuum of Minkowski space-time are the same as those of the Rindler space-time when the calculation is restricted to the same Rindler wedge in the Minkowski coordinate. So one may consider that the phase of the Minkowski vacuum cannot be changed in the Rindler vacuum since the VEV of any order parameter would be the same in both vacuums.

However, in the present paper, we show that the vacuum of the Rindler space-time is in the quark deconfinement phase in spite of the fact that the original theory in the Minkowski vacuum is in the confinement phase. Then our calculation seems to be inconsistent with the statement of [2]. However this point would be resolved as follows.

Since the confinement or deconfinement is discriminated by the VEV of the Wilson loop, we concentrate on this quantity here. In the field theory side, the corresponding operator would be given as,

$$\mathcal{O} = {\rm tr}(\mathcal{P} e^{ig \oint_C A_\mu(z) dz^\mu}) \tag{100}$$

where $\mathcal{P}$ denotes the path ordering of a closed path $\mathcal{C}$ in the line integration for the gauge field $A_\mu(z)$. Its VEV is written as

$$A = \langle 0|\mathcal{O}|0\rangle \tag{101}$$

for the Minkowski vacuum $|0\rangle$, and

$$B = \frac{{\rm Tr}(e^{-H^R/T}\mathcal{O})}{{\rm Tr} e^{-H^R/T}} \tag{102}$$

for the finite temperature (T) Rindler vacuum, respectively. The statement in [2] implies the equivalence of A and B when they are calculated within the same Rindler wedge.

In our holographic approach, the Wilson loop defined above is estimated in terms of the (static) string configurations, whose end points are on the path $\mathcal{C}$, for both A and B. The string configurations are obtained as the classical solutions of the Nambu-Goto action embedded in each background. Now, we perform the calculation for A for a fixed path $\mathcal{C}$. Then the calculation for B has been done by using the path and the string configurations, which are all obtained by ERT from those used in A. In this case, we will find $A = B$. However, we did not do the calculation in this way.

Our calculation of A and B do not lead to $A = B$ due to the following three reasons. First, the paths used in A and B are not related by ERT. In order to see the potential between the quark and the antiquark, we have performed the calculations for the rectangular path in the t-x plane for A and the one in the τ-β plane for B respectively. In this case, the rectangular path used in A cannot be transformed to the one used in B by ERT since it must be transformed to the τ dependent path.

Second, the static solutions used in the evaluation of A cannot be transformed to the static (τ independent) ones used in B since the static solutions given in the Minkowski vacuum are generally transformed by ERT to the τ dependent one. Then our Wilson-loop calculations in A and B are not related by ERT. We derived our result from them. Actually, we could obtain different results from the calculation of A and B, the linear confinement potential for A and the screening and the deconfinement for B respectively.

Third, we should give a comment for the quark string configuration in the Rindler vacuum. This point is also related to the fact that the quark in the Rindler vacuum is different from the one in the Minkowski vacuum. In estimating B, the essential string solution responsible for the proof of the deconfinement is the one which connects the boundary and the event horizon, because this solution can be interpreted as the free quark and this is possible only for the deconfinement phase. We could find such a solution only in the Rindler vacuum. The interesting point is that this free-quark configuration in the Rindler vacuum is obtained by ERT from the constantly accelerating quark string configuration given in the Minkowski vacuum as shown above. However this configuration is not used in the evaluation of A in our theoretical scheme. Because of these reasons, we are not seeing the relation $A = B$. We are examining the parts, which cannot be related by ERT, of A and B. Then our statement is not contradicting the one in [2].

Of course, there are problems remaining related to the coordinate transformation which is adopted in the present paper. We should study the other form of the coordinate transformation. For example, the coordinate transformation given in [2] can be considered as such a transformation, which does not include the fifth coordinate of the bulk. This is different from ERT used here. In the latter case, the transformation is performed in three dimensional coordinate including the fifth one. As a result, the event horizon appears in the bulk, then the infrared region is cut off in the dynamics of the dual 4D theory. Then the dynamical properties responsible to the long range force would be lost in the vacuum of the new coordinate.

We should study also the VEV of other physical quantities. In this context, we notice another work given in [16] from other 4D nonperturbative approach, and the author shows chiral symmetry restoration in the Rindler vacuum. So it would be necessary to proceed with work more in this direction in order to make clear the Rindler vacuum.

VI. SUMMARY AND DISCUSSIONS

We give here a constantly accelerated quark as a string solution of the Nambu-Goto action which is embedded in the supergravity background dual to the confining Yang-Mills theory. For this accelerated quark given in the zero-temperature Minkowski space-time, we find an event horizon in its world sheet metric. This horizon is also found in the case of AdS_5 background dual to the nonconfining theory. In any case, this fact can be considered as a clear signal of the radiation of gluons due to the acceleration of the color charged quark.

We consider the extended Rindler transformation proposed by Xiao in order to move to the comoving frame of the accelerated quark and to study the properties of the Rindler vacuum. The coordinate transformation is generalized to the 5D bulk theory, but the boundary is described by the usual 4D Rindler metric. In this case, the dual theory is found in a thermal medium with the Rindler temperature, so it is expected that the theory is in a different vacuum from the one in the inertial frame. To study this point, we have examined several dynamical properties of the new vacuum to compare them to those observed in the inertial coordinate.

We could find that the vacuum properties are changed from those seen in the inertial frame. In the Rindler vacuum, the color force is screened by the thermal effect at long distance. As a result, the confining property has been lost. The screening length depends on the temperature and also on the direction in the three space. As for the temperature dependence, we find some difference from the one observed previously in the AdS-Schwarzschild background, especially for the screening in the longitudinal direction. The reason of the anisotropy in the 3D space, longitudinal and transverse to the acceleration, can be reduced to the behavior of the dilaton. The dilaton expresses the gauge coupling constant, and it is deformed in the longitudinal direction due to the extended Rindler transformation. Then the potential between the quark and antiquark has different forms in the longitudinal and the transverse directions. This point has been assured through the direct observation of the potential between the quark and antiquark.

Even if we calculate the force in the transverse direction, we can see the effect of the longitudinally deformed force. For example, consider some excited bound state of the quark and antiquark. Then we could find a small repulsion in the direction (y) transverse to the accelerated direction (β). This repulsion is understood from the string configuration in the 3D space β-y-h. When the distance y decreases, the string is stretched in the β direction near the horizon $h \simeq a$. This phenomenon is understood as the remnant of the strong confining force near the horizon. On the other hand, the attractive force is exponentially enhanced in the longitudinal direction at fairly large distance. This behavior can be reduced to the deformed dilaton in the Rindler coordinate in the longitudinal direction.

We also discuss the drag force to see the thermal effects in the Rindler vacuum. We find the friction constant is related to the radiation power of the accelerated quark in the original vacuum. Then we recognize the difference of factor four between the friction constant in the Rindler vacuum and the one in the AdS_5-Schwarzschild background.

Finally, we give our main conclusion that we could find a new vacuum when we move to the comoving coordinate of an accelerating quark by the extended Rindler transformation considered here and find a kind of a high temperature theory of deconfinement phase. Then, in the new vacuum, the properties given by the long range force in the inertial frame have been lost. Thus our calculation seems to be inconsistent with the claim given in [2]. However, this point is resolved as explained in the previous section. The main reason is that we are considering the different VEV of Green's functions to decide the phase of the vacuum.

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