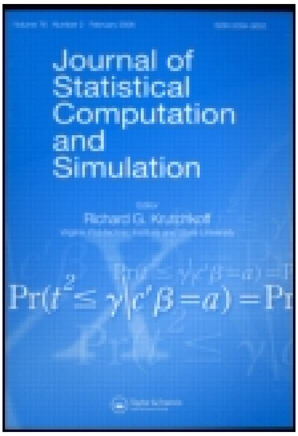

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An effective powerful test for one-sided supplier selection problem

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An effective powerful test for one-sided supplier selection problem

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In this paper, we consider the supplier selection problem, which deals with comparing two one-sided processes and selecting a better one that has a higher capability. We first review two existing approximation approaches, and an exact approach proposed which we refer to as the division method. We then develop a new exact approach called the subtraction method. We compare the two exact methods on the selection power. The results show that the proposed subtraction method is indeed more powerful than the division method. A two-phase selecting procedure is then developed based on the subtraction method for practical applications. Some computational results are tabulated for practitioners' convenience.

Keywords: supplier selection; process capability index; non-conformities

1. Introduction

Process capability indices, which are unitless numerical values, quantifying process performance, have been widely used in the manufacturing and service industries, to measure process reproduction capability of meeting the preset product quality requirement. Several basic process capability indices, including C_p, C_{PU}, C_{PL}, C_{pk}, C_{pm}, and C_{pmk}, have been developed for this purpose [1–3]. Those indices essentially compare the predefined product specifications with the actual production or service performance of the investigated quality characteristics. Those indices are defined as follows:

$$C_p = \frac{\mathrm{USL}-\mathrm{LSL}}{6\sigma}, \quad C_{\mathrm{PU}} = \frac{\mathrm{USL}-\mu}{3\sigma}, \quad C_{\mathrm{PL}} = \frac{\mu-\mathrm{LSL}}{3\sigma},$$

$$C_{\mathrm{pk}} = \min\left\{\frac{\mathrm{USL}-\mu}{3\sigma}, \frac{\mu-\mathrm{LSL}}{3\sigma}\right\}, \quad C_{\mathrm{pm}} = \frac{\mathrm{USL}-\mathrm{LSL}}{6\sqrt{\sigma^2+(\mu-T)^2}},$$

$$C_{\mathrm{pmk}} = \min\left\{\frac{\mathrm{USL}-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-\mathrm{LSL}}{3\sqrt{\sigma^2+(\mu-T)^2}}\right\},$$

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where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation (overall process variation), and T is the target value. While the indices C_p, C_{pk}, C_{pm}, and C_{pmk} are appropriate for statistically controlled normal processes with two-sided specification limits, the indices C_{PU} and C_{PL} are designed, specifically, for processes with one-sided specification limit. We note that if a process is under control, process mean μ and standard deviation σ are stable, but this does not imply that process quality is satisfactory, or meeting customers requirement, particularly, for processes with a very low fraction of defecting (where classical control charts cannot be used for controlling such stringent quality requirements).

Those indices have been implemented in many industry manufacturing applications, including the multi-process performance analysis chart for factory defective control [4–6], process performance analysis for multiple quality characteristics [7,8], better supplier selection [9–15], capability measures for multiple manufacturing streams [16,17], variables acceptance sampling plans for lot sentencing [18–20], tool replacement optimization [21–23] and many others. Pearn and Kotz [24] presented a thorough review on the development of process capability indices in the past 20 years.

We note that this paper deals with measuring process performance based on production yield rather than process loss, which is the focus of most current industrial applications. The purpose of the index C_{PU} is different from that of the index C_{pm}, and the focus of our paper is primarily on production yield (not process loss). The problem cast in decision theoretic terms based on loss functions has been considered by Chan *et al.* [2] who introduced the index C_{pm}.

1.1. *Fraction of non-conformities*

For normally distributed processes with one-sided specification limit USL or LSL, process yield, $\Pr(X < \mathrm{USL})$, or $\Pr(X > \mathrm{LSL})$ can be established as the following, where Z is the standard normal distribution,

$$\Pr(X < \mathrm{USL}) = \Pr\left(\frac{X-\mu}{\sigma} < \frac{\mathrm{USL}-\mu}{\sigma}\right) = \Pr\left(Z < 3C_{\mathrm{PU}}\right) = \Phi\left(3C_{\mathrm{PU}}\right),$$

$$\Pr(X > \mathrm{LSL}) = \Pr\left(\frac{\mu-X}{\sigma} < \frac{\mu-\mathrm{LSL}}{\sigma}\right) = \Pr\left(Z > -3C_{\mathrm{PU}}\right) = \Phi\left(3C_{\mathrm{PL}}\right).$$

For convenience of presentation, we define $C_I = C_{\mathrm{PU}}$ or C_{PL}. Therefore, the corresponding non-conformities in parts per million (NCPPM) for a well-controlled normally distributed process can be calculated exactly, as $\mathrm{NCPPM} = 10^6 \times [1 - \Phi(C_I)]$. For example, if $C_I = 1.00$, the corresponding NCPPM is 1350; if $C_I = 1.25$, the corresponding NCPPM is 88; if $C_I = 1.33$, the corresponding NCPPM is 32; if $C_I = 1.45$, the corresponding NCPPM is 6.8; if $C_I = 1.50$, the corresponding NCPPM is 3.4; if $C_I = 1.67$, the corresponding NCPPM is 0.27, and for $C_I = 2.00$, the corresponding NCPPM is 0.001.

1.2. *Discussion*

Many real-world processes are not normally distributed and this departure from normality could potentially affect the accuracy of the capability measure. In the normal case, the bootstrap resampling method can be used, which handles more general distributions. The bootstrap resampling method does not rely on any distributional assumptions about the underlying population, which has been proved useful in many existing research for those cases.

2. Supplier selection problem and existing selection methods

The supplier selection problem is a practical problem frequently occurred in the industry, particularly, in the top-down supply chain and logistic management. Some literatures have provided following criteria for selecting the best manufacturing supplier from several available candidates (Table 1).

From the above references, one may find that quality is a very important criterion. Recently, many studies have widely used the process capability indices as tools for comparing suppliers' capability. Examples include, Tseng and Wu [9], Chou [10], Huang and Lee [11], Hubele *et al.* [29], Chen and Chen [30] and Pearn *et al.* [15]. The results presented in this paper are useful for practitioners, which have received some research attentions.

For the supplier selection problem with measuring process performance based on C_{PU}, Chou [10] considered a likelihood ratio approximation approach (which is a division-oriented approach). A decision rule for testing the hypothesis H_0: $C_{\mathrm{PU1}} \geq C_{\mathrm{PU2}}$ *vs.* H_1: $C_{\mathrm{PU1}} < C_{\mathrm{PU2}}$ is proposed. The rule rejects H_0 if $\hat{C}_{\mathrm{PU1}} < \hat{C}_{\mathrm{PU2}}$ and satisfying the condition $A < \exp\{-\chi_1^2(1-2\alpha)/2\}$, where

$$A = \left\{ 2 \Big/ \left[\sqrt{a\hat{C}_{\mathrm{PU1}}^2 + 2}\,\sqrt{a\hat{C}_{\mathrm{PU2}}^2 + 2} - a\hat{C}_{\mathrm{PU1}}\hat{C}_{\mathrm{PU2}} \right] \right\}^n,$$

$a = 9n/(n-1)$ and $\chi_1^2(1-2\alpha)$ is the $(1-2\alpha)$th quantile of a chi-square distribution with one degree of freedom. The test statistic A is the log-likelihood ratio statistic, which is used to approximate an exponential function of the chi-square distribution with one degree of freedom using the large-sample theory. We note that this approximation method requires equal sample sizes, $n_1 = n_2 = n$, in all applications.

Hubele *et al.* [29] adopted the formulation of Chou [10] using the Wald test investigated in Nairy and Rao [31], and developed an approximation approach for testing whether k processes are equal (which is also a division-oriented approach). The hypothesis is H_0: $C_{\mathrm{PU1}} = C_{\mathrm{PU2}} = \cdots = C_{\mathrm{PU}k}$ *vs.* H_1: $C_{\mathrm{PU}i} = C_{\mathrm{PU}j}$, $i \neq j$ for at least one pair of (i, j), where $i, j \in \{1, 2, \ldots, k\}$. The approach does not require equal sample sizes.

2.1. *The division method*

Pearn *et al.* [15] developed an exact approach, which we refer to as the division method, to handle the supplier selection problem. The hypothesis testing considered is H_0: $C_{\mathrm{PU2}}/C_{\mathrm{PU1}} \leq 1$ *vs.* H_1: $C_{\mathrm{PU2}}/C_{\mathrm{PU1}} > 1$. The division method does not require equal sample sizes for the two processes. The probability density function of the test statistic $R = \hat{C}_{\mathrm{PU2}}/\hat{C}_{\mathrm{PU1}}$ is as follows:

$$\begin{aligned} f_R(r) = A \int_{-\infty}^{\infty} \left|\frac{1}{u}\right| I\left(\frac{r}{u} > 0\right) & \frac{(r/u)^{n_1-2}}{\left(1 + (n_1-1)/(n_2-1)(r/u)^2\right)^{((n_1+n_2)/2)-1}} \\ & \times \exp\left[\frac{9}{2}\frac{(n_1 C_{\mathrm{PU1}} + n_2 C_{\mathrm{PU2}} u)^2}{n_1 + n_2 u^2}\right] \left\{ \frac{2}{9(n_1+n_2u^2)} \exp\left[-\frac{9}{2}\frac{(n_1 C_{\mathrm{PU1}} + n_2 C_{\mathrm{PU2}} u)^2}{n_1 + n_2 u^2}\right] \right. \\ & \left. + \frac{n_1 C_{\mathrm{PU1}} + n_2 C_{\mathrm{PU2}} u}{n_1 + n_2 u^2} \sqrt{\frac{2\pi}{9(n_1+n_2u^2)}} \left[1 - 2\,\Phi\left(\frac{-3(n_1 C_{\mathrm{PU1}} + n_2 C_{\mathrm{PU2}} u)}{\sqrt{n_1 + n_2 u^2}} \right) \right] \right\} \mathrm{d}u, \end{aligned}$$

where

$$A = 2\left(\frac{n_1-1}{n_2-1}\right)^{(n_1-1)/2} \frac{\Gamma(((n_1+n_2)/2)-1)}{\Gamma((n_1-1)/2)\Gamma((n_2-1)/2)} \frac{9\sqrt{n_1 n_2}}{2\pi} \exp\left[-\frac{9}{2}(n_1 C_{\mathrm{PU1}}^2 + n_2 C_{\mathrm{PU2}}^2)\right],$$

Table 1. Some criteria for selecting the best supplier.

Author(s)	Criteria
Dickson [25]	(1) Quality; (2) delivery; (3) performance history
Krause *et al.* [26]	(1) Quality; (2) reliability; (3) flexibility; (4) cost; (5) originality
Stevenson [27]	(1) Lead time; (2) reputation; (3) flexibility; (4) delivery; (5) quality; (6) location; (7) price
Tracey and Tan [28]	(1) Quality; (2) price; (3) product performance; (4) delivery reliability

$\Phi(\cdot)$ is the cumulative function of the standard normal distribution, I is the indicator function, and $-\infty < r < \infty$.

Pearn *et al.* [15] demonstrated that the division method is indeed more accurate and powerful than Chou's approximation method (also a division method). A question remains unanswered is that whether the test used for selection is the uniformly most powerful test, which provides the maximal fair protection to the new competing supplier C_{PU2}. Consequently, the new development in this paper would answer that question.

3. The proposed subtraction method

In this section, we propose a new exact approach called the subtraction method. We consider the hypothesis testing for comparing the two C_{PU} values, H_0: $C_{\mathrm{PU2}} \le C_{\mathrm{PU1}}$ *vs.* H_1: $C_{\mathrm{PU2}} > C_{\mathrm{PU1}}$ (or equivalently, H_0: $C_{\mathrm{PU2}} - C_{\mathrm{PU1}} \le 0$ *vs.* H_1: $C_{\mathrm{PU2}} - C_{\mathrm{PU1}} > 0$). For test statistic

$$W = \hat{C}_{\mathrm{PU2}} - \hat{C}_{\mathrm{PU1}} = \frac{\mathrm{USL} - \bar{X}_1}{3S_1} - \frac{\mathrm{USL} - \bar{X}_2}{3S_2},$$

where $\bar{X}_1$ and $\bar{X}_2$ are sample mean, and S_1 and S_2 are sample standard deviation, we may obtain the probability density function as

$$\begin{aligned} f_W(w) &= \int_{-\infty}^{\infty} f_{Y_1}(y_1) f_{Y_2}(w + y_1)\,\mathrm{d}y_1 \\ &= A \times \int_{-\infty}^{\infty} g(w, y_1)\,\mathrm{d}y_1, \quad -\infty < w < \infty, \end{aligned}$$

where

$$\begin{aligned} A = {} & \frac{2}{\sqrt{2\pi/9n_1}\,\Gamma((n_1-1)/2)(2/(n_1-1))^{(n_1-1)/2}} \\ & \times \frac{2}{\sqrt{2\pi/9n_2}\,\Gamma((n_2-1)/2)(2/(n_2-1))^{(n_2-1)/2}}, \end{aligned}$$

and

$$\begin{aligned} g(w, y_1) = {} & \int_0^{\infty} \exp\left[-\frac{9n_1(v_1 y_1 - C_{\mathrm{PU1}})^2}{2}\right] v_1^{n_1-1} \mathrm{e}^{-((n_1-1)/2)v_1^2}\,\mathrm{d}v_1 \\ & \times \int_0^{\infty} \exp\left[-\frac{9n_2\,(v_2(w + y_1) - C_{\mathrm{PU2}})^2}{2}\right] v_2^{n_2-1} \mathrm{e}^{-((n_2-1)/2)v_2^2}\,\mathrm{d}v_2. \end{aligned}$$

Figure 1 plots the probability density function of W for $C_{\mathrm{PU1}} = 1.0, 1.5$, $C_{\mathrm{PU2}} = 1.0, 1.5$, and $n_1 = n_2 = 30, 50, 100, 150, 200$ (from the bottom to top in plots). From Figure 1, we can see that (1) the larger the value of $C_{\mathrm{PU2}} - C_{\mathrm{PU1}}$, the larger the variance of $W = \hat{C}_{\mathrm{PU2}} - \hat{C}_{\mathrm{PU1}}$, (2) the distribution of W is unimodal and is rather symmetric to $C_{\mathrm{PU2}} - C_{\mathrm{PU1}}$ even for small sample sizes.

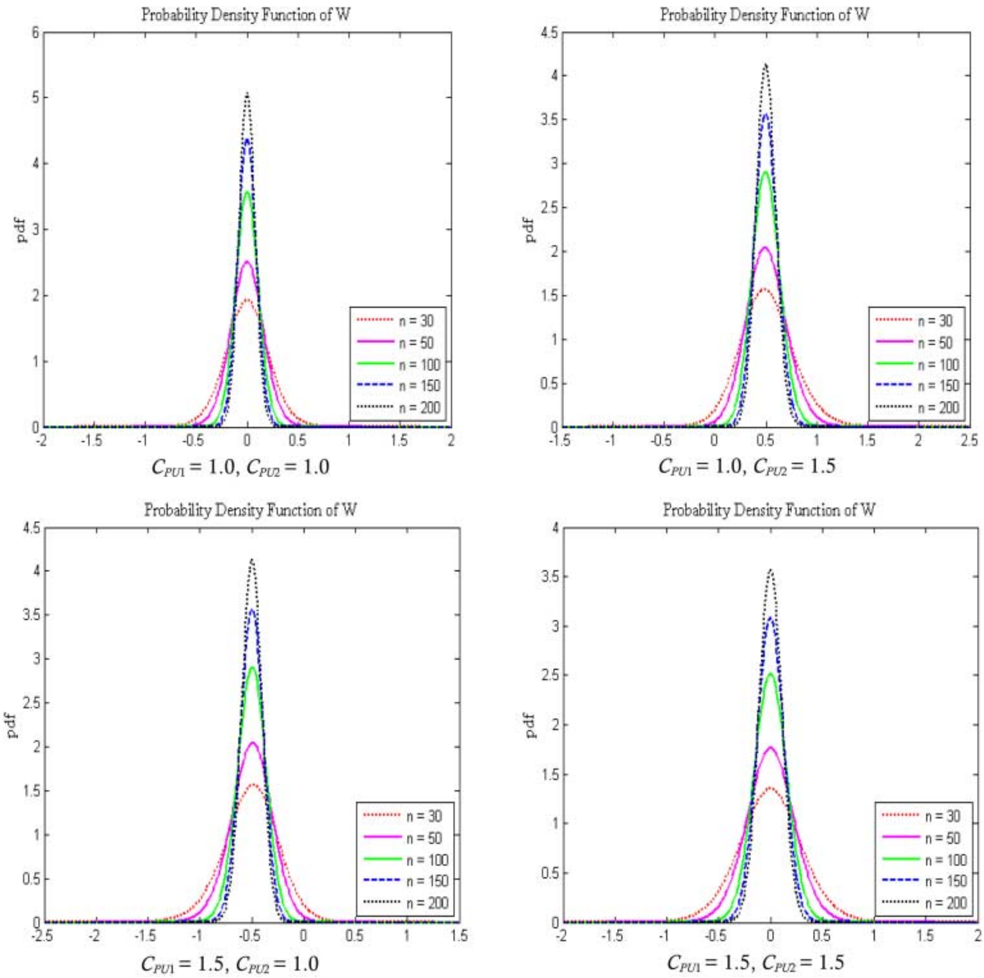

Figure 1. Probability density function plots of W for sample sizes $n_1 = n_2 = 30, 50, 100, 150, 200$ (from bottom to top in plots).

4. Selection procedure for the subtraction method

4.1. *Phase I: selection determination*

Assume that the minimum requirement of C_{PU} values for all candidate processes is C, and the existing supplier, supplier I, has achieved the requirement, i.e. $C_{\mathrm{PU1}} \geq C$. If a new supplier, supplier II, wants to compete for the orders by claiming that its capability is better than the existing supplier I, then the new supplier must furnish convincing information justifying the claim with a prescribed level of confidence. To test whether the new supplier, supplier II, has a better capability than the existing supplier I, we consider the hypothesis testing: H_0: $C_{\mathrm{PU2}} \leq C_{\mathrm{PU1}}$ *vs*. H_1: $C_{\mathrm{PU2}} > C_{\mathrm{PU1}}$, which is equivalent to

$$H_0: C_{\mathrm{PU2}} - C_{\mathrm{PU1}} \leq 0$$
$$H_1: C_{\mathrm{PU2}} - C_{\mathrm{PU1}} > 0.$$

Based on the probability density function of $W = \hat{C}_{PU2} - \hat{C}_{PU1}$ and a given level of Type I error α, the chance of incorrectly judging $C_{PU2} \le C_{PU1}$ as $C_{PU2} > C_{PU1}$, the decision rule is to reject H_0 if the test statistic $W = C_{PU2} - C_{PU1} \ge c_0$, where c_0 is the critical value that satisfies

$$\Pr\{W \ge c_0 | H_0 : C_{PU2} \le C_{PU1}, n_1, n_2, \text{ and } C_{PU1} \ge C\} \le \alpha.$$

Since the larger the $C_{PU2} - C_{PU1}$ value, the larger the critical value; we calculate the critical value c_0 with the conditions $C_{PU2} = C_{PU1} = C$, i.e.

$$\Pr\{W \ge c_0 | C_{PU2} = C_{PU1} = C, n_1, n_2\} = \alpha.$$

Note that our supplier selection procedure can be applied to cases with unequal sample sizes, $n_1 \ne n_2$. It is also noted that by replacing C_{PU} by C_{PL}, the procedure can be used to test the hypothesis: H_0: $C_{PL2} \le C_{PL1}$ *vs.* H_1: $C_{PL2} > C_{PL1}$. Table 2 displays the critical values for $C_{PU2} = C_{PU1} = 1.0(0.1)2.0$, $n_1 = n_2 = n = 30(10)200$ and $\alpha = 0.05$.

4.2. *Phase II: selection determination with designated outperformance*

In Phase I of the supplier selection procedure, the supplier selection decisions would be based solely on the hypothesis testing comparing the two C_{PU} values without investigating further the designated outperformance of the difference between the two suppliers. In practice, the customer would consider choosing the new supplier only if the new supplier's capability is significantly better than the existing supplier's by a designated outperformance $h > 0$, due to the high cost of the supplier replacement. Our exact approach, in this case, can be used to test the corresponding hypothesis:

$$H_0: C_{PU2} \le C_{PU1} + h$$
$$H_1: C_{PU2} > C_{PU1} + h,$$

Table 2. Critical values for rejecting $C_{PU2} \le C_{PU1}$ with $n_1 = n_2 = n = 30(10)200$ and $\alpha = 0.05$.

	$C_{PU1} = C_{PU2} = C$										
n	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
30	0.3512	0.3833	0.4124	0.4421	0.4725	0.5036	0.5355	0.5683	0.5978	0.6280	0.6591
40	0.2991	0.3232	0.3512	0.3761	0.4014	0.4272	0.4534	0.4802	0.5075	0.5355	0.5600
50	0.2651	0.2854	0.3094	0.3336	0.3547	0.3797	0.4014	0.4234	0.4496	0.4725	0.4958
60	0.2384	0.2584	0.2786	0.3025	0.3232	0.3441	0.3654	0.3833	0.4050	0.4272	0.4496
70	0.2219	0.2384	0.2584	0.2753	0.2956	0.3163	0.3336	0.3547	0.3725	0.3941	0.4124
80	0.2055	0.2219	0.2384	0.2584	0.2753	0.2922	0.3128	0.3301	0.3477	0.3654	0.3833
90	0.1924	0.2087	0.2252	0.2417	0.2584	0.2753	0.2922	0.3094	0.3267	0.3441	0.3618
100	0.1826	0.1989	0.2120	0.2285	0.2451	0.2618	0.2753	0.2922	0.3094	0.3267	0.3406
110	0.1729	0.1891	0.2022	0.2186	0.2318	0.2484	0.2618	0.2786	0.2956	0.3094	0.3267
120	0.1664	0.1794	0.1924	0.2087	0.2219	0.2384	0.2517	0.2651	0.2820	0.2956	0.3094
130	0.1600	0.1729	0.1859	0.1989	0.2120	0.2285	0.2417	0.2551	0.2685	0.2854	0.2991
140	0.1536	0.1664	0.1794	0.1924	0.2055	0.2186	0.2318	0.2451	0.2584	0.2719	0.2854
150	0.1471	0.1600	0.1729	0.8459	0.1989	0.2120	0.2252	0.2384	0.2517	0.2618	0.2753
160	0.1440	0.1536	0.1664	0.1794	0.1924	0.2055	0.2153	0.2285	0.2417	0.2551	0.2685
170	0.1375	0.1503	0.1600	0.1729	0.1859	0.1989	0.2087	0.2219	0.2351	0.2484	0.2584
180	0.1343	0.1439	0.1568	0.1697	0.1794	0.1924	0.2022	0.2153	0.2285	0.2384	0.2517
190	0.1311	0.1407	0.1536	0.1632	0.1761	0.1859	0.1989	0.2087	0.2219	0.2318	0.2451
200	0.1279	0.1375	0.1471	0.1600	0.1697	0.1826	0.1924	0.2055	0.2153	0.2285	0.2384

Table 3. Critical values for rejecting $C_{PU2} \leq C_{PU1} + h$ for various values of h with $\alpha = 0.05$.

	(C_{PU1}, C_{PU2})				
n	(1.25, 1.35)	(1.25, 1.45)	(1.25, 1.55)	(1.25, 1.65)	(1.25, 1.75)
30	0.5477	0.6727	0.7934	0.9186	1.0417
40	0.4802	0.5978	0.7146	0.8352	0.9540
50	0.4346	0.5518	0.6682	0.7833	0.9014
60	0.4050	0.5155	0.6324	0.7435	0.8423
70	0.3797	0.4918	0.6063	0.7194	0.8299
80	0.3582	0.4725	0.5808	0.6958	0.8089
90	0.3441	0.4534	0.5642	0.6773	0.7883
100	0.3301	0.4421	0.5518	0.6591	0.7732
110	0.3197	0.4272	0.5396	0.6457	0.7582
120	0.3094	0.4197	0.5275	0.6368	0.7435
130	0.3025	0.4087	0.5195	0.6280	0.7338
140	0.2922	0.4014	0.5075	0.6193	0.7241
150	0.2854	0.3941	0.5036	0.6106	0.7194
160	0.2786	0.3869	0.4958	0.6020	0.7099
170	0.2753	0.3833	0.4879	0.5978	0.7052
180	0.2685	0.3761	0.4841	0.5893	0.6958
190	0.2651	0.3725	0.4763	0.5851	0.6911
200	0.2618	0.3654	0.4725	0.5808	0.6865
	(1.45, 1.55)	(1.45, 1.65)	(1.45, 1.75)	(1.45, 1.85)	(1.45, 1.95)
30	0.6106	0.7338	0.8568	0.9782	1.1025
40	0.5315	0.6502	0.7682	0.8900	1.0031
50	0.4802	0.5978	0.7146	0.8299	0.9480
60	0.4459	0.5600	0.6727	0.7883	0.9014
70	0.4161	0.5315	0.6412	0.7533	0.8678
80	0.3941	0.5075	0.6193	0.7289	0.8406
90	0.3761	0.4879	0.5978	0.7099	0.8194
100	0.3618	0.4725	0.5808	0.6911	0.8037
110	0.3512	0.4572	0.5683	0.6773	0.7883
120	0.3371	0.4459	0.5560	0.6636	0.7732
130	0.3301	0.4383	0.5436	0.6546	0.7632
140	0.3197	0.4272	0.5355	0.6457	0.7533
150	0.3128	0.4197	0.5275	0.6368	0.7435
160	0.3059	0.4124	0.5195	0.6280	0.7338
170	0.2991	0.4050	0.5115	0.6193	0.7289
180	0.2922	0.4014	0.5075	0.6150	0.7194
190	0.2888	0.3941	0.4997	0.6063	0.7146
200	0.2820	0.3905	0.4958	0.6020	0.7099
	(1.60, 1.70)	(1.60, 1.80)	(1.60, 1.90)	(1.60, 2.00)	(1.60, 2.10)
30	0.6591	0.7782	0.9014	1.0222	1.1451
40	0.5725	0.6911	0.8089	0.9244	1.0483
50	0.5155	0.6324	0.7484	0.8623	0.9782
60	0.4763	0.5893	0.7052	0.8194	0.9361
70	0.4459	0.5600	0.6727	0.7833	0.8957
80	0.4234	0.5355	0.6457	0.7582	0.8678
90	0.4050	0.5155	0.6237	0.7338	0.8460
100	0.3869	0.4958	0.6063	0.7194	0.8299
110	0.3725	0.4802	0.5935	0.7005	0.8089
120	0.3618	0.4686	0.5767	0.6865	0.7986
130	0.3512	0.4572	0.5683	0.6773	0.7833
140	0.3406	0.4496	0.5559	0.6636	0.7732
150	0.3301	0.4383	0.5477	0.6546	0.7632
160	0.3232	0.4309	0.5396	0.6457	0.7533
170	0.3163	0.4234	0.5315	0.6368	0.7484
180	0.3094	0.4161	0.5235	0.6324	0.7386
190	0.3059	0.4124	0.5195	0.6237	0.7338
200	0.2991	0.4050	0.5115	0.6193	0.7241

The decision rule is similar to that in Phase I. We would reject H_0 and accept that $C_{\mathrm{PU2}} > C_{\mathrm{PU1}} + h$, if $W = \hat{C}_{\mathrm{PU2}} - \hat{C}_{\mathrm{PU1}}$ is larger than or equal to the critical value c_0, where c_0 satisfies

$$\Pr\{W \geq c_0 | H_0 : C_{\mathrm{PU2}} \leq C_{\mathrm{PU1}} + h,\ n_1, n_2,\ \text{and } C_{\mathrm{PU1}} \geq C\} \leq \alpha.$$

For all combinations of $(C_{\mathrm{PU1}}, C_{\mathrm{PU2}})$ under H_0, the maximal critical value occurs at $C_{\mathrm{PU1}} = C$ and $C_{\mathrm{PU2}} = C + h$, and the larger the α, the smaller the critical value. Thus, we calculate the

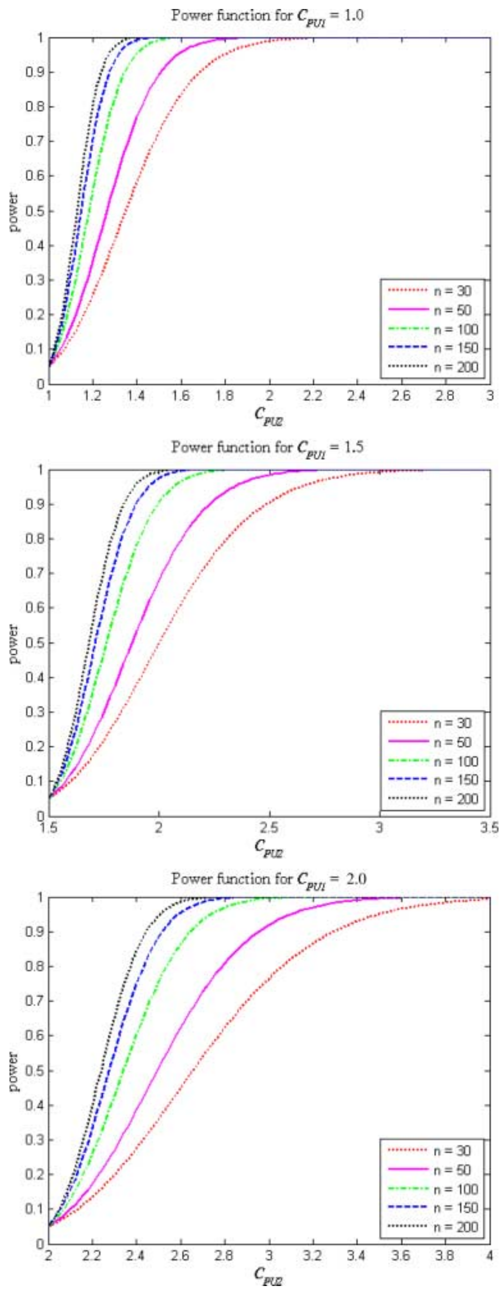

Figure 2. Power curves for $C_{\mathrm{PU1}} = 1.0$, 1.5, and 2.0, with sample sizes $n = 30, 50, 100, 150, 200$.

critical value c_0 with the probability

$$\Pr\{W \geqslant c_0 | C_{\mathrm{PU1}} = C,\ C_{\mathrm{PU2}} = C + h,\ n_1,\ n_2\} = \alpha.$$

If the test rejects the null hypothesis H_0, then there is sufficient information to conclude that supplier II is significantly better than supplier I by a designated outperformance h, and the replacement of supplier I by supplier II would be suggested. Table 3 shows the critical values for some minimum quality requirements C of C_{PU} suggested by Montgomery [32], with the designated outperformance h = 0.1(0.1)0.5, the difference between the two suppliers and $n_1 = n_2 = n = 30(10)200$ with $\alpha = 0.05$.

4.3. *Power analysis*

In Phases I and II, the supplier selection procedure is developed for given α risk, the probability of incorrectly judging H_0 as H_1, which does not take into account the β risk (Type II error), the probability of incorrectly judging H_1 as H_0, which is comparatively unfavourable to H_1 or the new supplier. This is due to the additional cost of replacing the existing supplier with the new one. Once the sample sizes and the α risk are defined, the power of test, $1 - \beta$, can be calculated. Figure 2 plots the power of the test for $C_{\mathrm{PU1}} = 1.0, 1.5, 2.0$ *vs*. various values of C_{PU2}, $n_1 = n_2 = n = 30$, 50, 100, 150, 200, and $\alpha = 0.05$. It can be seen that the larger the sample size, the larger the power of test, and consequently, the smaller the β risk.

$$\Pr\{W \geq c_0 | H_0 : C_{\mathrm{PU2}} \leq C_{\mathrm{PU1}}, n_1, n_2,\ \text{and}\ C_{\mathrm{PU1}} \geq C\} \leq \alpha,\ \text{and}$$

$$\Pr\{W \geq c_0 | H_1 : C_{\mathrm{PU2}} > C_{\mathrm{PU1}}, n_1, n_2,\ \text{and}\ C_{\mathrm{PU1}} \geq C\} \geq 1 - \beta.$$

To reduce the β risk and at the same time maintain the α risk at the required level, one could increase the sample sizes. By calculating the power for a specific value of C_{PU2}, we may obtain the minimal sample size required for designated power and α risk. The required sample size can be calculated using recursive search method with the following two probability equations.

5. Comparison between subtraction and division methods

Pearn *et al.* [15] developed an exact method for the supplier selection problem using test statistic $R = \hat{C}_{\mathrm{PU2}}/\hat{C}_{\mathrm{PU1}}$. Under the same significant level (Type I error), hypothesis tests are compared and evaluated through their power function, the probability of rejecting the null hypothesis correctly. Figures 3–18 display the power curves of the subtraction method (shown in — lines) and the division method (shown in - - - lines) for $C_{\mathrm{PU1}} = 1.00, 1.33, 1.67, 2.00$ *vs*. C_{PU2} with sample sizes $n_1 = n_2 = 30$, 50, 100, 150, respectively.

From those figures, it can be seen that the test using $W = \hat{C}_{\mathrm{PU2}} - \hat{C}_{\mathrm{PU1}}$ is significantly more powerful than the one based on $R = \hat{C}_{\mathrm{PU2}}/\hat{C}_{\mathrm{PU1}}$ as the power curves of the subtraction method are apparently higher than the power curves of the division method uniformly over all cases. Unfortunately, this result cannot be proved theoretically due to the complexity of their probability density functions.

In Table 4, we summarize the computational results in comparing the subtraction and the division methods for the sample sizes required to differentiate C_{PU1} and C_{PU2}. In Table 4, Type I error is set to $\alpha = 0.05$, the designated test power $1 - \beta$ is set to 0.90, 0.95, 0.975, 0.99, $C_{\mathrm{PU1}} = 1.00, 1.25, 1.45, 1.60$, and the designated outperformance of difference $C_{\mathrm{PU2}} - C_{\mathrm{PU1}} = 0.15(0.05)1.00$. For example, if the minimal capability requirement C_{PU} is 1.25, the designated α and β risks are 0.05 (i.e. power of the test = 0.95), and the expected designated outperformance of difference $C_{\mathrm{PU2}} - C_{\mathrm{PU1}}$ is 0.3, and then the required sample sizes for both suppliers is 233.

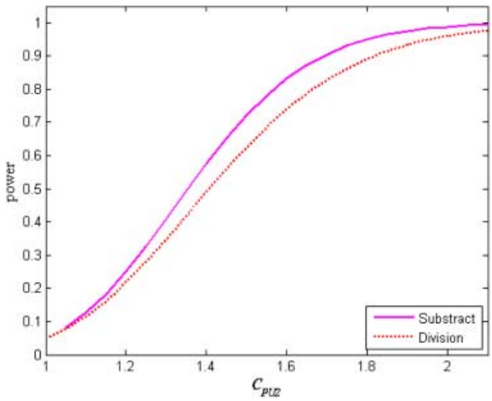

Figure 3. Power curves of the two methods for $C_{\mathrm{PU1}} = 1.00$ *vs*. C_{PU2} with $n_1 = n_2 = 30$.

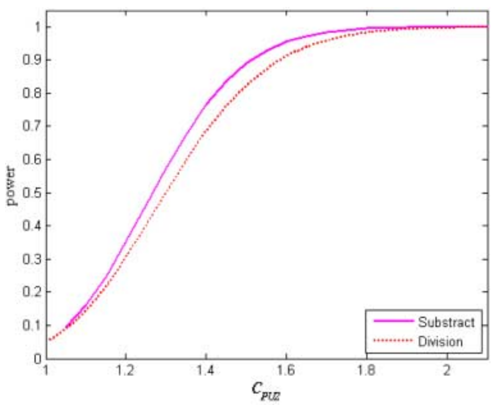

Figure 4. Power curves of the two methods for $C_{\mathrm{PU1}} = 1.00$ *vs*. C_{PU2} with $n_1 = n_2 = 50$.

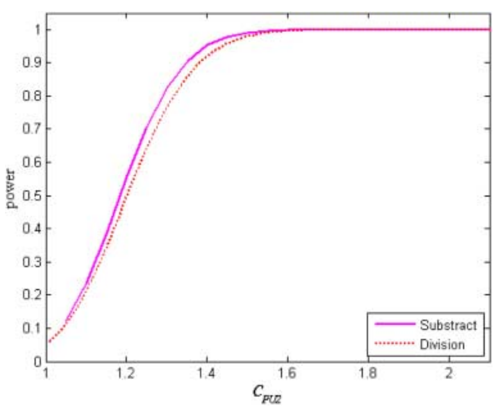

Figure 5. Power curves of the two methods for $C_{\mathrm{PU1}} = 1.00$ *vs*. C_{PU2} with $n_1 = n_2 = 100$.

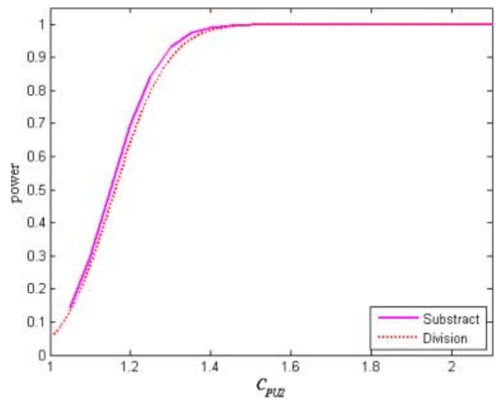

Figure 6. Power curves of the two methods for $C_{\text{PU1}} = 1.00$ *vs*. C_{PU2} with $n_1 = n_2 = 150$.

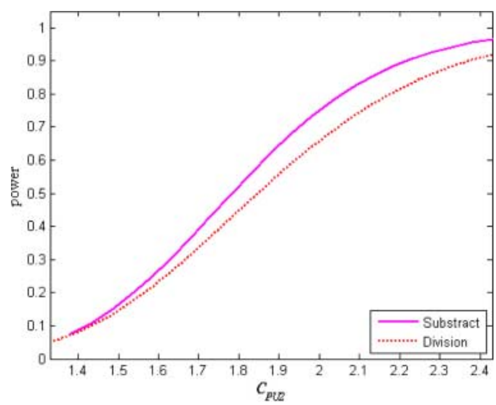

Figure 7. Power curves of the two methods for $C_{\text{PU1}} = 1.33$ *vs*. C_{PU2} with $n_1 = n_2 = 30$.

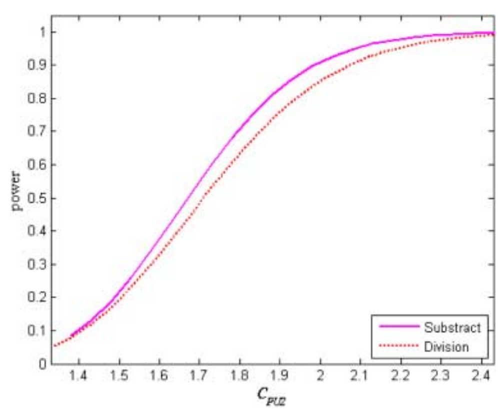

Figure 8. Power curves of the two methods for $C_{\text{PU1}} = 1.33$ *vs*. C_{PU2} with $n_1 = n_2 = 50$.

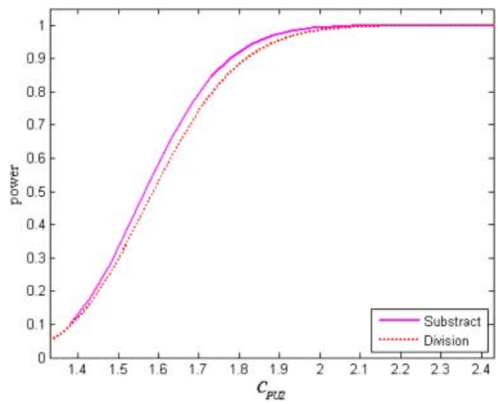

Figure 9. Power curves of the two methods for $C_{\text{PU1}} = 1.33$ *vs*. C_{PU2} with $n_1 = n_2 = 100$.

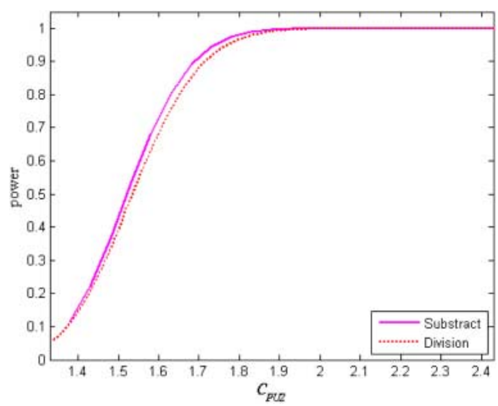

Figure 10. Power curves of the two methods for $C_{\text{PU1}} = 1.33$ *vs*. C_{PU2} with $n_1 = n_2 = 150$.

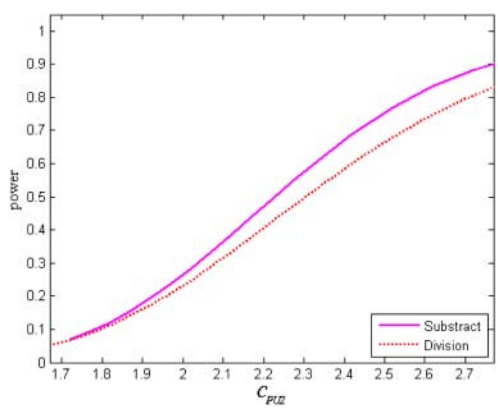

Figure 11. Power curves of the two methods for $C_{\text{PU1}} = 1.67$ *vs*. C_{PU2} with $n_1 = n_2 = 30$.

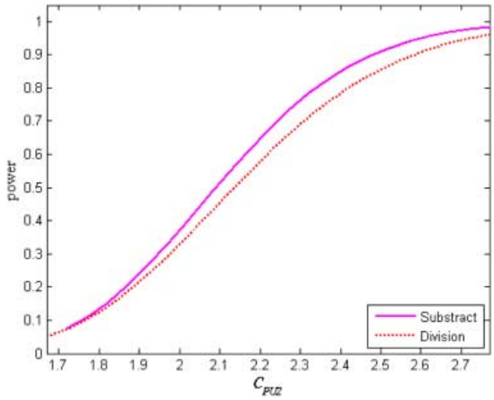

Figure 12. Power curves of the two methods for $C_{\mathrm{PU1}} = 1.67$ *vs*. C_{PU2} with $n_1 = n_2 = 50$.

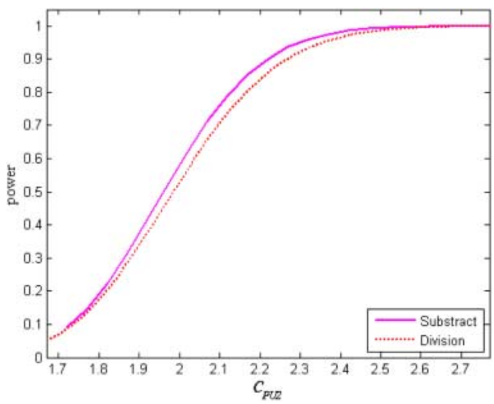

Figure 13. Power curves of the two methods for $C_{\mathrm{PU1}} = 1.67$ *vs*. C_{PU2} with $n_1 = n_2 = 100$.

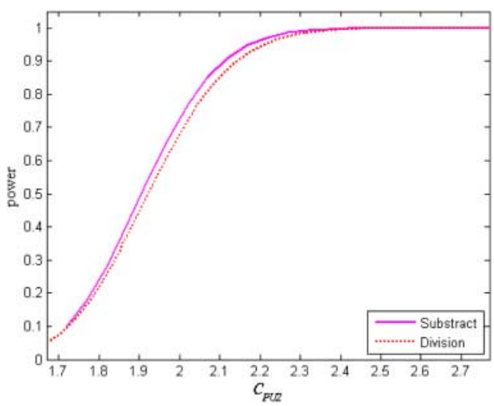

Figure 14. Power curves of the two methods for $C_{\mathrm{PU1}} = 1.67$ *vs*. C_{PU2} with $n_1 = n_2 = 150$.

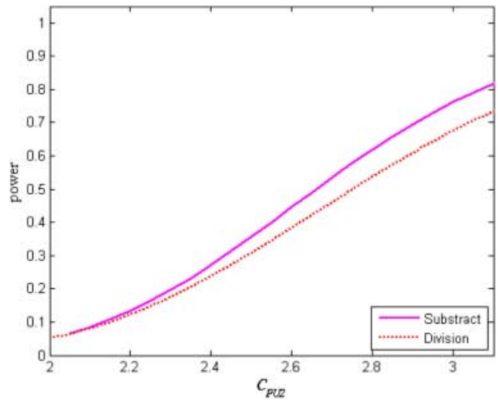

Figure 15. Power curves of the two methods for $C_{\mathrm{PU}1} = 2.00$ *vs*. $C_{\mathrm{PU}2}$ with $n_1 = n_2 = 30$.

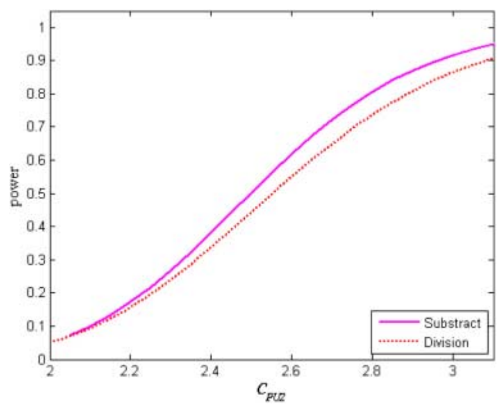

Figure 16. Power curves of the two methods for $C_{\mathrm{PU}1} = 2.00$ *vs*. $C_{\mathrm{PU}2}$ with $n_1 = n_2 = 50$.

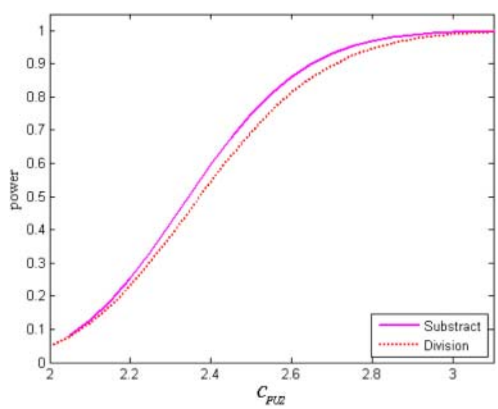

Figure 17. Power curves of the two methods for $C_{\mathrm{PU}1} = 2.00$ *vs*. $C_{\mathrm{PU}2}$ with $n_1 = n_2 = 100$.

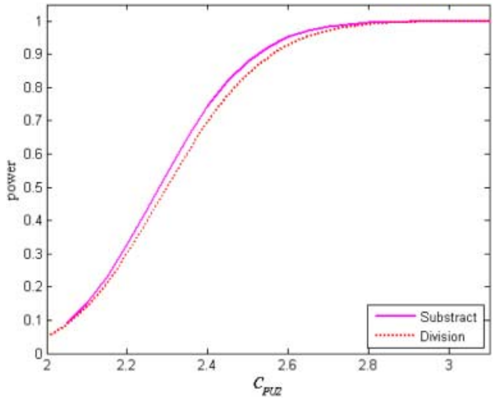

Figure 18. Power curves of the two methods for $C_{\mathrm{PU1}} = 2.00$ *vs*. C_{PU2} with $n_1 = n_2 = 150$.

Table 4. Sample size required for the subtraction (S) and the division (D) methods to differentiate C_{PU1} and C_{PU2} with power $1 - \beta = 0.90, 0.95, 0.975, 0.99$.

			Power							Power			
C_{PU1}	C_{PU2}		0.90	0.95	0.975	0.99	C_{PU1}	C_{PU2}		0.90	0.95	0.975	0.99
1.00	1.15	*S*	466	597	727	888	1.25	1.40	*S*	672	854	1043	1268
		D	535	675	810	980			*D*	763	965	1157	1403
	1.20	*S*	272	347	426	517		1.45	*S*	401	510	614	737
		D	314	396	474	574			*D*	445	561	673	816
	1.25	*S*	183	230	278	340		1.50	*S*	260	330	397	487
		D	210	264	316	383			*D*	295	372	447	540
	1.30	*S*	130	164	245	245		1.55	*S*	184	233	287	350
		D	152	191	229	277			*D*	212	267	320	388
	1.35	*S*	98	123	183	183		1.60	*S*	138	176	214	262
		D	116	146	175	211			*D*	161	203	243	295
	1.40	*S*	77	99	117	144		1.65	*S*	109	138	169	206
		D	93	116	139	168			*D*	128	161	192	233
	1.45	*S*	62	79	95	117		1.70	*S*	88	112	136	165
		D	76	96	114	138			*D*	104	131	157	190
	1.50	*S*	52	66	79	98		1.75	*S*	72	93	112	137
		D	64	81	96	116			*D*	87	110	131	159
	1.55	*S*	44	56	68	83		1.80	*S*	62	78	95	116
		D	55	69	82	99			*D*	75	94	112	135
	1.60	*S*	38	49	59	71		1.85	*S*	53	68	82	99
		D	48	60	72	87			*D*	65	81	97	117
	1.65	*S*	33	42	51	62		1.90	*S*	46	59	71	86
		D	43	53	63	77			*D*	57	71	85	103
	1.70	*S*	30	37	45	55		1.95	*S*	41	52	63	76
		D	38	48	57	68			*D*	51	64	76	92
	1.75	*S*	27	34	40	49		2.00	*S*	36	46	56	68
		D	35	43	51	62			*D*	46	57	68	82
	1.80	*S*	24	30	36	45		2.05	*S*	33	41	50	61
		D	31	39	47	56			*D*	41	52	62	74
	1.85	*S*	22	28	33	40		2.10	*S*	30	38	45	55
		D	29	36	43	51			*D*	38	47	56	68
	1.90	*S*	20	25	30	37		2.15	*S*	27	34	41	50
		D	27	33	39	47			*D*	35	43	52	62
	1.95	*S*	19	23	28	34		2.20	*S*	25	31	38	46
		D	25	31	36	44			*D*	32	40	48	57
	2.00	*S*	17	22	26	31		2.25	*S*	23	29	35	43
		D	23	29	34	41			*D*	30	37	44	53

(*Continued*)

Table 4. Continued

C_{PU1}	C_{PU2}		Power 0.90	0.95	0.975	0.99	C_{PU1}	C_{PU2}		Power 0.90	0.95	0.975	0.99
1.45	1.60	*S*	869	1099	1335	1639	1.60	1.75	*S*	1034	1368	1594	1947
		D	983	1241	1488	1805			*D*	1167	1475	1768	2145
	1.65	*S*	519	658	788	945		1.80	*S*	615	781	933	1123
		D	570	719	862	1046			*D*	674	852	1021	1238
	1.70	*S*	331	422	516	622		1.85	*S*	394	501	618	740
		D	376	474	569	690			*D*	444	560	672	815
	1.75	*S*	236	302	382	445		1.90	*S*	282	360	432	528
		D	269	339	407	493			*D*	317	400	479	581
	1.80	*S*	178	227	276	333		1.95	*S*	212	271	325	394
		D	204	257	308	373			*D*	239	302	362	438
	1.85	*S*	138	178	213	262		2.00	*S*	165	211	252	308
		D	161	203	243	294			*D*	189	238	285	345
	1.90	*S*	112	142	172	211		2.05	*S*	132	170	205	250
		D	131	165	197	239			*D*	153	193	231	280
	1.95	*S*	93	118	142	174		2.10	*S*	109	139	169	205
		D	109	137	164	199			*D*	127	160	192	232
	2.00	*S*	78	100	120	147		2.15	*S*	92	117	142	174
		D	93	117	140	169			*D*	108	136	163	197
	2.05	*S*	67	85	103	127		2.20	*S*	79	100	122	148
		D	80	101	121	146			*D*	93	117	140	170
	2.10	*S*	58	74	89	110		2.25	*S*	68	87	105	128
		D	71	88	106	128			*D*	82	103	123	148
	2.15	*S*	51	65	79	96		2.30	*S*	60	76	92	113
		D	63	78	94	113			*D*	72	91	109	131
	2.20	*S*	45	58	70	85		2.35	*S*	53	68	82	100
		D	56	70	84	101			*D*	65	81	97	117
	2.25	*S*	41	52	63	77		2.40	*S*	48	61	73	90
		D	51	63	76	91			*D*	58	73	87	106
	2.30	*S*	37	47	56	69		2.45	*S*	43	55	66	81
		D	46	58	69	83			*D*	53	66	79	96
	2.35	*S*	33	43	51	63		2.50	*S*	39	49	60	73
		D	43	53	63	76			*D*	48	61	72	87
	2.40	*S*	31	39	47	57		2.55	*S*	36	45	55	67
		D	40	49	58	70			*D*	45	56	67	80
	2.45	*S*	28	36	43	53		2.60	*S*	33	42	50	62
		D	37	45	54	65			*D*	41	52	62	74

6. A supplier selection example

The couplers and wavelength division multiplexers (WDM) have been widely used in high-speed, high-volume image data transmission systems to provide sufficient bandwidth and smaller channel spacing for greater throughput. The example was taken form a company located on the Science-Based Industrial Park, Hsinchu, Taiwan. This product has the multiple quality characteristics, including the polarization dependent loss (PDL), and the insertion loss. A detailed description of this product can be found in Wu and Pearn [8]. In this paper, the main contribution is to propose a simple and powerful procedure for the supplier selection problem, which outperforms the previous results. We considered the company importing WDM products requiring a minimal capability on the PDL characteristic, and adopted the recently published data given in Pearn *et al.* [15] to compare the selection power between the proposed subtraction method and the division method presented by Pearn *et al.* [15]. The minimal requirement of the PDL characteristic is $C_{PU1} = 1.25$. Kolmogorov–Smirnov normality tests for data of both suppliers are performed and confirmed to be normally distributed for both supplies. The test results in p-value are all greater than 0.15. Histograms of the data are shown in Figure 19. To determine if supplier II provides a better

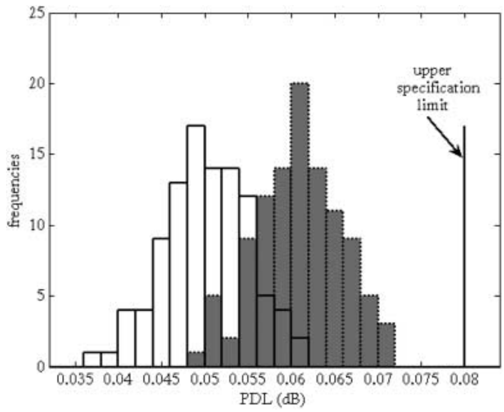

Figure 19. Histograms of the two PDL data [15].

process capability of the WDM products than supplier I, we perform the hypothesis testing: H_0: $C_{\mathrm{PU2}} \le C_{\mathrm{PU1}}$ *vs.* H_1: $C_{\mathrm{PU2}} > C_{\mathrm{PU1}}$. For the PDL data, the sample means, sample standard deviations, and the sample estimators for both suppliers are obtained as

$$\bar{x}_1 = 0.06079, \quad \bar{x}_2 = 0.05018.$$

$$s_1 = 0.00495, \quad s_2 = 0.00486.$$

$$\hat{C}_{\mathrm{PU1}} = 1.2936, \quad \hat{C}_{\mathrm{PU2}} = 2.04527.$$

We calculated $W = \hat{C}_{\mathrm{PU2}} - \hat{C}_{\mathrm{PU1}} = 0.75167$ for the proposed subtraction method, and $R = \hat{C}_{\mathrm{PU2}}/\hat{C}_{\mathrm{PU1}} = 1.58107$ for the division method. To compute the critical value, we used the 'Visual C++' computer program developed by Pearn *et al.* [15] to handle the complicated computations. The program reads the input parameters $C_{\mathrm{PU1}}, C_{\mathrm{PU2}}$, the corresponding sample sizes n_1, n_2, and α, the risk for wrongly rejecting $C_{\mathrm{PU2}} \le C_{\mathrm{PU1}}$ while actually $C_{\mathrm{PU2}} \le C_{\mathrm{PU1}}$ is true, and outputs with the critical value. We run the C++ program with $n_1 = 105$, $n_2 = 100$, $C_{\mathrm{PU2}} = C_{\mathrm{PU1}} = 1.25$ (the minimal capability requirement of C_{PU}) and $\alpha = 0.05$ to obtain the critical value as 0.2211 for the proposed subtraction method, and 1.1924 for the division method. Since the testing statistic $W = 0.75167 > 0.2211$, and $R = 1.58107 > 1.1924$, we therefore conclude that the supplier II is superior than supplier I with 95% confidence level (using either one of the two methods).

6.1. *Selection determination with designated outperformance*

To investigate the designated outperformance of the capability difference between the two suppliers, we perform the hypothesis testing: H_0: $C_{\mathrm{PU2}} \le C_{\mathrm{PU1}} + h$ *vs.* H_1: $C_{\mathrm{PU2}} > C_{\mathrm{PU1}} + h$. We run the C++ program with $n_1 = 105$, $n_2 = 100$, $C_{\mathrm{PU1}} = 1.25$ (the minimal capability requirement

Table 5. Critical values and decisions of testing the two WDM suppliers for $C_{\mathrm{PU1}} = 1.250$, $C_{\mathrm{PU2}} = 1.450, 1.550, 1.650, 1.660, 1.670$.

C_{PU1}	1.250	1.250	1.250	1.250	1.250
C_{PU2}	1.450	1.550	1.650	1.660	1.670
h	0.200	0.300	0.400	0.410	0.420
c_0	0.4412	0.5508	0.6625	0.6716	0.6830
S	Reject	Reject	Reject	Reject	Reject
D	Reject	Reject	Reject	Reject	Accept

Table 6. Critical values and decisions of testing the two WDM suppliers for $C_{PU1} = 1.250$, $C_{PU2} = 1.680(0.01)1.740$.

C_{PU1}	1.250	1.250	1.250	1.250	1.250	1.250	1.250
C_{PU2}	1.680	1.690	1.700	1.710	1.720	1.730	1.740
h	0.430	0.440	0.450	0.460	0.470	0.480	0.490
c_0	0.6946	0.7063	0.7182	0.7277	0.7398	0.7496	0.7521
S	Reject	Reject	Reject	Reject	Reject	Reject	Accept
D	Accept	Accept	Accept	Accept	Accept	Accept	Accept

of C_{PU}), $C_{PU2} = 1.25 + h$, $h = 0.2(0.1)0.4$, $0.41(0.01)0.49$ and $\alpha = 0.05$ to obtain the critical values for various h. The decisions of the hypotheses are shown in Tables 5 and 6. The results show that if the division method is used, we can only conclude that supplier II has a manufacturing capability better than supplier I by a designated outperformance $h = 0.41$, i.e. $C_{PU2} > C_{PU1} + 0.41$. If the subtraction method is used, however, we can conclude that supplier II is better than supplier I by a designated outperformance $h = 0.48$, i.e. $C_{PU2} > C_{PU1} + 0.48$.

7. Conclusions

For stable normal processes with one-sided specification limits, capability indices C_{PU} and C_{PL} have been widely used in the manufacturing industry to provide numerical measures on process performance. In this paper, we considered the supplier selection problem with two one-sided processes. We presented a new analytical exact approach to solve the problem. The proposed approach which we referred to as the subtraction method, indeed, is more powerful than the existing division method used in all existing research.

In practice, often the replacement of suppliers would be made only if the new supplier's capability is significantly better than the existing supplier's by a designated outperformance $h > 0$ due to the high cost for the replacement of a new supplier. The proposed exact approach, in this case, can be used to test the corresponding hypothesis H_0: $C_{PU2} \leq C_{PU1} + h$ *vs.* H_1: $C_{PU2} > C_{PU1} + h$. The investigation results showed that for the WDM example considered. If the existing division method is applied, supplier II outperforms supplier I by a designated outperformance 0.41. If the proposed subtraction method is used, however, supplier II outperforms supplier I with designated outperformance determined as 0.48 rather than 0.41.

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