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The domatic number problem*

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Abstract

A dominating set of a graph $G=(V,E)$ is a subset D of V such that every vertex not in D is adjacent to some vertex in D. The domatic number $d(G)$ of G is the maximum positive integer k such that V can be partitioned into k pairwise disjoint dominating sets. The purpose of this paper is to study the domatic numbers of graphs that are obtained from small graphs by performing graph operations, such as union, join and Cartesian product.

1. Introduction

A *dominating set* of a graph $G=(V,E)$ is a subset D of V such that every vertex not in D is adjacent to some vertex in D. The *domatic number* $d(G)$ of a graph $G=(V,E)$ is the maximum positive integer k such that V can be partitioned into k pairwise disjoint dominating sets $D_1, D_2, \ldots, D_k$. A partition of V into pairwise disjoint dominating sets is called a *domatic partition*. The concept of a domatic number was introduced in [5]. The word 'domatic' was created from the words 'dominating' and 'chromatic' in the same way the word 'smog' was created from the words 'smoke' and 'fog'. In a certain sense a domatic number is analogous to the chromatic number of a graph, which is the minimum positive integer k such that the vertex set can be partitioned into k pairwise disjoint stable sets.

Lower bounds, upper bounds and many propositions of domatic numbers were studied extensively in [3–6, 8–10, 12, 13, 15–23]. In particular, in [5] it was proved that for any graph G there is a natural primal dual weak inequality

$$d(G) \leqslant \delta(G)+1,$$

where $\delta(G)$ is the minimum degree of a vertex of G. Motivated from this, a graph G is called *domatically full* if $d(G)=\delta(G)+1$. For instance, the complete graph K_n of

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n vertices, the complement $\bar{K}_n$ of K_n, the cycle C_{3n} of $3n$ vertices, trees and maximal outerplanar graphs are all domatically full.

On the algorithmic side, the domatic number problem is NP-complete for general graphs [7] and circular arc graphs [2]. The problem has been solved in $O(n^2 \log n)$ time for proper circular arc graphs [2], $O(n^{2.5})$ time for interval graphs [1], and $O(n \log n)$ time for proper interval graphs [1], and has been improved by linear-time algorithms for interval graphs [11, 14].

The purpose of this paper is to study the domatic numbers of graphs that are obtained from small graphs by performing graph operations, such as union, join and Cartesian product. In particular, Section 2 gives solutions to the domatic number of the union of two graphs and the domatic number of the join of two or more graphs. Section 3 gives partial results of the domatic number of the Cartesian product of paths.

2. Graph union and join

Suppose $G_1=(V_1E_1)$ and $G_2=(V_2E_2)$ are two graphs with disjoint vertex sets V_1 and V_2 and disjoint edge sets E_1 and E_2. The *union* of G_1 and G_2 is the graph $G_1 \cup G_2=(V_1 \cup V_2, E_1 \cup E_2)$. The *join* of G_1 and G_2 is the graph G_1+G_2 that consists of $G_1 \cup G_2$ and all edges joining V_1 and V_2.

Proposition 2.1. *$d(G_1 \cup G_2)=\min\{d(G_1), d(G_2)\}$ for any two graphs G_1 and G_2.*

Proof. The proposition follows from the fact that D is a dominating set of $G_1 \cup G_2$ if and only if D is the union of a dominating set of G_1 and a dominating set of G_2. □

A *dominating vertex* is a vertex which forms a dominating set, i.e. a vertex adjacent to all other vertices. If x is a dominating vertex of a nontrivial graph G, then G is isomorphic to $(G-x)+K_1$.

Proposition 2.2. *If x is a dominating vertex of a graph G, then $d(G)=d(G-x)+1$.*

Proof. Since a domatic partition of $G-x$ together with $\{x\}$ forms a domatic partition of G, $d(G) \geqslant d(G-x)+1$. On the other hand, suppose $D_1, D_2, \ldots, D_k$ is a domatic partition of G, where $k=d(G)$. Assume $x \in D_1$. Note that $D_1 \cup D_2-\{x\}$, $D_3, \ldots, D_k$ is a domatic partition of $G-x$. So $d(G-x) \geqslant k-1=d(G)-1$. Thus, $d(G)=D(G-x)+1$. □

In the rest of this section, we give results for the domatic number of the join of graphs. By Proposition 2.2, from now on, we need only consider graphs without a dominating vertex. Let r be a positive integer greater than or equal to 2. If

$G_1, G_2, \ldots, G_r$ are graphs without a dominating vertex, then their join $G_1+G_2+\cdots+G_r$ also has no dominating vertex. For the domatic number of this join, there are two possible cases, which are solved in Theorem 2.3 and Corollary 2.6.

Theorem 2.3. *Suppose $r \geqslant 2$ and $G_1, G_2, \ldots, G_r$ are graphs with $n_1, n_2, \ldots, n_r$ vertices, respectively, and without a dominating vertex. If $1 \leqslant n_1 \leqslant n_2 \leqslant \cdots \leqslant n_r$ and $n_1+\cdots+n_{r-1} \geqslant n_r$, then $d(G_1+G_2+\cdots+G_r)=\lfloor (n_1+n_2+\cdots+n_r)/2 \rfloor$.*

Proof. Since $G_1+G_2+\cdots+G_r$ has no dominating vertex, each dominating set contains at least two vertices and so $d(G_1+G_2+\cdots+G_r) \leqslant \lfloor (n_1+n_2+\cdots+n_r)/2 \rfloor$.

On the other hand, we prove that $G_1+G_2+\cdots+G_r$ has a domatic partition of $\lfloor (n_1+n_2+\cdots+n_r)/2 \rfloor$ dominating sets such that each dominating set has exactly two vertices, except possibly one dominating set has three vertices. This assertion clearly implies the theorem. We prove the assertion by induction on $n=n_1+n_2+\cdots+n_r$. Note that the following argument is valid even when some G_i has a dominating vertex.

The assertion is clearly true for $n \leqslant 3$. It is also true for $r=2$ since $n_1=n_2$ and any vertex of G_1 together with any vertex of G_2 is a dominating set of G_1+G_2. Now, suppose $n \geqslant 4$, $r \geqslant 3$ and the assertion is true for $n'=n-2$. Choose a vertex x in G_{r-1} and a vertex y in G_r. Consider the graph $G'=G_1+\cdots+G_{r-2}+(G_{r-1}-x)+(G_r-y)$. For the case of $n_{r-2}<n_r$, we have

$$n_1 \leqslant \cdots \leqslant n_{r-2} \leqslant n_r-1, \; n_{r-1}-1 \leqslant n_r-1$$

and

$$n_1+\cdots+n_{r-2}+(n_{r-1}-1) \geqslant n_r-1.$$

For the case of $n_{r-2}=n_{r-1}=n_r$, we have

$$n_1 \leqslant \cdots \leqslant n_{r-3} \leqslant n_{r-2}, \; n_{r-1}-1=n_r-1<n_{r-2}$$

and

$$n_1+\cdots+n_{r-3}+(n_{r-1}-1)+(n_r-1) \geqslant n_{r-2}.$$

To see the last inequality: when $n_r=n_{r-1}=n_{r-2} \geqslant 2$, the left-hand side $\geqslant n_r=n_{r-2}$; when $n_r=n_{r-1}=n_{r-2}=1$, $n \geqslant 4$ implies that $r=n \geqslant 4$ and so the left-hand side $\geqslant n_{r-3}=1=n_{r-2}$. In either case, by the induction hypothesis, G' has a domatic partition of $\lfloor (n_1+n_2+\cdots+(n_{r-1}-1)+(n_r-1))/2 \rfloor$ dominating sets such that each dominating set has exactly two vertices, except possibly one dominating set has three vertices. These dominating sets together with $\{x, y\}$ form the desired domatic partition of $G_1+G_2+\cdots+G_r$. □

For the case of $n_1+n_2+\cdots+n_{r-1}<n_r$, we cannot get results similar to Theorem 2.3. To solve the problem for this case, we need a slightly more general concept, as follows. For any nonnegative integer m, an *m-domatic partition* of a graph $G=(V, E)$ is a collection $D_1, D_2, \ldots, D_k$ of k pairwise disjoint dominating sets such that

$|D_1 \cup D_2 \cup \cdots \cup D_k| \leqslant m$. The *m-domatic number* $d(G\,|\,m)$ of G is the maximum k such that an m-domatic partition of k dominating sets exists. Note that $d(G)=d(G\,|\,n)$ for any graph G of n vertices.

Proposition 2.4. *$d(G\,|\,m) \leqslant d(G\,|\,m')$ for any graph G and any nonnegative integers $m \leqslant m'$.*

Theorem 2.5. *Suppose $n_1 \leqslant n_2$ and $G_i=(V_i, E_i)$ is a graph of n_i vertices without a dominating vertex for $i=1,2$. Then*

$$d(G_1+G_2\,|\,m)=\begin{cases}[m/2] & \text{if } 0 \leqslant m \leqslant 2n_1,\\ n_1+d(G_2\,|\,m-2n_1) & \text{if } 2n_1 < m \leqslant n_1+n_2.\end{cases}$$

Proof. For the case when $0 \leqslant m \leqslant 2n_1$, there exist $\lfloor m/2 \rfloor$ pairs of vertices, each of them containing one vertex in G_1 and the other in G_2. So, each such pair is a dominating set of G_1+G_2 and $d(G_1+G_2\,|\,m) \geqslant \lfloor m/2 \rfloor$. On the other hand, since each G_i has no dominating vertex, neither does G_1+G_2. Consequently, each dominating set is of a size of at least two, and so $d(G_1+G_2\,|\,m) \leqslant \lfloor m/2 \rfloor$. Thus, $d(G_1+G_2\,|\,m) = \lfloor m/2 \rfloor$.

For the case when $2n_1 < m \leqslant n_1+n_2$, first of all, choose an $(m-2n_1)$-domatic partition $D_1, D_2, \ldots, D_k$ of G_2. These k dominating sets are also dominating sets of G_1+G_2. Note that G_2 has at least $n_2-(m-2n_1) \geqslant n_1$ vertices not in $D_1 \cup D_2 \cup \cdots \cup D_k$. By an argument similar to that in the first paragraph, n_1 of these vertices together with the n_1 vertices of G_1 form n_1 dominating sets of G_1+G_2. Thus, $d(G_1+G_2\,|\,m) \geqslant n_1 + d(G_2\,|\,m-2n_1)$. On the other hand, suppose $D_1, D_2, \ldots, D_r$ is an m-domatic partition of G_1+G_2, where $r=d(G_1+G_2\,|\,m)$. Note that each D_i contains at least two vertices, since G_1+G_2 has no dominating vertex. A dominating set is called *standard* if it contains exactly one vertex in V_1 and exactly one vertex in V_2. We claim that among these r dominating sets, there are exactly n_1 standard ones and the other $r-n_1$ sets are all subsets of V_2 by considering the following cases.

(1) Suppose some D_i contains at least one vertex x in V_1 and at least one vertex in V_2. We can replace D_i by a standard dominating set $\{x, y\}$.

(2) Suppose some D_i contains vertices only in V_1, say x and y, and some D_j contains only vertices in V_2, say z and w. We can replace D_i and D_j by two standard dominating sets $\{x, z\}$ and $\{y, w\}$.

(3) Suppose all nonstandard dominating sets are subsets of V_1. Since $n_1 \leqslant n_2$, we can replace each nonstandard dominating set by a standard one by taking a vertex from this set and a vertex of V_i, which is not in any D_i.

(4) Suppose there is a vertex x of V_1 not in any D_i. We can choose a vertex y in V_2, which either is in some nonstandard dominating set $D_j \subset V_2$ or is not in any D_i. In the former case, we can replace D_j by $\{x, y\}$ into the domatic partition.

The discussion of the above cases shows that the domatic partition has exactly n_1 standard dominating sets and $r-n_1$ nonstandard dominating sets that are subsets of V_2. These $r-n_1$ nonstandard dominating sets form an $(m-2n_1)$-domatic partition of

G_2. Therefore, $d(G_2|m-2n_1)\geqslant r-n_1$, i.e. $d(G_1+G_2|m)\leqslant n_1+d(G_2|m-2n_1)$. Thus, $d(G_1+G_2|m)=n_1+d(G_2|m-2n_1)$. □

Corollary 2.6. *Suppose $r\geqslant 2$ and $G_1, G_2, \ldots, G_r$ are graphs with $n_1, n_2, \ldots, n_r$ vertices, respectively, and without a dominating vertex. If $n_1+n_2+\cdots+n_{r-1}<n_r$, then $d(G_1+G_2+\cdots+G_r)=n_1+n_2+\cdots+n_{r-1}+d(G_r|n_r-n_1-\cdots-n_{r-1})$.*

Proof. The corollary follows from Theorem 2.5 by considering $G_1+\cdots+G_{r-1}$ as G_1, G_r as G_2, and $m=n_1+n_2+\cdots+n_r$. □

Corollary 2.7. *If $r\geqslant 2$ and $n_1+n_2+\cdots+n_{r-1}<n_r$, then $d(\bar{K}_{n_1}+\bar{K}_{n_2}+\cdots+\bar{K}_{n_r})=n_1+n_2+\cdots+n_{r-1}$.*

Proof. The corollary follows from Corollary 2.6 and the fact that $d(K_a|b)=0$ for $a>b$. □

3. Cartesian product

The *Cartesian product* of two graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ is the graph $G_1\times G_2=(V_1\times V_2,E)$ where

$$E=\{\{(a,c),(a,d)\}: a\in V_1 \text{ and } \{c,d\}\in E_2\}$$
$$\cup\{\{(a,c),(b,c)\}: \{a,b\}\in E_1 \text{ and } c\in V_2\}.$$

Denote by P_n the *path* of n vertices, i.e. P_n has vertex set $\{1,2,\ldots,n\}$ and edge set $\{\{i,i+1\}: 1\leqslant i\leqslant n-1\}$. The purpose of this section is to determine the domatic number of a *r-dimensional grid* $P_{n_1}\times P_{n_2}\times\cdots\times P_{n_r}$, where all $n_i\geqslant 2$. Note that $P_{n_1}\times P_{n_2}\times\cdots\times P_{n_r}$ has $n_1\cdot n_2\cdots n_r$ vertices of the form $(a_1,a_2,\ldots,a_r)$, where $1\leqslant a_i\leqslant n_i$ for $1\leqslant i\leqslant r$. Vertex $(a_1,a_2,\ldots,a_r)$ is adjacent to vertex $(b_1,b_2,\ldots,b_r)$ if and only if there is exactly one $|a_j-b_j|=1$ and all other $a_i=b_i$. Also $d(P_{n_1}\times P_{n_2}\times\cdots\times P_{n_r})\leqslant \delta(P_{n_1}\times P_{n_2}\times\cdots\times P_{n_r})+1=r+1$.

It is clear that P_n is domatically full for any $n\geqslant 1$.

For any 2-dimensional grid $P_{n_1}\times P_{n_2}$, $D_1=\{(a,b)$: a is odd$\}$ and $D_2=\{(a,b\}$: a is even$\}$ form a domatic partition. So $2\leqslant d(P_{n_1}\times P_{n_2})\leqslant 3$. It is clear that $d(P_2\times P_2)=2$ since $P_2\times P_2$ has only four vertices and no dominating vertex. It is also the case that $d(P_2\times P_4)=d(P_4\times P_2)=2$. However, $d(P_{n_1}\times P_{n_2})=3$ for all other 2-dimensional grids. To establish this result as well as others, we employ Propositions 2.1 and 3.1.

Proposition 3.1. *$d(H)\leqslant d(G)$ for any spanning subgraph $H=(V,E')$ of $G=(V,E)$.*

Proof. The proposition follows from the fact that a dominating set of H is also a dominating set of G. □

Theorem 3.2. *$d(P_{n_1} \times P_{n_2})=3$ for any 2-dimensional grid $P_{n_1} \times P_{n_2}$ except that $d(P_2 \times P_2)=d(P_2 \times P_4)=d(P_4 \times P_2)=2$.*

Proof. Assume (n_1, n_2) is not (2,2) (2,4) or (4,2). For the case when one of n_1 and n_2 is odd, say n_1, let

$$D_1=\{(a,b)\colon a\equiv 0 \pmod 2\},$$

$$D_2=\{(a,b)\colon a\equiv 1 \pmod 4 \text{ and } b\equiv 1 \pmod 2\},$$

$$\cup\{(a,b)\colon a\equiv 3 \pmod 4)\ b\equiv 0 \pmod 2\},$$

$$D_3=\{(a,b)\colon a\equiv 1 \pmod 4 \text{ and } b\equiv 0 \pmod 2\}$$

$$\cup\{(a,b)\colon a\equiv 3 \pmod 4\ b\equiv 1 \pmod 2\}.$$

Then D_1, D_2, D_3 form a domatic partition of $P_{n_1} \times P_{n_2}$. Thus $d(P_{n_1} \times P_{n_2})=3$. Fig. 1 shows a domatic partition of $P_5 \times P_4$.

For the case when both n_1 and n_2 are even: $d(P_4 \times P_4)=3$, shown in Fig. 2. Now, suppose at least one $n_i \geqslant 6$, say $n_1 \geqslant 6$. Since $(P_3 \times P_{n_2}) \cup (P_{n_1-3} \times P_{n_2})$ is a spanning subgraph of $P_{n_1} \times P_{n_2}$ and $d(P_3 \times P_{n_2})=d(P_{n_1-3} \times P_{n_2})=3$ by the above cases,

$$d(P_{n_1} \times P_{n_2}) \geqslant d((P_3 \times P_{n_2}) \cup (P_{n_1-3} \times P_{n_2}))$$

$$\geqslant \min\{d(P_3 \times P_{n_2}), d(P_{n_1-3} \times P_{n_2})\}=3$$

by Propositions 2.1 and 3.1. Thus, the theorem holds. □

For results on other grids, we need the concept about identifying two copies of a graph at a vertex set expressed in the following lemmas. More precisely, suppose $G=(V,E)$ is a graph and S a subset of V. Consider the graph $G \Delta S=(V^*, E^*)$ with $V^*=V \cup \{x^*\colon x \in V-S\}$ and

$$E^*=E \cup \{\{x^*, y\}\colon x \in V-S,\ y \in S,\ \{x,y\} \in E\} \cup \{\{x^*, y^*\}\colon x, y \in V-S,\ \{x,y\} \in E\}.$$

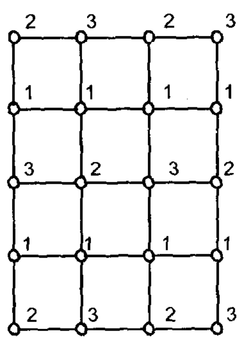

Fig. 1. $d(P_5 \times P_4)=3$.

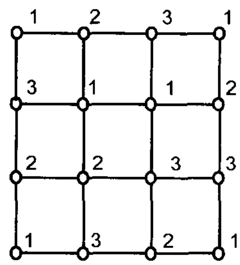

Fig. 2. $d(P_4 \times P_4)=3$.

Lemma 3.3. *If S is a subset of V in a graph $G=(V,E)$, then $d(G \Delta S) \geqslant d(G)$.*

Proof. The lemma follows from the fact that for any dominating set D of G, $D^*=D \cup \{x^*: x \in D-S\}$ is a dominating set of $G \Delta S$. □

Lemma 3.4. *If x is an end vertex of P_n, then $P_n \Delta \{x\}$ is isomorphic to P_{2n-1}.*

Lemma 3.5. *For any two graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$, if S is a subset of V_1 then $(G_1 \Delta S) \times G_2$ is isomorphic to $(G_1 \times G_2) \Delta (S \times V_2)$.*

Theorem 3.6. *If r and n are positive integers and $(n_1,n_2,\dots,n_r) \in \{n, 2n-1\}$, then $d(P_{n_1} \times P_{n_2} \times \cdots \times P_{n_r}) \geqslant d(P_n \times P_n \times \cdots \times P_n)$, where the grids are r-dimensional.*

Proof. By Lemmas 3.4 and 3.5, $P_{2n-1} \times P_{n_2} \times \cdots \times P_{n_r}$ is isomorphic to $(P_n \times P_{n_2} \times \cdots \times P_{n_r}) \Delta (\{x\} \times V_2 \times \cdots \times V_r)$. The theorem can be proved by induction on the number of n_i's that are equal to $2n-1$. □

For any positive integer n, since n and $2n-1$ are relatively prime, there exists some n_0 such that for any integer $m \geqslant n_0$, we can write $m=rn+s(2n-1)$ for some non-negative integers r and s. The minimum such n_0 is denoted by $M(n)$. For instance, $M(2)=2$ and $M(3)=8$.

Theorem 3.7. *If r and n are positive integers and $n_1,n_2,\dots,n_r \geqslant M(n)$ then $d(P_{n_1} \times P_{n_2} \times \cdots \times P_{n_r}) \geqslant d(P_n \times P_n \times \cdots \times P_n)$, where the grids are r-dimensional.*

Proof. Since for each n_i there exist r_i and s_i such that $n_i=r_in+s_i(2n-1)$, $P_{n_1} \times P_{n_2} \times \cdots \times P_{n_r}$ has a spanning subgraph which is the union of some grids $P_{m_1} \times P_{m_2} \times \cdots \times P_{m_r}$, where $m_1,m_2,\dots,m_r \in \{n, 2n-1\}$. The theorem follows from Propositions 2.1 and 3.1 and Theorem 3.6. □

Theorem 3.8 (Laborde, Zelinka [9, 21]). *If k is a positive integer and $r=2^k-1$, then the r-dimensional grid $P_2 \times P_2 \times \cdots \times P_2$ is domatically full.*

Colloary 3.9. *If k is a positive integer and $r=2^k-1$, then any r-dimensional grid $P_{n_1} \times P_{n_2} \times \cdots \times P_{n_r}$ is domatically full.*

We close this paper with the following conjecture: all r-dimensional grids, with finitely many exceptions, are domatically full. By Theorem 3.6, this conjecture is true if we can find some n such that the r-dimensional grid $P_n \times P_n \times \cdots \times P_n$ is domatically full. In fact a slight modification of the above arguments shows that the conjecture is true if we can find a domatically full r-dimensional grid.

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References

[1] A.A. Bertossi, On the domatic number of interval graphs, Inform. Process. Lett. 28 (1988) 275–280.
[2] M.A. Bonucelli, Dominating sets and domatic number of circular arc graphs, Discrete Appl. Math. 12 (1985) 203–213.
[3] E.J. Cockayne, Domination of undirected graphs — a survey, Lecture Notes in Mathematics 642 (Springer, Berlin, 1978) 141–147.
[4] E.J. Cockayne and S.T. Hedetniemi, Optimal domination in graphs, IEEE Trans. Circuits Systems CAS-22 (1975) 41–44.
[5] E.J. Cockayne and S.T. Hedetniemi, Towards a theory of domination in graphs, Networks 7 (1977) 247–261.
[6] M. Farber, Domination, independent domination, and duality in strongly chordal graphs, Discrete Appl. Math. 7 (1984) 115–130.
[7] M.R. Garey and D.S. Johnson, Computer and Intractability: A Guide to the Theory of NP-Completeness (Freeman, San Francisco, CA, 1979).
[8] F. Jaeger and C. Payan, Relations du type Nordhans–Gaddum pour le nombre d'absorption d'un graphe simple, C.R. Acad. Sci. Paris. Ser. A 274 (1972) 728–730.
[9] J.M. Laborde, Sur le nombre domatique du n-cube et une conjecture de Zelinka, European J. Combin. 8 (1987) 175–177.
[10] R. Laskar and H.B. Walikar, On domination related concepts in graph theory, Lecture Notes in Mathematics 885 (Springer, Berlin, 1980) 308–320.
[11] T.L. Lu, P.H. Ho and G.J. Chang, The domatic number problem in interval graphs, SIAM J. Discrete Math. 3 (1990) 531–536.
[12] O. Ore, Thery of Graphs, Amer. Math. Soc. Coll. Publ. 38, Providence, RI, 1962.
[13] P.J. Slater, R-domination in graphs, J. ACM 23 (1976) 446–450.
[14] A. Srinivasa Rao and C. Pandu Rangan, Linear algorithm for domatic number problem on interval graphs, Inform. Process. Lett. 33 (1989/1990) 29–33.
[15] B. Zelnika, Domatically critical graphs, Czech. Math. J. 30 (1980) 486–489.
[16] B. Zelinka, On domatic numbers of graphs (Russian summary). Math. Slovaca 31 (1981) 91–95.
[17] B. Zelinka, Some remarks on domatic numbers of graphs, Časopis Pěst Mat. 106 (1981) 373–375.
[18] B. Zelinka, Domatic numbers of cube graphs (Russian summary), Math. Slovaca 32 (1982) 117–119.
[19] B. Zelinka, Domatically cocritical graphs, Časopis Pěst Mat. 108 (1983) 82–88.
[20] B. Zelinka, Domatic number and degrees of vertices of a graph, Math. Slovaca 33 (1983) 145–147.
[21] B. Zelinka, Domatic number and bichromaticity of a graph, Graph Theory Lagow 1981, Lecture Notes in Mathematics 1018 (Springer, Berlin, 1983) 278–285.
[22] B. Zelinka, Domatic numbers of uniform hypergraphs, Arch. Math. 21 (1985) 129–134.
[23] B. Zelinka, Regular totally domatically full graphs, Ann. Discrete Math., to appear.