



Optimizing time limits for maximum sales response in Internet shopping promotions

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ABSTRACT

Sellers usually set a promotional time limit to ensure that products can be sold as soon as possible in Internet markets. This research attempts to build a decision support system that optimizes the time limit for maximum sales response or profit in Internet shopping promotions. We decompose the effect of time limits into two opposing forces, which are the awareness and urgency of a promotional offer that are depicted as hyperbolic S-shaped functions. Using the decision calculus approach, we can determine the optimal promotional time limit with different input parameters. We illustrate the use of the proposed system with real world examples and conduct some sensitivity analyses. We compare our numerical results from hyperbolic functions with those from simple exponential functions; we find that hyperbolic functions yield more appropriate promotional time limits on Internet. This research contributes to the field of decision support by providing a new approach to determining the optimal time limit for online sales promotions.

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1. Introduction

Selling products on Internet is a popular and effective method. Yahoo, Amazon online shopping, and other shopping websites exist in many countries. With increasing consumer interest in online shopping, the amount of trade has seen extraordinary growth. For example, UK shoppers spent £26.5 billion online in the first six months of 2008, up 38% on the £19.2 billion recorded for the same period last year, and the researchers have predicted that between 30% and 50% of retail sales will be online in five years (Clark, 2008). Sigue (2008) also stated that the effect of promotions on sales is increasing, and Nair and Tarasewich (2003) stressed the need of the optimal design of a series of promotions mailed to potential customers. Changchien, Lee, and Hsu (2004) proposed an online personalized sales promotion decision support system.

Many Internet shopping websites have set the time limit or deadline for sales promotions. It is interesting and important for sellers to think about the question: How long shall we promote a particular product in the Internet market? Kogan and Herbon (2008) observed that there exists a maximum time length for promotions. On the other hand, the promotional time limit cannot be too short, for sales might not be increased significantly. It seems that there are two different opposing forces in Internet shopping

promotions, which are the awareness and urgency of an offer. The awareness force represents that the buyers become aware of the promotional information (e.g. price discounts, free gifts, coupons or special services) in online shopping. The urgency force indicates that the promotional information will encourage the buyers to purchase the product now rather than later. These forces are similar to those observed by Hanna, Berger, and Abendroth (2005) in an email promotion. How to balance these two forces and find an appropriate time limit is an important issue for online marketing managers.

The purpose of this research is to find the optimal time limit for a product promoted in Internet shopping. We model the awareness and urgency forces as hyperbolic functions with an S-shaped curve, which may be more appropriate than simple exponential functions (as used by Hanna et al.) in most of the online promotions. We further apply the decision calculus methodology to determine the parameters in each of the awareness and urgency functions. We then can solve the proposed system to find the optimal time limit for maximum sales response in Internet shopping.

The contribution of this paper is twofold. From theoretical a viewpoint, we propose a decision support system (that consists of a mathematical model) with two opposing forces, which are hyperbolic functions with an S-shaped curve. This is never seen in past research in sales response. From practical a viewpoint, our system yields promotional time limits that are less than a week, which agrees with online Yahoo (and other shopping websites) shopping promotions.

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The remaining of this paper is organized as follows. We first briefly review the literature in Internet shopping promotions as well as the concept of decision calculus. Then we propose a decision support system to determine the optimal time limit for maximum sales response. Next, we illustrate our system with a real world example. We also conduct sensitivity analyses with different input parameters of the system. Finally, we conclude our research and point out some future research directions.

2. Literature review

Doukidis, Pramatar, and Lekakos (2008) summarized the various aspects in which operations research may support the management of electronic services. Many promotional activities can be used to increase sales response in Internet marketing. Zhang and Krishnamurthi (2004) formulated a customer response model and developed an optimization procedure for customized promotions in Internet shopping. Saunderson, Jobber, and Mitchell (2006) pointed out that monetary incentives can increase response to mail surveys. Sen, King, and Shaw (2006) suggested that understanding the buyers' choice of online search strategies can help an online seller to optimize its online pricing and improve its online promotional activities. Berger, Lee, and Weinberg (2006) indicated that profit would be greater when a firm integrated the Internet into its retailing process. Hruschka (2006) focused on the comparison of two sales response models, a strict parametric and a flexible model. Manfreda, Bosnjak, Berzelak, Haas, and Vehovar (2008) compared the response rates from web surveys with those from other survey modes. Romaniuk and Wight (2009) examined how using the brand influences consumer responses. All of the above researchers rarely explore the issue of setting the time limit for a product promoted in Internet shopping.

Hanna et al. (2005) first addressed the issue of the optimal time limit or deadline in an email marketing application. They identified two opposing forces of an email promotional offer: awareness and urgency, and used a simple exponential function to model these two forces. The use of exponential functions to model sales response (or market share) can be traced back to Parfitt and Collins (1968), Little and Lodish (1969), Rangan (1987), and recently Yi (2008), etc. Little (1970) suggested a different sales response function that was concave up at low promotion levels and concave down at high promotion levels, which was an S-shaped curve. The use of S-shaped functions to model response can also be found in Johansson (1979), Hill (1981), Parasuraman (1982), Austin (2004), Shore and Benson-Karhi (2007), and Agrah and Geunes (2009). DiClemente and Hantula (2003) found that a hyperbolic discount function best described the amount of entries into each online store and relative number of purchases in each store. In addition, Chang (2008) applied two S-shaped desirability functions in experimental parameter design, and Baylari and Montazer (2009) developed an e-learning system based on an S-shaped item characteristic curve.

Little (1970) developed the decision calculus methodology to assist marketing managers in setting advertising budgets. Decision calculus is typically applied in the form of decision support systems to solve various marketing issues (e.g. Wierenga & van Bruggen, 1997). Tversky and Kahneman (1974) showed that biases in judgments reveal some heuristics of thinking under uncertainty. Chakravarti, Mitchell, and Staelin (1979) made an experimental investigation of the decision calculus approach and pointed out that judgmental inputs might substantially affect model-based decisions. Chakravarti, Mitchell, and Staelin (1981) argued that there is a need to understand and take into account the cognitive abilities of managers in designing marketing decision models and their support systems. Little and Lodish (1981) made a commen-

tary on judgment-based marketing decision models. As a note, Tutkun (2009) suggested using the real coded genetic algorithm approach for parameter estimation in mathematical models.

3. Proposed decision support system

3.1. System architecture

We propose a decision support system with three major elements as shown in Fig. 1. The first element is the generation of input parameters by the decision calculus approach. The second element is the mathematical model which uses the input generated to derive and solve the sales response function (by a computer program using Excel or Matlab). The third element is the output which gives the optimal promotional time limit and the corresponding maximum sales response. The proposed decision system also consists of a feedback mechanism if the optimal time limit obtained is apparently incorrect. Then we go back to check and discuss with marketing managers or sellers to see whether or not the questions asked were not appropriately addressed or the decision calculus approach was misunderstood.

3.2. The system hypotheses are summarized as follows

1. A seller wants to promote a product with some price discount in Internet shopping markets.
2. Time limit, as we define it, begins with the time the product is promoted on Internet.
3. Both the awareness and urgency of an offer are hyperbolic functions with an S-shaped curve.
4. Typical expert copy and graphics are assumed and held constant.
5. Competition does not vary their prices during the offer period.
6. This is a one-time offer, single-period maximization decision system.
7. Time limit offered as well as all other parameters of the offer is constant.

The proposed system is concerned with setting the optimal time limit for a product promoted on Internet. We model the awareness and urgency of a promotional offer as hyperbolic functions with an S-shaped curve. Awareness is an increasing function with time, while urgency is a decreasing function with time. As time passes by (since the promotion), awareness increases quickly with concave up and urgency decreases quickly with concave down. However, after a reflection point, awareness increases slowly with concave down and urgency decreases slowly with concave up. In other words, awareness increases with an increasing rate and urgency decreases with an increasing rate; but, after the reflection point, awareness increases at a decreasing rate and urgency decreases at a decreasing rate. We note that assumptions 4–7 are similar to those of Hanna et al. (2005) (in an email promotion), who modeled awareness only as a concave function and urgency only as a convex function. Our awareness and urgency functions agree more with real-world situations (explained in detail below).

3.3. System development

Our goal is to maximize the profit for a product promoted on Internet. Let t be the time limit in days for the product promoted, D the unit discount, M the unit profit margin or the unit gross margin in dollars. The unit net profit for the seller as a function of time limit, $\Pi(t)$, can be calculated by multiplying the unit profit margin, $(M - D)$, times the response rate or the proportion of those

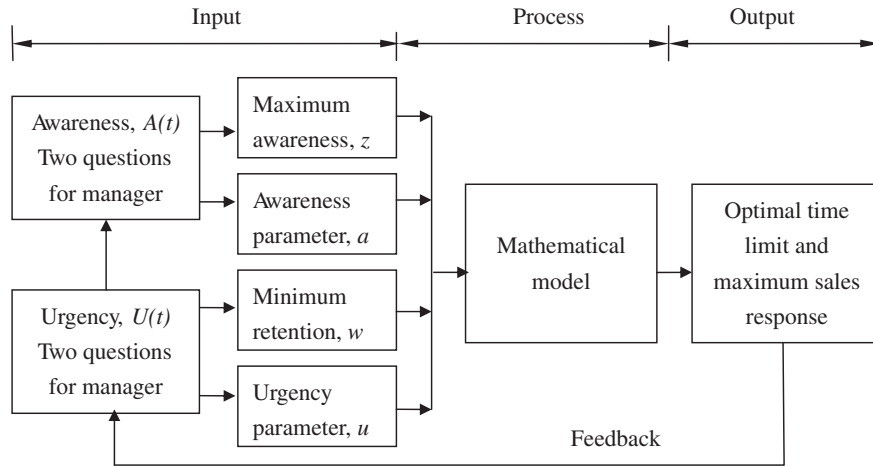


Fig. 1. Proposed decision support system architecture.

receiving the promotion that make a purchase as a function of the time limit, $R(t)$. This may be expressed in expression (1), as in Hanna et al.

$$\Pi(t) = (M - D) \cdot R(t) \tag{1}$$

To determine the optimal time limit for the promotional offer in Internet shopping, we consider two different forces acting on the customers' response: the awareness of an offer and the urgency of the offer. The awareness force is defined as the customers reading the product promotion in the Internet shopping, while the urgency force represents that the potential buyers may lose their sense of urgency of purchasing the product as time passes by. These two forces are basically defined in the same manner as in Hanna et al., except that we will express them quite differently, which is discussed below.

3.4. The awareness function

The first force of our system is the awareness of an offer $A(t)$ which is an increasing function with respect to the promotional time limit. When more time is passing, potential buyers usually are more aware of product information in Internet shopping. If there is only one force in Internet promotions and other conditions do not change, the response (or purchase) is affected by the awareness in Eq. (2). There exists a coefficient c that converts awareness to response, where $0 < c < 1$. The coefficient c depends on product, price, discount, and other promotional conditions

$$R(t) = c \cdot A(t) \tag{2}$$

$A(t)$, initially, is a convex function that increases with an increasing rate. After some point (i.e. the inflection point), $A(t)$ becomes a concave function that increases with a decreasing rate. Hanna et al. considered $A(t)$ only as a concave function without the convex part (as mentioned above). However, we observe that when the product is promoted on Internet, customers usually become aware of this information more rapidly at the beginning of the promotion; as time passes by (beyond the inflection point), customers' (cumulative) awareness still increases but with a decreasing rate. It should be noted that our S-shaped curve of $A(t)$ is not unprecedented. Little (1970) also considered a market share function that exhibits this kind of shape (i.e. market share first increases with an increasing rate as advertising is increased; then it increases with a decreasing rate with more advertising). Explicitly, we model $A(t)$ as a hyperbolic function given by

$$A(t) = z / [(1 + \tanh(t - a)) / 2] \tag{3}$$

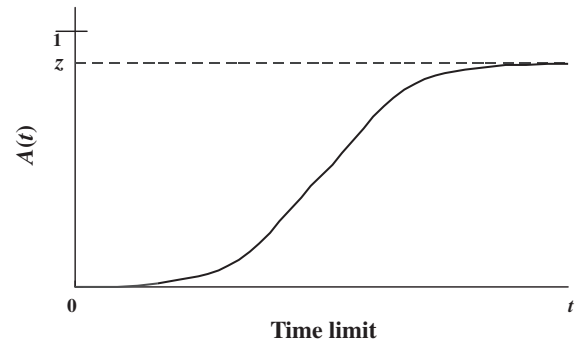


Fig. 2. Awareness as a function of time limit.

where z is the maximum proportion of awareness (i.e. the saturation point) and a is the awareness parameter, If $t \rightarrow \infty$, then $A(t) = z$. $A(t)$ is depicted in Fig. 2.

3.5. The urgency function

The second force of our system is the urgency of an offer $U(t)$ which is a decreasing function with respect to the promotional time limit t . Dhar and Nowlis (1999) observed that time pressure decreases choice deferral. A longer time causes customers to postpone a purchase and even never buy it forever. Buyers feel that it is not urgent to buy the product promoted if t is long. Blattberg and Neslin (1990) pointed out that a lack of urgency will reduce the response. Hanna et al. (2005) explicitly modeled the urgency of an offer. As in Hanna et al., let $U(t)$ be the proportion of those aware that are retained due to urgency (i.e. not lost in spite of the lack of urgency), where $0 \leq U(t) \leq 1$. Thus, $R(t)$ is given by Eq. (4) with two opposing forces

$$R(t) = c \cdot A(t) \cdot U(t) \tag{4}$$

$U(t)$, initially, is a concave function that decreases with an increasing rate. After some point (i.e. the inflection point), $U(t)$ becomes a convex function that decreases with a decreasing rate. Hanna et al. considered $U(t)$ only as a convex function without the concave part. However, we observe that customers' sense of urgency usually decreases more rapidly at the beginning of the promotion; as the promotion becomes longer (beyond the inflection point), customers' sense of urgency still decreases but with a slower rate. Hence, we model $U(t)$ as a hyperbolic function given by

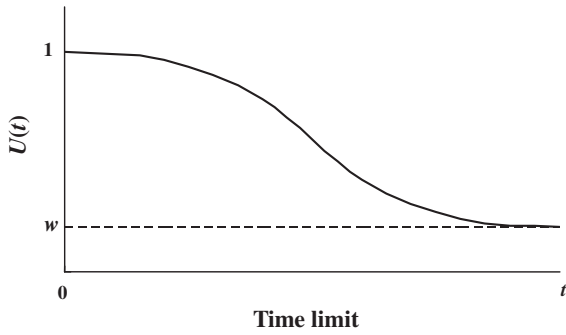


Fig. 3. Urgency as a function of time limit.

$$U(t) = (1 - w)[(1 - \tanh(t - u))/2] + w \tag{5}$$

where w is the proportion not lost (i.e. retained) if $t \rightarrow \infty$, where $0 < w < 1$, and u is the urgency parameter. $U(t)$ is depicted in Fig. 3.

3.6. The response function

We express $R(t)$ explicitly with the two opposing forces $A(t)$ and $U(t)$ in Eq. (6), which is depicted in Fig. 4. If $t \rightarrow \infty$, then $R(t) = czw$

$$\begin{aligned} \Pi'(t) &= k \cdot z \left[\frac{\exp(t-a)[\exp(t-a) + \exp(-t+a)] - \exp(t-a)[\exp(t-a) - \exp(-t+a)]}{[\exp(t-a) + \exp(-t+a)]^2} \right] \cdot \left[(1-w) \left(\frac{\exp(-t+u)}{\exp(t-u) + \exp(-t+u)} \right) + w \right] \\ &\quad + k \cdot z \left[\frac{\exp(t-a)}{\exp(t-a) + \exp(-t+a)} \right] \cdot \left[(1-w) \left(\frac{-\exp(-t+u)[\exp(t-u) + \exp(-t+u)] - \exp(-t+u)[\exp(t-u) - \exp(-t+u)]}{[\exp(t-u) + \exp(-t+u)]^2} \right) \right] \\ &= k \cdot z \left[\frac{2}{[\exp(t-a) + \exp(-t+a)]^2} \right] \cdot \left[(1-w) \left(\frac{\exp(-t+u)}{\exp(t-u) + \exp(-t+u)} \right) + w \right] + k \cdot z \left[\left(\frac{\exp(t-a)}{\exp(t-a) + \exp(-t+a)} \right) \right] \\ &\quad \cdot \left[(1-w) \left(\frac{-2}{[\exp(t-u) + \exp(-t+u)]^2} \right) \right] = 0 \end{aligned}$$

$$R(t) = c \cdot z[(1 + \tanh(t - a))/2] \cdot [(1 - w)[(1 - \tanh(t - u))/2] + w] \tag{6}$$

Substituting Eq. (6) into (1), we get

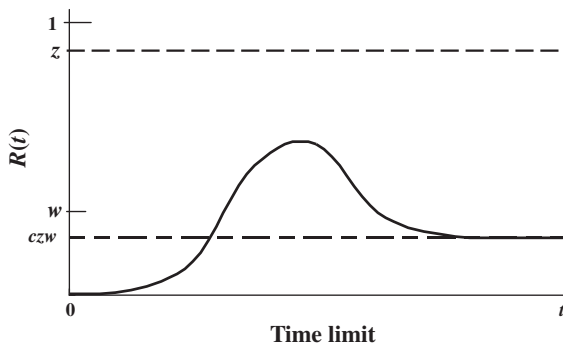


Fig. 4. Response as a function of awareness and urgency.

$$\Pi(t) = (M - D) \cdot c \cdot z[(1 + \tanh(t - a))/2] \cdot [(1 - w)[(1 - \tanh(t - u))/2] + w] \tag{7}$$

Let $k = (M - D) \cdot c$. Then we have

$$\Pi(t) = k \cdot z[(1 + \tanh(t - a))/2] \cdot [(1 - w)[(1 - \tanh(t - u))/2] + w] \tag{8}$$

By the definition of hyperbolic tangents, we can rewrite $A(t)$ and $U(t)$ as follows

$$\begin{aligned} A(t) &= z[(1 + \tanh(t - a))/2] \\ &= z \left[\frac{\exp(t-a)}{\exp(t-a) + \exp(-t+a)} \right] \\ U(t) &= (1 - w)[(1 - \tanh(t - u))/2] + w \\ &= (1 - w) \left[\frac{\exp(-t+u)}{\exp(t-u) + \exp(-t+u)} \right] + w \end{aligned}$$

Hence, we can express Eq. (8) as

$$\Pi(t) = k \cdot z \left[\frac{\exp(t-a)}{\exp(t-a) + \exp(-t+a)} \right] \cdot \left[(1 - w) \left(\frac{\exp(-t+u)}{\exp(t-u) + \exp(-t+u)} \right) + w \right] \tag{9}$$

Differentiating $\Pi(t)$ with respect to t and setting the derivative to zero, we obtain

If we multiply both sides by $[\exp(t - a) + \exp(-t + a)]^2[\exp(t - u) + \exp(-t + u)]^2$, we get

$$\begin{aligned} \Pi'(t) &= 2kz\{(1 - w) \exp(-t + u)[\exp(t - u) + \exp(-t + u)] \\ &\quad + w[\exp(t - u) + \exp(-t + u)]^2\} \\ &\quad - 2kz(1 - w) \exp(t - a)[\exp(t - a) + \exp(-t + a)] \\ &= 2kz\{(1 - w)[1 + \exp(-2t + 2u)] + w[\exp(2t - 2u) \\ &\quad + 2 + \exp(-2t + 2u)]\} - 2kz(1 - w)[\exp(2t - 2a) + 1] \\ &= 2kz[1 + \exp(-2t + 2u) - w - w \exp(-2t + 2u) \\ &\quad + w \exp(2t - 2u) + 2w + w \exp(-2t + 2u)] \\ &\quad - 2kz[\exp(2t - 2a) + 1 - w \exp(2t - 2a) - w] \\ &= 2kz[2w + \exp(-2t + 2u) + w \exp(2t - 2u) \\ &\quad - \exp(2t - 2a) + w \exp(2t - 2a)] = 0 \end{aligned} \tag{10}$$

After some algebra, we can write the above expression by:

$$2w + \frac{\exp(2u)}{\exp(2t)} = \exp(2t) \left(\frac{1}{\exp(2a)} - \frac{w}{\exp(2a)} - \frac{w}{\exp(2u)} \right) \tag{11}$$

We can see that if $w = 0$, then $t^* = (u + a)/2$. If $w = 1$, then $U(t) = 1$, ie, $R(t)$ is decided by $A(t)$ only, so $t^* = \infty$. Hence, w is assumed to be smaller than one in the following. If $0 < w < 1$, there is no closed form solution for the optimal time limit t^* in Eq. (11). However, we can solve (11) for t^* using computer software such as Microsoft Excel or Matlab. In this research we used Microsoft Excel to obtain t^* .

4. Illustrative real world example

We illustrate our system with a real world example in the Yahoo shopping market in Taiwan. Buying or selling products on Internet is very convenient and popular in many countries. From computers to toys anything you want, Internet shopping markets can afford any seller or buyer to enter a transaction platform easily. We asked a fashion dress seller in the Yahoo shopping market some questions that were related to his selling experience of a popular product. We then used his answers as input to our system to determine the product’s promotional time limit (by the decision calculus methodology).

The product unit profit margin was TWD 600 with a discount TWD 200 and thus unit net profit margin was TWD 400. In order to obtain the two parameters in the awareness function, we asked two different questions. The first question was addressed as follows. What proportion of potential customers would become aware of the price discount offer in the Internet shopping promotion if the offer had no deadline (i.e. $A(t) = ?$, if $t \rightarrow \infty$)? The seller told us that the awareness was about 99% if the offer had no deadline, i.e. $z = 99\%$.

Table 1
Sensitivity of t^* with changes of $A(t)$ and $U(t)$.

| Retention, $U(t = 4)$ | Reduction, $1 - U(t = 4)$ | Awareness, $A(t = 4)$ | | | | |
|--------------------------|------------------------------|-----------------------|------|------|------|------|
| | | 0.10 | 0.37 | 0.65 | 0.77 | 0.90 |
| 0.90 | 0.10 | 5.1 | 4.7 | 4.4 | 4.2 | 4.0 |
| 0.70 | 0.30 | 4.8 | 4.3 | 4.1 | 3.9 | 3.6 |
| 0.50 | 0.50 | 4.6 | 4.1 | 3.8 | 3.7 | 3.4 |
| 0.30 | 0.70 | 4.4 | 3.9 | 3.6 | 3.5 | 3.2 |
| 0.10 | 0.90 | 4.7 | 3.6 | 3.3 | 3.1 | 2.9 |

Holding $z = 0.99$ and $w = 0.01$.

Table 2
Sensitivity of $\Pi(t^*)$ with changes of $A(t)$ and $U(t)$.

| Retention, $U(t = 4)$ | Reduction, $1 - U(t = 4)$ | Awareness, $A(t = 4)$ | | | | |
|--------------------------|------------------------------|-----------------------|---------|---------|---------|---------|
| | | 0.10 | 0.37 | 0.65 | 0.77 | 0.90 |
| 0.90 | 0.10 | 9.9999 | 19.3378 | 25.7140 | 28.5115 | 32.4000 |
| 0.70 | 0.30 | 4.5865 | 11.6014 | 18.2054 | 21.7263 | 27.1504 |
| 0.50 | 0.50 | 2.5351 | 7.5249 | 13.2988 | 16.7871 | 22.8103 |
| 0.30 | 0.70 | 1.3100 | 4.4535 | 8.8731 | 11.9106 | 17.8924 |
| 0.10 | 0.90 | 0.4193 | 1.5928 | 3.7509 | 5.5611 | 9.9834 |

Holding $z = 0.99$ and $w = 0.01$.

Table 3
 $\Pi(t)$ using sub-optimal $t = 3.8$.

| Retention, $U(t = 4)$ | Reduction, $1 - U(t = 4)$ | Awareness, $A(t = 4)$ | | | | |
|--------------------------|------------------------------|-----------------------|---------|---------|---------|---------|
| | | 0.10 | 0.37 | 0.65 | 0.77 | 0.90 |
| 0.90 | 0.10 | 2.5813 | 10.5303 | 20.7006 | 25.8401 | 32.0698 |
| 0.70 | 0.30 | 2.1540 | 8.7870 | 17.2735 | 21.5622 | 26.7605 |
| 0.50 | 0.50 | 1.6583 | 6.7650 | 13.2988 | 16.6005 | 20.6027 |
| 0.30 | 0.70 | 1.0766 | 4.3919 | 8.6336 | 10.7771 | 13.3753 |
| 0.10 | 0.90 | 0.3842 | 1.5673 | 3.0810 | 3.8460 | 4.7732 |

Holding $z = 0.99$ and $w = 0.01$.

The second question to the manager was: What proportion of the customers would become aware of the price discount offer if the promotional time limit was 4 days? The reason we chose 4 days to ask the seller about awareness was because sellers usually chose no more than a week as the deadline in most Internet shopping promotions. The seller’s answer for our second question was about 65% with a 4-day time limit. According to this information, we can obtain the awareness parameter a as follows:

$$A(t) = z[(1 + \tanh(t - a))/2] = 65\% = 0.99[(1 + \tanh(4 - a))/2] = 0.65$$

After some algebra, we have:

$$a = 4 - \tanh^{-1}[(2 \cdot 0.65/0.99) - 1] = 3.6760$$

Next, we could determine the two parameters in the urgency function by asking the manager two different questions. The first question was addressed as follows. What proportion of those aware would be retained to buy the product if the promotional offer had no deadline? The seller told us that the retention was about 1% without deadline, i.e. $w = 1\%$.

To get the other parameter, we asked the manager: What proportion of those who were aware would be retained to buy if the promotional time limit was 4 days? The seller answered us that the proportion retained would be 50% with a time limit of 4 days; namely, the reduction in awareness is 50% (i.e. retention = 1 - reduction). From the answer we could determine the urgency parameter u as follows:

$$U(t) = (1 - w)[(1 - \tanh(t - u))/2] + w = 50\% = (1 - 0.01)[(1 - \tanh(4 - u))/2] + 0.01 = 0.50$$

After some algebra, we have:

$$u = 4 - \tanh^{-1}(1 - 2[(0.5 - 0.01)/(1 - 0.01)]) = 3.9899$$

Assume that conversion coefficient $c = 0.1$ (the value of c is not central to decision of the optimal time limit) and unit net profit margin $(M - D) = 400$. Using the values of $z = 0.99$, $a = 3.6760$, $w = 0.01$, and $u = 3.9899$ in Eq. (10), we find that the optimal time limit $t^* = 3.8$ days, the maximum response $R(t^*) = 0.3325$, and the maximum profit $\Pi(t^*) = 13.2988$.

Table 4
Percent decrease between Tables 2 and 3.

| Retention, $U(t = 4)$ | Reduction, $1 - U(t = 4)$ | Awareness, $A(t = 4)$ | | | | |
|--------------------------|------------------------------|-----------------------|-------|-------|-------|-------|
| | | 0.10 | 0.37 | 0.65 | 0.77 | 0.90 |
| 0.90 | 0.10 | 74.19 | 45.55 | 19.50 | 9.37 | 1.02 |
| 0.70 | 0.30 | 53.04 | 24.26 | 5.12 | 0.76 | 1.44 |
| 0.50 | 0.50 | 34.58 | 10.10 | 0.00 | 1.11 | 9.68 |
| 0.30 | 0.70 | 17.82 | 1.38 | 2.70 | 9.52 | 25.25 |
| 0.10 | 0.90 | 8.37 | 1.60 | 17.86 | 30.84 | 52.19 |

Holding $z = 0.99$ and $w = 0.01$.

Table 5
Optimal t (\hat{t}) by running Hanna et al.'s model.

| Retention, $U(t = 4)$ | Reduction, $1 - U(t = 4)$ | Awareness, $A(t = 4)$ | | | | |
|--------------------------|------------------------------|-----------------------|------|------|------|------|
| | | 0.10 | 0.37 | 0.65 | 0.77 | 0.90 |
| 0.90 | 0.10 | 26.4 | 14.5 | 9.0 | 7.3 | 5.3 |
| 0.70 | 0.30 | 9.9 | 7.2 | 5.2 | 4.4 | 3.4 |
| 0.50 | 0.50 | 5.4 | 4.4 | 3.5 | 3.1 | 2.5 |
| 0.30 | 0.70 | 3.2 | 2.8 | 2.4 | 2.2 | 1.8 |
| 0.10 | 0.90 | 1.7 | 1.6 | 1.4 | 1.3 | 1.2 |

Holding $z = 0.99$ and $w = 0.01$.

Notice that in the above base case, we asked the manager: What proportion of the customers would become aware of the offer and what proportion of those aware would be retained if the promotional time limit was 4 days? Suppose, instead, that we asked the manager: What proportion of the customers would become aware and what proportion of those aware would be retained if the time limit was 7 days? And their answers were still 65% and 50%, respectively. Then the optimal time limit t^* is 6.8 days (i.e. increased by $7 - 4 = 3$ days), $R(t^*) = 0.3325$, and $\Pi(t^*) = 13.2988$ (i.e. the same as before).

5. Sensitivity analyses

Using the above real world example as a base case, we next carry out sensitivity analyses of several parameters of our decision system. We first conduct the sensitivity of t^* and Π^* to $A(t)$ and

$U(t)$, as the functions change with the input values of $A(t = 4)$ and $U(t = 4)$ while holding $z = 0.99$ and $w = 0.01$. Unlike Hanna et al. (2005), we have more variable $A(t)$ and $U(t)$ values (ranging from 0.10 to 0.90). As we see from Table 1, the optimal time limit t^* decreases as $A(t)$ increases or $U(t)$ decreases, which agree with our intuition and Hanna et al. The optimal t^* varies from 2.9 to 5.1 days. In Table 2, we see that the corresponding $\Pi(t^*)$ increases as $A(t)$ or $U(t)$ increases, which also intuitively makes sense.

Next, we examine sensitivity analyses to the correctness of the seller's opinion/input; that is, how sensitive is the optimal profit Π^* to the exact values put forth by the seller using the decision calculus approach. What if the t^* we determined, based on the input of the seller, is not the true optimal t but is instead some other value because the seller's $A(t = 4)$ and $U(t = 4)$ are not accurate? In Table 3 we see the values of $\Pi(t = 3.8)$ for different (true) values of $A(t = 4)$ and $U(t = 4)$. Table 3 reports the profit achieved by using the wrong t^* (the true value of t^* for each combination is in Table 1). The values in Table 3 are of course lower than their counterpart in Table 2 (except the central cell). In Table 4 we show the percentage decrease of $\Pi(t)$ from Tables 2 and 3. As we see, the percentage decrease in profit is larger when $A(t = 4)$ is overestimated and $U(t = 4)$ underestimated, or the opposite, $A(t = 4)$ is underestimated and $U(t = 4)$ overestimated. When both $A(t)$ and $U(t)$ are overestimated or both underestimated, the percentage reduction in profit was relatively low. This result agrees with Hanna et al.

In addition, we investigate what results Hanna et al.'s model could obtain, if our decision system is indeed correct for setting the time limit for Internet shopping promotions. We run Hanna et al.'s model by using our data in Table 1. The optimal t obtained

Table 6
 $\Pi(t)$ using the sub-optimal \hat{t} in Table 5.

| Retention, $U(t = 4)$ | Reduction, $1 - U(t = 4)$ | Awareness, $A(t = 4)$ | | | | |
|--------------------------|------------------------------|-----------------------|--------|---------|---------|---------|
| | | 0.10 | 0.37 | 0.65 | 0.77 | 0.90 |
| 0.90 | 0.10 | 0.3960 | 0.3960 | 0.4118 | 0.8646 | 15.8799 |
| 0.70 | 0.30 | 0.3967 | 0.5437 | 6.8393 | 18.0086 | 26.3215 |
| 0.50 | 0.50 | 0.1534 | 7.0633 | 11.9329 | 12.4398 | 12.5325 |
| 0.30 | 0.70 | 0.5935 | 1.6721 | 2.6090 | 3.2448 | 4.2068 |
| 0.10 | 0.90 | 0.0406 | 0.1790 | 0.3919 | 0.5899 | 1.3623 |

Holding $z = 0.99$ and $w = 0.01$.

Table 7
Percent decrease between Tables 2 and 6.

| Retention, $U(t = 4)$ | Reduction, $1 - U(t = 4)$ | Awareness, $A(t = 4)$ | | | | |
|--------------------------|------------------------------|-----------------------|-------|-------|-------|-------|
| | | 0.10 | 0.37 | 0.65 | 0.77 | 0.90 |
| 0.90 | 0.10 | 96.04 | 97.95 | 98.40 | 96.97 | 50.99 |
| 0.70 | 0.30 | 91.35 | 95.31 | 62.43 | 17.11 | 3.05 |
| 0.50 | 0.50 | 33.43 | 6.13 | 10.27 | 25.90 | 45.06 |
| 0.30 | 0.70 | 54.70 | 62.45 | 70.60 | 72.76 | 76.49 |
| 0.10 | 0.90 | 90.31 | 88.76 | 89.55 | 89.39 | 86.35 |

Holding $z = 0.99$ and $w = 0.01$.

by using their model, denoted by \hat{t} , is reported in Table 5, the resulting profit by substituting \hat{t} into Eq. (7) is recorded in Table 6, and the percent decrease in profit of using Hanna et al.'s model is given in Table 7. As we see from Table 5, \hat{t} can be as high as 26.4 days, which seems unlikely for most Internet shopping promotions, and the decrease in profit from Table 7 can be over 96%. Hanna et al. (p. 16) indicated themselves that consumers may put off the purchase or even never buy if an online offer lasts seven days or longer. Thus, hyperbolic functions are more appropriate for setting the time limit in Internet shopping promotions.

6. Conclusion and discussion

In this paper, we propose a decision support system with two opposing forces to determine the optimal time limit for maximum sales response or profit in Internet shopping promotions. One force is the awareness of an offer which is an increasing function of the time limit. The other force is the urgency which is a decreasing function of the time limit. The awareness and urgency forces exhibit a hyperbolic function with an S-shaped curve that is appropriate for many real-world situations in Internet shopping promotions.

Under the assumptions made, the proposed decision system decides the promotional time limit with the seller's input of estimation of sales response. We illustrate the use of the system with an example which has an optimal time limit of 3.8 days. In many shopping websites, marketing managers set about four days to promote electronic products like mobile phones, notebooks, and digital cameras. We also conduct some sensitivity analyses when the seller's input values change. We compare our numerical results from hyperbolic functions with those from simple exponential functions. We show that the percent decrease in profit could be very high if simple exponential functions are used.

To summarize, this paper contributes to the field of decision support systems both in theoretical development and in practical applications. For the former, we model the awareness and urgency of a promotional offer as hyperbolic functions with an S-shaped curve. This is never seen in past research in sales response. For the latter, Yahoo or other shopping websites typically choose less than a week as the promotional time limit. Our numerical results agree with this real-world situation.

There are some suggestions for further research. The discount rate may affect the promotional time limit. How to include the discount variable into the proposed decision support system is worth to investigate. Also, it is interesting to explore whether the seller can use the proposed system to determine the auction time of products (sellers usually set few days in online auction to sell products as soon as possible). For multi-criteria decision support one may further consider entering other variables into the decision system.

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