

failure. From this a 'required crossconnections' matrix, $P_{[r,s]}$, was formed for each node to represent the involvement of each of the γ span pairings at a node in the specific pathset for restoration of each of the S single-span failures that are possible.

(ii) The co-ordinate-constrained gradient search was applied at each node, iteratively inspecting $P_{[r,s]}$ to determine $\delta C_u / \delta X_{[r,s]}^a$ in eqn. 5 and to implement the summations in eqn. 4.

(iii) Using the optimised preconnection vectors, $\hat{X}_{[r,s]}^a$, $a = 1, \dots, N$, that resulted from (ii), the readiness bound N_r/N_p was evaluated for each network, again using $P_{[r,s]}$, this time to obtain N_p . The results are summarised in Table 1.

Discussion: Results show that, with the above procedure to pre-configure spare links, an average of 79% of the span-to-span pairings required for restoration in the 10 test networks could be preconnected in advance of failure. This implies that we could reasonably expect to have a useful fraction of the end-to-end paths required for restoration also available instantaneously, without any crossconnect deployment delays. However, to maximise the latter potential we must further devise a means of co-ordinating the total quantities of span-to-span preconnection in the $X_{[r,s]}^a$ vectors at all nodes so that individual link crossconnections are most coherently related to those at other nodes, thereby forming the maximum number of fully preconnected end-to-end paths for the average restoration case. This is a topic of ongoing work. These first results, however, give significant early motivation, and indicate considerable practical benefit from the overall strategy of preconnection in mesh-restorable networks. Similar results may be obtained with the procedure given for other distributed restoration

protocols and networks.

© IEE 1994

22 November 1993

Electronics Letters Online No: 19940131

W. D. Grover and M. H. MacGregor (TRILabs & University of Alberta, Edmonton, T6G 2E1, Canada)

References

- GROVER, W.D., VENABLES, B.D., MACGREGOR, M.H., and SANDHAM, J.H.: 'Development and performance verification of a distributed asynchronous protocol for real-time network restoration', *IEEE J. Sel. Areas Commun.*, 1991, SAC-9, (1), pp. 112-125
- SAKAUCHI, H., NISHIMURA, Y., and HASEGAWA, S.: 'A self-healing network with economical spare-channel assignment'. *IEEE Globecom '91*, pp. 438-443
- YANG, C.H., and HASEGAWA, S.: 'FITNESS: A failure immunization technology for network service survivability'. *IEEE Globecom '88*, pp. 1549-1554
- SR-NWT-002514: 'Digital cross-connect systems in transport network survivability', *Bellcore*, 1, January 1993
- WU, T.-H., and KOBRINSKI, H.: 'A service time restoration study for distributed control SONET digital cross-connect system self-healing networks'. *IEEE ICC '93*, pp. 893-899
- GROVER, W.D., BILODEAU, T.D., and VENABLES, B.D.: 'Near-optimal spare capacity planning in a mesh restorable network'. *IEEE Globecom '91*, pp. 2007-2012

Power control algorithm for cellular radio systems

J.-C. Lin, T.-H. Lee and Y.-T. Su

Indexing terms: Cellular radio, Algorithms

In a recent paper, Zander presented an optimum transmitter power control scheme to minimise the outage probability for cellular systems and proposed the stepwise removal algorithm (SRA) for use in practical implementations. In the Letter, the authors present a new power control algorithm which considers the transmitter power together with link gains. The algorithm provides better outage probability with only a negligible increase in computational complexity.

Introduction: Frequency reuse is the core concept in cellular mobile systems. The interference due to common use of the same channel limits the system capacity. Transmitter power control is a technique which can be used to reduce cochannel interference and allow as many receivers as possible to obtain a satisfactory reception.

Aein [1] proposed controlling the transmitter power to achieve a balanced carrier-to-interference ratio (CIR). Under the balancing scheme, all receivers experience the same CIR. Unfortunately, it is possible that for a balanced CIR the reception may be unsatisfactory for all receivers. To achieve satisfactory reception, some transmitters may need to be prohibited from transmitting. Zander [2] studied this problem and derived an optimum power control scheme which minimises the outage probability. However, the computational complexity makes the optimum power control scheme impractical. For this reason, Zander proposed a simple procedure, called the stepwise removal algorithm (SRA), for practical implementations.

In this Letter, we propose a new power control scheme that takes into account both the transmitter power and the link gains. Numerical results show that, compared with the SRA algorithm, our scheme achieves a smaller outage probability with only a negligible increase in computational complexity.

System model: The cellular system investigated in this Letter consists of a finite number of cells. Cells using the same channel are placed symmetrically in a hexagonal grid. Base stations use omnidirectional antennas and are located at the centre of the cells. The mobiles are assumed to be uniformly distributed over the cell area. The set of cells using a certain channel at some given instant is called the cochannel set and the size of the cochannel set is denoted by N . All cochannels are assumed to be in use. The total interference power is modelled as the sum of the powers of all active interferers and the transmission quality is assumed to depend only on CIR. Under these assumptions, the CIR at mobile i is given by

$$\begin{aligned} CIR_i &= \frac{R_i}{\sum_{j=1}^N R_j - R_i} \\ &= \frac{T_i L_{ii}}{\sum_{j=1}^N T_j L_{ij} - T_i L_{ii}} \\ &= \frac{T_i}{\sum_{j=1}^N T_j Z_{ij} - T_i} \end{aligned}$$

where R_j is the power received from the j th transmitter, T_j is the transmitter power used by the base station in cell j , L_{ij} is the gain on the link between the mobile in cell i and the base station in cell j at some given moment, and $Z_{ij} = L_{ij}/L_{ii}$ is the normalised link gain. The link gain L_{ij} is modelled as $L_{ij} = A_{ij}/d_{ij}^\nu$ where A_{ij} is the attenuation factor, d_{ij} is the distance between the mobile in cell i and the base station in cell j , and ν is a constant that models the large scale propagation loss. The attenuation factor models the power variation due to shadowing. We assume that A_{ij} , $1 \leq i, j \leq N$, are independently log-normally distributed random variables with 0dB expectation and σ dB log variance.

A CIR is defined to be achievable in the cochannel set if there exists a power vector $\mathbf{T} = [T_1, T_2, \dots, T_N]^T$ such that $CIR_i \geq CIR$ for all i , $1 \leq i \leq N$. Let CIR_0 be the system protection ratio or the minimum CIR required for a satisfactory reception. It was shown [2] that the largest achievable CIR, denoted by CIR^* , is given by $CIR^* = 1/(\lambda^* - 1)$, where λ^* is the largest real eigenvalue of the positive link gain matrix $\mathbf{Z} = [Z_{ij}]$. Moreover, the optimum power vector \mathbf{T}^* is the eigenvector of \mathbf{Z} corresponding to the eigenvalue λ^* . All mobiles experience the same CIR^* when \mathbf{T}^* is used.

Power control algorithms: An optimum power control algorithm (PCA) will, by removing as few cells as possible, find the largest submatrix of Z for which CIR_0 is achievable. The following brute force algorithm is an optimum PCA.

(a) **Algorithm 1: Brute force algorithm (BFA) [2]:**

(i) **Step 1:** Determine CIR^* corresponding to Z . If $CIR^* \geq CIR_0$, then use the eigenvector T^* and stop, else perform step 2.

(ii) **Step 2:** Remove all combinations of at most $N - 2$ cells and compute the eigenvalues of each reduced system. If for some i , $1 \leq i \leq N - 2$, $CIR^* \geq CIR_0$, then use the corresponding eigenvector and stop, else remove $N - 1$ cells and stop.

When using the BFA algorithm to remove any combination of n cells ($1 \leq n \leq N - 2$), we need to determine the eigenvalue and eigenvector pair

$$\binom{N}{n} = \frac{N!}{n!(N-n)!}$$

times. Therefore, in the worst case, we have to determine the eigenvalue and eigenvector pair

$$\binom{N}{1} + \binom{N}{2} + \dots + \binom{N}{N-2} = 2^N - N - 2$$

times to remove $N - 1$ cells to achieve CIR_0 .

(b) **Algorithm 2: Stepwise removal algorithm (SRA) [2]:**

(i) **Step 1:** Determine CIR^* corresponding to Z . If $CIR^* \geq CIR_0$, then use the eigenvector T^* and stop; else set $N' = N$ and perform step 2.

(ii) **Step 2:** Remove cell k for which the maximum of the row and column sums, $R_k = \sigma_{j=1}^{N'} Z_{kj}$ and $R_k^T = \sum_{j=1}^{N'} Z_{kj}$, is maximised and form the $(N' - 1) \times (N' - 1)$ matrix Z' . Determine CIR^* corresponding to Z' . If $CIR^* \geq CIR_0$, then use the corresponding eigenvector and stop; else set $N' = N' - 1$ and repeat step 2.

For the SRA algorithm, to remove one cell, we need to find the eigenvalue and eigenvector pair once. So, in the worst case, we need to solve the eigen system $N - 2$ times which is far less than the $2^N - N - 2$ times required for the BFA algorithm. However, the SRA algorithm is not an optimum PCA. To improve the performance of the SRA algorithm, we propose algorithm 3.

(c) **Algorithm 3: Stepwise maximum-interference removal algorithm (SMIRA):**

The SMIRA algorithm considers transmitter power to be an important factor for the criterion of removing cells. The idea is that the larger the transmitter power, the greater the interference it causes in mobiles in the other cells. Therefore, if a cell which uses a high transmitting power is removed, it is likely that the remaining cells can achieve CIR_0 . Define

$$R_l = \sum_{j=1, j \neq l}^{N'} T_j * Z_{lj} \quad R_l^T = T_l * \sum_{j=1, j \neq l}^{N'} Z_{jl}$$

and $R_{max,l} = \max(R_l, R_l^T)$

The SMIRA algorithm is the same as the SRA algorithm except that in step 2 cell k is removed if $R_{max,k} = \max_l R_{max,l}$. The term 'maximum interference' is used because R_l represents the total interference received by the mobile in cell l and R_l^T represents the total interference to other mobiles caused by base station l . Clearly, the SMIRA algorithm requires N^2 more multiplications than the SRA algorithm each time a cell is removed. However, the computational complexity is dominated by solving the eigensystem and thus the N^2 multiplications are negligible.

Numerical results and discussion: Numerical results were obtained for a system containing 19 cochannel cells that uses a seven-cell cluster (see Fig. 1). All the values were obtained by means of Monte Carlo simulation for 500 independent configurations. In our study, we used $\sigma = 6$ and $\nu = 4$.

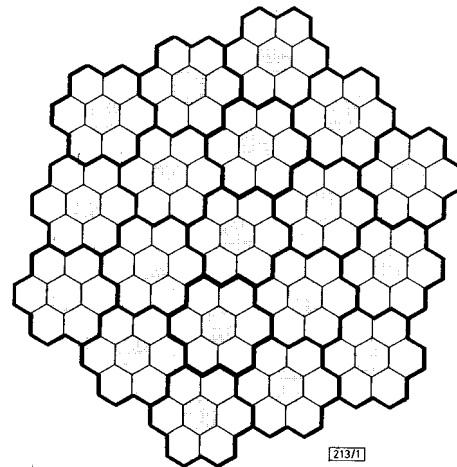


Fig. 1 19 cochannel cells with seven-cell cluster

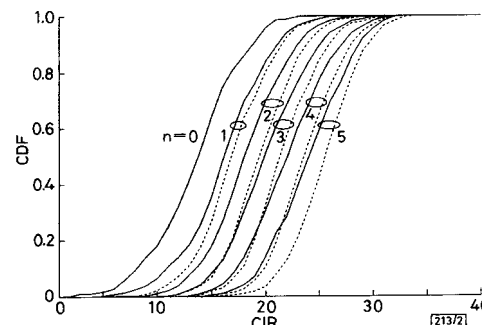


Fig. 2 Distribution of maximum achievable CIR for SRA and SMIRA

— SRA with n cells removed
 - - - SMIRA with n cells removed

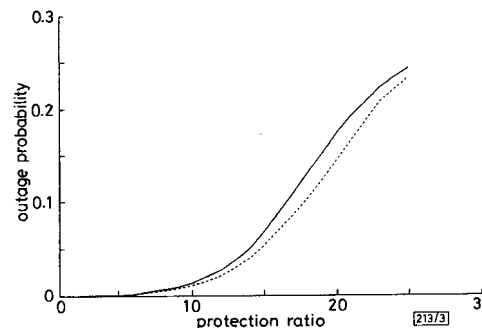


Fig. 3 Outage probability comparison for SRA and SMIRA

— SRA
 - - - SMIRA

Fig. 2 shows the maximum achievable CIR after a number of cells were removed using the SRA and the SMIRA algorithms. It can be seen that the SMIRA algorithm is consistently better than the SRA algorithm. In Fig. 3 we compare the outage probability. The improvement of SMIRA over SRA can be as high as 22% when the protection ratio is ~15dB. When only one cell is removed, our examples show that the SMIRA algorithm removes the cell in an optimum way with probability 64.8% (324 out of 500), whereas the SRA algorithm achieves probability 44% (220 out of 500). When two cells are removed, the probabilities for the

SMIRA algorithm and the SRA algorithm are 48.8 and 22.2%, respectively.

© IEE 1994

22 November 1993

Electronics Letters Online No: 19940140

J.-C. Lin (Institute of Electronics, National Chiao Tung University, Hsinchu, Taiwan 30035, Republic of China)

T.-H. Lee and Y.-T. Su (Department of Communications Engineering, National Chiao Tung University, Hsinchu, Taiwan 30035, Republic of China)

References

- 1 AFIN, J.M.: 'Power balancing in systems employing frequency reuse', *COMSAT Tech. Rev.*, 1973, 3, (2), pp. 277-99
- 2 ZANDER, J.: 'Performance of optimum transmitter power control in cellular radio systems', *IEEE Trans.*, 1992, VT-41, (1), pp. 57-62

Recursive algorithm for the discrete cosine transform with general lengths

L.-P. Chau and W.-C. Siu

Indexing terms: Discrete cosine transforms, Algorithm theory

A novel VLSI algorithm for computing the discrete cosine transform of variable length is proposed. By using some mathematical techniques, any general length DCT can be converted into a recursive equation and this structure can be realised using software, hardware and VLSI techniques. The formulation is particularly suitable for the real-time computation of the DCT and this algorithm can be implemented using regular and parallel VLSI structures, so that the computational complexity is greatly reduced. It can also be extended to implement a two-dimensional DCT in a straightforward way.

Introduction: The discrete cosine transform is widely used in the area of image processing and other digital signal processing applications. Hence, the search for fast algorithms for computing dedicated lengths of DCT [1-6] is an active area of research. However there has not been much work carried out on the computation of the transform of any length.

In [7], Goertzel proposed an algorithm to evaluate a finite trigonometric series and the algorithm requires only N multiplications and $\sim 2N$ additions to obtain the results. This algorithm can easily be implemented to compute the discrete fourier transform and it has also been extended to calculate the DCT [8]. The advantage of this algorithm is its regular structure and parallelism, which make it suitable for implementation using VLSI techniques.

We present a novel algorithm to convert the DCT into a more efficient recursive form, which is suitable for parallel computation and no permutation is required for the input/output sequences.

Derivation of algorithm: The DCT of a sequence $\{y(i) : i = 0, 1, \dots, N-1\}$ can be written as

$$Y(k) = \sum_{i=0}^{N-1} y(i) \cos\left(\frac{\pi}{2N}(2i+1)k\right) \quad (1)$$

for $k = 0, 1, \dots, N-1$

To introduce a time recursive kernel for the DCT, we define Hence, we have

$$V_m = y(m) + 2 \cos x V_{m+1} - V_{m+2} \quad (3)$$

We intend to obtain a simplified expression for the DCT, let $x = \pi k/N$, then

$$= y(0) \cos\left(\frac{x}{2}\right) + \sum_{i=1}^{N-1} y(i) \left(\cos(x i) \cos\left(\frac{x}{2}\right) - \sin(x i) \sin\left(\frac{x}{2}\right) \right)$$

$$\begin{aligned} V_m \sin x &= \sum_{i=m}^{N-1} y(i) \sin(i-m+1)x \\ &= y(m) \sin x + \sum_{i=m+1}^{N-1} y(i) \sin(i-m+1)x \\ &= y(m) \sin x + \sum_{i=m+1}^{N-1} y(i) (2 \cos x \sin(i-m)x - \sin(i-m-1)x) \\ &= y(m) \sin x + 2 \cos x \sum_{i=m+1}^{N-1} y(i) \sin(i-m)x - \sum_{i=m+2}^{N-1} y(i) \sin(i-m-1)x \\ &= y(m) \sin x + 2 \cos x V_{m+1} \sin x - V_{m+2} \sin x \quad (2) \end{aligned}$$

$$\begin{aligned} Y(k) &= \sum_{i=0}^{N-1} y(i) \cos\left(\frac{x}{2}(2i+1)\right) \\ &= y(0) \cos\left(\frac{x}{2}\right) + \sum_{i=1}^{N-1} y(i) \cos\left(\frac{x}{2}(2i+1)\right) \end{aligned}$$

To obtain the time recursive transfer function of $Y(k)$, we multiply both sides by $\sin x$:

$$\begin{aligned} Y(k) \sin x &= y(0) \cos\left(\frac{x}{2}\right) \sin x \\ &+ \sum_{i=1}^{N-1} y(i) \left(\cos(ix) \cos\left(\frac{x}{2}\right) \sin x - \sin(ix) \sin\left(\frac{x}{2}\right) \sin x \right) \\ &= y(0) \cos\left(\frac{x}{2}\right) \sin x \\ &+ \sum_{i=1}^{N-1} y(i) \left(\left(\cos x \cos\left(\frac{x}{2}\right) - \sin x \sin\left(\frac{x}{2}\right) \right) \sin(ix) \right) \\ &- \sum_{i=1}^{N-1} y(i) \left(\left(\sin(ix) \cos x - \cos(ix) \sin x \right) \cos\left(\frac{x}{2}\right) \right) \\ &= y(0) \cos\left(\frac{x}{2}\right) \sin x \\ &+ \sum_{i=1}^{N-1} y(i) \left(\cos\left(\frac{3x}{2}\right) \sin(ix) - \cos\left(\frac{x}{2}\right) \sin(i-1)x \right) \\ &= y(0) \cos\left(\frac{x}{2}\right) \sin x \\ &+ \sum_{i=1}^{N-1} y(i) \cos\left(\frac{3x}{2}\right) \sin(ix) - \sum_{i=2}^{N-1} y(i) \cos\left(\frac{x}{2}\right) \sin(i-1)x \\ &= y(0) \cos\left(\frac{x}{2}\right) \sin x \\ &+ \cos\left(\frac{3x}{2}\right) \sin x V_1 - \cos\left(\frac{x}{2}\right) \sin x V_2 \quad (4) \end{aligned}$$

After the above obvious steps, we finally obtain

$$Y(k) = y(0) \cos\left(\frac{\pi k}{2N}\right) + \cos\left(\frac{3\pi k}{2N}\right) V_1 - \cos\left(\frac{\pi k}{2N}\right) V_2 \quad (5)$$

From eqns. 3 and 5, we know that no complex multiplication is required during the recursive computation. Eqn. 5 is a discrete time recursive transfer function of finite duration sequence, $y(i)$, $i = N-1, N-2, \dots, 1, 0$. As a consequence, $Y(k)$ is obtained as the output of a finite impulse response system. Fig. 1 shows a recursive structure for the realisation of $Y(k)$.

In the literature, not much work has been reported on the computation of the DCT using a recursive filter structure; we have only found one paper on this topic. One possible realisation [8] has been suggested recently, which uses the Goertzel [7] algorithm