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Development of a semi-implicit fluid modeling code using finite-volume method based on Cartesian grids

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ABSTRACT

Presented is the HLLG (Harten, Lax and van Leer with Gradient inclusion) method for application to the numerical solution of general Partial Differential Equations (PDEs) in conservation form. The HLLG method is based on the traditional HLL method with formal mathematical inclusion of gradients of conserved properties across the control volume employed for flux derivation. The simple extension demonstrates that conventional higher extensions of the HLL method are mathematically inconsistent and produce various numerical instabilities. The HLLG method, with higher order extensions consistent with the flux derivation, is absent of (or less affected by) the said numerical instabilities. The HLLG method is then applied to solutions of the Euler Equations and the simulation of 1D argon RF plasma simulation.

1. Introduction

The finite volume method for solution to partial differential equations forms the mainstay of modern Computational Fluid Dynamics (CFD). One of the early pioneering methods of the Finite Volume Method (FVM) was the HLL (Harten, Lax and van Leer) method. The original HLL method was developed by Harten, Lax and van Leer [1] as an approximate Riemann solver for use in a Godunov solver. Rather than solving the Riemann problem analytically with knowledge of the behavior of the system (in many cases, an ideal gas), the HLL method solves for the flux in an intermediate region (or *star region*) between cells directly from the governing partial differential equations. Through the introduction of the *star region* bounded by two propagating waves, the flux across the interface can be mathematically determined. This allows for the flexible solution of a various number of conservative hyperbolic systems, such as the Shallow Water Equations and the Euler Equations.

Presented here is a modification to the HLL method where allowance is made for the mathematical inclusion of gradient terms within the flux expressions. The integral form of the HLL expressions is presented and then re-evaluated allowing conserved quantities to vary linearly in space. The resulting fluxes form the basis of the HLLG scheme, where the G represents the inclusion of gradient terms. This method is then applied to the solution of the Euler Equations and finally the fluid modeling equations for a 1D Argon RF discharge. The results show an improvement in the numerical diffusion associated with the traditional HLL higher order extensions without significantly increasing computational expense.

2. Harten, Lax and van Leer (HLL) method

The fluxes developed by Harten, Lax and van Leer [1] are presented here in their complete integral form. Fig. 1 shows a control volume in x–t space covering the region between cells i and $i+1$ centered on the interface separating the cells at $x=0$. The region is temporally bound by the limits $t=0$ and $t=T$. At $t=0$, waves moving at velocities S_L (<0) and S_R (>0) move away from the discontinuity between the cell interface. The conditions inside the control volume in the region between $[0,T]$ and $[x_L, x_R]$ can be described by the integral:

$$\int_{S_L T}^{S_R T} U_*^T\,dx = \int_{x_L}^{0} U_L^0\,dx + \int_{0}^{x_R} U_R^0\,dx + \int_0^T F_L\,dt - \int_0^T F_R\,dt - \int_{x_L}^{S_L T} U_L^T\,dx - \int_{S_R T}^{x_R} U_R^T\,dx \tag{1}$$

where the subscripts L and R represent conditions in the left and right cells respectively, while the superscripts 0 and T represent

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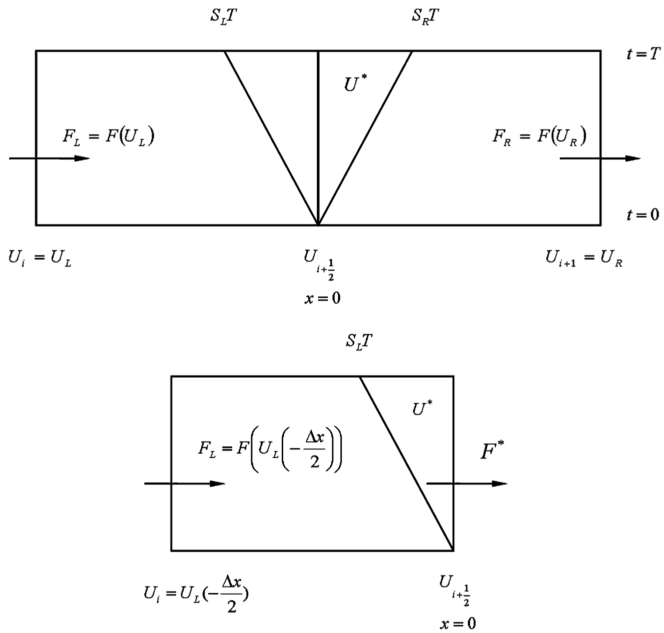

Fig. 1. Control Volumes (CVs) employed in HLL flux derivation.

conditions at time $t = 0$ and $t = T$ respectively. The subscript $*$ represents conditions in the star region between the propagating waves. This equation assumes nothing regarding variation of fluxes F or conserved quantities U within space and time. The only assumptions made are in the presence of the two propagating waves surrounding a single intermediate region at $x = 0$. Fig. 1 shows the revised x–t diagram focusing on the left cell only. Using the same method of integrating conserved quantities over space and fluxed quantities over time obtains:

$$\int_{x_L}^{0} U_L^0\,dx + \int_0^T F_L\,dt - \int_0^T F_* = \int_{x_L}^{S_L T} U_L^T\,dx + \int_{S_L T}^{0} U_*^T\,dx \tag{2}$$

By assuming that the average state over the region between $x = \text{SLT}$ and $x = \text{SRT}$ is the same as the average between the region $x = \text{SLT}$ and $x = 0$, we can substitute the equations together to obtain the expression for the interface flux:

$$\begin{aligned}\int_0^T F_* = {} & \int_{x_L}^{0} U_L^0\,dx - \int_{x_L}^{S_L T} U_L^T\,dx + \int_0^T F_L\,dt \\ & + \frac{S_L}{S_R - S_L}\Bigg(\int_{x_L}^{0} U_L^0\,dx - \int_{x_L}^{S_L T} U_L^T\,dx + \int_0^{x_R} U_R^0\,dx \\ & - \int_{S_R T}^{x_R} U_R^T\,dx + \int_0^T F_L\,dt - \int_0^T F_R\,dt\Bigg)\end{aligned} \tag{3}$$

By assuming that the quantities U remain spatially constant (i.e. monotonic in nature) and the fluxes are temporally constant we recover the original HLL flux expressions:

$$\overline{F_*} \approx \frac{F(U_L)S_R - F(U_R)S_L - S_R S_L(U_R - U_L)}{S_R - S_L} \tag{4}$$

3. Extension of HLL to HLLG

Formal inclusion of gradients into Eqs. (1)–(3) provides the HLLG flux expressions, which can be written in the form:

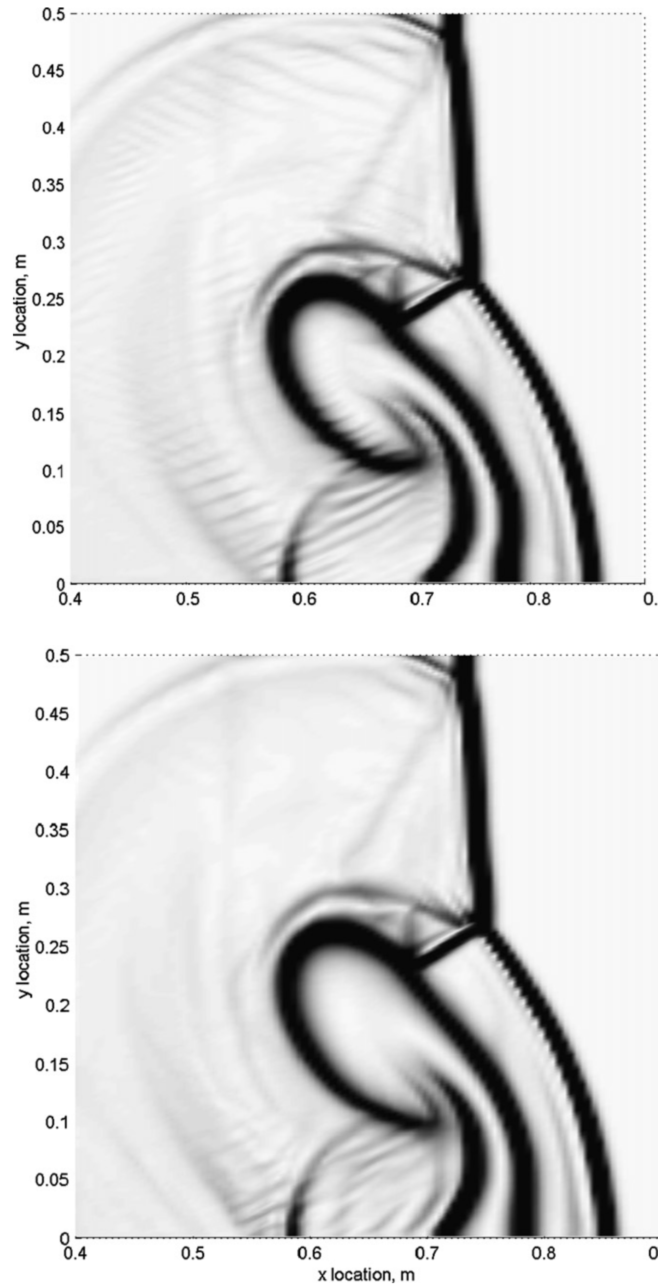

Fig. 2. Numerical schlierens for the 2D shock-bubble interaction problem, [top] HLL, [bottom] HLLG. Both simulations employed the MINMOD limiter with a constant maximum CFL = 0.5.

$$\overline{F_*} \approx \frac{F(U_L^1)S_R - F(U_R^1)S_L - S_R S_L(U_R^2 - U_L^2)}{S_R - S_L} \tag{5}$$

where superscripts 1 and 2 indicate spatial reconstruction at distances ST and $ST/2$ away from the cell interfaces respectively. The resulting flux expressions differ from those of traditional higher order extension which conventionally is performed at the cell interfaces [2].

4. Results

Results are presented for the simulation of a shock wave passing over a low density bubble [3] and for a 1D RF argon plasma problem. Numerical schlierens demonstrating spurious oscillations in the conventional HLL result and the (less affected) HLLG result are shown in Fig. 2. The solution of the 1D argon plasma sim-

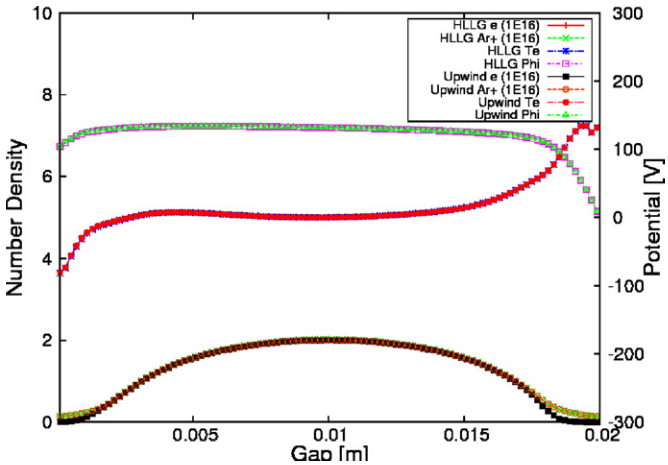

Fig. 3. Sample output from 1D simulation of an RF plasma showing number densities, electron temperatures and potential field. A voltage (100 V, sin wave 13.56 MHz) is applied over two plates separated by distance 2 cm.

ulation is shown in Fig. 3. Solutions are comparable against the traditional HLL results and a conventional TVD upwind scheme due to the large Peclet numbers involved.

5. Conclusion

A recently developed finite volume method (HLLG) has been applied to the solution of the Euler Equations and charged particle (plasma) transport equations. The HLLG scheme is based on the consistent inclusion of gradients during the flux derivation. The HLLG scheme demonstrates improved stability with fewer spurious oscillations. Results for the simulation of the 2D Euler Equations and a 1D Argon RF discharge are presented.

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