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Glass transition and the replica symmetry breaking in vortex matter: MC study

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ABSTRACT

We investigate effects of disorder and thermal fluctuations on the Abrikosov state of type II superconductors applying the Monte Carlo method to Ginzburg–Landau theory to confirm earlier replica calculation. The vortex phase diagram has two transition lines, the melting line and the vortex glass transition lines in both crystalline and homogeneous states. The glass line is always a continuous transition, while the melting line is the first order.

Combination of disorder, interactions and thermal fluctuation in condensed matter systems often results in a complicated phase diagram containing a variety of glassy states which are notoriously difficult to describe theoretically. Systems of Abrikosov vortices created by magnetic field in type II superconductors offer a unique testing ground for experimental verification of theoretical methods attempting to describe these complex states. In this system all three ingredients are naturally present. Interaction between vortices is quite strong and at zero temperature and without disorder creates highly correlated systems like Abrikosov lattice and impurities are present naturally. In highly anisotropic (quasi 2D) high T_c superconductors like *BSCCO* (*BSCCO*) the Ginzburg number *Gi* characterizing the strength of thermal fluctuations is not small and thereby thermal fluctuations are strong. They compete with interactions leading to the lattice melting into a homogeneous vortex liquid state and also effectively reducing disorder leading to thermal depinning of vortices and demise of the glassy state in large portions of the H–T phase diagram. The *BSCCO* phase diagram obtained recently using the magnetization method revealed four distinct phases: Bragg glass (BG), vortex glass (VG), liquid (L) and the Abrikosov lattice (A) [1]. The last phase which exists at elevated temperatures, weaker disorder and relatively strong interactions is interesting, since theoretically there is no clear agreement even on its existence [2,3,7].

Since in most relevant experiments on strongly type II ($\kappa = \lambda/\xi \gg 1$ λ-penetration depth, ξ-coherence length) high T_c superconductors fields are much larger than the lower critical field $H_{c1}(T)$, magnetic fields of the vortices overlap and induction B becomes essentially homogeneous. Instead of flux lines one should return to a less phenomenological Ginzburg–Landau model in constant field.

Numerical simulation of the XY model generally have not detected the Abrikosov lattice phase, [4], although very recent numerical dynamic simulation is consistent with the four phase diagram with the Abrikosov lattice (A) phase called "floating solid" phase. However it is interpreted as a finite size effect [5]. Disordered GL model was simulated only in 2D and within the lowest Landau level (LLL) approximation discussed below [6], exhibit the first order homogeneous–crystalline transition, but no clear evidence of the glassy behavior was demonstrated. Analytical methods include the replica [2,3] and dynamic approach [8] and are summarized below.

In this note we study the disordered 2D Ginzburg–Landau model within the lowest Landau level (LLL) approximation numerically using the Monte Carlo (MC) simulation. Our starting point is the Gibbs energy:

$$G = L_z \int d^2 r \, \frac{\hbar^2}{2m^*} |\mathbf{D}\psi|^2 + \alpha T_c (1-t) \times \left[1 + W(r)\right] |\psi|^2 + \frac{\beta}{2} |\psi|^4, \tag{1}$$

where covariant derivative is defined by $\mathbf{D} \equiv \nabla - i \frac{e^*}{\hbar c \Phi_0} \mathbf{A}$, with $\mathbf{A}$ being the vector potential in Landau gauge $\mathbf{A} = (By, 0)$, $t \equiv T/T_c$ and L_z is the thickness. Mesoscopic thermal fluctuations are accounted for via statistical sum $Z = \int_\psi \exp\{-G[\psi^*, \psi]/T\}$. Point-like ($\delta T_c$) quenched disorder on the mesoscopic scale is described by the random potential with variance $\overline{W(r)W(r')} = R' \xi^2 \delta(r - r')$. In wide range of fields and temperatures, see Ref. [9] for details, the model can be simplified by retaining just the lowest Landau level. Unit of magnetic field will be $H_{c2}(T=0)$ so that $b = B/H_{c2}$

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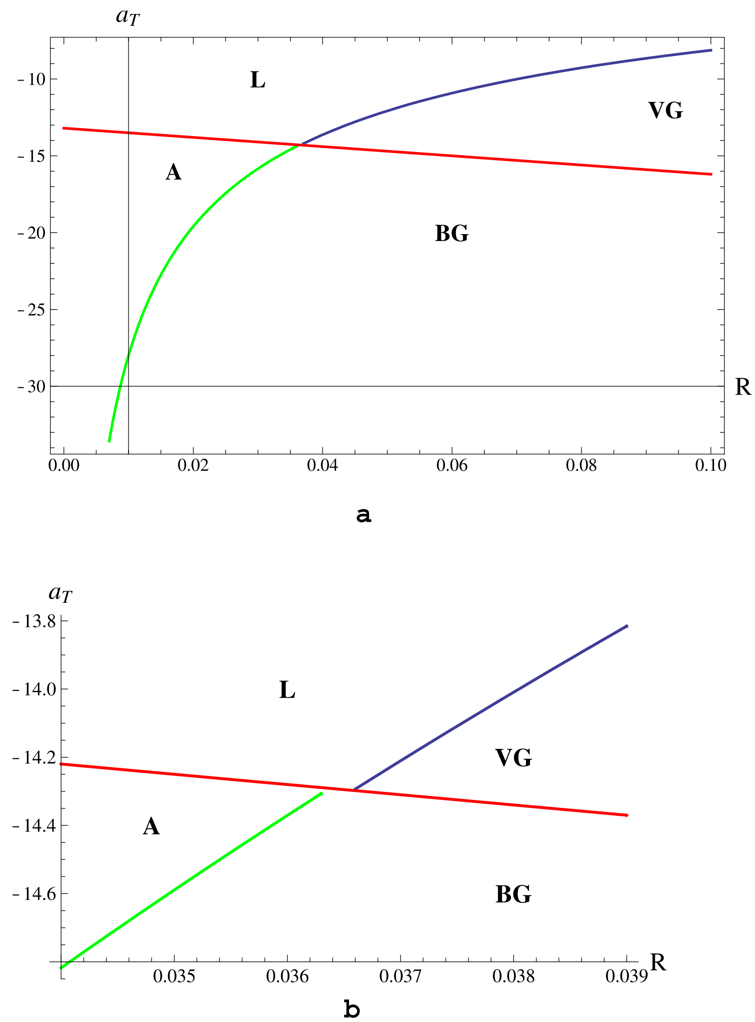

Fig. 1. Generic phase diagram of the vortex matter. (a) The order–disorder line (red) separates the crystalline phases (A and BG) from the homogeneous phases (L and VG). The glass transition line is the blue line in homogeneous phase and the light green line in the crystalline phase. (b) is an enlarged region near the crossing point. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

and unit of length—magnetic length $l_H = \xi/b^{1/2}$. We will use the quasimomentum basis of Ref. [9], that has an advantage for the MC simulation. Our sample has he following dimensions: $L\mathbf{d}_1, L\mathbf{d}_2$ (area is $2\pi L^2$ containing $N = L^2$ vortices), where Abrikosov lattice vectors are $\mathbf{d}_1 = (a, 0), \mathbf{d}_2 = a(\frac{1}{2}, \frac{\sqrt{3}}{2})$ with $a = \sqrt{4\pi/\sqrt{3}}$. Periodic boundary conditions make quasimomentum discrete taking values $\mathbf{k} = \frac{n_1}{L}\widetilde{\mathbf{d}}_1 + \frac{n_2}{L}\widetilde{\mathbf{d}}_2$, $n_i = 0, \ldots, L-1$.

The Boltzmann weight of the reduced (LLL) model is

$$g = \frac{1}{4\pi}\int_{x,y} a_T\left|\varphi(x,y)\right|^2 + w(r)\left|\varphi(x,y)\right|^2 + \frac{1}{2}\left|\varphi(x,y)\right|^4,$$

$$\left\langle w(x,y)w\left(x',y'\right)\right\rangle = 4\pi R\delta\left(x-x'\right)\delta\left(y-y'\right), \tag{2}$$

where the 2D LLL reduced temperature $a_T \equiv -\frac{H_{c2}}{\kappa}\sqrt{\frac{\Phi_0 L_z}{4\pi HT}}(1 - t - h)$, and $R = \frac{(1-t)^2}{4\omega t}R'$, $w(x,y) = -\sqrt{\frac{\pi}{\omega bt}}(1-t)W(r)$ where $\omega = \frac{m^*\beta}{2\hbar^2\alpha L_z}$. Our simulation has no spatial grid avoiding the problem of the artificial pinning by the grid [4,5]. We use Metropolis algorithm to simulate up to 100 different disorder configurations for sizes up to $N = 16 \times 16$. Thermalization was achieved by 4×10^5–4×10^6 sweeps and results were obtained from runs up to 10^7 sweeps. Clean system was simulated with results similar to those obtained in other simulations and is consistent with theory presented [3] and plotted in [1]. The generic phase diagram is given in Fig. 1. Homogeneous phases (L and VG) appear at larger scaled temperature a_T than the crystalline ones (A, BG), while pinned (glassy) phases (VG, BG) appear at larger disorder R than the unpinned ones (L, A). In particular Fig. 1(b) shows that there is a tiny split in the glass line into two tricritical points, see below for explanation.

In Fig. 2 points in the (a_T, R) parameter space in which the MC simulations were performed. We fix $R = 0.001$, a_T is varied from $-15, -16, 17, 18, 19, 20$. Structure functions are presented in Fig. 2. One clearly observes that AL with clear Bragg peaks becomes a Bragg lattice with diffuse peaks and become homogeneous glassy state at large disorder.

The glass line in the homogeneous phase was calculated using the replica method in [3] and using the dynamic approach in [8]:

$$a_T^g = 4\frac{R-1}{\sqrt{2R}}. \tag{3}$$

The glass transition in the crystalline phase follows a more complicated formula;

$$a_T^g = -e^{-1} - Re/2 - 4(2R)^{-1/2} + 3(2R)^{1/2}, \tag{4}$$

where e is a solution of cubic equation,

$$-R^2e^3 + Re^3 - (2R)^{1/2}e^2 + (2R)^{3/2}e^2 - 2e + 4(2R)^{-1/2} - 4(2R)^{1/2} = 0. \tag{5}$$

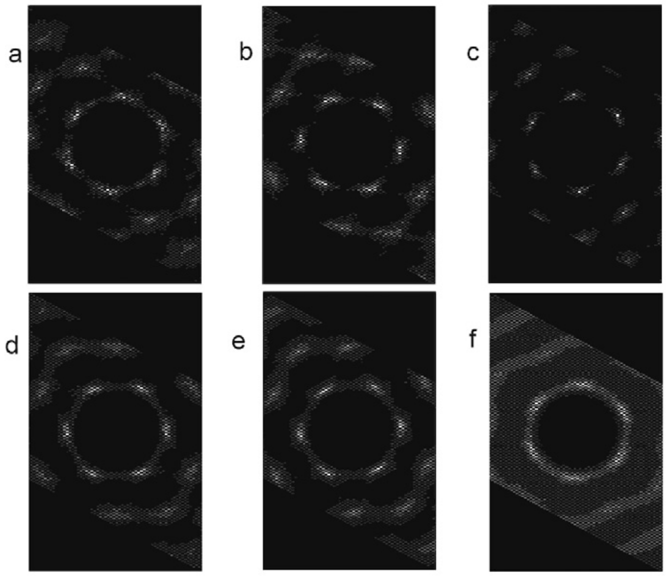

Fig. 2. Structure functions with disorder at different temperature. (a)–(c) represent the structure functions at $a_T = 15, 16, 17$ respectively and they have sharp Bragg peaks and they belong to Abrikosov states. (d), (e) have smeared Bragg peak and they belong to Bragg glass. (f) restores rotational symmetry and it belongs to vortex glass.

At small R, $e_O = (2R)^{-1/2} + \cdots$ and the correction to the line is of order $R^{3/2}$ [1], $a_T^g = 4\frac{R-1}{\sqrt{2R}} + O(R^{3/2})$. There are two tricritical points separated only slightly (since at that point R is small), so that the crystalline glass line is at higher temperatures compared to the homogeneous one.

To summarize, the MC simulation confirms the replica results for the four phase picture of vortex matter phase diagram in 2D far from the $H_{c1}(T)$ line. The glassy phases in both the homogeneous and the crystalline segments are established, although it is difficult at this stage to confirm details of the phase boundaries.

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