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# Aircraft replacement scheduling: A dynamic programming approach

Chaug-Ing Hsu [a,*], Hui-Chieh Li [b], Su-Miao Liu [a], Ching-Cheng Chao [c]

[a] *Department of Transportation Technology and Management, National Chiao Tung University, Hsinchu 300, Taiwan, ROC*
[b] *Marketing and Logistics Management, Ta Hwa Institute of Technology, Qionglin, Hsinchu County 307, Taiwan, ROC*
[c] *Department of Shipping and Transportation Management, National Kaohsiung Marine University, Kaohsiung 81143, Taiwan, ROC*

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ABSTRACT

This study developed a stochastic dynamic programming model to optimize airline decisions regarding purchasing, leasing, or disposing of aircraft over time. Grey topological models with Markov-chain were employed to forecast passenger traffic and capture the randomness of the demand. The results show that severe demand fluctuations would drive the airline to lease rather than to purchase its aircrafts. This would allow greater flexibility in fleet management and allows for matching short-term variations in the demand. The results of this study provide a useful reference for airlines in their replacement decision-making procedure by taking into consideration the fluctuations in the market demand and the status of the aircraft.



## 1. Introduction

The ability to match fleet capacity to passenger demand is one of the crucial factors deciding the profitability of an airline. The extent to which economic cycles influence air transportation demand is quite apparent. An economic recession usually accompanies reduced air demand, resulting in insufficient revenue and surplus capacity that further burdens the airlines with fleet idle costs, thereby lowering profits. On the other hand airlines also suffer a great profit loss under a quick economic recovery, when the fleet capacity may not be able to expand in time to satisfy the high demands, due to the time lag between ordering, receiving and operating of extra aircraft. Although aircraft replacement decisions can be made in advance in order to match future demand, the fluctuating and cyclical nature of passenger demand complicates the fleet capacity management problem.

Decisions about fleet capacity management are classified under airline strategic planning, which involves decisions such as when to purchase, lease or dispose of aircraft. Fleet expansions and reductions are achieved through aircraft purchase, lease or by disposing of the surplus airplanes. Leasing an airplane gives the airlines flexibility in capacity management. However, airlines must pay a risk premium to leasing companies for bearing the risks (Oum et al., 2000). Also, the lease cost for an airplane may be very high when there is a high demand for them in the market. The scrapping and replacing of an existing aircraft is generally motivated by the physical deterioration of the aircraft or the availability of newer, more efficient ones. However, the decision to replace can be scheduled in advance to coincide when the airline market is forecasted to going into downward trend, thereby reducing the operating and maintenance costs. How to schedule capacity expansion or reduction decisions in advance is an essential and critically important task for the airlines, since the aircraft fleet must not only serve current but also future demands. Although any particular replacement decision is necessarily influenced by the current fleet composition as well as any possible future demand, it still has a long-term impact on the airline fleet. Under these

* Corresponding author. Address: Department of Transportation Technology and Management, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 300, Taiwan, ROC. Tel.: +886 3 573 1672; fax: +886 3 572 0844.
*E-mail address:* cihsu@mail.nctu.edu.tw (C.-I. Hsu).

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circumstances, accurate demand forecasts are required to enable the airlines to properly schedule their aircraft replacement decisions in response to the fluctuating and cyclical demands.

Past studies have investigated the issues in the context of fleet capacity problems, such as decisions on aircraft type, flight frequency (e.g. Kanafani and Ghobrial, 1982; Teodorovic and Krcmar-Nozic, 1989) and optimal combinations of owned and leased capacity (Oum et al., 2000). Researchers have studied fleet management problems at operational and tactical levels in addition to the strategic level (e.g. Powell and Carvalho, 1997; Jin and Kite-Powell, 2000). There is scant literature available on replacement cost in relation to fleet capacity management over different time periods, or for revenue loss associated with dynamic and cyclical demand.

In this study, the cost of operating an aircraft is dependent upon its status, as defined by type of aircraft, age and total mileage traveled. The fleet is composed of different number and status of purchased and leased aircraft. On the demand side, this study employs the Grey topological forecasting method combined with the Markov-chain model to forecast passenger traffic and capture the random and cyclic demand. The decision periods are identified according to the pattern of the passenger demand cycles over the length of the study period. For each decision period, the airline makes decisions not only on whether and which aircraft to be replaced with a purchased or leased one, but also on whether or not to purchase or lease an aircraft as an entirely new addition to the fleet.

This study aims to determine an optimal replacement schedule for an airline by considering the randomness in airline operations and the cyclical demand through the use of stochastic dynamic programming. This study will also determine the optimal candidate aircraft to be recruited or disposed of. The stochastic dynamic programming method is solved with backward dynamic programming in which the impact of replacement decisions made at a specific period under uncertain passenger demand on airline operation can be fully considered. This study first formulates airline cost function of a decision period assuming independent decision-making results between periods. These costs include operating cost, replacement cost and penalty cost. The operating cost is the cost related to the operation of the existing fleet. The replacement costs arise from the replacement decisions made at a specific period. In addition, a penalty cost is introduced to reflect losses in revenue associated with the difference between the forecasted and realized passenger demand. The expected cost function of the period is further formulated by taking into consideration the cost dependent relationship between decisions made in neighboring periods and the probabilities of different variations in the forecasted and realized passenger demand. Then, the stochastic dynamic programming model for the replacement schedule can be formulated to determine the optimal replacement schedule by minimizing the total expected cost of each period over the study period.

The remainder of this paper is organized as follows: Section 2 reviews the literature on fleet capacity and equipment replacement problems. Section 3 formulates the cost functions based on a single period operation. Section 4 provides the stochastic dynamic programming model for determining the optimal schedule of the replacement decisions. A numerical example is provided in Section 5, to illustrate the application of the models and the effects of changes in key parameters on the optimal solutions. In section 6, we make our concluding remarks.

## 2. Literature review

The fleet capacity of an airline is the total number of different types of aircraft purchased, leased and scrapped over a period of time. Relevant studies have focused mainly on choosing the right type of aircraft, route, and flight frequency using deterministic mathematical programming methods (e.g. Kanafani and Ghobrial, 1982; Teodorovic and Krcmar-Nozic, 1989; Yan et al., 2006). Wei and Hansen (2007) considered the factors of competition in the decisions on both aircraft size and service frequency. They examined the impact of these decisions on both the cost and the demand of air transportation.

Equipment replacement problems in industries with high capital assets have been widely discussed in industrial engineering and operations research literature (e.g. Hartman, 2004, 1999; Rajagopalan, 1998; Jones et al., 1991). Hartman (2001) examined the effect of probabilistic asset utilization on the replacement decision making process, using dynamic programming. Powell and Carvalho (1997) dealt with the multi-commodity fleet problem and formulated the problem as a dynamic control problem. Jin and Kite-Powell (2000) explored the replacement problem for a fleet of ships for a profit-maximizing operator, assuming a homogenous fleet and uniform demand. Wu et al. (2005) addressed a rental fleet-sizing problem in the truck-rental industry. They combined both operational and tactical decision levels, subject to uncertain customer travel time and non-stationary customer demand. Oum et al. (2000) developed a model for the airlines to determine the optimal mix of leased and owned capacity, taking into consideration that the demand for air transportation is uncertain and cyclical. The empirical results suggested that the optimal demand for the airlines would range between 40% and 60% of their total fleet. The financial status of the airline and the passenger demand are critical factors when it comes to leased and owned capacity decisions. Although the uncertainty in demand has been included and investigated in the literature, research regarding the schedules of the above decisions and their impacts on airline operation and the total cost over a time horizon is scant. Furthermore, the cost dependent relationships between subsequent periods due to replacement decisions made in previous periods have not been discussed yet.

When it comes to methods to forecast airline passenger demand, the multi-regression model and the time-series model are the most widely employed. Horonjeff and McKelvey (1994) generalized past literature and classified airline passenger traffic forecasting models into four categories: judgment prediction, trend projection and speculation, market analysis and econometric modeling method. However, the number of available traffic observations has usually not been large enough due to a short accumulation time, particularly city-pair data (Horonjeff and McKelvey, 1994). Collecting a large number of data to develop a conventional statistical forecasting model is difficult.

The Grey topological forecasting model was developed based on the Grey system theory (Deng, 1985, 1986), and is also called the Grey pattern prediction or system trend prediction model. The Grey theory deals with systems with poor information. Other related models have also been used in many applications (e.g. Deng and Guo, 1996; Deng, 1999; Hsu and Wen, 1998). Hsu and Wen (1998) applied the Grey theory to forecast airline passenger traffic. They constructed an improved GM(1, 1) time-series model and showed that the forecasted result from the Grey model is more accurate than those predicted by the regression model or the ARIMA model. However, there is no literature available that applies the Grey topological model for forecasting airline passenger traffic influenced by the economic cycle. The advantage of employing the Grey topological model lies not only in that it requires only little historic data to formulate a prediction model, it is also constructed to forecast system data with pattern development, making it suitable for pattern or economic cycle forecasting. This makes the Grey topological forecasting model suitable for predicting airline passenger traffic, since international city-pair air passenger data is usually not sufficient, and airline market traffic shows a pattern of being influenced by the economic cycle. Passenger demand forecasts are inherently uncertain because of assumptions about random future demand. Any forecast result involves a potential variance or bias.

In sum, few have combined the Grey topological forecasting model with the Markov-chain to investigate the demand fluctuations and the stochastic demand realizations. This study integrates Grey topological forecasting model, Markov-chain model and dynamic programming method to investigate the replacement schedule for an airline by considering the randomness in airline operations and the cyclical demand.

## 3. Cost function

Consider an airline that operates various routes, with $R$ and $r$ representing the set of these routes and a particular route, respectively, $r \in R$. Let $T$ be the study period with $n$ number of decision periods $t$, $t = 0, 1, 2, \ldots, n$. The duration of the decision periods may vary from each other and from different routes due to different economic cycles. Let's suppose that there are three possible future demand trends forecasted by the Grey topological forecasting model, upward, equal and downward, respectively. Let $w$ represent three possible fluctuations for the demand, with $w = 1$, 2 and 3. We let $w = 1$ represent a rising demand; $w = 2$ a similar demand; and $w = 3$ a declining demand of the period, as compared with that of the previous period. Let $p_w^t$ represent the probability of the demand fluctuation labeled as $w$ at period $t$. It must be noted that $p_w^t \geqslant 0$ and $\sum_{w=1}^{3} p_w^t = 1$. In addition, $F_r^t$ represents the forecasted passenger demand on route $r$ at period $t$. The values of $p_w^t$ and $F_r^t$ are then determined by the Grey topological forecasting method combined with the Markov chain and they are summarized in Appendix A.

Let $N_{qym}^{Bt}$ and $N_{qym}^{Lt}$ be the number of aircraft associated with the replacement decisions made at period $t$, where superscripts $B$ and $L$ represent the aircraft being purchased and leased, while the subscripts $q$, $y$ and $m$ describe the status of an aircraft as its type, remaining available years and mileage traveled, respectively. The remaining available years of an aircraft, $y$, is determined by its number of years of maximum usage, $Y$, and the age of the aircraft, $y'$, such that $y = Y - y'$. Note that $N_{qym}^{Bt}$ and $N_{qym}^{Lt}$ are both integers. The decision of whether the fleet being expanded or reduced in terms of aircraft being recruited or disposed of, is judged by the value of $N_{qym}^{Bt}$ and $N_{qym}^{Lt}$. When the variables $N_{qym}^{Bt}$ and $N_{qym}^{Lt}$ are positive, the airlines will decide to expand their fleet capacity through purchasing and/or leasing, and the numbers of recruited aircraft with status $(q, y, m)$ are $N_{qym}^{Bt}$ and $N_{qym}^{Lt}$, respectively. Otherwise, the optimal decision will result in a capacity reduction with negative values for $N_{qym}^{Bt}$ and $N_{qym}^{Lt}$. Since the total number of aircraft scrapped cannot be larger than the existing scale, the following inequalities hold:

$$\begin{cases} E_{qym}^{Bt} \geqslant |N_{qym}^{Bt}| & \text{if} \quad N_{qym}^{Bt} < 0 \\ E_{gym}^{Lt} \geqslant |N_{gym}^{Lt}| & \phantom{\text{if}} \quad N_{gym}^{Lt} < 0 \end{cases} \tag{1}$$

where $E_{qym}^{Bt}$ and $E_{qym}^{Lt}$ represent the total number of purchased and leased aircraft with status $(q, y, m)$ at period $t$, respectively, in the airline fleet. Let $S_t$ be the set of all aircraft operated by the airline during period $t$, $S_t \equiv \{E_{qym}^{Bt}, E_{qym}^{Lt}, \forall q, y, m\}, E_{qym}^{Bt}, E_{qym}^{Lt} \in I^+ \cup \{0\}$. Let $d_{t-1}$ denote the set of aircraft recruited or disposed of at period $(t-1)$ and these aircraft will be operated at period $t$, $d_{t-1} \equiv \{N_{qym}^{B(t-1)}, N_{qym}^{L(t-1)}, \forall q, y, m\}, N_{qym}^{Bt}, N_{qym}^{Lt} \in I$ and $t = 1, 2, \ldots, n$. Then, the fleet operated at period $t$, $S_t$, can be specifically formulated as follows:

$$S_t = S_{t-1} + d_{t-1} \quad t = 1, 2, \ldots, n \tag{2a}$$

$$E_{qym}^{Bt} = E_{qym}^{B(t-1)} + N_{qym}^{B(t-1)} \quad t = 1, 2, \ldots, n \tag{2b}$$

$$E_{qym}^{Lt} = E_{qym}^{L(t-1)} + N_{qym}^{L(t-1)} \quad t = 1, 2, \ldots, n \tag{2c}$$

which show that the fleet capacity and composition of period $t$ are the result of the replacement decisions made at period $(t-1)$.

The total fleet capacity of an airline can change according to the different numbers of seats offered by different aircraft types. Let $Q_q$ represent the capacity of aircraft type $q$ and let $K^{tr}_{qym}$ be the total flight frequencies on route $r$ offered by the aircraft with status $(q, y, m)$ during period $t$. Then the total capacity, i.e. the number of seats on route $r$ during period $t$, $A^t_r$ can be formulated as

$$A^t_r = \sum_{\forall q}\sum_{\forall y}\sum_{\forall m}\delta^{tr}_{qym}Q_q(E^{Btr}_{qym} + E^{Ltr}_{qym})K^{tr}_{qym} \quad \forall r \tag{3}$$

where $\delta^{tr}_{qym}$ is an indicator variable; and $\delta^{tr}_{qym} = 1$ for an aircraft with status $(q, y, m)$ during period $t$ serving route $r$; otherwise, $\delta^{tr}_{qym} = 0$. Moreover, the inequality $\sum_{\forall q}\sum_{\forall y}\sum_{\forall m}\delta^{tr}_{qym} \geqslant 1 \quad \forall r$ must hold to ensure every route is being served by at least one aircraft. In practice, the airline may set an ideal load factor on each route, and then the minimized fleet capacity can be obtained. The realized fleet capacities on the routes, depending on the average load factor, must be equal to or larger than the forecasted demand of decision period $t$, which yields

$$l^t_r A^t_r \geqslant F^t_r \quad \forall r \tag{4}$$

where $l^t_r$ denotes the average load factor on route $r$ during period $t$. From Eq. (3), Eq. (4) can be further revised as $\sum_{\forall q}\sum_{\forall y}\sum_{\forall m}\delta^r_q Q_q(E^{Bt}_{qym} + E^{Lt}_{qym})K^{tr}_{qym} \geqslant \frac{F^t_r}{l^t_r}$. The fact that not all aircraft can be assigned to a flight due to factors such as maintenance and turnover accounts, should be considered in the aircraft utilization model. Let $B^r_q$ denote the block time of type $q$ aircraft on route $r$, including the time spent in various aircraft trip modes, and let $u^t_{qym}$ represent the maximum possible utilization of the aircraft with status $(q, y, m)$ during period $t$, respectively. A maximum possible utilization also implies a maximum possible daily use of the aircraft for a certain period of time (Kane, 1990; Teodorovic, 1983). For all aircraft with different status, the total aircraft utilization must be less than or equal to the maximum possible utilization. This study uses the relation of Teodorovic et al. (1994), such as $\sum_{\forall r}\sum_{\forall q}\sum_{\forall y}\sum_{\forall m}B^r_q K^{tr}_{qym} \leqslant \sum_{\forall q}\sum_{\forall y}\sum_{\forall m}(E^{Bt}_{qym} + E^{Lt}_{qym})u^t_{qym}$. For a specific aircraft type with status $(q, y, m)$, the inequality $\sum_{\forall r}B^r_q K^{tr}_{qym} \leqslant u^t_{qym}$ must hold. Any surplus capacity from an aircraft not reaching the maximum possible operation time can be relocated to routes with an aircraft of insufficient capacity. Therefore, an aircraft might be shared on two routes.

The direct operating costs are all those expenses associated with operating a fleet of aircraft, including depreciation costs, maintenance costs and flying costs. The depreciation costs reflect the reduction in the value of the existing fleet and can be calculated based on the purchase or lease price of the aircraft. In some ways, the depreciation costs depend on the market demand when the aircraft is originally purchased or leased. For instance, when most airlines forecast an upward trend in future demand, the original purchase or lease cost will be high, resulting in a high depreciation cost. However, since the total lease expense decreases with the total duration of the lease period, the foregoing can be neglected when the leased period is contracted for a long time, thereby yielding a constant average lease cost. Let $P_{qym}$ represent the average purchase cost for an aircraft with status $(q, y, m)$ and $R^{td}_{qym}$ denote the average lease cost for an aircraft with status $(q, y, m)$ with a total leased period $d$ at period $t$, respectively. Then, the depreciation cost related to the existing fleet of period $t$ can be formulated as

$$\sum_{\forall q}\sum_{\forall y}\sum_{\forall m}E^{Bt}_{qym}P_{qym}X^t_g + \sum_{\forall q}\sum_{\forall y}\sum_{\forall m}E^{Lt}_{qym}R^{td}_{qym} \quad \forall t \tag{5}$$

where $X^t_g$ denotes the average remaining resale ratio of the original purchase price with an average yearly interest rate $g$ of period $t$.

Maintenance cost can be further divided into fixed maintenance cost and variable maintenance cost. Fixed maintenance costs includes maintenance overhead including the maintenance of the building and equipment as well as land rental, none of which vary with the number of aircraft. On the other hand, variable maintenance costs change with the status of the aircraft, and the number of aircraft. Generally speaking, the running and preventive maintenance costs increase with the age of the aircraft and the mileage traveled. In addition, there are economies of scale that allow airlines with many aircraft of a similar type in their fleet to operate more efficiently than those with several different types. The maintenance cost of period $t$ can then be expressed as

$$M^t + \sum_{\forall q}\sum_{\forall y}\sum_{\forall m}V^t_{qym}(E^{Bt}_{qym} + E^{Lt}_{qym}) \tag{6}$$

where $M^t$ represents the fixed maintenance cost (overhead) of period $t$ and $V^t_{qym}$ denotes the variable maintenance cost of the aircraft with status $(q, y, m)$ during period $t$. The flying cost related to the total flight frequencies on all routes is

$$\sum_{\forall r}\sum_{\forall q}\sum_{\forall y}\sum_{\forall m}b^t_{qr}\delta^r_q K^{tr}_{qym} \tag{7}$$

where $b^t_{qr}$ represents the average flying cost of an aircraft of type $q$ on route $r$ during period $t$. The total direct operating cost of the airline for operating the existing fleet during period $t$, $C^t_D$, can be formulated as

$$C^t_D = \sum_{\forall q}\sum_{\forall y}\sum_{\forall m}(E^{Bt}_{qym}P_{qym}X^t_g + E^{Lt}_{qym}R^{td}_{qym} + V^t_{qym}(E^{Bt}_{qym} + E^{Lt}_{qym})) + \sum_{\forall r}\sum_{\forall q}\sum_{\forall y}\sum_{\forall m}b^t_{qr}\delta^r_q K^{tr}_{qym} + M^t \tag{8}$$

The total indirect operating cost as a result of serving passengers at period $t$, $C_I^t$ can be expressed as follows:

$$C_I^t = \sum_{\forall r} F_r^t H^r \tag{9}$$

where $H^r$ denotes the average indirect cost per passenger on route $r$. Summing up the total direct and indirect operating costs in Eqs. (8) and (9) yields the total operating cost of the airline during period $t$, $C^t$.

When disposing of a purchased aircraft, the airline will receive the salvage value of the aircraft, which is its remaining value after depreciation. The salvage value is inversely related to the age and mileage traveled. When terminating the contract of a leased aircraft, the airline has to pay a penalty for returning the aircraft earlier than stipulated in the lease contract. The longer the remaining lease period, the higher the penalty will be. Moreover, both salvage value and penalty cost as a result of fleet reduction are dependent upon the demand for aircraft in the market. If most airlines forecast a boom in demand in the near future, the tendency towards expanding fleet capacity will be high, resulting in a higher price for aircraft, i.e. lower salvage cost borne by the airline. Conversely, it costs the airline a lot of time and effort to dispose of their excess capacity when the demand is low, resulting in an increased loss of salvage value. Let $D_{qym}^t$ and $Z_{qym}^{te}$ represent the salvage value and penalty cost of an aircraft with status $(q, y, m)$ and with a remaining lease period $e$ at period $t$, respectively. Let $P_{qym}^t$ and $Y_{qym}^t$ denote the original purchase price and total depreciation cost of an aircraft with status $(q, y, m)$ at period $t$, respectively. The airline suffers a loss if it disposes of an aircraft when $D_{qym}^t < P_{qym}^t - Y_{qym}^t$, while on the other hand $D_{qym}^t > P_{qym}^t - Y_{qym}$ would imply a revenue gain. The total replacement cost for disposing of an aircraft during period $t$ can be expressed as $\sum_{\forall q}\sum_{\forall y}\sum_{\forall m}|N_{qym}^{Bt}|(P_{qym}^t - Y_{qym}^t - D_{qym}^t)$, where $|N_{qym}^{Bt}|$ is the number of purchased aircraft to be disposed of. On the other hand, the penalty cost for disposing of an aircraft during period $t$ can be expressed as $\sum_{\forall q}\sum_{\forall y}\sum_{\forall m}|N_{qym}^{Lt}|Z_{qym,e}^t$, where $|N_{qym}^{Lt}|$ is the number of aircraft whose lease will be terminated. Taking into consideration both salvage and penalty costs, the replacement cost during period $t$ with demand fluctuations labeled $w$ can be expressed as

$$U^t = \sum_{\forall q}\sum_{\forall y}\sum_{\forall m}\alpha_{qym}^{Bt}|N_{qym}^{Bt}|(P_{qym}^t - Y_{qym}^t - D_{qym}^t) + \sum_{\forall q}\sum_{\forall y}\sum_{\forall m}\beta_{qym}^{Lt}|N_{qym}^{Lt}|Z_{qym}^{te} \tag{10}$$

Indicators $\alpha_{qym}^{Bt}$ and $\beta_{qym}^{Lt}$ are both binary variables, and their relationship with the replacement decisions are as follows:

$$\begin{cases} \alpha_{qym}^{Bt} = 1 \\ \beta_{gym}^{Lt} = 1 \end{cases} \text{if} \begin{matrix} N_{qym}^{Bt} < 0 \\ N_{gym}^{Lt} < 0 \end{matrix} \text{ else } \begin{matrix} \alpha_{qym}^{Bt} = 0 \\ \beta_{gym}^{Lt} = 0 \end{matrix} \tag{11}$$

In the study, the decisions on whether or not, and which aircraft should be disposed of depend mainly on the sum of operating cost, replacement cost and penalty cost. However, an airline that has safety as its highest priority should immediately dispose of or terminate the lease of any aircraft once its age and mileage traveled has reached the safety threshold. The utilization of an aircraft is only influential if the two factors of remaining years and mileage traveled, are within the safety parameters. The relationship between the optimal candidate aircraft to be disposed of and its remaining years as well as its mileage traveled can be expressed as

$$W_{qym}^t = \begin{cases} 0 \\ 1 \end{cases} \text{if} \begin{matrix} \min\left\{\frac{A_q}{y}, \frac{G_q}{m}\right\} \leqslant 1 \\ \min\left\{\frac{A_q}{y}, \frac{G_q}{m}\right\} > 1 \end{matrix} \tag{12}$$

where $W_{qym}^t$ is an indicator variable; and where $W_{qym}^t = 0$ refers to the aircraft with status $(q, y, m)$ being disposed of at period $t$, otherwise, $W_{qym}^t = 1$. And, $A_q$ and $G_q$ represent, respectively, the maximum years of expected service and the maximum allowable mileage to be traveled by a type $q$ aircraft.

The actual demand may be underestimated, overestimated or be correct, regardless of the demand fluctuation labeled as $w$, since label $w$ represents the cyclical demand fluctuation. Let $f_r^t$ be the actual passenger demand on route $r$ during period $t$. If the actual demand is less than the forecasted result, i.e. $f_r^t - F_r^t < 0$, then the airline bears an increased total indirect operating cost for serving their passengers due to the unsold seats. The punishment associated with an overestimation is included in Eq. (9). On the contrary, there will be unsatisfied passengers for $f_r^t - F_r^t \geqslant 0$ due to insufficient fleet capacity as determined in accordance with the forecasted demand. Let $I_r^t$ represent the average revenue loss associated with one unit of insufficient seats on route $r$ during period $t$, which can be estimated by the average fare on the route. The penalty cost function due to the inaccurate forecast on route $r$ at period $t$, $\ell^t$, can then be formulated as

$$\ell_r^t = (f_r^t - F_r^t)I_r^t \tag{13}$$

The total penalty cost of the airline during period $t$ is $L^t = \sum_{\forall r}\ell_r^t$. The total cost during period $t$, $Q^t$, given by the operating cost, the replacement cost and the penalty cost can be formulated as follows:

$$Q^t = C^t + U^t + L^t \tag{14}$$

Note that $Q^t$ is independent of the fleet operated in previous period and depends on the fleet being operated and the decisions made in period $t$.

## 4. Stochastic dynamic programming model

Section 3 formulates the cost function of the airline for a single period. From Eqs. (2a)–(2c), the fleet operated during period $t$ is the result of the replacement decisions made at period $(t-1)$ and in addition, the fleet capacity during period $t$ is determined based on the demand forecast at period $(t-1)$ towards period $t$. That is to say, the replacement decisions made and the demand forecast executed at period $(t-1)$ have a certain level of involvement with the operating cost, $C^t$, and the penalty cost, $L^t$, of period $t$. Similarly, the demand forecast result for period $(t+1)$ served as the reference of the replacement decisions made at period $t$, which resulted in the replacement cost, $U^t$ in Eq. (14). These cost dependent relationships between decisions made at neighboring periods are explained as "recursions", and are depicted graphically in Fig. 1 by taking into consideration the demand fluctuation. The circular node represents the set of aircraft operated during period $t$, $S_t$, while the square node is the set of aircraft recruited or disposed of at period $t$ when the demand of period $(t+1)$ is forecasted as label $w$, $d_t^w$, respectively. As shown in Fig. 1, the resulting fleet $S_t$ is the result of the decision made during period $(t-1)$ with respect to different demand fluctuations labeled $w$.

For a given period $t$, the airline makes the replacement decisions in accordance with the forecasted result for period $(t+1)$, including the three possible demand trends, the demand of period $(t+1)$ forecasted to be upward, equal and downward compared with the demand of period $t$. However, the realization of the demand might fall short of the forecasted result. In other words, the total cost of period $(t+1)$, given by the sum of operating, replacement and penalty costs, is directly affected by the decision made at period $t$ and the forecasted demand for period $(t+1)$.

As for dynamic programming, the stage and the state in this study refer to decision period $t$ and operating fleet $S^t$, respectively. Let $C^t(S^t, d^t)$ represent the total cost from period $t$ forward, where $d^t$ denotes the replacement decision. Given $S^t$ and $t$, let $d_*^t$ denote any value of $d^t$ that minimizes $C^t(S^t, d^t)$, and let $C_*^t(S^t)$ be the corresponding minimum value of $C^t(S^t, d^t)$. Then,

$$C_*^t(S^t) = \min_{d^t} C^t(S^t, d^t) = C^t(S^t, d_*^t) \quad (15)$$

In order to consider the stochastic feature of future demand even further, the minimum expected sum from period $t$ forward, $C^t(S^t, d^t)$, given that the fleet and replacement decision in period $t$ are $S^t$ and $d^t$, can be formulated as follows:

$$C^t(S^t, d^t) = \sum_{w=1}^{w=3} p_w^t [Q^t + C_*^{t+1}(S^{t+1})] \quad (16)$$

where $C_*^{t+1}(S^{t+1}) = \min_{d^{t+1}} C^{t+1}(S^{t+1}, d^{t+1})$ is the recursive relationship that identifies the optimal decision for period $(t+1)$, given that the optimal decision for period $(t+2)$ has been made.

The objective for the aircraft replacement schedule problem is to determine $\pi = [d^1, d^2, \ldots, d^t, \ldots, d^T]$ so as to

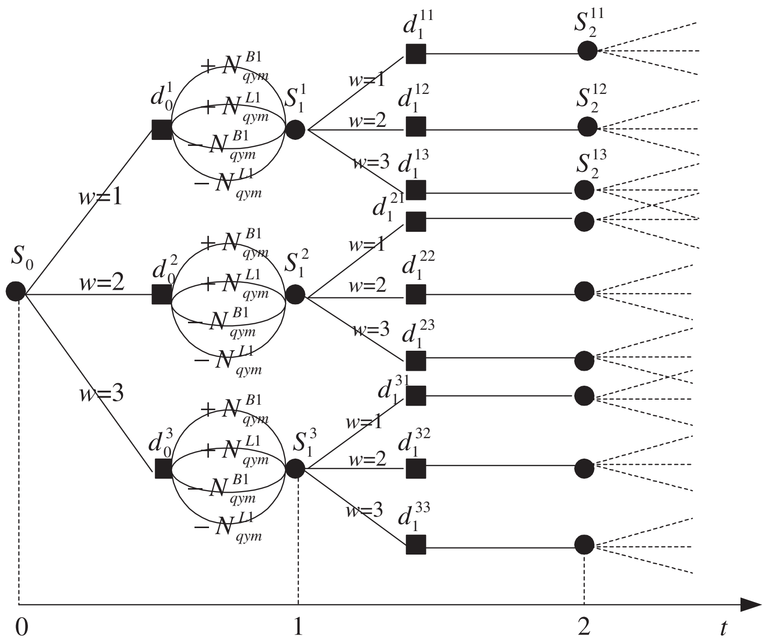


**Fig. 1.** A stochastic dynamic programming network.

$$\min_{\pi} \quad E\left(\sum_{t} Q^{t}\right) \tag{17a}$$

$$\text{s.t.} \quad \sum_{\forall q}\sum_{\forall y}\sum_{\forall m} \delta_q^r Q_q (E_{qym}^{Bt} + E_{qym}^{Lt}) K_{qym}^{tr} \geqslant \frac{F_r^t}{l_r^t} \tag{17b}$$

$$N_{qym}^{Bt} \text{ and } N_{qym}^{Lt} \quad \text{integers } \forall q, y, m \ \forall t \tag{17c}$$

The recursive relationship for the problem is $C_*^{t+1}(S^{t+1}) = \min_{d^{t+1}} C^{t+1}(S^{t+1}, d^{t+1})$. The optimal decision at period $t$ is found by solving by backwards induction starting at $t = n$ and using Eq. (16) at each step to find the optimal decision for the periods. The replacement decisions include when, how many, and how many different types of aircraft are to be recruited through lease or purchase, as well as when and which aircraft, leased and purchased, with various statuses are to be disposed of. Moreover, the duration of each period may be different for different city-pairs due to variations in the length and trend of the economic cycle. The fleet operating on the routes, depending on the average load factor, must be equal or larger than the forecasted demand of the period. Only two routes with identical duration and number of periods can share an aircraft.

## 5. Example

This study further presents a case study to demonstrate applications of the models, based on available data from EVA Airways (EVA). For the sake of simplification, nine cities in seven countries were selected from all the cities currently being served by EVA. The eight city-pairs (routes) are Taipei (TPE)–Los Angeles (LAX), –Seattle (SEA), –San Francisco (SFO), –Tokyo (TYO), –Hong Kong (HKG), –Singapore (SIN), –Bangkok (BKK) and –Sydney (SYD). There are 15 wide-body aircraft including 6 Boeing 747–400 combi, 4 Boeing 767–300, 4 Boeing 747–400 and 1 MD11 flying on these routes. Tables 1 and 2 list the basic data of the fleet and the aircraft in the fleet, respectively.

In the present study, the forecast results from the Grey topological forecasting model represent the demands on the routes carried by all airlines on the market. This study further estimates the demand carried by EVA based on their market share. According to Teodorovic and Krcmar-Nozic (1989), the market share of airline $i$ on route $r$, $MS_{ir}$ can be estimated by

$$MS_{ir} = \frac{K_{ir}^{\alpha}}{\sum_{\forall i} K_{ir}^{\alpha}} \tag{18}$$

**Table 1**
Basic data of the fleet. Source: http://www.evaair.com/html/b2c/english/

| Aircraft, $r$ | Number | Average capacities (numbers of seat) | Average age (year) | Number of purchased and leased aircrafts (purchased/leased) |
|---|---|---|---|---|
| B747–400 combi | 6 | 272 | 7.5 | 0/6 |
| B767–300 | 4 | 221 | 10.1 | 0/4 |
| B747–400 | 4 | 363 | 6 | 0/4 |
| MD11 | 1 | 271 | 7.3 | 1/0 |

**Table 2**
Basic data of aircrafts in the fleet. Source: http://www.evaair.com/html/b2c/english/

| Leased aircraft | | Average lease cost per month (US$) | Contracted lease period |
|---|---|---|---|
| B767–300 | | 600,000 | 1998/07–2004/03 |
| B767–300 | | 600,000 | 1998/08–2004/04 |
| B767–300 | | 550,000 | 1997/12–2002/12 |
| B767–300 | | 550,000 | 1997/12–2002/12 |
| B747–400 combi | | 1,300,000 | 2000/10–2008/04 |
| B747–400 combi | | 1,300,000 | 2000/10–2007/10 |
| B747–400 combi | | 1,100,000 | 1997/12–2002/12 |
| B747–400 combi | | 1,200,000 | 1997/12–2002/12 |
| B747–400 combi | | 1,130,000 | 1999/07–2007/01 |
| B747–400 combi | | 1,040,000 | 1999/08–2006/08 |
| B747–400 | | 1,125,000 | 1997/12–2002/12 |
| B747–400 | | 1,290,000 | 1997/12–2002/12 |
| B747–400 | | 1,400,000 | 1998/04–2005/04 |
| B747–400 | | 1,200,000 | 1998/05–2005/11 |
| Purchased aircraft | Purchase date | Total purchase cost (US$) | Salvage value at the end of 2000 (US$) |
| MD11 | 1994/08 | 3,345,313,186 | 2,220,935,549 |

where $K_{ir}$ represents the flight frequencies of airline $i$ on route $r$ and $\alpha$ is an empirically obtained constant which value is approximately 1.2 (Teodorovic and Krcmar-Nozic, 1989). Table 3 describes the supply parameters for the routes as related to aircraft type, frequencies, block time and fares. The parameter values related to the market shares and the load factors on the different routes are also shown in Table 3, where $MS_r$ represents the market share of EVA. For the sake of simplification, the impacts of the newly developed aircraft types such as Boeing 787 and Airbus A380 on the optimal replacement decisions are not discussed in the current case study.

The study period in this study totals eight years, from 2002 to 2009. Table 4 shows the optimal replacement decisions made in the first period, including the duration of the decision periods and the fleet compositions.

As shown in Table 4, the airline tends to simplify the fleet compositions to three types of aircraft, such that each of the routes is served by only one type of aircraft, although with different numbers of aircraft. Through this strategy, operating and maintenance costs are decreased due to the realization of economies of scale. Moreover, as shown in Table 4, during the second period the aircraft serving these 8 routes are all leased. The reason for this is that the severe demand fluctuation encourages airlines to choose lease arrangements for their aircraft. This allows them to manage their fleet size and composition, in as flexible a manner as possible to match the demand. It should be noted that there is a time lag between purchase/lease and the delivery of these aircraft. Hence, after having determined the fleet compositions for each period using our proposed model, airlines can then estimate the time lag using past experience, and include that in their plan for purchasing, leasing or disposing of aircraft thereby satisfying the demand for aircraft in different periods.

Although the replacement decisions made for each period are affected by the forecasted demand, the total expected cost of the airline is increased with each inaccurate forecasted result. Fig. 2 shows the cost of route TPE–BKK with different fluctuating demands for different periods, where the first and second number in the parentheses represent label $w$ and the probability, $p_w^t$, respectively. From left to right, the costs represent the total expected cost over the study period, and the expected cost of the first and second decision periods, respectively.

As shown in Fig. 2, there are three demand forecast results, but with different probabilities. For each decision period, there is a minimized cost when the forecasted demand is totally matched to the realized one, i.e. $w = 2$, and while there are

**Table 3**
Parameter values related to routes. Source: http://www.evaair.com/html/b2c/english/

| Route, $r$ | Aircraft type, $q$ | Weekly flight frequencies (one-direction) | Block time (hours) | Fare (US$/person-trip) | Market share, $MS_r$ (%) | Load factor, $l_r^t$ (%) |
|---|---|---|---|---|---|---|
| TPE–LAX | B747–400, B747–400 combi | 14 | 12.67 | 820 | 100 | 80.69 |
| TPE–SEA | B747–400 combi | 7 | 11.67 | 882 | 18.94 | 76.03 |
| TPE–SFO | B747–400 combi | 10 | 12.00 | 882 | 30.68 | 78.90 |
| TPE–TYO | B767–300 | 21 | 3.08 | 423 | 5.62 | 77.43 |
| TPE–HKG | B747–400 combi, B767–300 | 26 | 1.75 | 341 | 23.70 | 73.90 |
| TPE–SIN | B747–400 combi | 7 | 4.33 | 417 | 22 | 67 |
| TPE–BKK | B747–400, MD11 | 25 | 3.58 | 402 | 12.7 | 70.98 |
| TPE–SYD | B767–300 | 2 | 9.25 | 817 | 77.09 | 35.99 |

**Table 4**
The optimal purchase and replacement decisions made in the first period.

| Route, $r$ | The first period, $t = 1$ | | | | | The second period, $t = 2$ | | |
|---|---|---|---|---|---|---|---|---|
| | | Fleet composition | | Purchase and replacement decisions | | | Fleet composition | |
| | Duration | Aircraft type | Number | Aircraft type | Number | Duration | Aircraft type | Number |
| TPE–LAX | 2002–2005 | B747–400 | 3 (leased) | – | – | 2005–2009 | B747–400 | 4 (leased) |
| TPE–SEA | 2002–2004 | B747–400 combi | 2 (leased) | – | – | 2004–2007 | B747–400 combi | 2 (leased) |
| TPE–SFO | 2002–2005 | B747–400 combi | 2 (leased) | B747–400 combi | 1 (leasing) | 2005–2009 | B747–400 combi[1] | 3 (leased) |
| TPE–TYO | 2002–2005 | B767–300 | 2 (leased) | B767–300 | 1 (leasing) | 2005–2007 | B767–300[2] | 3 (leased) |
| TPE–HKG | 2002–2005 | B767–300 | 1 (leased) | B767–300 | 1 (disposing of) | 2005–2007 | B747–400 combi | 1 (leased) |
| | | B747–400 combi | 1 (leased) | | | | | |
| TPE–SIN | 2002–2006 | B747–400 combi | 1 (leased) | – | – | 2006–2009 | B747–400 combi | 1 (leased) |
| TPE–BKK | 2002–2004 | MD11 | 1 (purchased) | MD11 | 1 (disposing of) | 2004–2007 | B747–400 | 2 (leased) |
| | | B747–400 | 1 (leased) | B747–400 | 1 (leasing) | | | |
| TPE–SYD | 2002–2005 | B767–300 | 1 (leased) | – | – | 2005–2009 | B767–300 | 2 (leased) |
| Total | | B747–400 combi | 6 (leased) | B747–400 combi | 1 (leasing) | | 747–400 combi | 7 (leased) |
| | | B747–400 | 4 (leased) | B747–400 | 1 (leasing) | | B747–400 | 5 (leased) |
| | | 767–300 | 4 (leased) | B767–300 | 1 (leasing) | | B767–300 | 4 (leased) |
| | | MD11 | 1 (purchased) | MD11 | 1 (disposing of) | | | |
| | | | | B767–300 | 1 (disposing of) | | | |

[1] 8% of the capacities from the B747–400 combi are shared with route TPE–LAX.
[2] 11% of the capacities from the B767–300 are shared with route TPE–SYD.

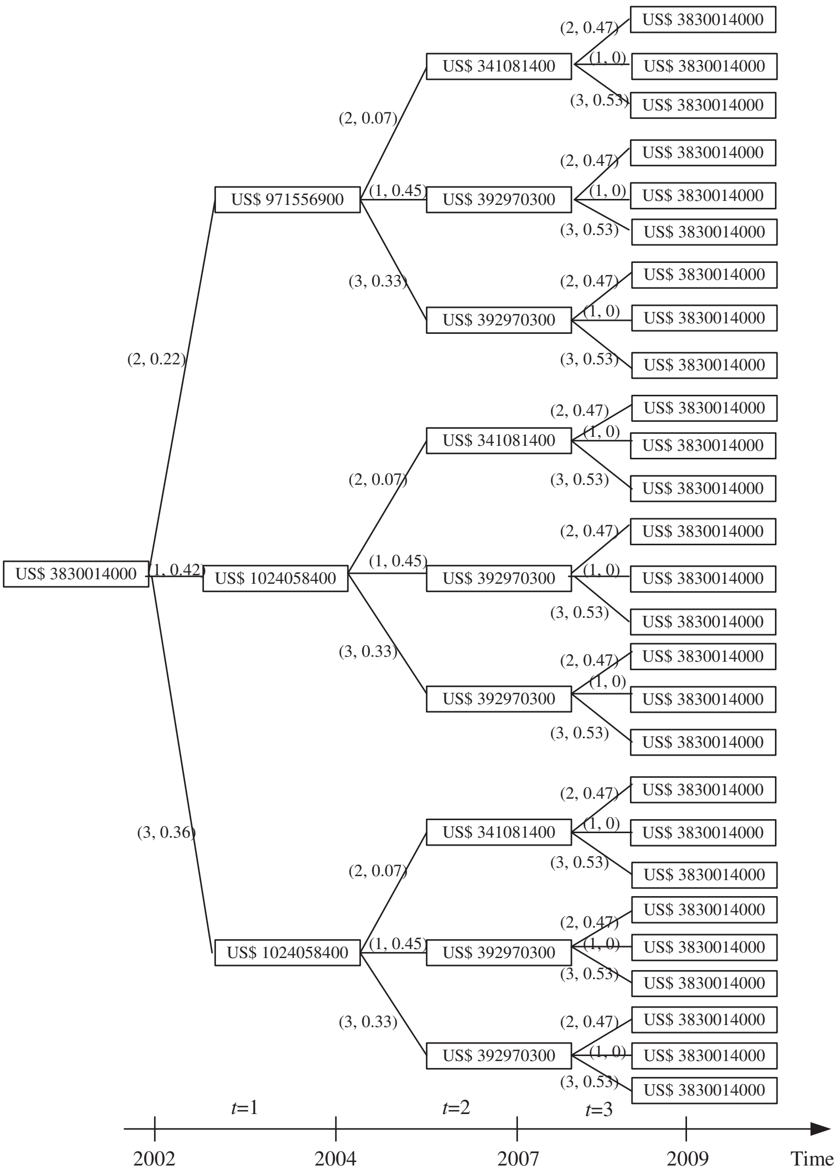


**Fig. 2.** The costs of route TPE–BKK with different fluctuated demand at different periods.

increased costs with either an overestimated or an underestimated demand, i.e. $w = 1$ and $w = 3$. However, the impact of the forecast results on total cost rely not only on the difference between forecasted and realized demand, but also on the probability that the forecast results in fact occur. As shown in Fig. 2, the high probability that the forecasts are an increasing or decreasing demand, $p^1_{w=1}$ and $p^1_{w=3}$, respectively, combined with the increased costs leads to a relatively high expected cost over the study period.

In the present study, the airline serves the routes entirely with leased aircraft because that way the airline is exempt from the high depreciation cost and only needs to pay the lease cost. However, high maintenance cost places a heavy financial burden on the airline when the aircraft become older and have high mileage. When that happens, the airline may prefer to purchase rather than lease these older aircraft since the flexibility of leasing may not compensate for the high maintenance cost. Next we perform a sensitivity analysis to investigate how changes in the age of the aircraft and the average lease cost per year affect the decisions to purchase or lease. Fig. 3 shows the threshold of the purchase and lease decision by comparing various lease costs and the age of the B747–400 combi aircraft.

Compared to just leasing or terminating the lease of an aircraft, the purchase or the disposal of an aircraft requires a much longer time. Hence, airlines tend to lease aircraft rather than purchasing them in order to satisfy short-term fluctuations in demand. To simplify the problem, the time until receiving a new aircraft that has been purchased or leased is neglected in this study because aircraft replacement decisions are made earlier in order to match future demand. The benefits of leasing include savings in depreciation cost and greater flexibility in matching the demand in the short run. Moreover, the older the aircraft the less the benefits of leasing, and the higher the costs of leasing. As seen in Fig. 3, the threshold of leasing an aircraft decreases with the increase in the age of the aircraft. Nevertheless, leasing is an optimal alternative if there is a substantial decrease in lease cost. Also, the effect of lease cost on purchase and lease decisions is marginal for B747–400 combi aircraft if they are older than five years, as shown in Fig. 3. The value of an aircraft depreciates exponentially from the moment the aircraft has been manufactured and is being operated. In other words, the purchase cost of the aircraft decreases with its age, and so does the depreciation cost. In addition, a low salvage price is of no consequence to the airline once the aircraft is scrapped because it is being replaced or simply because of fleet reduction. When the airline expands the fleet capacity by adding a used but not aged aircraft, these advantages explain why an airline may prefer to purchase rather than lease. The results provide a reference for the airline in their decision making process of replacement decisions in accordance with the aircraft age and in negotiating with the leasing company for the lease price of different aircraft.

The maintenance cost depends on the status of the aircraft, including type, age and mileage. In this study, the cost of operating an aircraft is dependent on its status, as defined by type, age and total mileage traveled. Given the passenger demand, an aircraft should be disposed of and replaced with a new aircraft when the maintenance requirements become excessive. On the other hand, all things being equal, it pays to keep an aircraft if it is in good condition, i.e. low maintenance cost. Fig. 4

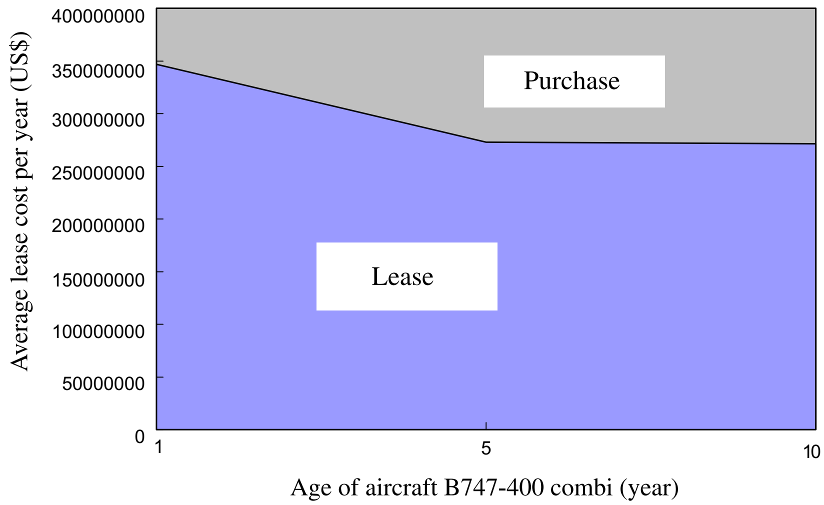


**Fig. 3.** The threshold of purchase and lease decisions by comparing lease cost and the age of aircraft B747–400 combi.

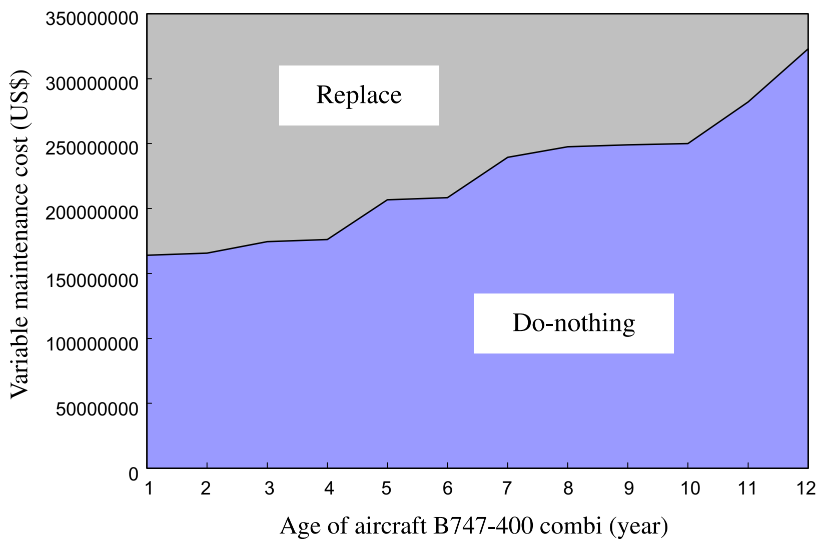


**Fig. 4.** The threshold of whether or not disposing of the aircraft by comparing the variable maintenance cost and the age of aircraft B747–400 combi.

shows the threshold of whether or not to dispose of an older aircraft and replace it with a new one by comparing the maintenance cost and the age of the B747–400 combi aircraft. The left-hand and right-hand sides of the solid line in Fig. 4 represent the decisions regarding disposing of the existing aircraft and replacing with a new one, and do-nothing, respectively.

As shown in Fig. 4, the threshold of the replacement increased with the increased age of the aircraft. As an aircraft becomes older, the annual depreciation decreases with the accumulated depreciation being spread over an increasing number of service years. Hence, airlines are more inclined to retain older aircraft despite their high maintenance cost. As shown in Fig. 4, the threshold for a replacement increases with the age of the aircraft. If the maintenance cost of an aircraft does not exceed the threshold, the airline should retain the aircraft; while if the maintenance cost keeps increasing, the tendency to dispose of the aircraft will also increase. With other words, if the maintenance cost of the aircraft does not exceed the threshold, the result suggests that the airline keeps the aircraft. However, the tendency towards disposing of the aircraft is high once the aircraft has a high maintenance cost.

In this study, the total cost of the airline is affected by disturbance of demand fluctuations. An overestimated demand leads to excess capacity, while an underestimated demand results in insufficient capacity. Variables $p^t_{w=1}, p^t_{w=2}$ and $p^t_{w=3}$ represent, respectively the probabilities that the following occur, the demand of period $t$ is fluctuated to be increasing, the same and decreasing, as compared with that of period $(t-1)$. Fig. 5 shows the total expected cost of the routes under different occurrence of forecast results. The $X$-axis in Fig. 5 represents different criteria regarding the combinations of the three probabilities, and from left to right, the $X$-axis indicates (1) the original probabilities from the Grey topological model and the Markov-chain; (2) three forecast results exist evenly, i.e. $p^t_{w=1} = p^t_{w=2} = p^t_{w=3} = 0.33$; (3) the future demand is exclusively increasing, i.e. $p^t_{w=1} = 1$; (4) the future demand is exclusively decreasing, i.e. $p^t_{w=3} = 1$; and (5) the future demand is the same with that of the previous period, i.e. $p^t_{w=2} = 1$.

As shown in Fig. 5, there is a similar cost pattern among the routes, where the future demand is the same with that of the previous period shows the lowest, i.e. label (5) in $X$-axis while the demand being exclusively increasing and decreasing are the highest. The total cost when the three forecast results exist equally is moderate between all criteria. The results demonstrate the importance of stochastic future demand. Accurate demand forecasts will enable the airline not only to schedule aircraft replacement decisions in response to fluctuating and cyclical demands, but will also achieve an overall minimized cost. The forecasted demand of the route is calculated based on the market share of that route. The market share is positively affected by the flight frequencies provided. An increased flight frequency leads to a higher market share and a higher passenger demand carried by the airline. Supposing EVA intends to increase its market share of route TPE–HKG from 5.62% up to 30%, by increasing its flight frequency. Table 5 shows the optimal replacement decisions of route TPE–HKG with market shares of 5.62% and 30%, respectively.

Due to the limited utilization of the aircraft, the total number of aircraft should be increased with the increased total flight frequencies. As shown in Table 5, the fleet should obtain 3 additional leased B747–400 combi if the airline expects an increase in market share from 5.62% to 30%. The additional costs, such as the costs related to the aircraft and the costs originated from the passengers, the total expected cost is substantially increased as shown in Table 5.

In this study, the realized fleet capacities, i.e. the numbers of seats on the routes are influenced by the load factor, which is determined based on historical data. Assume the service performance remains the same under different settings of the load factor. As Eq. (4) shows, under a constant demand, a larger value of the load factor leads to a lower capacity requirement, thus a lower number of aircraft and lower total expected cost. Supposing EVA decides to increase the average load factor of route TPE–SYD from 71% to 80%. Table 6 shows the optimal replacement decisions of route TPE–SYD with the average load factors of 71% and 80%, respectively.

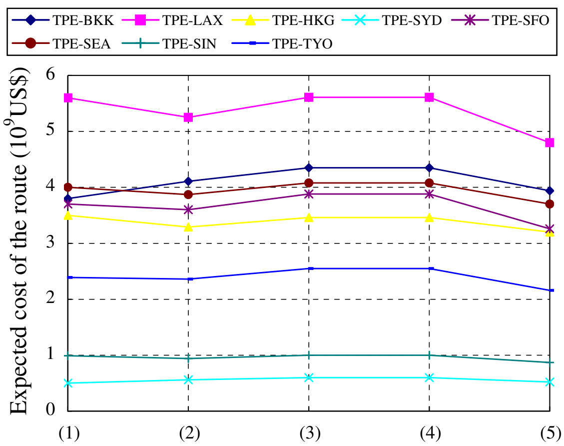


**Fig. 5.** The total expected cost of the routes under different occurrence of forecast results.

**Table 5**
The optimal purchase and replacement decisions of route TPE–HKG with market shares of 5.62% and 30%.

| Market share | The first period, $t = 1$ | | | | The second period, $t = 2$ | | Total expected cost (US$) |
|---|---|---|---|---|---|---|---|
| | Fleet composition | | Purchase and replacement decisions | | Fleet composition | | |
| | Aircraft type | Number | Aircraft type | Number | Aircraft type | Number | |
| 5.62% | B767–300<br>B747–400 combi | 1 (leased)<br>1 (leased) | B767–300 | 1 (disposing of) | B747–400 combi | 1 (leased) | 3,460,846,000 |
| 30% | B767–300<br>B747–400 combi | 1 (leased)<br>1 (leased) | B767–300<br>B747–400 combi | 1 (disposing of)<br>3 (leasing) | B747–400 combi | 4 (leased) | 6,453,704,000 |

**Table 6**
The optimal purchase and replacement decisions of route TPE–SYD with the average load factors of 71% and 80%.

| Average load factor (%) | The first period, $t = 1$ | | | | The second period, $t = 2$ | | Total expected cost (US$) |
|---|---|---|---|---|---|---|---|
| | Fleet composition | | Purchase and replacement decisions | | Fleet composition | | |
| | Aircraft type | Number | Aircraft type | Number | Aircraft type | Number | |
| 71 | B767–300 | 1 (leased) | – | – | B767–300[a] | 2 (leased) | 450,932,700 |
| 80 | B767–300 | 1 (leased) | – | – | B767–300 | 1 (leased) | 433,195,100 |

[a] One of which is shared with route TPE–TYO.

Because the load factor is as low as 71%, route TPE–SYD requires 2 leased B767–300 to provide the services. One of these aircraft is shared by route TPE–TYO with 89% of its capacity. As Table 6 shows, there is only 1 leased B767–300 required to serve the route under the increased load factor, i.e. 80%. Because there are fewer aircraft being operated, the total expected cost is correspondingly reduced.

## 6. Conclusions

Past studies have investigated the equipment replacement problems in the field of industrial engineering and operations. Other studies have discussed fleet management problems at both operational and tactic levels, in addition to the strategic level. However, there is scant literature available on replacement cost in relation to fleet capacity management over different time periods, or for revenue loss associated with dynamic and cyclical demand. Therefore, the contribution of this paper to the literature is to fill in the above gap. Moreover, the decision on whether to expand a fleet by purchasing new aircraft or lease them, or to reduce a fleet through disposal of the purchased or leased aircraft are also investigated.

The application of our proposed dynamic programming model is illustrated with a case study involving EVA airlines. It was found that EVA tends to simplify its fleet composition by using a single type of aircraft for each route served. To maximize capacity utilization and reduce any related costs, some aircraft are assigned to two routes. In addition, severe demand fluctuations have driven EVA to lease rather than purchase their aircraft. This is allowing EVA greater flexibility in fleet management and in matching short-term variations in demand. In addition, the total cost for a particular decision period can be minimized by providing a perfect match of the forecasted demand with the actual demand, instead of overestimated or underestimated forecasts that will lead to increased costs. However, the impact of forecasted results for total cost varies not only with the difference between forecasted and actual demands, but also on the probability that a demand forecast will occur. In other words, although an accurate demand forecast avoids a penalty cost, the total cost will still be high if the precise estimation occurs only rarely. Hence, the total cost for the airline can only be minimized if all the impacts of the demand fluctuations and cyclic demands on the airline's fleet management are fully captured.

As a leased aircraft becomes older, the benefits of leasing will decline further, resulting in a smaller tendency towards leasing the aircraft. Leasing an older aircraft is an optimal alternative only if there is a substantial reduction in lease cost. In addition, the threshold of the replacement decision increases with the increase in age of the aircraft. In other words, if the increased maintenance cost of an older aircraft does not exceed the threshold, the aircraft should be retained and vice versa. The results of this study provide a useful reference for airlines in their airplane replacement decision-making taking into account the fluctuations in market demand and the status of the aircraft.

The study period in the case study is set to be eight years, and involves only replacement scheduling for a short run. Future studies can extend the study period to explore medium- and long-term replacement scheduling. A limitation of our study is the fact that it considers only passenger demand while neglecting the demand for air cargo, which makes up a very important portion of the demand for air transport. To get an overall picture of the actual operation of an airline it is worth exploring the replacement scheduling considering both passenger and air cargo demands. The case study in this research is focused on a single airline, and the effect of strategic alliances with other airlines has been neglected. It would be interesting to examine if airlines that have formed strategic alliances have a different approach to optimizing their replacement scheduling. This study employs the Grey topological forecasting method combined with the Markov-chain model to forecast passenger traffic and to capture the random and cyclic demands. Nevertheless, air passenger demand is not only affected by the economic situation but also by the threat of terrorism, airplane crashes, and the development of new routes and markets. The impact of all these issues must be taken into account when assessing the fluctuation in passenger demand when deciding on a replacement schedule. The computational difficulties are of the most challenges when solving larger scale-instances of the problem. To make backwards computing possible, at each step the decision functions must be included in the computations and stored until the end. Considerable storage capacity is therefore required, because these functions are, as a rule, obtained only in tabular form (Bronshtein and Semendyayev, 1985).

**Acknowledgement**

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## Appendix A. Grey topological model and Markov-chain

### A.1. Grey topological model

The steps of constructing a Grey topological model are described as follows:

1. Plot a series of $X^{(0)}$ in two-dimensional $X$, $Y$-plane. Every $X$ in $X^{(0)}$ has its own $Y$-axis coordinate, while a $Y$-axis coordinate may be mapped to several $X$-axis coordinates. Let $k$ represent the order number of the $X$-axis coordinates sharing the same $Y$-axis coordinate and $x^{(0)}(k)$ represent the $X$-axis coordinates mapped by that $Y$-axis coordinate. Plot the curve, $X^{(0)}$, in $X$, $Y$-plane, using $[k, x^{(0)}(k)]$.
2. According to the sequence $x^{(0)}(k)$, find the maximum value Max $X^{(0)}$ and minimum value min $X^{(0)}$. Select many reference values $\zeta_i$ at $Y$-axis, $i = 1, 2, \ldots, m$. Note that the domain of the reference value is $\min X^{(0)} \leqslant \zeta_i \leqslant MaxX^{(0)}, i = 1, 2, \ldots, m$.
3. Find the corresponding $Y$-axis coordinate of $\zeta_i$, as $\zeta_i : \{X^{(0)}\} \to \{mt_i^{(0)}\}$. Let $mt_i^{(0)}(k)$ represent the $k$ th tangent point of the horizontal line, $\zeta_i$ passing curve $X^{(0)}$. Then, all $X$-axis coordinates can form a set of $P : \{(mt_i^{(0)}(k), \zeta_i)\} \to \{mt_i^{(0)}(k)\}, \quad k = 1, 2, \ldots, n_i$ and $mt_i^{(0)} = \{mt_i^{(0)}(1), mt_i^{(0)}(2), \ldots, mt_i^{(0)}(n_i)\}$.
4. Every fixed reference value should map to a coordinate set $W_i^{(0)}$, composed by $i$ number of X-axis coordinates, it's $mt_i^{(0)}(k) = W_i^{(0)}(k)$, and $P(mt_i^{(0)}(k), \zeta_i) = W_i^{(0)}(k)$, therefore $W_i^{(0)} = \{W_i^{(0)}(1), W_i^{(0)}(2), \ldots, W_i^{(0)}(n_i)\}$. And, $W_i^{(0)}$ represents a set of $X$-axis coordinates, which map to a fixed reference value $\zeta_i$ in the $Y$-axis.
5. Perform an accumulated generating operation for set $W_i^{(0)}$, and obtain a new generating series $W_i^{(1)}$, it's $AGO : W_i^{(0)} \to W_i^{(1)}$.
6. Construct a GM(1, 1) model for each new generating series $W_i^{(1)}$, represented as $GM : W_i^{(1)} \to \widehat{W}_i^{(1)}$.

Perform an inverse accumulated generating operation to each new GM model and obtain the predicting model $IAGO : \widehat{W}_i^{(1)} \to \widehat{W}_i^{(0)}$. The whole procedure can be represented as follows:

$$GM \cdot AGO \cdot P \cdot \zeta_i(\{X^{(0)}\}) = \widehat{W}_i^{(1)} \tag{A1}$$

$$IAGO \cdot GM \cdot AGO \cdot P \cdot \zeta_i(\{X^{(0)}\}) = \widehat{W}_i^{(0)} \tag{A2}$$

7. Every fixed reference value can develop a particular forecasting model as shown in step 6. According to these forecasting models, find the $X$-axis coordinate corresponding to the fixed reference value $\zeta_i$ in the $Y$-axis for $i = 1, 2, \ldots, m$, then these $X$-axis coordinates are

$$\widehat{W}_1^{(0)}(n_1 + 1), \widehat{W}_2^{(0)}(n_2 + 1), \ldots, \widehat{W}_m^{(0)}(n_m + 1) \tag{A3}$$

The forecasting value $\widehat{W}_i^{(0)}(n_i + 1)$ represents the distance from the origin to the $(n_i + 1)$th data in the $X$-axis. The coordinate is represented as $(\widehat{W}_i^{(0)}(n_i + 1), \zeta_i)$ in a two-dimensional plane. By linking these coordinates as a curve and the Topological forecasting curve, $\widehat{X}^{(0)}$ can be obtained

$$\widehat{X}^{(0)} = \{(\widehat{W}_i^{(0)}(n_i + 1), \zeta_i) | i = 1, 2, \ldots, m\} \tag{A4}$$

### A.2. Markov-chain

The Markov-chain theory is widely applied to predict a dynamic random system. A Markov-chain describes the states of a system at successive times. At these times the system may have changed from the state it was in the moment before to another or remained in the same state. The changes of state are called transitions. The Markov property means that the conditional probability distribution of the state in the future, given the state of the process currently and in the past, depends only on its current state and not on its state in the past. A $n$-step Markov-chain is composed of a set of $n$-state and one set of transition probability. There is only one state at one moment, and any further changes in the system can be determined by the transition probability in each state at different moments. The transition probability of each state represents the level of effects incorporating every random factor. Therefore the Markov-chain is suitable for forecasting random series.

This study combines the Grey topological and Markov-chain models for forecasting airline passenger demand with respect to different economical situations. Previous literature has forecasted gross national product (GNP) based on Grey predicting GM (1, 1) combined with the Markov-chain model, and the result was shown to be more accurate than GM(1, 1) alone. The implementation steps of the Markov-chain model are listed below.

*A.2.1. Categorize the states*

Categorize every moment in the Grey topological model into $k$ states. Let the result of the Grey topological forecasting model, $\widehat{W}_i^{(0)}$ in moment $i$ be the central point of every state. Determine a proper percentage $P$% of $\widehat{W}_i^{(0)}$ to be the upper and lower bounds of every moment in each state. Then, the boundary of the $j$th state in moment $i$, $E_{ij}$ can be represented as

$$E_{ij} \in [A_{ij}, B_{ij}], j = 1, 2, \dots, k \tag{A5}$$

where $A_{ij}$ and $B_{ij}$ represent the upper and lower bounds of the $j$th state in moment $i$, respectively. Linking the boundary of the same states in every moment results in a function curve which is nearly parallel with the curve of the Grey topological forecasting model. The zone between every two adjacent curves form a state zone, so we can determine the state in which every Grey topological predicting result will be at each moment. Classify those predicting results which are less than $A_{i1}$ as state one, those which are larger than $B_{ik}$ as state k. The values of $A_{ij}$, $B_{ij}$, and $k$ can be decided by research subject and the amount of original data.

*A.2.2. Establish a matrix of state transition probability*

The state transition probability can be formulated as

$$P_{ab}^{(m)} = \frac{M_{ab}^{(m)}}{M_a} \tag{A6}$$

where $P_{ab}^{(m)}$ represents the probability of transition from state $a$ to state $b$ after $m$ steps, $M_{ab}^{(m)}$ represents the frequency of transition from state $a$ to state $b$ after $m$ steps, $M_a$ is the frequency of state $a$. Due to the unknown transition from the last state to its next state of the original series, the data of the last $(m - 1)$ steps will be eliminated when calculating $M_a$. The state transition probability matrix, $R^{(m)}$ can be written as

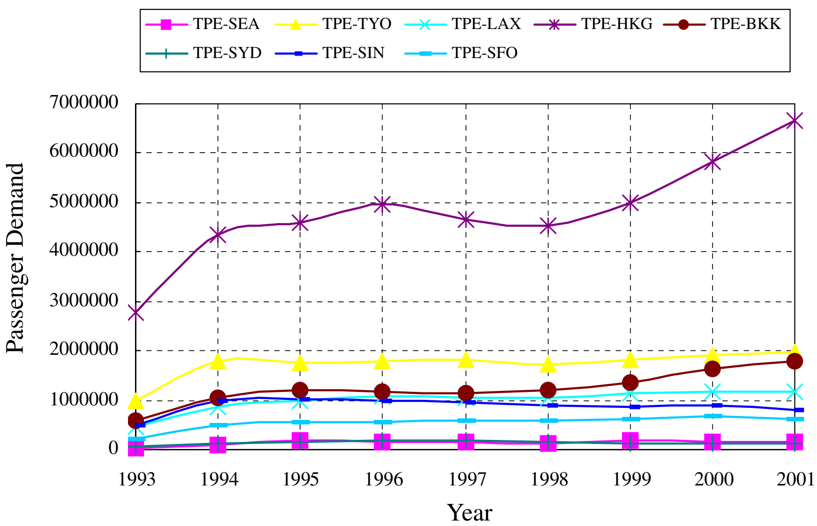



**Fig. 6.** Passenger demand of the routes from 1993 to 2001.

$$R^{(m)} = \begin{bmatrix} P_{11}^{(m)}, P_{12}^{(m)}, \ldots, P_{1k}^{(m)} \\ P_{21}^{(m)}, P_{22}^{(m)}, \ldots, P_{2k}^{(m)} \\ \vdots \\ P_{k1}^{(m)}, P_{k2}^{(m)}, \ldots, P_{kk}^{(m)} \end{bmatrix} \tag{A7}$$

where $P_{ab}^{(m)}$ represents the probability of transition from state $a$ to state $b$ after $m$ steps. Considering $m = 1$, if the forecasted data falls in the $a$th state, then check the $a$th row of matrix $R^{(1)}$. If $Max_b P_{ab}^{(1)} = P_{aL}$, then state $L$ is the most likely state that the series transfer to at the next moment.

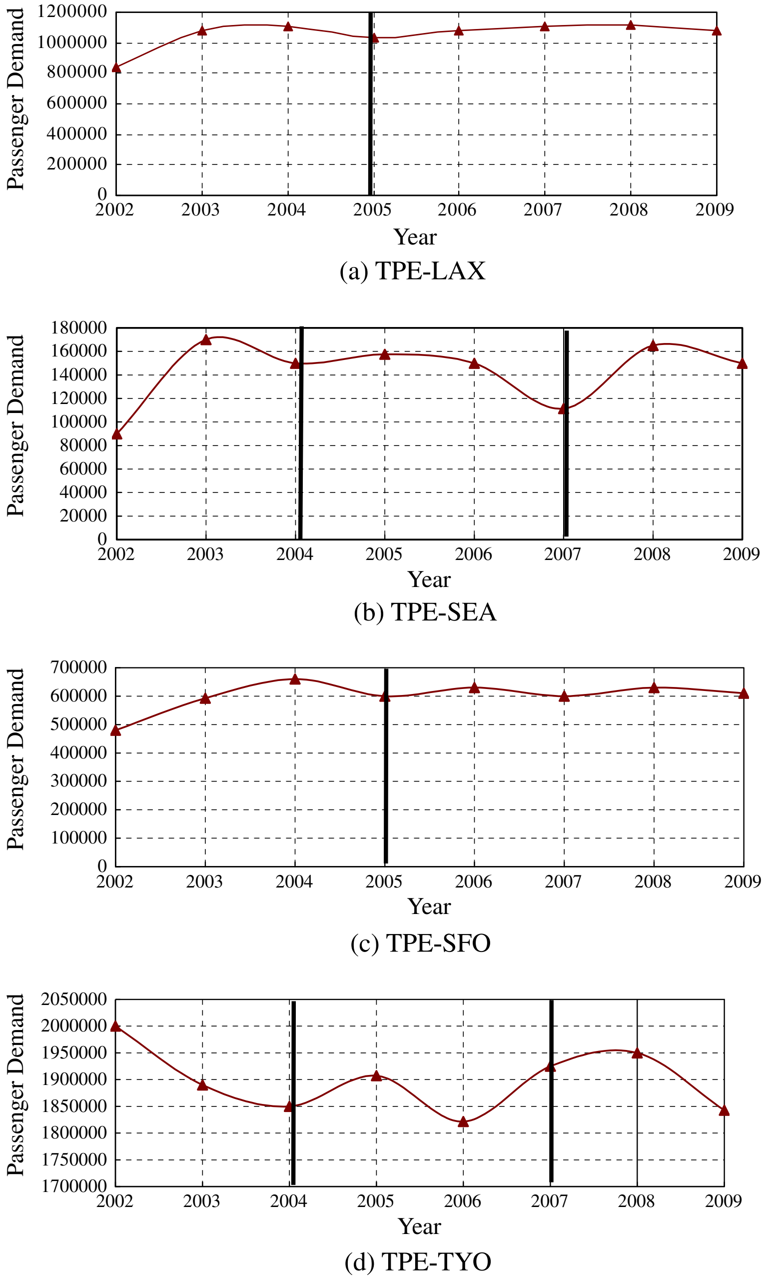
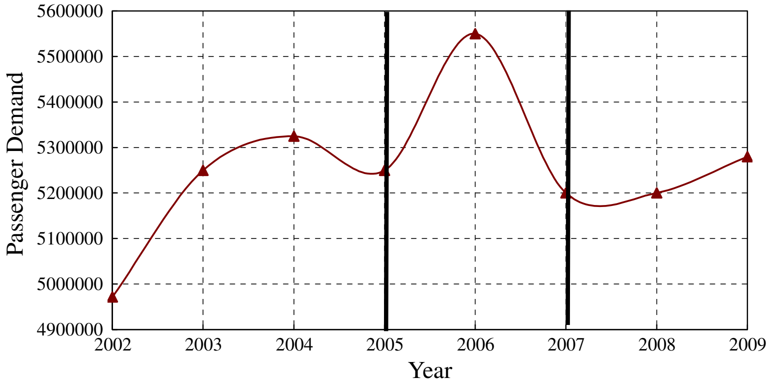


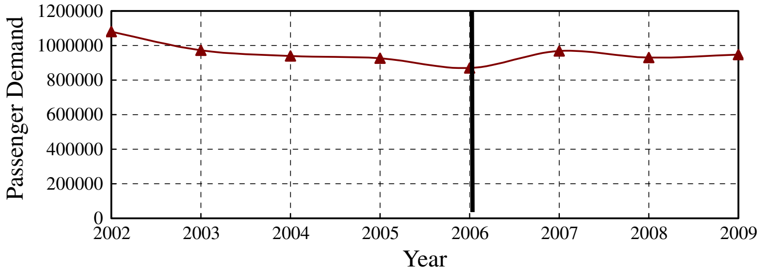

**Fig. 7.** Forecasted yearly demand of the routes.

*A.2.3. Compose a transition probability matrix table*

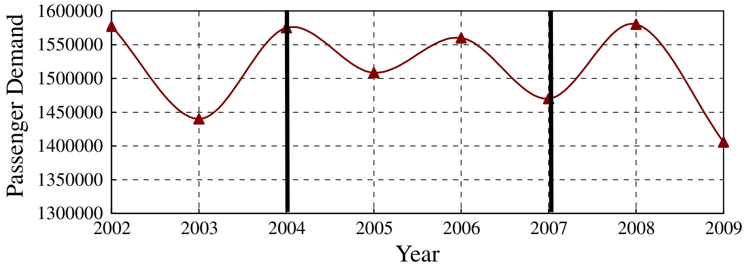
Determine a number of $r$ as the number of moments from the past to the forecasted moment. From the nearest to furthest moments, the transition steps from the past moments to the forecasted moment are $1, 2, \ldots, r$. For all transition probability matrices of the steps, extract the vector rows from the transition probability matrix mapped by the beginning state and compose those as a new transition probability matrix. By summing up all the vectors in the column, the state of the forecasted moment can be obtained as the state with the maximum value.



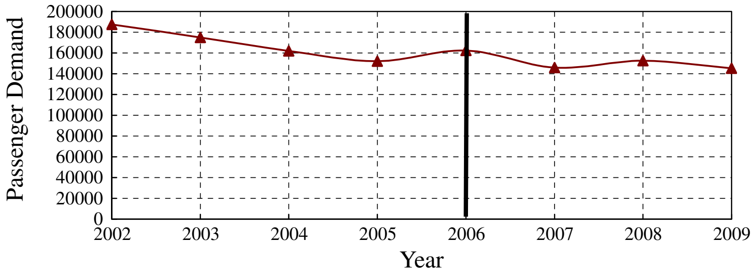

(e) TPE-HKG




(f) TPE-SIN




(g) TPE-BKK




(h) TPE-SYD

**Fig. 7** (*continued*)

*A.2.4. Calibrate the forecasted values*

After obtaining the transition state of the future moment of the series by step 3 the upper and lower bounds of the state can be further determined. The forecasted value will be the average of the upper and lower bounds.

This study further applies the Grey topological model and the Markov-chain model formulated above to forecast the future passenger traffic of the routes in the case study. The implementation steps are described as follows:

*Step* 1. Collect historical passenger traffic, construct a passenger demand forecasting model using the Grey topological model. Compare the forecasted result with the actual data to verify its accuracy. Moreover, determine the decision periods for the routes according to the economic cycle.

**Table 7**
Comparison between forecasted demand based on Grey Topological model and the actual data.

| Route | Average yearly demand from 1993 to 2001 | | Average difference (%) |
|---|---|---|---|
| | Actual | Forecasted | |
| TPE–LAX | 994,253 | 989,203 | 0.47 |
| TPE–SEA | 139,766 | 141,034 | 5.51 |
| TPE–SFO | 547,829 | 557,762 | 3.64 |
| TPE–TYO | 1,727,317 | 1,718,283 | 0.26 |
| TPE–HKG | 4,816,613 | 4,798,122 | 0.36 |
| TPE–SIN | 876,060 | 883,171 | 1.05 |
| TPE–BKK | 1,241,295 | 1,218,530 | 0.95 |
| TPE–SYD | 134,629 | 139,271 | 4.05 |

**Table 8**
The boundaries of states and the state of the forecasted demand on route TPE–BKK.

| Year | Actual demand | Forecasted demand | A1 | A2 = B1 | A3 = B2 | B3 | State |
|---|---|---|---|---|---|---|---|
| 1993 | 598,107 | 602,000 | 566,361 | 589,960 | 614,040 | 638,601 | 2 |
| 1994 | 1,060,692 | 1,100,000 | 1,034,880 | 1,078,000 | 1,122,000 | 1,166,880 | 1 |
| 1995 | 1,215,534 | 1,164,000 | 1,095,091 | 1,140,720 | 1,187,280 | 1,234,771 | 3 |
| 1996 | 1,178,071 | 1,188,000 | 1,117,670 | 1,164,240 | 1,211,760 | 1,260,230 | 2 |
| 1997 | 1,147,768 | 1,213,333 | 1,141,504 | 1,189,066 | 1,237,600 | 1,287,104 | 1 |
| 1998 | 1,201,234 | 1,230,000 | 1,157,184 | 1,205,400 | 1,254,600 | 1,304,784 | 1 |
| 1999 | 1,363,266 | 1,346,667 | 1,266,944 | 1,319,733 | 1,373,600 | 1,428,544 | 2 |
| 2000 | 1,626,603 | 1,550,769 | 1,458,963 | 1,519,753 | 1,581,784 | 1,645,056 | 3 |
| 2001 | 1,780,378 | 1,572,000 | 1,478,937 | 1,540,560 | 1,603,440 | 1,667,577 | 3 |

**Table 9**
Transition probability matrix of route TPE–BKK.

| Original state | | State after transition | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| R1 | 1 | 0.33 | 0.33 | 0.33 |
| | 2 | 0.67 | 0.00 | 0.33 |
| | 3 | 0.00 | 0.50 | 0.50 |
| R2 | 1 | 0.00 | 0.67 | 0.33 |
| | 2 | 0.33 | 0.00 | 0.67 |
| | 3 | 1.00 | 0.00 | 0.00 |
| R3 | 1 | 0.33 | 0.00 | 0.67 |
| | 2 | 0.00 | 1.00 | 0.00 |
| | 3 | 1.00 | 0.00 | 0.00 |
| R4 | 1 | 0.50 | 0.00 | 0.50 |
| | 2 | 0.50 | 0.00 | 0.50 |
| | 3 | 1.00 | 1.00 | 0.00 |
| R5 | 1 | 0.00 | 1.00 | 0.00 |
| | 2 | 0.50 | 0.00 | 0.50 |
| | 3 | 0.00 | 0.00 | 1.00 |
| R6 | 1 | 0.00 | 0.00 | 1.00 |
| | 2 | 0.00 | 1.00 | 0.00 |
| | 3 | 0.00 | 0.00 | 1.00 |

**Table 10**
The state transition probability matrix from year 2002–2009 on route TPE–BKK.

| Year | State | Step | State | | |
|---|---|---|---|---|---|
| | | | 1 | 2 | 3 |
| *Predicted year: 2002* | | | | | |
| 1996 | 2 | 6 | 0.00 | 1.00 | 0.00 |
| 1997 | 1 | 5 | 0.00 | 1.00 | 0.00 |
| 1998 | 1 | 4 | 0.50 | 0.00 | 0.50 |
| 1999 | 2 | 3 | 0.00 | 1.00 | 0.00 |
| 2000 | 3 | 2 | 1.00 | 0.00 | 0.00 |
| 2001 | 3 | 1 | 0.00 | 0.50 | 0.50 |
| Total | | | 1.50 | 3.50 | 1.00 |
| *Predicted year: 2003* | | | | | |
| 1997 | 1 | 6 | 0.00 | 0.00 | 1.00 |
| 1998 | 1 | 5 | 0.00 | 1.00 | 0.00 |
| 1999 | 2 | 4 | 0.50 | 0.00 | 0.50 |
| 2000 | 3 | 3 | 1.00 | 0.00 | 0.00 |
| 2001 | 3 | 2 | 1.00 | 0.00 | 0.00 |
| 2002 | 2 | 1 | 0.67 | 0.00 | 0.33 |
| Total | | | 3.17 | 1.00 | 1.83 |
| *Predicted year: 2004* | | | | | |
| 1998 | 1 | 6 | 0.00 | 0.00 | 1.00 |
| 1999 | 2 | 5 | 0.50 | 0.00 | 0.50 |
| 2000 | 3 | 4 | 0.00 | 1.00 | 0.00 |
| 2001 | 3 | 3 | 1.00 | 0.00 | 0.00 |
| 2002 | 2 | 2 | 0.33 | 0.00 | 0.67 |
| 2003 | 1 | 1 | 0.33 | 0.33 | 0.33 |
| Total | | | 2.17 | 1.33 | 2.50 |
| *Predicted year: 2005* | | | | | |
| 1999 | 2 | 6 | 0.00 | 1.00 | 0.00 |
| 2000 | 3 | 5 | 0.00 | 0.00 | 1.00 |
| 2001 | 3 | 4 | 0.00 | 1.00 | 0.00 |
| 2002 | 2 | 3 | 0.00 | 1.00 | 0.00 |
| 2003 | 1 | 2 | 0.00 | 0.67 | 0.33 |
| 2004 | 3 | 1 | 0.00 | 0.50 | 0.50 |
| Total | | | 0.00 | 4.17 | 1.83 |
| *Predicted year: 2006* | | | | | |
| 2000 | 3 | 6 | 0.00 | 0.00 | 1.00 |
| 2001 | 3 | 5 | 0.00 | 0.00 | 1.00 |
| 2002 | 2 | 4 | 0.50 | 0.00 | 0.50 |
| 2003 | 1 | 3 | 0.33 | 0.00 | 0.67 |
| 2004 | 3 | 2 | 1.00 | 0.00 | 0.00 |
| 2005 | 2 | 1 | 0.67 | 0.00 | 0.33 |
| Total | | | 2.50 | 0.00 | 3.50 |
| *Predicted year: 2007* | | | | | |
| 2001 | 3 | 6 | 0.00 | 0.00 | 1.00 |
| 2002 | 2 | 5 | 0.50 | 0.00 | 0.50 |
| 2003 | 1 | 4 | 0.50 | 0.00 | 0.50 |
| 2004 | 3 | 3 | 1.00 | 0.00 | 0.00 |
| 2005 | 2 | 2 | 0.33 | 0.00 | 0.67 |
| 2006 | 3 | 1 | 0.00 | 0.50 | 0.50 |
| Total | | | 2.33 | 0.50 | 3.17 |
| *Predicted year: 2008* | | | | | |
| 2002 | 2 | 6 | 0.00 | 1.00 | 0.00 |
| 2003 | 1 | 5 | 0.00 | 1.00 | 0.00 |
| 2004 | 3 | 4 | 0.00 | 1.00 | 0.00 |
| 2005 | 2 | 3 | 0.00 | 1.00 | 0.00 |
| 2006 | 3 | 2 | 1.00 | 0.00 | 0.00 |
| 2007 | 3 | 1 | 0.00 | 0.50 | 0.50 |
| Total | | | 1.00 | 4.50 | 0.50 |

**Table 10** (*continued*)

| Year | State | Step | State | | |
|---|---|---|---|---|---|
| | | | 1 | 2 | 3 |
| *Predicted year: 2009* | | | | | |
| 2003 | 1 | 6 | 0.00 | 0.00 | 1.00 |
| 2004 | 3 | 5 | 0.00 | 0.00 | 1.00 |
| 2005 | 2 | 4 | 0.50 | 0.00 | 0.50 |
| 2006 | 3 | 3 | 1.00 | 0.00 | 0.00 |
| 2007 | 3 | 2 | 1.00 | 0.00 | 0.00 |
| 2008 | 2 | 1 | 0.67 | 0.00 | 0.33 |
| Total | | | 3.17 | 0.00 | 2.83 |

*Step* 2. Combine the Grey topological model with the Markov-chain model, to estimate the state transition probability matrix between the decision periods. Then, the probability of the forecasted passenger traffic fluctuation status for every period can be obtained from this state transition probability matrix. The obtained probability will be the probability of the forecasted passenger demand fluctuation combinations of two adjacent decision periods in the case study.

Fig. 6 shows the passenger traffic data from June, 1993 to December, 2001 of the eight routes in the case study. As shown in Fig. 6, although the total passenger traffic of these eight routes differs from each other, there is an upward trend that began in 1993, reached a peak in 1995 or 1996, and started to fall starting in 1997 and 1998. After 1999, the passenger traffic increased again. The results show that the airline passenger traffic indeed exhibits an economic cycle trend.

Fig. 7 represents the forecasted passenger demand of the routes from January, 2002 to December, 2009, showing the duration of the decision periods. Table 7 calibrates the forecast results by comparing the forecasted results of the Grey topological model with the actual data from June, 1993 to December, 2001.

As shown in Table 7, the maximum difference between the forecasted demand from the Grey topological forecasting model and the actual data is less than 6%. Moreover, the average differences on routes TPE–LAX, TPE–HKG, TPE–TYO and TPE–BKK are less than 1%. It can be concluded that overall the result from the Grey topological forecasting model is accurate. This study further combines the Grey topological forecasting results with the Markov-chain model, to investigate the demand fluctuations and to determine the probability of the three demand realizations.

Let the forecasted results from the Grey topological model be the middle value and be denoted by *X*. The boundary values of the three states, i.e. upward fluctuating demand, a demand similar to that of the previous period and a decreasing demand, can be determined according to the middle value, *X* and a difference rate of 4%. The boundary values of these three states, A1, A2, A3, and A4 can be expressed as follows:

$$A1 = A2 * (1 - 0.04) \tag{A8a}$$

$$A2 = X * (1 - 0.04/2) \tag{A8b}$$

$$A3 = X * (1 + 0.04/2) \tag{A8c}$$

$$A4 = A3 * (1 + 0.04) \tag{A8d}$$

For the demand fluctuation labeled as $w = 1$, 2 and 3, the future demand may lie between A1 and A2, A2 and A3, and A3 and A4 and the realized demand is $\frac{1}{2}$(A1 + A2), $\frac{1}{2}$(A2 + A3) and $\frac{1}{2}$(A3 + A4), respectively. Take route TPE–BKK as an example. Table 8 shows the boundaries of the states and the results of the state of the forecasted demand. The transition probability of route TPE–BKK can be further calculated based on Eqs. (A6) and (A7) and is shown in Table 9.

As shown in Table 9, R1, R2, R3, R4 and R5 represent the steps required for state *i* transferring to state *j*. For example, the probability of transition from state 1 to state 2 by 1 step is 0.33. Table 10 shows the state transition probability matrix from 2002 to 2009 on route TPE–BKK.

**Table 11**
Transition probability of states of route TPE–BKK.

| Year (State) | State | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 2002 (2) | 0.27 | 0.47 | 0.27 |
| 2003 (1) | 0.50 | 0.25 | 0.25 |
| 2004 (1) | 0.63 | 0.06 | 0.31 |
| 2005 (1) | 0.56 | 0.13 | 0.31 |
| 2006 (3) | 0.30 | 0.15 | 0.55 |
| 2007 (3) | 0.14 | 0.31 | 0.56 |
| 2008 (2) | 0.18 | 0.51 | 0.31 |
| 2009 (2) | 0.27 | 0.47 | 0.27 |

Take year 2002 as an example. As shown in Table 10, the largest total probability of state 2 of 3.5 shows that there is every likelihood that the forecasted demand is precise without fluctuation. The probabilities of the demand being overestimated, precisely estimated and underestimated can be further calculated as 1.5/(1.5 + 3.5 + 1.0), 3.5/(1.5 + 3.5 + 1.0), and 1.0/(1.5 + 3.5 + 1.0). Then, the probability of the transition from the current state of the year to different states can be restated as shown in Table 11.

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## Glossary of symbols

*Notation:* Definition
$p_w^t$: the probability of demand fluctuation labeled ***w*** at period ***t***
$F_r^t$: the forecasted passenger demand on route ***r*** at period ***t***
$f_r^t$: the actual passenger demand on route ***r*** at period ***t***
$N_{qym}^{Bt}$: the number of purchased aircraft with status (***q***, ***y***, ***m***) associated with the replacement decisions made at period ***t***
$N_{qym}^{Lt}$: the number of leased aircraft with status (***q***, ***y***, ***m***) associated with the replacement decisions made at period ***t***
$E_{qym}^{Bt}$: the total number of purchased aircraft with status (***q***, ***y***, ***m***) at period ***t*** in the airline fleet
$E_{qym}^{Lt}$: the total number of leased aircraft with status (***q***, ***y***, ***m***) at period ***t*** in the airline fleet
$\boldsymbol{Q_q}$: the capacity of a ***q*** type aircraft
$K_{qym}^{tr}$: the total flight frequencies on route ***r*** offered by aircraft with status (***q***, ***y***, ***m***) during period ***t***
$\delta_{qym}^{tr}$: an indicator variable denoting whether the aircraft with status (***q***, ***y***, ***m***) during period ***t*** is serving route ***r*** or not
$B_q^r$: the block time of a ***q*** type aircraft on route ***r***
$u_{qym}^t$: the maximum possible operating time of an aircraft with status (***q***, ***y***, ***m***) during period ***t***
$P_{qym}^t$: the average purchase cost for an aircraft with status (***q***, ***y***, ***m***) at period ***t***
$X_g^t$: the average remaining resale ratio of the original purchase price with an average annual interest rate ***g*** in period ***t***
$R_{qym}^{td}$: the average lease cost for an aircraft with status (***q***, ***y***, ***m***) with a total leased period ***d*** in period ***t***
$V_{qym}^t$: the variable maintenance cost of the aircraft with status (***q***, ***y***, ***m***) during period ***t***
$\boldsymbol{M^t}$: the fixed maintenance cost (overhead) of period ***t***
$b_{qr}^t$: the average flying cost of an aircraft of type ***q*** on route ***r*** during period ***t***
$O_D^t$: the total direct operating cost for the airline for operating the existing fleet during period ***t***
$O_I^t$: the total indirect operating cost as a result of serving passengers at period ***t***
$\boldsymbol{H^r}$: the average indirect cost per passenger on route ***r***
$\boldsymbol{C^t}$: the total operating cost of the airline during period ***t***
$A_{qym}^t$: the maximum usage of the aircraft with status (***q***, ***y***, ***m***) at period ***t***
$G_{qym}^t$: the maximum allowable mileage traveled of an aircraft with status (***q***, ***y***, ***m***) at period ***t***
$D_{qym}^t$: the salvage cost of an aircraft with status (***q***, ***y***, ***m***) during period ***t***
$Z_{qym}^{te}$: the penalty cost of an aircraft with status (***q***, ***y***, ***m***) and with a remaining lease period ***e*** during period ***t***
$\boldsymbol{U^t}$: the replacement cost during period ***t***
$W_{qym}^t$: the indicator variable representing whether the aircraft with status (***q***, ***y***, ***m***) should be disposed of at period ***t***
$I_r^t$: the average revenue loss associated with one unit of insufficient seats on route ***r*** during period ***t***