Alternative method for measuring the full-field refractive index of a gradient-index lens with normal incidence heterodyne interferometry

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A linearly/circularly polarized heterodyne light beam coming from a heterodyne light source with an electro-optic modulator in turn enters a modified Twyman–Green interferometer to measure the surface plane of a GRIN lens. Two groups of periodic sinusoidal segments recorded by a fast complementary metal-oxide semiconductor camera are modified, and their associated phases are derived with the unique technique. The data are substituted into the special equations derived from the Fresnel equations, and the refractive index can be obtained. When the processes are applied to other pixels, the full-field refractive-index distribution can be obtained similarly. Its validity is demonstrated. © 2010 Optical Society of America

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1. Introduction

The gradient-index (GRIN) lens is widely used in many fields, such as in fiber communications and biomedical and industrial applications [1,2]. The correctness and symmetry of the refractive-index distribution directly influence its product quality. Therefore, it is necessary to identify the full-field refractive-index distribution of the GRIN lens in advance. Several methods [3–9] have been proposed to measure the full-field refractive index of the optical waveguide or the GRIN lens, and they have good results. In our previous paper [10], we developed a method to measure the full-field refractive-index distribution of a GRIN lens based on the oblique incident approach without immersing it in the matching liquid. However, the camera must be set obliquely to match the image plane in order to correct the ratio of lateral scales, and the depth of field of the imaging lens limits the oblique angle of the measured sample. For a larger oblique angle, the images are easily obscured. To overcome these drawbacks, an alternative method for measuring the full-field refractive-index distribution of the GRIN lens based on the normal incident approach is presented in this paper. A linearly/circularly polarized heterodyne light beam in turn enters a modified Twyman–Green interferometer, in which a GRIN lens is located in one arm for test. Two groups of full-field interference signals are taken by a fast complementary metal-oxide semiconductor (CMOS) camera. The sampling intensities recorded at each pixel can be fitted to derive sinusoidal signals, and their associated phase can be calculated with the unique technique [11]. Then, substituting these data into the special equations derived from Fresnel equations, the refractive index at that pixel can be estimated. The processes are applied to other pixels,

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then full-field refractive-index distribution can be obtained similarly. Its validity is demonstrated. Although it is easier to align the imaging system and get clear images in this method, two phase measurement procedures need to be operated. Moreover, its measurement error is smaller than that of the oblique incidence method. This method measures only the refractive index in the end surface plane from which the light in the interferometric method is reflected. Thus, it does not in any way measure the refractive index in any internal plane perpendicular to the waveguide axis; i.e., the uniformity of the axial index distribution is not probed by this method.

2. Principle

A. Phase of the Interference Signal

A modified Twyman–Green interferometer for testing a GRIN lens is shown in Fig. 1. It consists of a heterodyne light source (HLS), beam splitter (BS), quarter-wave plates (Q_1 and Q_2), reference mirror (M), test GRIN lens (G), analyzer (AN), imaging lens (IL), and CMOS camera (C). In this interferometer, two optical paths are (1) BS $\rightarrow$ Q_2 $\rightarrow$ M $\rightarrow$ Q_2 $\rightarrow$ BS $\rightarrow$ AN $\rightarrow$ IL $\rightarrow$ C (the reference beam) and (2) BS $\rightarrow$ G $\rightarrow$ BS $\rightarrow$ AN $\rightarrow$ IL $\rightarrow$ C (the test beam). The lights reflected from the M and the G pass through the AN and interfere with each other. For convenience, the $+z$ axis is chosen to be along the light propagation direction and the y axis is along the direction perpendicular to the paper plane. If the HLS [10] with a frequency difference f between the x and y polarizations is used, its Jones vector can be written as [12]

$$E_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\pi ft} \\ e^{-i\pi ft} \end{pmatrix}. \tag{1}$$

Let the fast axis of the Q_2 and the transmission axis of the AN be at 45° and 0° to the x axis, respectively, and then the Jones vectors of the reference beam and the test beam can be derived [12] and expressed as

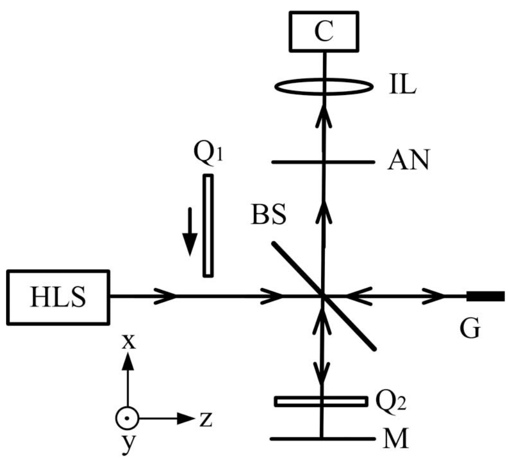

Fig. 1. Schematic diagram of the modified Twyman–Green interferometer.

$$\begin{aligned} E_{r1} &= AN(0°)\cdot Q_2(-45°)\cdot M\cdot Q_2(45°)\cdot R_{BS}\cdot E_1\cdot e^{i\phi_{d1}} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} -r_m & 0 \\ 0 & r_m \end{pmatrix}\frac{1}{\sqrt{2}} \\ &\times \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} e^{-i\phi_r/2} & 0 \\ 0 & e^{i\phi_r/2} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\pi ft} \\ e^{-i\pi ft} \end{pmatrix} e^{i\phi_{d1}} \\ &= \frac{ir_m e^{-i(\pi ft-\phi_{d1}-\phi_r/2)}}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \end{aligned} \tag{2}$$

$$\begin{aligned} E_{t1} &= AN(0°)\cdot R_{BS}\cdot G\cdot E_1\cdot e^{i\phi_{d2}} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} e^{-i\phi_r/2} & 0 \\ 0 & e^{i\phi_r/2} \end{pmatrix}\begin{pmatrix} -r & 0 \\ 0 & r \end{pmatrix} \\ &\times \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\pi ft} \\ e^{-i\pi ft} \end{pmatrix} e^{i\phi_{d2}} = -\frac{re^{i(\pi ft+\phi_{d2}-\phi_r/2)}}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix}. \end{aligned} \tag{3}$$

Here, R_{BS}, M, and G are the reflection matrix of the BS, M, and G; r_m and r are the reflection coefficients of the M and G; and ϕ_{d1} and ϕ_{d2} are the phase variations due to the optical path lengths of the reference and the test beams, respectively. ϕ_r is the phase difference between the x and the y polarizations coming from the reflection at the BS. The associated interference signal is

$$\begin{aligned} I_A &= |E_{r1}+E_{t1}|^2 = I_{01}+\gamma_1\cdot\cos(2\pi ft+\phi_1) \\ &= \frac{1}{2}\left\{r^2+{r_m}^2-2rr_m\cos\left[2\pi ft+\frac{\pi}{2}\right.\right. \\ &\left.\left.-(\phi_{d1}-\phi_{d2}+\phi_r)\right]\right\}, \end{aligned} \tag{4}$$

where I_{01}, γ_1, and ϕ_1 are the mean intensity, visibility, and phase of the interference signal, respectively. From Eq. (4), we have

$$\phi_1 = \frac{\pi}{2}-(\phi_{d1}-\phi_{d2}+\phi_r). \tag{5}$$

Next, Q_1 with the fast axis at 45° to the x axis is inserted into the optical setup, as shown in Fig. 1, and we have a circularly polarized heterodyne light beam. Consequently, the Jones vectors of the reference beam and the test beam become

$$\begin{aligned} E_{r2} &= (AN(0°)\cdot Q_2(-45°)\cdot M\cdot Q_2(45°) \\ &\times\cdot R_{BS}\cdot Q_1(45°)\cdot E_1)\cdot e^{i\phi_{d1}} \\ &= \frac{e^{i2\pi ft}+i}{\sqrt{2}} r_m e^{i(\phi_{d1}-\pi ft+\phi_r/2)}\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \end{aligned} \tag{6}$$

$$\begin{aligned} E_{t2} &= (AN(0°)\cdot R_{BS}\cdot G\cdot Q_1(45°)\cdot E_1)\cdot e^{i\phi_{d2}} \\ &= -\frac{e^{i2\pi ft}-i}{\sqrt{2}} re^{i(\phi_{d2}-\pi ft-\phi_r/2)}\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \end{aligned} \tag{7}$$

respectively. The interference signals are

$$I_B = |E_{r2} + E_{t2}|^2 = I_{02} + \gamma_2 \cdot \cos(2\pi f t + \phi_2)$$
$$= A \cdot \cos(2\pi f t) + B \cdot \sin(2\pi f t) + C, \quad (8)$$

where I_{02}, γ_2, and ϕ_2 are the mean intensity, visibility, and phase of the interference signals, respectively:

$$A = r r_m \sin(\phi_{d1} - \phi_{d2} + \phi_r) \quad \text{and} \quad B = \frac{1}{2}({r_m}^2 - r^2). \quad (9)$$

So we have

$$\phi_2 = \tan^{-1}\left(-\frac{B}{A}\right) = \cot^{-1}\left[\frac{2 r r_m \sin(\phi_{d1} - \phi_{d2} + \phi_r)}{({r_m}^2 - r^2)}\right]. \quad (10)$$

Substituting Eq. (5) into Eq. (10), it becomes

$$r = \frac{-\cos\phi_1 + \sqrt{\cos^2\phi_1 + \cot^2\phi_2}}{\cot\phi_2} r_m, \quad (11a)$$

where n is the refractive index of the G. According to Fresnel equations [13], r can be rewritten as

$$r = \frac{n-1}{n+1}. \quad (11b)$$

From Eqs. (11a) and (11b), we have

$$n = \frac{\cot\phi_2 - r_m \cos\phi_1 + r_m\sqrt{\cos^2\phi_1 + \cot^2\phi_2}}{\cot\phi_2 + r_m \cos\phi_1 - r_m\sqrt{\cos^2\phi_1 + \cot^2\phi_2}}. \quad (12)$$

It is obvious from Eq. (12) that n can be calculated with the measurement of phases ϕ_1 and ϕ_2, assuming the experimental condition r_m is specified.

B. Phase Measurements with Heterodyne Interferometry

Because the reference signals used in conventional heterodyne interferometry for evaluating the absolute phases are difficult to apply to a two-dimensional measurement, our previously proposed technique [11] with an electro-optic modulator is introduced into this method. The configuration of the HLS is shown in Fig. 2, and it consists of a linearly polarized laser source (LS), an electro-optic modulator (EO), a linear voltage amplifier (LVA), and a function generator (FG). An external sawtooth voltage signal with period $T (T = 1/f)$ and amplitude V coming from the FG and the LVA is applied to drive the EO. The C with a frame frequency of f_c and a frame exposure time Δt is used to take s frames. We choose the condition that V is smaller than the half-wave voltage V_π, so each pixel records a group of periodic sinusoidal segment and each segment has an initial phase ψ, as shown in Fig. 3(a). Here, we have $\psi = \phi - \phi_0$, where $\phi_0 = \frac{V}{V_\pi}\pi$ is the characteristic phase and ϕ is the absolute phase to be measured. Next, let the T be lengthened to $T + (V_\pi - V)T/V$ as shown in Fig. 3(b), the interference signal can be modified to a continuous sinusoidal signal and represented as

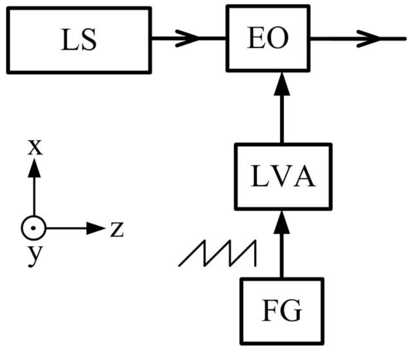

Fig. 2. Configuration of the heterodyne light source.

$$I_c(t_k) = I_0\left[1 + \gamma\cos\left(2\pi\frac{V}{V_\pi T}t_k + \psi\right)\right]. \quad (13)$$

Then, Eq. (13) is processed by using the three-parameter sine wave fitting [14] to get the phase data, and the fitted equation has the form of

$$I(t_k) = A_0 \cdot \cos(2\pi f t_k) + B_0 \cdot \sin(2\pi f t_k) + C_0$$
$$= \sqrt{{A_0}^2 + {B_0}^2} \cdot \cos(2\pi f t_k + \psi) + C_0, \quad (14)$$

where

$$\psi = \tan^{-1}\left(-\frac{B_0}{A_0}\right). \quad (15)$$

where A_0, B_0, and C_0 are real numbers and they can be derived with the least-squares method [14]. Finally, the phase ϕ can be obtained by calculating under the condition that V and V_π are specified. If these processes are applied to all other pixels, then the associated data $\phi(x, y)$ can be obtained similarly.

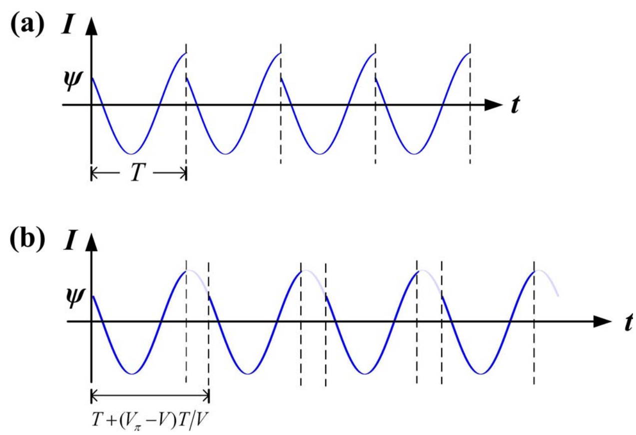

Fig. 3. (Color online) (a) Sampled interference signal as $V < V_\pi$ and (b) corresponding modified interference signal with lengthened period.

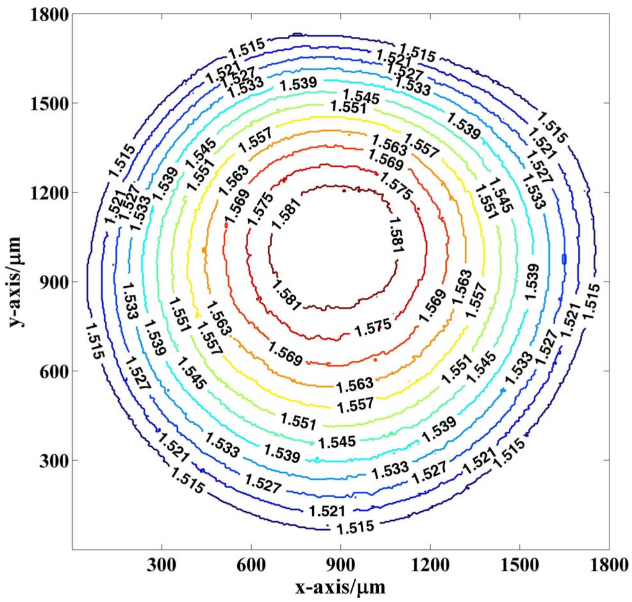

Fig. 4. (Color online) Refractive-index contour of the GRIN lens with this method.

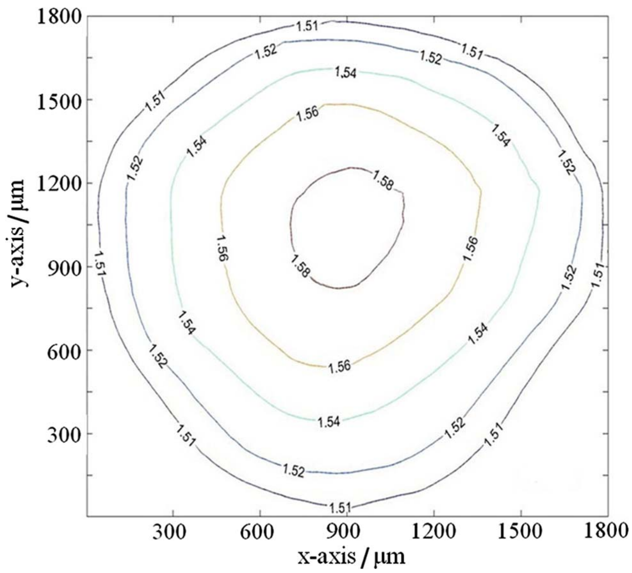

Fig. 5. (Color online) Refractive-index contour of the same GRIN lens with our previous method.

3. Experiments and Results

To show the feasibility of this method, a GRIN lens (AC Photonics, Inc./ALC-18) with a NA of 0.46, a diameter of 1.8 mm, and a length of 0.25 pitch is tested. A He–Ne laser with 632.8 nm wavelength, an electro-optic modulator (New Focus/Model 4002), a CMOS camera (Basler/A504K) with 8 bit gray levels and 600×600 pixels, a reference mirror with $r_m = 99\%$, and a 4× image lens IL were used. Under the conditions $f = 20$ Hz, $V_\pi = 144$ V, $V = 120$ V, $f_c = 300$ frames/s and $s = 300$ frames were taken in 1 s each time. The measurement was applied in turn before/after inserting the quarter-wave plate Q_1; the associated phase distributions $\phi_1(x,y)$ and $\phi_2(x,y)$ were obtained with MATLAB software (MathWorks, Inc.). Substituting them into Eq. (12), $n(x,y)$ can be estimated. For easier reading, the contour of the refractive index $n(x,y)$ is depicted in Fig. 4. For comparison, this GRIN lens was also tested with our previous method [10], and the measured contour of the refractive index is shown in Fig. 5. From these two figures, we can see that they have the same eccentric distances and a similar refractive-index distribution. Hence, both results are consistent. The deviations from axial symmetry in both results might come from the fabrication error.

4. Discussion

The errors in the phase measurements in this method may be influenced by the characteristic phase errors $\Delta\phi_0$, the sampling error $\Delta\phi_s$, and the polarization-mixing error $\Delta\phi_p$. The errors in V and V_π directly introduce a systematic error $\Delta\phi_0$ to the characteristic phase ϕ_0. The resolution of V from the power supplier is $\Delta V = 0.016$ V. Also, V_π can be measured [15], and its error is estimated as $\Delta V_\pi = 0.015$ V. Hence the error of ϕ_0 can be estimated and expressed as $\Delta\phi_0 = \phi_0 \cdot [(\Delta V/V)^2 + (\Delta V_\pi/V_\pi)^2]^{1/2}$. Substituting our measurement results $\phi_1 \approx 0°$ and $63.5° \le \phi_2 \le 67.5°$ in turn into ϕ_0, the associated errors can be derived as $\Delta\phi_{01} = 0°$ and $\Delta\phi_{02} \approx 0.01°$, respectively. Besides, $\Delta\phi_s = 0.036°$ and $\Delta\phi_p = 0.03°$ can be estimated in our previous paper [11].

Consequently, the total errors of $\Delta\phi_1$ and $\Delta\phi_2$ are $\Delta\phi_1 = \Delta\phi_{01} + \Delta\phi_s + \Delta\phi_p = 0.066°$ and $\Delta\phi_2 = \Delta\phi_{02} + \Delta\phi_s + \Delta\phi_p = 0.076°$, respectively. From Eq. (12), the error of refractive-index measurement in this method can be derived and expressed as

$$\Delta n = \left|\left(\frac{\partial n}{\partial \phi_1}\right) \cdot \Delta\phi_1\right| + \left|\left(\frac{\partial n}{\partial \phi_2}\right) \cdot \Delta\phi_2\right| = \frac{r_m |\sin\phi_1 \sin 2\phi_2 \Delta\phi_1 + 2\cos\phi_1 \Delta\phi_2| \left|\sqrt{\cos^2\phi_1 + \cot^2\phi_2} - \cos\phi_1\right|}{\sqrt{\cos^2\phi_1 + \cot^2\phi_2}\left[\cos\phi_2 + r_m \sin\phi_2\left(\cos\phi_1 - \sqrt{\cos^2\phi_1 + \cot^2\phi_2}\right)\right]^2}. \tag{16}$$

Substituting $r_m = 99\%$, $\phi_1 \approx 0°$, and $63.5° \leq \phi_2 \leq 67.5°$ into Eq. (16), $\Delta n \approx 0.002$ can be obtained. Besides, the reflectance of the mirror M might influence the measurement resolution. Substituting $\phi_1 \approx 0°$, $\phi_2 = 64°$, and $90\% \leq r_m \leq 99\%$ into Eq. (16), the variation of Δn does not exceed 10^{-5}; therefore, this issue can be neglected.

In our previous oblique incidence method [10], the camera should be set obliquely to match the image plane. Consequently, it may have the magnification error and low image quality. In addition, it is difficult to set the camera properly. Although it is easier to align the imaging system and get clear images in this method, two phase measurement procedures need to be operated. Moreover, its measurement error is smaller than that of the oblique incidence method ($\Delta n = 0.021 - 0.025$).

5. Conclusion

An alternative method for measuring the full-field refractive-index distribution of the GRIN lens has been presented in this paper. A linearly/circularly polarized heterodyne light beam in turn enters a modified Twyman–Green interferometer in which a GRIN lens is located in one arm for test. Two groups of full-field interference signals are taken by a fast CMOS camera. The sampling intensities recorded at each pixel are fitted to derive a sinusoidal signal, and its associated phase can be calculated with the unique technique. Then, substituting these two groups of phase distribution data into the special equations derived from the Fresnel equations, the refractive index at that pixel can be estimated. The processes are applied to other pixels, and then the full-field refractive-index distribution can be obtained similarly. The validity has been demonstrated.

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