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Semiclassical analysis of string-gauge duality on noncommutative space

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We use semiclassical methods to study closed strings in the modified $AdS_5 \times S^5$ background with constant B fields. The pointlike closed strings and the stretched closed strings rotating around the big circle of S^5 are considered. Quantization of these closed string leads to a time-dependent string spectrum, which we argue corresponds to the renormalization-group flow of the dual noncommutative Yang-Mills theory.

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I. INTRODUCTION

The duality between string theories in AdS space and conformal field theories on the boundary of the AdS space (AdS/CFT) [1] is a powerful method to study field theories by studying their dual string theories. Usually, full string theories in the curved background, such as $AdS_5 \times S^5$, are not solvable. We have to study their low energy effective theories, i.e., supergravity theories. According to infrared/ultraviolet (IR/UV) duality, the low energy region in the string theories is dual to the high energy region in the corresponding field theories side. However, the low energy region of the field theory is more attractive for a practical reason. To study the low energy region of these field theories, we need to solve the full string theory. It was shown that, by taking the Penrose limit, $AdS_5 \times S^5$ geometry reduces to a pp-wave geometry [2], which is solvable for the full string theory.

The success of the pp-wave method for $AdS_5 \times S^5$ background led many authors to use it to study some more complicated geometry backgrounds, e.g. [3]. For example, the near horizon geometry of NS5-branes is the linear dilaton background. String theory in this background was conjectured to be dual to a nonlocal field theory [4], so-called little string theory. It was shown that the pp-wave limit of this background is the Nappi-Witten background. String theory in Nappi-Witten background is again exactly solvable and has been widely studied; see, for example, [5,6]. A more complicated example is the pp-wave limit of the near horizon geometry of nonextremal NS5-branes, which leads to a time-dependent background. Luckily, strings in this time-dependent background are also exactly solvable and have been studied in [5,7]. Another nonlocal gauge theory example is noncommutative Yang-Mills theories (NCYM), which are conjectured to be dual to the near horizon geometry of N coinciding Dp-branes in the presence of the background constant B fields. Geometries with different number p have been studied in [5] and more extensively in [8], in which the authors claimed that, only for D6-brane, the pp-wave limit of the geometry leads to a *time independent* background and is solvable. In this work, we will show that, for general Dp-branes, the pp-wave limit of the geometry leads to time-dependent backgrounds which are solvable in certain limits. Studying string theories in these backgrounds will enable us to understand more about the dual theory-noncommutative Yang-Mills theory, such as IR/UV mixing, nonlocal behavior, etc. In addition, through the holography relationship, we can use the time-dependent pp-wave background to study the features of renormalization-group (RG) flow in NCYM.

In the next section, we study the supergravity solution with constant Neveu-Schwarz (NS) B fields. In Sec. III, the semiclassical method is used to study the first order fluctuation around some classical string configurations. The Hamiltonian will be obtained for the fluctuation. We then quantize the model to get the string spectrum in Sec. IV. Section V contains discussion on the results.

II. SUPERGRAVITY SOLUTION WITH CONSTANT B FIELDS

In this section we consider a system of Dp-branes with a constant NS B along their world-volume directions. For simplicity, we consider the B field only along the directions of x_B^2 and x_B^3 in the D-brane; the supergravity solution can be obtained by performing a T duality along x^3 and then T dualize back. The solution in string metric is [9,10]

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$$ds_B^2 = f^{-1/2}[-dx_0^2 + dx_1^2 + h(dx_{B2}^2 + dx_{B3}^2)] + f^{1/2}(dr^2 + r^2 d\Omega_5^2), \quad (1)$$

where

$$f = 1 + \frac{\alpha'^2 R^4}{r^4}, \qquad h^{-1} = \sin^2\theta f^{-1} + \cos^2\theta,$$
$$B_{23} = \frac{\sin\theta}{\cos\theta} f^{-1} h, \qquad e^{2\phi} = g^2 h, \quad (2)$$
$$F_{01r} = \frac{1}{g}\sin\theta \partial_r f^{-1}, \qquad F_{0123r} = \frac{1}{g}\cos\theta h \partial_r f^{-1}.$$

To study the dual noncommutative Yang-Mills theory on the boundary, we need to take the background B field to infinity $B_{23} \to \infty$. Following the procedure in [9,10], we first rescale the parameters as follows:

$$x_{0,1} \to x_{0,1}, \quad x_{B2,3} \to \left(\frac{\alpha'}{b}\right)^{-1} x_{B2,3}, \quad B_{23} \to \left(\frac{\alpha'}{b}\right)^2 B_{23},$$
$$\tan\theta = \frac{b}{\alpha'}, \quad r = \alpha' R^2 u, \quad g \to \frac{\alpha'}{b} g, \quad (3)$$

then we take the decoupling limit $\alpha' \to 0$ with b fixed. The metric now can be written as

$$ds_B^2 = \alpha' R^2 \left[u^2(-dx_0^2 + dx_1^2) + \frac{u^2}{1 + a^4u^4} \times (dx_{B2}^2 + dx_{B3}^2) + \frac{du^2}{u^2} + d\Omega_5^2 \right], \quad (4)$$

where

$$a^2 = bR^2, \quad B_{23} = \frac{\alpha'}{b}\frac{a^4u^4}{1+a^4u^4}, \quad e^{2\phi} = \frac{g^2}{1+a^4u^4},$$
$$A_{01} = \alpha' \frac{b}{g} u^4 R^4, \quad F_{0123u} = \frac{\alpha'^2 \partial_u(u^4R^4)}{g(1+a^4u^4)}. \quad (5)$$

We will consider the sigma model in the above background

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma (\sqrt{h} h^{ab} G_{mn} \partial_a X^m \partial_b X^n + \epsilon^{ab} B_{ij} \partial_a X_B^i \partial_b X_B^j), \quad (6)$$

where the metric G_{mn} and the constant B_{ij} are given by Eqs. (4) and (5), respectively. Let us make some transformations to bring the metric to a more convenient form. First of all, we set $R = 1$ for simplicity and define the variables

$$x_B^i = \sqrt{1 + a^4u^4} x^i, \qquad i = 2, 3. \quad (7)$$

In these notations the metric can be written as

$$ds_B^2 = ds^2 + \frac{4a^8 x^{i2} u^8}{(1 + a^4u^4)^2} du^2 + \frac{4a^4u^5x^i}{1 + a^4u^4} dx^i du, \quad (8)$$

where

$$ds^2 = u^2(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{du^2}{u^2} + d\Omega_5^2 \quad (9)$$

is the standard $AdS_5 \times S^5$ metric. We call the above metric (8) modified $AdS_5 \times S^5$ metric because the only deviation of (9) from the standard $AdS_5 \times S^5$ metric is given by some B field dependent terms. Later we will consider the case of $a \to 0$, where the modification from the standard $AdS_5 \times S^5$ metric is small provided u is not too large.

Next, to write the above metric in "global" coordinates, we use the transformations

$$u = \cosh\rho \cos t - \sinh\rho \Omega_4,$$
$$x^0 = \frac{\cosh\rho \sin t}{\cosh\rho \cos t - \sinh\rho \Omega_4}, \quad (10)$$
$$x^\mu = \frac{\sinh\rho \Omega_\mu}{\cosh\rho \cos t - \sinh\rho \Omega_4},$$

where the four-vector Ω satisfies the normalization condition $\Omega^2 = 1$ and the explicit form of its components is

$$\Omega_1 = \cos\beta_1 \cos\beta_2 \cos\beta_3,$$
$$\Omega_2 = \cos\beta_1 \cos\beta_2 \sin\beta_3,$$
$$\Omega_3 = \cos\beta_1 \sin\beta_2, \quad (11)$$
$$\Omega_4 = \sin\beta_1.$$

The metric now takes the form as

$$ds_B^2 = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\Omega_3^2 + d\psi_1^2 + \cos^2\psi_1(d\psi_2^2 + \cos^2\psi_2 d\Omega_3'^2) + \frac{4a^8x^{i2}u^8}{(1 + a^4u^4)^2} du^2 + \frac{4a^4u^5x^i}{1 + a^4u^4} dx^i du, \quad (12)$$

where

$$d\Omega_3^2 = d\beta_1^2 + \cos^2\beta_1(d\beta_2^2 + \cos^2\beta_2 d\beta_3^2),$$
$$d\Omega_3'^2 = d\psi_3^2 + \cos^2\psi_3(d\psi_4^2 + \cos^2\psi_4 d\psi_5^2). \quad (13)$$

In the final form of the metric (12), to keep the metric neat, we did not write dx^i and du in terms of the new coordinates explicitly in the last two terms. The reader should be careful that there are only ten independent coordinates in the metric (12) since u and x^i $(i = 2, 3)$ are not independent coordinates.

III. SEMICLASSICAL ANALYSIS

In this section we will study two classical solutions of the sigma model (6) in the modified $AdS_5 \times S^5$ background (12) which we discussed in the previous section. One classical solution corresponds to pointlike closed strings rotating around the big circle of S^5, and the other corresponds to stretched closed strings rotating around the big circle of S^5. We then consider the first order fluctuations around these classical solutions to get the transverse Hamiltonian.

A. Pointlike closed string rotating in S^5

It is easy to verify that

$$t = \nu\tau, \qquad \rho = 0, \qquad \beta_l = 0 \qquad (l = 1, 2, 3),$$
$$\varphi(\equiv \psi_5) = \nu\tau, \qquad \psi_s = 0 \qquad (s = 1, 2, 3, 4) \tag{14}$$

is a solution of the sigma model (6) corresponding to the metric (12). This solution describes a pointlike ($\rho = 0$) closed string boosting around the big circle of S^5.

To find one-loop approximation, we will consider the fluctuations around the above classical solution and expand them to the first order. It is useful to replace (ρ, β_l) by four Cartesian coordinates $\eta_k (k = 1, 2, 3, 4)$ as in [11,12]

$$\eta_k = 2r\Omega_k, \qquad \frac{2r}{1 - r^2} = \sinh\rho. \tag{15}$$

Written in these Cartesian coordinates, the metric reads

$$ds_B^2 = -\left(\frac{1 + \eta^2/4}{1 - \eta^2/4}\right)^2 dt^2 + \frac{1}{(1 - \eta^2/4)^2} d\eta_k^2 + d\psi_1^2 + \cos^2\psi_1 (d\psi_2^2 + \cos^2\psi_2 d\Omega_3'^2) + \frac{4a^8 x^{i2} u^8}{(1 + a^4 u^4)^2} du^2 + \frac{4a^4 u^5 x^i}{1 + a^4 u^4} dx^i du, \tag{16}$$

and the transformations (10) become

$$u = \frac{1 + \eta^2/4}{1 - \eta^2/4} \cos t - \frac{\eta_4}{1 - \eta^2/4},$$
$$x^0 = \frac{1 + \eta^2/4 \sin t}{(1 + \eta^2/4)\cos t - \eta_4}, \tag{17}$$
$$x^\mu = \frac{\eta_\mu}{(1 + \eta^2/4)\cos t - \eta_4}.$$

Now, we are ready to consider the fluctuations around the above classical solution (14) with large sigma model coupling constant λ,

$$t = \nu\tau + \frac{1}{\lambda^{1/4}} \tilde{t}, \qquad \varphi = \nu\tau + \frac{1}{\lambda^{1/4}} \tilde{\varphi},$$
$$\eta_k = \frac{1}{\lambda^{1/4}} \tilde{\eta}_k, \qquad \psi_s = \frac{1}{\lambda^{1/4}} \tilde{\psi}_s. \tag{18}$$

Up to quadratic fluctuations, the Virasoro constraints of the sigma model can be written as

$$T_{aa} = G_{mn} \partial_a X^m \partial_a X^n = \frac{1}{\sqrt{\lambda}} \Big[-2\lambda^{1/4} \nu \partial_\tau (\tilde{t} - \tilde{\varphi}) - \partial_a \tilde{t} \partial^a \tilde{t} + \partial_a \tilde{\varphi} \partial^a \tilde{\varphi} - \nu^2 (\tilde{\eta}^2 + \tilde{\psi}_s^2) + \partial_a \tilde{\eta}_k \partial^a \tilde{\eta}_k + \partial_a \tilde{\psi}_s \partial^a \tilde{\psi}_s - \frac{4a^4 \sin^2\nu\tau \cos^2\nu\tau}{(1 + a^4 \cos^4\nu\tau)^2} \nu^2 \tilde{\eta}_i^2 - \frac{4a^4 \sin\nu\tau \cos^3\nu\tau}{1 + a^4 \cos^4\nu\tau} \nu \tilde{\eta}_i \partial_\tau \tilde{\eta}_i \Big] = 0. \tag{19}$$

The energy and angular momentum then are given as

$$E = P_t = 2 \int_0^{2\pi} \frac{d\sigma}{2\pi} \Big[\sqrt{\lambda}\nu + \lambda^{1/4} \partial_\tau \tilde{t} + \nu \tilde{\eta}^2 + \frac{4a^4 \sin^2\nu\tau \cos^2\nu\tau}{(1 + a^4 \cos^4\nu\tau)^2} \nu \tilde{\eta}_i^2 + \frac{2a^4 \sin\nu\tau \cos^3\nu\tau}{1 + a^4 \cos^4\nu\tau} \tilde{\eta}_i \partial_\tau \tilde{\eta}_i + \frac{4\alpha'}{b} \frac{a^8 \sin\nu\tau \cos^5\nu\tau}{1 + a^4 \cos^4\nu\tau} (\tilde{\eta}^2 \partial_\sigma \tilde{\eta}^3 - \tilde{\eta}^3 \partial_\sigma \tilde{\eta}^2) \Big], \tag{20}$$

$$J = P_\varphi = 2 \int_0^{2\pi} \frac{d\sigma}{2\pi} [\sqrt{\lambda}\nu + \lambda^{1/4} \partial_\tau \tilde{\varphi} - \nu \tilde{\psi}_s^2], \tag{21}$$

and the difference between the energy and angular momentum can be obtained as

$$E - J = \int_0^{2\pi} \frac{d\sigma}{2\pi} \Big[\lambda^{1/4} \partial_\tau (\tilde{t} - \tilde{\varphi}) + \nu (\tilde{\eta}^2 + \tilde{\psi}_s^2) + \frac{4a^4 \sin^2\nu\tau \cos^2\nu\tau}{(1 + a^4 \cos^4\nu\tau)^2} \nu \tilde{\eta}_i^2 + \frac{2a^4 \sin\nu\tau \cos^3\nu\tau}{1 + a^4 \cos^4\nu\tau} \tilde{\eta}_i \partial_\tau \tilde{\eta}_i + \frac{4\alpha'}{b} \times \frac{a^8 \sin\nu\tau \cos^5\nu\tau}{1 + a^4 \cos^4\nu\tau} (\tilde{\eta}^2 \partial_\sigma \tilde{\eta}^3 - \tilde{\eta}^3 \partial_\sigma \tilde{\eta}^2) \Big]. \tag{22}$$

Now we can solve for $\partial_\tau(\tilde{t} - \tilde{\varphi})$ from the constraint (19), and plug it into the above expression (22). After rescaling the fields by $\sqrt{2}\lambda^{1/4}$, we end up with

$$E - J = \frac{1}{2\nu} \int_0^{2\pi} \frac{d\sigma}{2\pi} \Big[-\partial_a \tilde{t} \partial^a \tilde{t} + \partial_a \tilde{\varphi} \partial^a \tilde{\varphi} + \nu^2 (\tilde{\eta}^2 + \tilde{\psi}_s^2) + \partial_a \tilde{\eta}_k \partial^a \tilde{\eta}_k + \partial_a \tilde{\psi}_s \partial^a \tilde{\psi}_s + \frac{4a^4 \sin^2\nu\tau \cos^2\nu\tau}{(1 + a^4 \cos^4\nu\tau)^2} \nu^2 \tilde{\eta}_i^2 + \frac{4\alpha'}{b} \frac{a^8 \sin\nu\tau \cos^5\nu\tau}{1 + a^4 \cos^4\nu\tau} \times (\tilde{\eta}^2 \partial_\sigma \tilde{\eta}^3 - \tilde{\eta}^3 \partial_\sigma \tilde{\eta}^2) \Big] \equiv \frac{1}{\nu} \int_0^{2\pi} \frac{d\sigma}{2\pi} \mathcal{H}^{(2)}. \tag{23}$$

Thus, $E - J$ is given by the expectation value of the transverse Hamiltonian

$$\mathcal{H}^{(2)} = \frac{1}{2}\Big[-\partial_a\tilde{t}\partial^a\tilde{t} + \partial_a\tilde{\varphi}\partial^a\tilde{\varphi} + \nu^2(\tilde{\eta}^2 + \tilde{\psi}_s^2) + \partial_a\tilde{\eta}_k\partial^a\tilde{\eta}_k + \partial_a\tilde{\psi}_s\partial^a\tilde{\psi}_s + \frac{4a^4\sin^2\nu\tau\cos^2\nu\tau}{(1+a^4\cos^4\nu\tau)^2}\nu^2\tilde{\eta}_i^2 + \frac{4\alpha'}{b}\frac{a^8\sin\nu\tau\cos^5\nu\tau}{1+a^4\cos^4\nu\tau} \times(\tilde{\eta}^2\partial_\sigma\tilde{\eta}^3 - \tilde{\eta}^3\partial_\sigma\tilde{\eta}^2)\Big]. \quad (24)$$

It is clear that there are eight massive bosonic coordinates with two of them ($\tilde{\eta}_i$, $i = 2, 3$) time dependent. Defining $\tilde{x}^\pm = \tilde{t} \pm \tilde{\varphi}$, the transverse Hamiltonian in Eq. (24) can be written in ''light-cone-like'' form

$$\mathcal{H}^{(2)} = \frac{1}{2}\Big[-\partial_a\tilde{x}^+\partial^a\tilde{x}^- - \frac{1}{4}(\tilde{\eta}^2 + \tilde{\psi}_s^2)\partial_a\tilde{x}^+\partial^a\tilde{x}^+ + \partial_a\tilde{\eta}_k\partial^a\tilde{\eta}_k + \partial_a\tilde{\psi}_s\partial^a\tilde{\psi}_s + \frac{4a^4\sin^2\nu\tau\cos^2\nu\tau}{(1+a^4\cos^4\nu\tau)^2}\nu^2\tilde{\eta}_i^2 + \frac{4\alpha'}{b}\frac{a^8\sin\nu\tau\cos^5\nu\tau}{1+a^4\cos^4\nu\tau} \times(\tilde{\eta}^2\partial_\sigma\tilde{\eta}^3 - \tilde{\eta}^3\partial_\sigma\tilde{\eta}^2)\Big]. \quad (25)$$

A similar Hamiltonian can be also obtained by taking the Penrose limit as in [5].

B. Closed string rotating in S^5

In this section we will consider another classical solution of a closed string whose center of mass is not moving on S^5, but spins around that point and is correspondingly stretched. Again, it is easy to verify that the ansatz

$$t=\nu\tau, \quad \rho=0, \quad \beta_l=0 \quad (l=1,2,3), \quad \theta(\equiv\psi_1)=\theta(\sigma), \quad \varphi(\equiv\psi_5)=\nu\tau, \quad \psi_s=0 \quad (s=2,3,4) \quad (26)$$

is a solution for the sigma model (6) associated with the modified $AdS_5\times S^5$ background (12).

The constraint for θ can be found from the Virasoro constraint $T_{ab}=0$, which is

$$(\theta')^2 = \nu^2 - \nu^2\cos^2\theta = \nu^2\sin^2\theta. \quad (27)$$

Now we would like to repeat the analysis of the previous section in the case of the more general solution (26). To determine the quantum string spectrum to the leading order in the large sigma model coupling constant λ, we consider the quantum fluctuation around the classical solution (26)

$$t = \nu\tau + \frac{1}{\lambda^{1/4}}\tilde{t}, \qquad \varphi = \nu\tau + \frac{1}{\lambda^{1/4}}\tilde{\varphi}, \qquad \theta = \theta(\sigma) + \frac{1}{\lambda^{1/4}}\tilde{\theta}, \qquad \eta_k = \frac{1}{\lambda^{1/4}}\tilde{\eta}_k, \qquad \psi_s = \frac{1}{\lambda^{1/4}}\tilde{\psi}_s \qquad (s=2,3,4). \quad (28)$$

Up to quadratic fluctuations, the constraints of the sigma model can be written as

$$T_{aa} = G_{mn}\partial_aX^m\partial_aX^n = \frac{1}{\sqrt{\lambda}}\Big[-2\lambda^{1/4}\nu(\partial_\tau\tilde{t} - \cos^2\theta\partial_\tau\tilde{\varphi}) - \partial_a\tilde{t}\partial^a\tilde{t} + \cos^2\theta\partial_a\tilde{\varphi}\partial^a\tilde{\varphi} + 2\lambda^{1/4}\nu\sin\theta\partial_\sigma\tilde{\theta} - 2\lambda^{1/4}\nu^2\sin\theta\cos\theta\cdot\tilde{\theta} - 4\nu\sin\theta\cos\theta\cdot\tilde{\theta}\partial_\tau\tilde{\varphi} - \nu^2(\cos^2\theta - \sin^2\theta)\tilde{\theta}^2 - \nu^2(\tilde{\eta}^2 + \cos^2\theta\cdot\tilde{\psi}_s^2) + \partial_a\tilde{\eta}_k\partial^a\tilde{\eta}_k + \partial_a\tilde{\theta}\partial^a\tilde{\theta} + \cos^2\theta\partial_a\tilde{\psi}_s\partial^a\tilde{\psi}_s - \frac{4a^4\sin^2\nu\tau\cos^2\nu\tau}{(1+a^4\cos^4\nu\tau)^2}\nu^2\tilde{\eta}_i^2 - \frac{4a^4\sin\nu\tau\cos^3\nu\tau}{1+a^4\cos^4\nu\tau}\nu\tilde{\eta}_i\partial_\tau\tilde{\eta}_i\Big] = 0, \quad (29)$$

where we have used the constraint (27) to eliminate θ'.

The energy and angular momentum are correspondingly

$$E = P_t = 2\int_0^{2\pi}\frac{d\sigma}{2\pi}\Big[\sqrt{\lambda}\nu + \lambda^{1/4}\partial_\tau\tilde{t} + \nu\tilde{\eta}^2 + \frac{4a^4\sin^2\nu\tau\cos^2\nu\tau}{(1+a^4\cos^4\nu\tau)^2}\nu\tilde{\eta}_i^2 + \frac{2a^4\sin\nu\tau\cos^3\nu\tau}{1+a^4\cos^4\nu\tau}\tilde{\eta}_i\partial_\tau\tilde{\eta}_i + \frac{4\alpha'}{b}\frac{a^8\sin\nu\tau\cos^5\nu\tau}{1+a^4\cos^4\nu\tau}(\tilde{\eta}^2\partial_\sigma\tilde{\eta}^3 - \tilde{\eta}^3\partial_\sigma\tilde{\eta}^2)\Big], \quad (30)$$

$$J = P_\varphi = 2\int_0^{2\pi}\frac{d\sigma}{2\pi}[\sqrt{\lambda}\nu\cos^2\theta + \lambda^{1/4}\cos^2\theta\partial_\tau\tilde{\varphi} - 2\lambda^{1/4}\nu\sin\theta\cos\theta\cdot\tilde{\theta} - 2\sin\theta\cos\theta\cdot\tilde{\theta}\partial_\tau\tilde{\varphi} - \nu(\cos^2\theta - \sin^2\theta)\tilde{\theta}^2 - \nu\cos^2\theta\tilde{\psi}_s^2], \quad (31)$$

therefore the difference between energy and angular momentum can be easily calculated:

$$E - J = \int_0^{2\pi} \frac{d\sigma}{2\pi}\Big[\sqrt{\lambda}\nu\sin^2\theta + \lambda^{1/4}(\partial_\tau \tilde{t} - \cos^2\theta\partial_\tau\tilde{\varphi}) + 2\lambda^{1/4}\nu\sin\theta\cos\theta\cdot\tilde{\theta} + 2\sin\theta\cos\theta\tilde{\theta}\partial_\tau\tilde{\varphi} + \nu(\cos^2\theta - \sin^2\theta) \times\tilde{\theta}^2 + \nu(\tilde{\eta}^2 + \cos^2\theta\tilde{\psi}_s^2) + \frac{4a^4\sin^2\nu\tau\cos^2\nu\tau}{(1+a^4\cos^4\nu\tau)^2}\nu\tilde{\eta}_i^2 + \frac{2a^4\sin\nu\tau\cos^3\nu\tau}{1+a^4\cos^4\nu\tau}\tilde{\eta}_i\partial_\tau\tilde{\eta}_i + \frac{4\alpha'}{b}\frac{a^8\sin\nu\tau\cos^5\nu\tau}{1+a^4\cos^4\nu\tau} \times(\tilde{\eta}^2\partial_\sigma\tilde{\eta}^3 - \tilde{\eta}^3\partial_\sigma\tilde{\eta}^2)\Big]. \quad (32)$$

As before, we can solve for $(\partial_\tau\tilde{t} - \cos^2\theta\partial_\tau\tilde{\varphi})$ from the constraint (29), and plug it into the above expression (32). After rescaling the fields by $\sqrt{2}\lambda^{1/4}$, we obtain

$$E - J = \frac{1}{2\nu}\int_0^{2\pi}\frac{d\sigma}{2\pi}\Big[-\partial_a\tilde{t}\partial^a\tilde{t} + \cos^2\theta\partial_a\tilde{\varphi}\partial^a\tilde{\varphi} + \nu^2(\cos^2\theta - \sin^2\theta)\tilde{\theta}^2 + \partial_a\tilde{\theta}\partial^a\tilde{\theta} + \cos^2\theta(\nu^2\tilde{\psi}_s^2 + \partial_a\tilde{\psi}_s\partial^a\tilde{\psi}_s) + \nu^2\tilde{\eta}^2 + \partial_a\tilde{\eta}_k\partial^a\tilde{\eta}_k + \frac{4a^4\sin^2\nu\tau\cos^2\nu\tau}{(1+a^4\cos^4\nu\tau)^2}\nu^2\tilde{\eta}_i^2 + \frac{4\alpha'}{b}\frac{a^8\sin\nu\tau\cos^5\nu\tau}{1+a^4\cos^4\nu\tau}(\tilde{\eta}^2\partial_\sigma\tilde{\eta}^3 - \tilde{\eta}^3\partial_\sigma\tilde{\eta}^2) + 2\nu^2\sin\theta\partial_\sigma\tilde{\theta} + 2\nu^2\sin\theta\cos\theta\cdot\tilde{\theta} + 2\nu^2\sin^2\theta\Big] \equiv \frac{1}{\nu}\int_0^{2\pi}\frac{d\sigma}{2\pi}\mathcal{H}^{(2)}. \quad (33)$$

Thus, $E - J$ is given by the expectation value of the transverse Hamiltonian

$$\mathcal{H}^{(2)} = -\partial_a\tilde{t}\partial^a\tilde{t} + \cos^2\theta\partial_a\tilde{\varphi}\partial^a\tilde{\varphi} + \nu^2(\cos^2\theta - \sin^2\theta)\tilde{\theta}^2 + \partial_a\tilde{\theta}\partial^a\tilde{\theta} + \cos^2\theta(\nu^2\tilde{\psi}_s^2 + \partial_a\tilde{\psi}_s\partial^a\tilde{\psi}_s) + \nu^2\tilde{\eta}^2 + \partial_a\tilde{\eta}_k\partial^a\tilde{\eta}_k + \frac{4a^4\sin^2\nu\tau\cos^2\nu\tau}{(1+a^4\cos^4\nu\tau)^2}\nu^2\tilde{\eta}_i^2 + \frac{4\alpha'}{b}\frac{a^8\sin\nu\tau\cos^5\nu\tau}{1+a^4\cos^4\nu\tau}(\tilde{\eta}^2\partial_\sigma\tilde{\eta}^3 - \tilde{\eta}^3\partial_\sigma\tilde{\eta}^2) + 2\nu^2\sin\theta\partial_\sigma\tilde{\theta} + 2\nu^2\sin\theta\cos\theta\cdot\tilde{\theta} + 2\nu^2\sin^2\theta. \quad (34)$$

The following transformation

$$\bar{t} = \tilde{t}, \quad \bar{\varphi} = \cos\theta\tilde{\varphi}, \quad \bar{\psi}_s = \cos\theta\tilde{\psi}_s (s = 2, 3, 4), \quad \bar{\theta} = \tilde{\theta}, \quad \bar{\eta}_k = \tilde{\eta}_k \quad (k = 0, 1, 2, 3), \quad (35)$$

will bring the kinetic terms to canonical form. The resulting Hamiltonian is

$$\mathcal{H}^{(2)} = -\partial_a\bar{t}\partial^a\bar{t} + \partial_a\bar{\varphi}\partial^a\bar{\varphi} + \partial_a\bar{\theta}\partial^a\bar{\theta} + \partial_a\bar{\eta}_k\partial^a\bar{\eta}_k + \partial_a\bar{\psi}_s\partial^a\bar{\psi}_s + m_\varphi^2\bar{\varphi}^2 + m_\theta^2\bar{\theta}^2 + m_\psi^2\bar{\psi}_s^2 + m_\lambda^2\bar{\eta}_\lambda^2 + m_i^2\bar{\eta}_i^2 + \frac{4\alpha'}{b}\frac{a^8\sin\nu\tau\cos^5\nu\tau}{1+a^4\cos^4\nu\tau}(\bar{\eta}^2\partial_\sigma\bar{\eta}^3 - \bar{\eta}^3\partial_\sigma\bar{\eta}^2) + \frac{2\nu\sin^2\theta}{\cos\theta}(\bar{\varphi}\partial_\sigma\bar{\varphi} + \bar{\psi}_s\partial_\sigma\bar{\psi}_s) + 2\nu^2\sin\theta(1 + \cos\theta\cdot\bar{\theta} + \partial_\sigma\bar{\theta}), \quad (36)$$

where

$$m_\varphi^2 = \frac{\nu^2\sin^4\theta}{\cos^2\theta}, \quad (37)$$

$$m_\theta^2 = \nu^2(\cos^2\theta - \sin^2\theta), \quad (38)$$

$$m_\psi^2 = \nu^2\left(1 + \frac{\sin^4\theta}{\cos^2\theta}\right), \quad (39)$$

$$m_\lambda^2 = \nu^2, \qquad \lambda = 0, 1, \quad (40)$$

$$m_i^2 = \nu^2\left(1 + \frac{4a^4\sin^2\nu\tau\cos^2\nu\tau}{(1+a^4\cos^4\nu\tau)^2}\right), \qquad i = 2, 3. \quad (41)$$

It is clear that when $\theta(\sigma) = 0$, which corresponds to the pointlike closed string case in the previous section, the Hamiltonian (36) reduces to the Hamiltonian (24) as expected.

IV. QUANTIZATION

In this section, we will quantize the classical systems we studied in the previous sections to compute the full closed string spectrum. We see that, when $a = 0$, which corresponds to the case of the vanishing background B field, the Hamiltonians (24) and (36) will reduce to the closed strings moving in the standard $AdS_5 \times S^5$ background. The quantization of the closed strings moving in the standard $AdS_5 \times S^5$ background has been widely studied by many authors, for example, in [11,12]. To quantize the classical closed string systems in the modified $AdS_5 \times S^5$ background (12), we need to solve the equations of motion corresponding to the Hamiltonians (24) and (36), respectively. Since the difference between our current case and the case of the closed strings moving in the standard $AdS_5 \times S^5$ background is the a dependent terms, we need only to solve for the fields whose equations of motion are affected by a. It is easy to see in the Hamiltonians (24) and (36) that all the fields have the exact same equations of motion as the case without the background B field except for the fields $\tilde{\eta}_i$, $i = 2, 3$, which are the fields along the directions with the B field turned on. The Hamiltonians for the fields $\tilde{\eta}_i$, $i = 2, 3$ coming from (24) and (36) turn out to be in the same form, which is

$$\mathcal{H}_{\tilde{\eta}} = \frac{1}{2}\int \frac{d\sigma}{2\pi}\bigg[\partial_\tau\tilde{\eta}_i\partial_\tau\tilde{\eta}_i + \partial_\sigma\tilde{\eta}_i\partial_\sigma\tilde{\eta}_i + \bigg(1 + \frac{4a^4\sin^2\nu\tau\cos^2\nu\tau}{(1 + a^4\cos^4\nu\tau)^2}\bigg)\nu^2\tilde{\eta}_i^2 + \frac{4\alpha'}{b} \times \frac{a^8\sin\nu\tau\cos^5\nu\tau}{1 + a^4\cos^4\nu\tau}(\tilde{\eta}^2\partial_\sigma\tilde{\eta}^3 - \tilde{\eta}^3\partial_\sigma\tilde{\eta}^2)\bigg]. \tag{42}$$

It is hard to solve the equations of motion corresponding to the above Hamiltonian (42) because of the complicated time-dependent coefficient. To proceed, we make an approximation by letting $a \to 0$ but still *finite*. Notice that according to the decoupling limit we took in (3) and the definitions (2) and (5), any *finite* value of a, no matter how small it is, corresponds to the infinite value of the background B field, i.e., $B_{23} \to \infty$. This is important for us because only for the infinite background B field the dual field theory on the boundary is noncommutative field theory. Under the limit $a \to 0$, we expand the Hamiltonian (42) in a and keep the lowest order. The Hamiltonian now is simplified to the form

$$\mathcal{H}_{\tilde{\eta}} = \int \frac{d\sigma}{2\pi}[\partial_\tau\tilde{\eta}_i\partial_\tau\tilde{\eta}_i + \partial_\sigma\tilde{\eta}_i\partial_\sigma\tilde{\eta}_i + (1 + a^4\sin^2 2\nu\tau)\nu^2\tilde{\eta}_i^2]. \tag{43}$$

It is easy to get the equations of motion for the fields $\tilde{\eta}_i$:

$$\partial_\tau^2\tilde{\eta}_i + \partial_\sigma^2\tilde{\eta}_i + (1 + a^4\sin^2 2\nu\tau)\nu\tilde{\eta}_i^2 = 0. \tag{44}$$

We expand[1] the field $\tilde{\eta}$ in the different modes as $\tilde{\eta} = \sum \tilde{\eta}_n e^{in\sigma}$; the equation of motion for each mode $\tilde{\eta}_n$ can be obtained as

$$\frac{\partial^2\tilde{\eta}_n}{\partial z^2} + (\lambda_n - 2q\cos 2z)\tilde{\eta}_n = 0, \tag{45}$$

where

$$\lambda_n = \frac{1}{4\nu^2}\bigg(n^2 + \nu^2 + \frac{\nu^2}{2}a^4\bigg), \qquad q = \frac{a^4}{16}, \qquad z = 2\nu\tau. \tag{46}$$

Equation (45) is a typical Mathieu equation. It is worth noting that the similar Mathieu equation has been obtained when people studied the scalar field in the same NS B field background, but here we are considering the full closed string fields. We refer the reader to [10,13] for the general method to solve the Mathieu equation.

[1]We ignore the subindex $i(= 2, 3)$ from now for neat expression.

Let us define $Z_n^\pm$ as the two independent solutions of Eq. (45). Then the mode expansion for the field $\tilde{\eta}$ can be written as

$$\tilde{\eta} = \sum_{n=1}^{\infty}[C_n Z_n^-(\alpha_n e^{in\sigma} + \tilde{\alpha}_n e^{-in\sigma}) + C_{-n}Z_n^+(\alpha_n^+ e^{-in\sigma} + \tilde{\alpha}_n^+ e^{in\sigma})], \tag{47}$$

where $\alpha_n/\tilde{\alpha}_n$ and $\alpha_n^+/\tilde{\alpha}_n^+$ are annihilation and creation operators; C_n/C_{-n} are constant coefficients.

To quantize the field $\tilde{\eta}$, we define the conjugate momentum

$$\Pi_{\tilde{\eta}} = \partial_\tau\tilde{\eta} = \sum_{n=1}^{\infty}[C_n\dot{Z}_n^-(\alpha_n e^{in\sigma} + \tilde{\alpha}_n e^{-in\sigma}) + C_{-n}\dot{Z}_n^+ \times(\alpha_n^+ e^{-in\sigma} + \tilde{\alpha}_n^+ e^{in\sigma})], \tag{48}$$

and impose the quantization condition

$$[\tilde{\eta}^i(\sigma, \tau), \Pi_{\tilde{\eta}}^j(\sigma', \tau)] = i\pi\delta^{ij}\delta(\sigma - \sigma'). \tag{49}$$

The constants C_n can be determined using the quantization condition (49) as the normalization condition

$$C_n = C_{-n} = \sqrt{\frac{i}{(Z_n^-\dot{Z}_n^+ - Z_n^+\dot{Z}_n^-)}}, \tag{50}$$

where $(Z_n^-\dot{Z}_n^+ - Z_n^+\dot{Z}_n^-)$ is the Wronskian of the Mathieu equation and therefore it is a constant.

The substitution of the solution (47) into the Hamiltonian (43) leads to the expression

$$\mathcal{H}_{\tilde{\eta}} = \sum_{n=0}^{\infty} C_n^2\{[(n^2 + \nu^2 + \nu^2a^4\sin^2 2\nu\tau)Z_n^-Z_n^+ + \dot{Z}_n^-\dot{Z}_n^+](\alpha_n^\dagger\alpha_n + \tilde{\alpha}_n^\dagger\tilde{\alpha}_n) + [(n^2 + \nu^2 + \nu^2a^4\sin^2 2\nu\tau)Z_n^-Z_n^- + \dot{Z}_n^-\dot{Z}_n^-]\alpha_n\tilde{\alpha}_n + [(n^2 + \nu^2 + \nu^2a^4\sin^2 2\nu\tau)Z_n^+Z_n^+ + \dot{Z}_n^+\dot{Z}_n^+]\alpha_n^\dagger\tilde{\alpha}_n^\dagger\}. \tag{51}$$

It is easy to verify that when $a = 0$ the last two terms vanish and the above Hamiltonian reduces to the original one without the background B field.

To understand the Hamiltonian better, we need the explicit expression of the functions $Z_n^\pm$. But, in general, the solutions of the Mathieu equation cannot be written as any known function which is easy to work with. However, in the limit of $a \to 0$, we can use WKB approximation. In this case the solutions have a very simple form:

$$Z_n^{\pm} = \exp\left\{\pm i\sqrt{\lambda_n}\left(z - \frac{q}{2\lambda_n}\sin 2z\right)\right\}. \tag{52}$$

The constant C_n can then be determined as[2]

$$C_n^2 = \frac{\sqrt{n^2 + \nu^2 + \frac{\nu^2 a^4}{2}}}{2(n^2 + \nu^2 + \frac{\nu^2 a^4}{4})} \approx \frac{1}{2\sqrt{n^2 + \nu^2}}. \tag{53}$$

With these functions $Z_n^{\pm}$ at hand, the Hamiltonian can be expressed as

$$\mathcal{H}_{\tilde{\eta}} = 2\sum_{n=0}^{\infty} C_n^2(n^2 + \nu^2 + \nu^2 a^4 \sin^2 2\nu\tau) \times(\alpha_n^\dagger \alpha_n + \tilde{\alpha}_n^\dagger \tilde{\alpha}_n). \tag{54}$$

Finally, the difference between energy and angular momentum $E - J$ is given by the expectation value of the above Hamiltonian, which is

$$\begin{aligned} E - J &= \frac{1}{\nu}\sum_{n=0}^{\infty}\left(\sqrt{n^2 + \nu^2} + \frac{\nu^2 a^4 \sin^2 2\nu\tau}{2\sqrt{n^2 + \nu^2}}\right)(N_n + \tilde{N}_n) \\ &= \sum_{n=0}^{\infty}\left(\sqrt{1 + \frac{\lambda n^2}{J^2}} + \frac{a^4 \sin^2 2\nu\tau}{2\sqrt{1 + \frac{\lambda n^2}{J^2}}}\right)(N_n + \tilde{N}_n) \\ &= \sum_{n=0}^{\infty}\left(\sqrt{1 + \frac{\lambda n^2}{J^2}} + \frac{2a^4 u^2 |1 - u^2|}{\sqrt{1 + \frac{\lambda n^2}{J^2}}}\right)(N_n + \tilde{N}_n), \end{aligned} \tag{55}$$

where we used $J = \sqrt{\lambda}\nu$ and $u = \cos\nu\tau \in [-1, 1]$.

V. HOLOGRAPHIC NONCOMMUTATIVITY

String field in the modified $AdS_5 \times S^5$ background (4) is conjectured to be dual to the noncommutative Yang-Mills theory on the boundary [9,10]. Thus, similar to the analysis in [2], the string spectrum should correspond one-to-one-to certain operators in the noncommutative Yang-Mills theory. However, the closed string spectrum (55), which we found in the previous section, is time dependent. At first glance it seems strange that the spectrum depends on time. In fact, this kind of time-dependent spectrum has been studied in [5,7] when the authors tried to quantize the string fields in the nonextremal $NS5$-brane background. The crucial point is that the time τ in the string spectrum (55) is not the space-time time t, but the world-sheet time. Actually, the world-sheet time τ is related to the space-time u direction, which measures the energy scale in the holographic description of the boundary field theory [5,14]. Therefore, the world-sheet time-dependent string spectrum corresponds to the operators at the different energy scales and can be interpreted as the RG flow in the dual field theories.

When we consider the closed string states created by the string fluctuation around the classical solution, a string state $a^\dagger|0\rangle$ corresponds to an operator $\mathcal{O}$ in the dual field theory (at certain energy scale Λ) [2]. In the absence of the background B field, the Hamiltonians of the quadratic fluctuations are time independent, so that the corresponding string state is the same at different world-sheet time τ along the classical path, as in Fig. 1(a). Thus, the string state corresponds to an operator at the same energy scale in the dual field theory. In the presence of the B field, however, the Hamiltonians of the quadratic fluctuations (24) and (36) are time dependent. The string state could absorb or lose energy to change to another string state over time, so that the string state $a^\dagger(\tau)|0\rangle$ is different at different time τ along the classical path, as indicated in Fig. 1(b). The claim is that a string state varying with the world-sheet time $a^\dagger(\tau)|0\rangle$ corresponds to a scale dependent operator $\mathcal{O}(\Lambda)$ in the dual field theory running over varying energy scales $\Lambda(\tau)$, i.e., the RG flow in the dual field theory. The correlation functions of the operators $\mathcal{O}(\Lambda)$ should satisfy the standard Callan-Symanzik equations. In our case, the dual field theory is conjectured to be the noncommutative Yang-Mills theory [9,10].

For small u, the spectrum (55) reduces to the string spectrum without the background B field, which corresponds to the IR regime of the gauge theory. This is consistent with the expectation that noncommutative Yang-Mills theory reduces to ordinary Yang-Mills theory at long distances. The other important regime of the dual gauge theory is the UV regime. Unfortunately, our result is not able to make any prediction about the UV regime due to the feature of the string spectrum (55) that u has a finite range and the energy oscillates between $u_{\min} = -1$ and $u_{\max} = 1$, so that the strings never get to the boundary at $u \to \infty$ in our configuration. This is because of the special classical solution (14) and (26) we used in our analysis. Both of these solutions only cover the region $[-1, 1]$ in the u direction. To study the UV regime of the dual gauge theory, we need to find another classical path which reaches the boundary of AdS space at $u \to \infty$. However, we can read some information around $u \to \infty$ directly from the original metric (4). The coefficient of the terms along the B-field directions is invariant under the transformation $u \to 1/(a^2 u)$. Thus, the physics at $u \to \infty$ is equivalent to the physics at $u \to 0$. This suggests that we cannot reach the boundary of AdS at $u \to \infty$ and are not able to define the UV fixed point in the dual non-

[2] Directly putting the solutions (52) into (50) will produce a time-dependent C_n since (52) are not exact solutions of the equation of motion (45). We need to use the fact that, when $\tau \to 0$, the solutions should reduce to the ones without background B field to determine C_n.

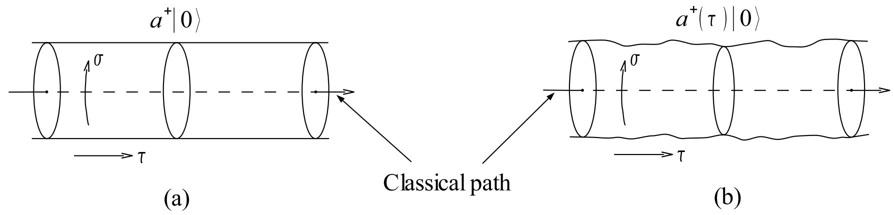

FIG. 1. Closed string fluctuation about the classical solution: (a) Without the background B field, the fluctuated string state $a^\dagger|0\rangle$ is independent of the world-sheet time τ; (b) with the background B field, the fluctuated string state $a^\dagger(\tau)|0\rangle$ dependents on the world-sheet time.

commutative Yang-Mills theory. This makes perfect sense since a local field theory is defined at short distances, and in terms of the AdS/CFT correspondence this means that the microscopic structure of the theory is encoded on the boundary of the AdS space; on the other hand, the noncommutative Yang-Mills theory is a nonlocal theory with UV/IR mixing, which implies that we should not be able to define the theory at short distances.

VI. DISCUSSION

We studied closed strings in the background $D3$-brane with constant NS B field, which is conjectured to be dual to the noncommutative Yang-Mills theory. The closed string spectrum $E - J$ has been computed in the limit $a \to 0$, where a is a noncommutativity parameter. When $a = 0$, (55) reduces to the standard Berenstein-Maldacena-Nastase formula corresponding to a $D3$-brane without the background B field as expected. When $a \neq 0$, the fluctuation string spectrum becomes time dependent and can be interpreted as the RG flow in the dual noncommutative Yang-Mills theory.

A direct check of this duality would be to compare the closed string spectrum (55) with the anomalous dimension of certain operators in the dual noncommutative Yang-Mills theory. In the case of $\mathcal{N} = 4$ Yang-Mills theory, the dual operators are [2]

$$\mathcal{O}(x) = \mathrm{Tr}(Z^J), \qquad \mathrm{Tr}(\phi Z^J)\ldots \tag{56}$$

The naive guess of the dual operators in the noncommutative theory would be to replace the ordinary products in the operators (56) by the Moyal star products as

$$\mathcal{O}_{\mathrm{NC}}(x) = \mathrm{Tr}(Z^{*J}), \qquad \mathrm{Tr}(\phi^r * Z^{*J})\ldots \tag{57}$$

where

$$\mathrm{Tr} Z^{*J} \equiv \mathrm{Tr}(\underbrace{Z * Z * \cdots * Z}_{J}), \tag{58}$$

and $Z = \phi^5 + i\phi^6$, ϕ^r, $r = 1, 2, 3, 4$ are six transverse scalars in the field theory on the D-brane.

However, the operators in (57) are not gauge invariant. To construct gauge-invariant operators in coordinate space, we define [15]

$$\hat{\mathcal{O}}_{\mathrm{NC}}(k) = \int d^4x \mathcal{O}_{\mathrm{NC}}(x) * W(x, C) * e^{ik\cdot x}, \tag{59}$$

where

$$W(x, C) = P_* \exp\biggl(ig \int_C d\sigma \frac{d\zeta^\mu}{d\sigma} A_\mu[x + \zeta(\sigma)]\biggr). \tag{60}$$

Such an operator will be gauge invariant.

Gauge-invariant operators in coordinate space can also be defined by a noncommutative Fourier transformation as[3] [16–18]

$$\hat{\mathcal{O}}_{\mathrm{NC}}(\hat{x}) = \int d^4y \mathcal{O}_{\mathrm{NC}}(y)\delta^{(4)}(\hat{x} - y), \tag{61}$$

where the noncommutative δ function

$$\begin{aligned}\delta^{(4)}(\hat{x} - y) &\equiv \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot y} e^{ik\cdot\hat{x}} \\ &= \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot y} W(x, C) * e^{ik\cdot x}.\end{aligned} \tag{62}$$

The next job is to calculate the correlation function of the operators $\hat{\mathcal{O}}_{\mathrm{NC}}$ in (59) or (61) to get their anomalous dimensions, and compare them to the string spectrum (55). We will postpone this to future work.

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[3] However, the above procedure of defining gauge-invariant operators in coordinate space is not so clear since these operators require momentum dependent regularization in perturbation theory [15].

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