ground water

Computer Note/

A Simple Approach Using Bouwer and Rice's Method for Slug Test Data Analysis

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Abstract

Slug test data obtained from tests performed in an unconfined aquifer are commonly analyzed by graphical or numerical approaches to determine the aquifer parameters. This paper derives three fourth-degree polynomials to represent the relationship between Bouwer and Rice's coefficients and the ratio of the screen length to the radius of the gravel envelope. A numerical approach using the nonlinear least squares and Newton's method is used to determine hydraulic conductivity from the best fit of the slug test data. The method of nonlinear least squares minimizes the sum of the squares of the differences between the predicted and observed water levels inside the well. With the polynomials, the hydraulic conductivity can be obtained by simply solving the nonlinear least squares equation by Newton's method. A computer code, SLUGBR, was developed from the derived polynomials using the proposed numerical approach. The results of analyzing two slug test datasets show that SLUGBR can determine hydraulic conductivity with very good accuracy.

Introduction

Hydraulic conductivity and storativity are two major hydrogeologic parameters required for quantitative analysis of ground water problems. A slug test involves instantaneous injection (or removal) of a small volume of water into (or from) a well; aquifer parameters can be obtained by analyzing water levels measured during the slug test. The slug test method is simple, quick, and economical.

Ferris and Knowles (1954) originally introduced the slug test approach to find the transmissivity of confined aquifers. Transmissivity was estimated from a straight-line plot of the residual head response vs. the inverse of time. Later, Bredehoeft et al. (1966) demonstrated that Ferris and Knowles' approximation is valid only at the time when the ratio of the water level in the well to the initial water level is very small. Using the modified Thiem equation for unconfined and steady state conditions, Bouwer and Rice (1976) presented a procedure for determining hydraulic conductivity or transmissivity for unconfined aquifers. Using results from an electric analog model, they obtained two empirical formulas related to the effective radius for partially and fully penetrating wells. Bouwer (1989) provided information on using Bouwer and Rice's method for testing the validity of falling level tests, the application of the method to confined aquifers, the effect of well diameter, and the computer processing of field data. Black (1978) employed this method and used the procedure suggested by Cooper Jr. et al. (1967) to obtain similar curves for an unconfined aquifer. These graphical methods require data plotting and subjective judgment during the curve-fitting procedure, making the data analyses cumbersome and time-consuming. Moreover, errors may be introduced in the process.

Dagan (1978) presented a simple numerical method that can be used to estimate hydraulic conductivity for data obtained from packer, recovery, and slug tests. The method involves solving the flow problem for source distributions along the well axis using the Green function. While his method is versatile, it has the limitation that the active portion of the well length should be much larger (say, 50 times) than the well radius. Kemblowski and Klein (1988) used the least squares approach and sensitivity analysis to estimate hydraulic conductivity. Their approach requires

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reading a coefficient from Figure 3 in Bouwer and Rice's paper. Errors could be incurred due to the probable bias in estimating the coefficient. Chirlin (1989) gave a brief review of various mathematical models that represent the slug test experiment for estimating hydraulic parameters such as hydraulic conductivity, storage coefficient, and skin factor. He also explored the relation between the effective radius and the storage coefficient. Hyder et al. (1994) gave a semianalytical solution to a mathematical model describing ground water flow in response to a slug test in a confined or unconfined aquifer. Their model incorporates the effects of partial penetration, anisotropy, finite-radius well skin, and upper and lower boundaries of either a constant-head or an impermeable layer. Their solution was employed to quantify the error introduced in parameter estimates using slug test response data. Hyder and Butler Jr. (1995) suggested using the semianalytical solution derived from the model given by Hyder et al. (1994) for parameter estimation, noting that conventional approaches such as Bouwer and Rice's method gave large error. Rupp et al. (2001) employed a two-dimensional, radially symmetric and variable saturated, ground water model to simulate well recovery given a range of well and aquifer geometries and unsaturated soil properties. Then, they modified Bouwer and Rice's method to explain the recovery rates based on the well geometry and soil type. Their modification introduced a parameter related to soil capillarity to improve the estimation accuracy of saturated hydraulic conductivity for slug test data. Butler Jr. (2002) gave a simple procedure for correcting hydraulic conductivity estimates obtained from slug tests performed in small-diameter installations screened in highly permeable aquifers. Jiao and Leung (2003) gave a brief review on the key features, advantages, and disadvantages of spreadsheets for the analysis of aquifer test and slug test data.

This study proposes a numerical approach combined with three fourth-degree polynomials to determine automatically Bouwer and Rice's coefficients and hydraulic conductivity. The numerical approach including the nonlinear least squares and Newton's method is used to solve for the best-fit value of aquifer parameters to the slug data. This approach has been used successfully for identifying the parameters of a confined aquifer (Yeh 1987). A computer code written in Fortran, called SLUGBR, was developed from the derived polynomials. Illustrations for input data format and variable definitions are given at the beginning of the code. The code is available from the authors upon request. The main advantage of using SLUGBR to estimate hydraulic conductivity is that errors caused by data plotting and estimating values from the plots can be avoided and the time-consuming labor in graphical works can be saved. Commercially available software packages such as AquiferWin32 and AQTESOLV also provide a way to use Bouwer and Rice's method without the hand plotting of data and subjective reading of coefficients

Bouwer and Rice's Method

Bouwer and Rice (1976) used a modified Thiem equation to estimate the hydraulic conductivity as

$$K = \frac{r_c^2 \, ln(R_e/r_w)}{2L}\frac{1}{t}\, ln\frac{H_0}{H(t)} \tag{1}$$

where K is the hydraulic conductivity of the aquifer, L is the height of the well through which water enters, H(t) is the vertical distance between the water level inside the well and the equilibrium water table in the aquifer, H_0 is the initial water level inside the well, R_e is the effective radius over which H(t) is dissipated, r_c is the radius of well casing, and r_w is the radial distance from the well center to the undisturbed aquifer. A plot of lnH(t) vs. t shows a straight line since K, r_c, r_w, R_e, and L in Equation 1 are constants. Bouwer and Rice suggested using a graphical curve-fitting approach to determine the value of K from the slug test data.

Bouwer and Rice (1976) used an electric analog model for different values of r_w, L, L_w, and D to determine the value of R_e. Two empirical equations relating $\ln(R_e/r_w)$ in Equation 1 to the geometry of aquifer system are

$$\ln\frac{R_e}{r_w} = \left[\frac{1.1}{\ln(L_w/r_w)} + \frac{A + B\ln\left|(D - L_w)/r_w\right|}{L/r_w}\right]^{-1} \tag{2}$$

for a partially penetrating well, and

$$\ln\frac{R_e}{r_w} = \left[\frac{1.1}{\ln(L_w/r_w)} + \frac{C}{L/r_w}\right]^{-1} \tag{3}$$

for a fully penetrating well. The dimensionless parameters A and B in Equation 2 and C in Equation 3 are functions of $\log(L/r_w)$ (Bouwer and Rice 1976).

Proposed Numerical Approach

Nonlinear Least Squares

The value of H(t) according to Equation 1 may be written as

$$H(t) = H_0 \exp\left[-\frac{2LKt}{r_c^2\ln(R_e/r_w)}\right] = H_0\exp(-C'Kt) \tag{4}$$

where

$$C' = \frac{2L}{r_c^2\ln(R_e/r_w)} \tag{5}$$

To get the best value of hydraulic conductivity, the partial derivative of the sum of squares of the differences between the predicted and observed water levels inside the well with respect to K is set to be zero. Thus,

$$\frac{\partial}{\partial K}\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} 2C'\left|H_e(t_i) - H(t_i)\right|\left|t_i H(t_i)\right| = 0 \tag{6}$$

where e_i is the prediction error, $H_e(t_i)$ represents the observed water level at time t_i, $H(t_i)$ represents the predicted water level in the well, and n is the total number of observations. The nonlinear least squares equation, Equation 6, can be used to determine the best value of hydraulic conductivity.

Newton's Method

Newton's method, which can be used to find solutions of nonlinear equations, may be expressed as (Gerald and Wheatley 1989)

$$K_{j+1} = K_j - \frac{F(K_j)}{F'(K_j)} \qquad (7)$$

where $F(K_j)$ is the nonlinear equation, Equation 6, $F'(K_j)$ is the derivative of the function in the denominator with respect to K, and j represents the number of iterations. Note that $F'(K_j)$ must be nonzero. The derivative term may be approximated by a finite-difference formula or obtained by taking the derivative directly. The result of differentiation is

$$F'(K) = \frac{\partial F(K)}{\partial K} = \sum_{i=1}^{n} C' t_i^2 H(t_i)\left[2H(t_i) - H_e(t_i)\right] \qquad (8)$$

The tolerance used to stop the iteration is

$$\left|K_{j+1} - K_j\right| < KTOL \qquad (9)$$

and/or

$$\left|F(K_{j+1})\right| < FTOL \qquad (10)$$

where the values of KTOL and FTOL depend on the desired accuracy of the result.

Several error criteria may be used to assess the goodness-of-fit during curve fitting or the performance of parameter identification by different methods (Yeh 1987). The mean error (ME) is defined as the sum of the errors divided by the number of data points. The mean absolute error (MAE) is defined as the sum of the absolute errors divided by the number of data points. The standard error of estimate (SEE) is defined as the square root of the sum of squared errors divided by n – m – 1 where m is the degree of the polynomial and n is the number of data points.

Curves Relating Bouwer and Rice's Coefficients A, B, and C

The curves relating coefficients A, B, and C to $\log(L/r_w)$ given in Bouwer and Rice (1976) are read by a digitizer and expressed in the polynomial equations. The best-fit equations for those three curves can be found using the least squares approach. Functions more complex than fourth-degree polynomials are rarely needed (Gerald and Wheatley 1989). Thus, the best-fit equations for these three curves are expressed in terms of fourth-degree polynomials as

$$A(x) = 1.353 + 2.157x - 4.027x^2 + 2.777x^3 - 0.460x^4 \qquad (11)$$

$$B(x) = -0.401 + 2.619x - 3.267x^2 + 1.548x^3 - 0.210x^4 \qquad (12)$$

and

$$C(x) = -1.605 + 9.496x - 12.317x^2 + 6.528x^3 - 0.986x^4 \qquad (13)$$

where x is the value of $\log(L/r_w)$ in Equations 11, 12, and 13. For any given x, the values of coefficients A, B, and C can be calculated from these three equations. The prediction errors of ME, MAE, and SEE for Equations 11, 12, and 13 are listed in Table 1. The values of Bouwer and Rice's coefficients A, B, and C range from zero to 13. Generally speaking, the estimation error for each coefficient is < 10%. By using these three polynomials, the parameter for an unconfined aquifer can be easily estimated.

As reported by Butler Jr. (1998), Van Rooy (1988) also used a regression method to develop a set of polynomial functions in terms of L/r_w for Bouwer and Rice's coefficients A, B, and C. Figure 1 shows the curves plotted using Equations 11, 12, and 13, and Van Rooy's polynomial functions. For the coefficient B, Van Rooy's curve starts to deviate from Bouwer and Rice's curve when L/r_w is over 600. In addition, Van Rooy's curve for coefficient A has a dip near $L/r_w = 1000$ and his curves for coefficients A and C give very large extrapolation errors when L/r_w is beyond 1500. Note that our polynomials and Van Rooy's polynomials are expressed in terms of $\log(L/r_w)$ and L/r_w, respectively. Bouwer and Rice (1976) presented the value of L/r_w from 4 to 1500 as indicated in Figure 1. However, the range of $\log(L/r_w)$ is from 0.60 to 3.18. Such a smaller range of $\log(L/r_w)$ may make the regression curves smoother and result in fewer errors when extrapolation is required for $L/r_w > 1500$.

Data Analyses

In this study, the value of KTOL (Equation 9) is set to three orders of magnitude less than that of the initial guess of the unknown parameter. The value of FTOL (Equation 10) is set to one order of magnitude less than KTOL. The maximum number of iterations is set to 20.

A digitizer was used to read two sets of slug test data from figures given in Kemblowski and Klein (1988). The first dataset comprises computer-generated data from a fully penetrating well with internal radius $r_c = 0.05$ m and external radius $r_w = 0.1$ m. The initial penetrated thickness of the aquifer is $L_w = 15$ m, the screen height is L = 10 m, and the hydraulic conductivity is assumed to be 0.288 m/d. The initial drawdown is $H_0 = 1$ m. The coefficient C is estimated to be 1.5 and that calculated by Equation 13 is 4.57. In fact, the correct value of C should be ~4.6 for $L/r_w = 100$ as estimated from the curve provided by Bouwer and Rice (1976). The hydraulic conductivity estimated by the

Table 1
Prediction Errors of Polynomial Equations Representing Bouwer and Rice's Coefficients A, B, and C

	Prediction Errors		
Coefficient	**ME**	**MAE**	**SEE**
A	–0.00	0.09	0.11
B	–0.02	0.05	0.07
C	0.01	0.18	0.22

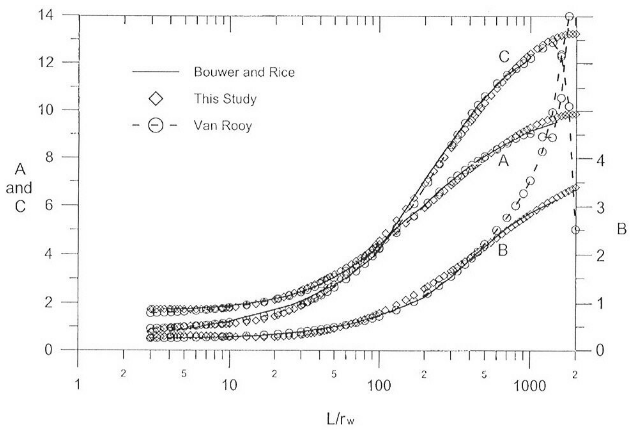

Figure 1. Curves relating coefficients A, B, and C to L/r_w.

proposed approach with C = 1.5 is 0.287 m/d, which is very close to the assumed value of Kemblowski and Klein (1988). However, the hydraulic conductivity estimated by the proposed approach with C = 4.57 is 0.254 m/d. The second dataset was taken at a site in Kalkaska, Michigan. The field test was performed in a sandy aquifer using a partially penetrating well with internal radius $r_c = 0.032$ m and external radius $r_w = 0.086$ m. The well penetration depth L_w and the screen height L are equal to 0.949 m. The saturated thickness is 30.48 m and the initial drawdown is $H_0 = 0.207$ m. The coefficients, A and B, estimated from Kemblowski and Klein (1988) are 1.80 and 0.25, respectively. Coefficients estimated by Equations 11 and 12 are 1.83 and 0.28, respectively. The two methods give the same value of hydraulic conductivity, which is 16.09 m/d. The prediction errors of ME, MAE, and SEE are 0.002, 0.007, and 0.01, respectively. The results of data analyses indicate that our proposed approach can obtain Bouwer and Rice's coefficients A, B, and C in a very efficient way and determine hydraulic conductivity with good accuracy.

Summary and Conclusions

This study employs the linear least squares approach to approximate three curves representing the relation between coefficients A, B, and C, and L/r_w using fourth-degree polynomials. A numerical approach including the nonlinear least squares and Newton's method is used to determine hydraulic conductivity from two slug test datasets, one from a fully penetrating well and another from a partially penetrating well. The nonlinear least squares method is employed to minimize the sum of squares of the differences between the predicted and observed water levels inside the well. Newton's method is used to solve the nonlinear least squares equation when combined with three fourth-degree polynomials to estimate the Bouwer and Rice coefficients. A computer code, SLUGBR, was developed from the derived polynomials using the proposed numerical approach. The code can automatically identify the hydraulic conductivity without involving data plotting, graphs, and curve reading. The results of the slug test data analyses from two case studies demonstrate that the code can automatically find the best-fit value of hydraulic conductivity. In addition, the code has the merits of giving accurate results and is easy to use.

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