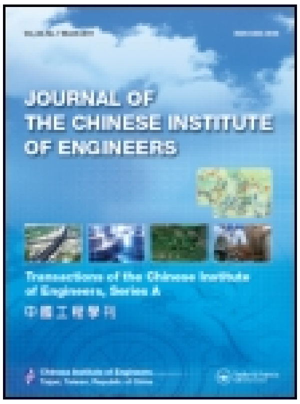

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Evaluation of dynamic vehicle load on bridge decks

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EVALUATION OF DYNAMIC VEHICLE LOAD ON BRIDGE DECKS

Jeng-Hsiang Lin* and Cheng-Chiang Weng

ABSTRACT

Developed herein is a spectral approach for evaluating the dynamic vehicle load due to the passage of a vehicle moving at constant speed along a rough bridge surface. Based on the approach, a simple closed-form solution for predicting the variation of dynamic vehicle load on a bridge deck is derived. Numerical examples of the application of the solution to a simply-supported bridge are presented. Four different classes of pavement roughness (including: very good, good, average, and poor pavements) and three different vehicle speeds (speeds of 60, 100, and 140 km/h) are used in numerical analysis. The Dynamic Load Coefficient (DLC), a parameter used to characterize the magnitude of dynamic vehicle load, is estimated. The effects of vehicle speed and pavement roughness on the variation of dynamic vehicle load are investigated. It is concluded that if the effect of engine motion on vehicle vibration is disregarded, both the DLC and standard deviation of dynamic vehicle load are proportional to the square root of the pavement roughness coefficient $S(n_0)$ for a specified vehicle speed.

Key Words: dynamic vehicle load, bridge, pavement roughness, DLC.

I. INTRODUCTION

The dynamic force induced by vehicle-bridge interaction resulting from the passage of vehicles plays a significant role in the design of a bridge structure. In practice, to allow for such a dynamic effect, it is required that the static vehicle force be increased by a dynamic allowance factor, called the impact factor, in design. Many codes, including AASHTO, specify the factor as a function of span length only (AASHTO, 1992). However, it has been observed that the dynamic force, called the dynamic vehicle load on a bridge, depends on dynamic properties of the vehicle, dynamic properties of the bridge, vehicle speed, and bridge-surface roughness. Recently, there has been an increasing interest in and concern about bridge design forces. The dynamic force is an important parameter in bridge design and evaluation. In addition to the importance in design, the dynamic vehicle load causes subtle problems and contributes to fatigue, surface wear, and cracking of concrete that leads to corrosion (Anon, 1992). It continually degrades bridges and increases the necessity of regular maintenance. Thus, the determination of the dynamic vehicle load resulting from the passage of a vehicle across a span of a bridge is a problem of great interest for bridge engineers. The need to develop an approach and derive a simple closed-form solution to predict the dynamic vehicle load for applications of bridge design is apparent. To solve the problem of vehicle-bridge interaction, two sets of equations of motion can be written, one for the vehicle and the other for the bridge. To couple mathematically the motion of the vehicle and the bridge, the interactive force existing at the contact point between the two subsystems is considered in analysis.

The main objective of this study is to develop a spectral approach for evaluating the dynamic vehicle load due to the passage of a vehicle moving at constant speed along a rough bridge surface. A simple closed-form solution for predicting the variation of dynamic vehicle load on bridge pavement is derived. In this study, the vehicle is replaced by a simple, linear, damped spring-mass system which moves on a bridge

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at constant speed. The vehicle is in contact with the bridge surface. A displacement is imposed at the lower end of the vehicle system to model the bridge surface roughness and the deflection of the bridge due to dynamic vehicle load. A harmonic engine-induced force is applied to the mass to model the effect of engine motion. Numerical examples of the application of the proposed solution to a simply-supported bridge are presented. Comparisons of the numerical results and the available experimental results are made to validate the accuracy of the developed approach.

II. LITERATURE REVIEW

The subject of dynamic responses of bridges to the passage of vehicles has been studied for many years. Numerous research results, including theoretical and numerical results (Jeffcott, 1929; Huang and Veletsos, 1960; Wen and Veletsos, 1962; Luthe-Garcia *et al*., 1964; Tan and Shore, 1968; Timoshenko *et al*., 1974; Warburton, 1976; Sridharan and Mallik, 1979; Blejwas *et al*., 1979; Inbanathan and Wieland, 1987; Fryba, 1987; Smith, 1988; Akin and Mofid, 1989; Hwang and Nowak, 1991; Wang *et al*., 1991; Yang and Lin, 1995), and results of laboratory and field tests (Biggs *et al*., 1959; Fenves *et al*., 1962a;1962b; Walker, 1968; Swannell and Miller, 1987; Mitchell and Gyenes, 1989; Green, 1990) have been proposed.

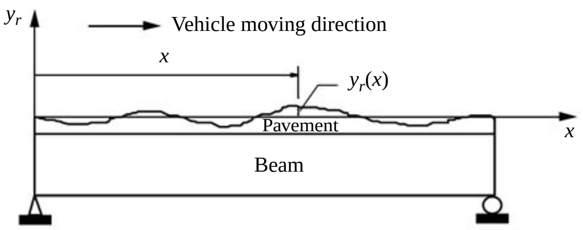
Research on the dynamic responses of bridges subjected to a moving vehicle load dates back to the work of Jeffcott (1929). In the mid-twentieth century, approximate solutions were developed for the particular problem of idealized beam structures. Several of these classical solutions have been summarized (Ayre *et al*., 1951). In past decades, for simplicity, the weight of a vehicle was taken to be the only external force acting on the bridge. A moving vehicle force traveling along a bridge has been modeled as a moving "constant" force. The inertia force resulting from vibrations of vehicle mass is neglected (Tan and Shore, 1968; Fryba, 1999; Timoshenko *et al*., 1974; Warburton, 1976; Sridharan and Mallik, 1979; Mackertich, 1990; Pesterev and Bergman, 1998a). It is noted that the results are only valid for a case when the bridge surface is smooth or very good. For cases when the vehicle is moving along a rough bridge surface, the inertia of a vehicle is significant and cannot be ignored. A moving-mass model has to be used instead (Blejwas *et al*., 1979; Inbanathan and Wieland, 1987; Sadiku and Leipholz, 1987; Akin and Mofid, 1989; Pesterev and Bergman, 1998b). The moving mass contains a term that depends on the location of the moving vehicle mass in order to take care of inertial interaction between vehicle and bridge. Although most research has focused on the moving force problem or the moving mass problem, the moving oscillator problem has been addressed relatively infrequently. It should be noted that neither the moving force solution nor the moving force/moving mass solution could adequately account for the complex and important dynamic effects caused by the compliance of the moving oscillator.

Recently, more realistic and sophisticated models that consider various dynamic characteristics of the moving vehicle have been used to solve the problem of vehicle-bridge interaction (Chu *et al*., 1986; Hwang and Nowak, 1991; Wang *et al*., 1991; Yang and Lin, 1995). Valuable insights on the behaviors of vehicle-bridge interaction have been proposed. However, most investigators have focused their attentions on the deterministic aspect of the problem (Smith, 1988; Timoshenko *et al*., 1974; Sridharan and Mallik, 1979; Warburton, 1976; Wu and Dai, 1987). It has been recognized that the load process of a vehicle moving along a rough pavement surface is stochastic (LaBarre *et al*., 1970; Dodds and Robson, 1973; Inbanathan and Wieland, 1987; Marcondes *et al*., 1991) and depends on characteristics of vehicles, vehicle speed, and pavement roughness (Mannering and Kilareski, 1990; Ullidtz, 1987). Dynamic interactive forces between a vehicle and a rough pavement surface are essentially random in nature and are assumed to have properties of a stationary process. The forces can be experimentally determined for a particular stretch of pavement (Warburton, 1976). Very few models have been proposed for evaluating the dynamic force due to the complexity of analysis.

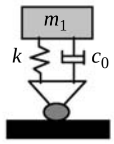
III. EQUATION OF MOTION OF VEHICLE

Figure 1(a) shows a possible profile of the irregularities on a fixed surface, for instance, a pavement surface on a bridge. The height, y_r, of the surface above a fixed datum is plotted as a function of distance x along the bridge. An idealized vehicle model of mass m_1, spring constant k, and damping coefficient c_0 moving from left to right with constant speed V along a rough bridge pavement is considered. Fig. 1(b) shows the simplified vehicle system whose behavior is used to model the behavior of a moving vehicle. The pavement-surface elevation $\tilde{y}(d, t)$ imposed at the lower end of the vehicle can be expressed as the sum of the pavement roughness and displacement of the bridge. With respect to an observer on the moving vehicle, the pavement-surface elevation $\tilde{y}(d, t)$ and the absolute displacement of the vehicle $\tilde{Z}(t)$ are functions only of time. The equation of motion of the vehicle is then in the form

$$m_1\ddot{\tilde{Z}}(t)+c_0(\dot{\tilde{Z}}(t)-\dot{\tilde{y}}_{x=d})+k(\tilde{Z}(t)-\tilde{y}_{x=d})=f(t)-m_1g \quad (1)$$

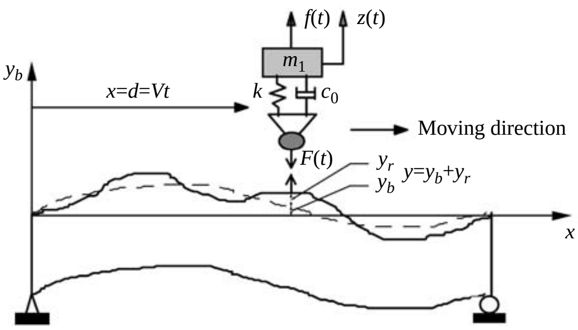


(a)



Simplified vehicle model

(b)



(c)

Fig. 1 Road roughness and analytical model: (a) probable profile of road roughness; (b) simplified vehicle model; (c) analytical model of a bridge subjected to dynamic vehicle load

where $\ddot{\tilde{Z}}$, $\dot{\tilde{Z}}$, and $\tilde{Z}$ are absolute acceleration, velocity, and displacement of the vehicle, respectively. $f(t)$ and $m_1 g$ are the engine-induced force and the vehicle gravity force, respectively. The symbol d expresses the moving distance of the vehicle on the bridge from left side, i.e.,$d=Vt$.

Rearranging the order of Eq. (1), the equation of motion of the vehicle then becomes the form

$$m_1\ddot{\tilde{Z}}(t)+c_0\dot{\tilde{Z}}(t)+k\tilde{Z}(t)=f(t)-m_1g+c_0\dot{\tilde{y}}_{x=d}+k\tilde{y}_{x=d} \tag{2}$$

Since $\tilde{y}(x, t)=\tilde{y}(x=d=Vt, t)$, Eq. (2) falls into the typical form of a linear single-degree-of-freedom system. The inputs to the suspension system include the time varying parameter $\tilde{y}(d, t)$, the engine-induced force $f(t)$, and the vehicle gravity force m_1g. Note that $\tilde{y}(d, t)$ can be expressed as the sum of the bridge deflection $y_{b1}(d, t)$ due to the moving vehicle gravity force (a moving constant force), the bridge deflection $y_b(d, t)$ due to the dynamic vehicle load $F(t)$ (a moving random force with zero mean), and the pavement roughness $y_r(t)$. Let

$$\tilde{Z}(t)=Z_1(t)+Z(t) \tag{3}$$

$$\tilde{y}(d, t)=y_{b1}(d, t)+y_r(t)+y_b(d, t) \tag{4}$$

and

$$y(d, t)=y_r(t)+y_b(d, t) \tag{5}$$

Eq. (2) can be further decomposed into two equations:

$$m_1\ddot{Z}_1(t)+c_0\dot{Z}_1(t)+kZ_1(t)=c_0\dot{y}_{b1}(d, t)+ky_{b1}(d, t) \tag{6}$$

$$m_1\ddot{Z}(t)+c_0\dot{Z}(t)+kZ(t)=f(t)-m_1g+c_0\dot{y}(d, t)+ky(d, t) \tag{7}$$

Note that the bridge deflection $y_{b1}(d, t)$ due to the moving constant force m_1g and the vehicle displacement $Z_1(t)$ in Eq. (6) are deterministic functions; $f(t)$, $y(d, t)$, and $Z(t)$ in Eq. (7) are a random function. If $y_{b1}(d, t)$ is known, $Z_1(t)$ of Eq. (6) can readily be solved by any available method.

IV. POWER SPECTRAL DENSITY FUNCTION OF VEHICLE DISPLACEMENT

If the equation of motion of a vehicle is expressed with reference to the static-equilibrium position, the vehicle displacements in future discussions will be referenced from the position, and the response of the vehicle that is determined will be the dynamic “random” response. The analytical model of the dynamic vehicle-bridge interactive system is shown in Fig. 1(c). Therefore, total response of the vehicle, such as: displacement, spring force, etc., can be obtained only by adding the appropriate static quantities to the results of the dynamic analysis. If the static responses due to the vehicle gravity force are temporarily ignored in the analysis, Eq. (7) then becomes the form

$$m_1\ddot{Z}(t)+c_0\dot{Z}(t)+kZ(t)=f(t)+c_0\dot{y}(d, t)+ky(d, t) \tag{8}$$

If $y(d, t)$ and $f(t)$ have respectively power spectral density function $S_{yy}(d, \omega)$ and $S_{ff}(\omega)$ with respect to time, the relation of the power spectral density function of the vehicle response $S_{zz}(d, \omega)$ and of the inputs is then given by

Table 1 Pavement classes based on principal road spectra

Pavement class		$S(n_0)$	ω_1 Mean	ω_1 Standard deviation	ω_2 Mean	ω_2 Standard deviation
Principal roads	Very good	2-8	2.05	0.487	1.44	0.266
	Good	8-32				
	Average	32-128				
	Poor	128-512				

$S(n)$ measured in units of 10^{-6} (m^3/cycle), $n_0=1/2\pi$ (cycle/m)

$$S_{ZZ}(d,\omega)=\sum_r\sum_s H_r^*(\omega)H_s(\omega)S_{rs}(d,\omega)\,,\quad r,\,s=y(d,t),\,f(t) \tag{9}$$

where $H_r{}^*(\omega)$ is the complex conjugate of $H_r(\omega)$.

In general, the engine-induced force $f(t)$ exhibits a harmonic form and little correlative with the pavement roughness and the moving distance of vehicle.

For uncorrelated inputs, $S_{zz}(d,\omega)$ can be expressed by the relation

$$S_{ZZ}(d,\omega)=\left|H_y(\omega)\right|^2S_{yy}(d,\omega)+\left|H_f(\omega)\right|^2S_{ff}(\omega) \tag{10}$$

where

$$H_y(\omega)=\frac{k+ic_0\omega}{(k-m_1\omega^2)+ic_0\omega} \tag{11}$$

and

$$H_f(\omega)=\frac{1}{(k-m_1\omega^2)+ic_0\omega} \tag{12}$$

The time spectral density function $S_{yy}(d,\omega)$ is a Fourier transform of the time autocorrelation function $R_{yy}(d,\tau)$ and can be expressed by the form

$$S_{yy}(d,\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}R_{yy}(d,\tau)e^{-i\omega\tau}d\tau \tag{13}$$

where the time autocorrelation function $R_{yy}(d,\tau)$ is defined by

$$R_{yy}(d,\tau)=E[y(d,t)y(d,t+\tau)] \tag{14}$$

To find $R_{yy}(d,\tau)$, substitute Eq. (5) into Eq. (14) to obtain

$$R_{yy}(d,\tau)=E[(y_b(d,t)+y_r(t))(y_b(d,t+\tau)+y_r(t+\tau))] \tag{15}$$

where y_b and y_r represent the deflection of bridge due to dynamic vehicle load $F(t)$ and the pavement roughness, respectively.

V. PAVEMENT ROUGHNESS

In general, the manner of variation of a bridge surface as a function of distance is assumed to be a zero-mean stationary random process. The power spectral density function of y_r is approximated by an equation of the form (LaBarre *et al*., 1970)

$$S_{y_ry_r}(n)=\begin{cases}S(n_0)(\frac{n}{n_0})^{-\omega_1}\,, & n\le n_0\\ S(n_0)(\frac{n}{n_0})^{-\omega_2}\,, & n\ge n_0\end{cases} \tag{16}$$

where n is the spatial frequency and $S(n_0)$ is the pavement roughness coefficient, which is suggested by LaBarre. n_0, ω_1, and ω_2 are the parameters of spectral shape. The pavement roughness is determined by surface condition of the approach and the bridge. According to Dodds's research (Dodds, 1973), the parametric values for typical principal roads were given for four different classes of pavement (Table 1).

Sun and Deng (1996) proved that there exits a definite relationship between the two kinds of spectra expressed by distinct frequencies, that is

$$S_{y_ry_r}(n)=2\pi VS_{y_ry_r}(\omega) \tag{17}$$

and the time angular frequency ω can be expressed by

$$\omega=2\pi nV \tag{18}$$

Thus, the spectral density $S_{y_ry_r}(\omega)$ can then be expressed by

$$S_{y_ry_r}(\omega)=\begin{cases}\frac{1}{2\pi V}S(n_0)(\frac{\omega}{2\pi Vn_0})^{-\omega_1}\,, & \omega\le 2\pi Vn_0\\ \frac{1}{2\pi V}S(n_0)(\frac{\omega}{2\pi Vn_0})^{-\omega_2}\,, & \omega\ge 2\pi Vn_0\end{cases} \tag{19}$$

VI. ENGINE-INDUCED FORCE

The engine-induced force exhibits generally a harmonic form and can be expressed as

$$f(t)=A_0\sin(\omega_0 t+\theta) \tag{20}$$

where A_0 and ω_0 are the amplitude and the circular frequency of engine force, respectively. The phase angle θ is a random variable in the range of 0 to 2π.

The one-sided spectral density of engine force $f(t)$ can be expressed by the form

$$S_{ff}(\omega)=\frac{A_0^2}{2}\delta(\omega-\omega_0) \tag{21}$$

VII. DYNAMIC RESPONSE OF BRIDGE

1. Equation of Motion of Bridge

Consider an elastic uniform straight bridge of length L and mass per unit length $\overline{m}$ subjected to a viscous damping force c per unit length per unit velocity and a transverse force $p(x, t)$ per unit length. The end-support conditions for the bridge are arbitrary. The equation of motion of this bridge system can readily be derived by considering the equilibrium of forces acting on the differential segment of the bridge and introducing the basic moment-curvature relationship of elementary beam theory. Thus, the transverse deflection $\tilde{y}_b(x, t)$ of the bridge satisfies the following partial differential equation

$$\overline{m}\frac{\partial^2\tilde{y}_b}{\partial t^2}+c\frac{\partial\tilde{y}_b}{\partial t}+EI\frac{\partial^4\tilde{y}_b}{\partial x^4}=p(x,t) \tag{22}$$

where $\tilde{y}_b(x, t)$ is the transverse deflection of the bridge at time t and distance x from its left-hand end and EI is the constant bending stiffness of the bridge.

For a moving vehicle load at constant speed V, $p(x, t)$ can be replaced by

$$p(x,t)=\delta(x-d)P(t)=\delta(x-d)(m_1g+F(t))$$

$$=\delta(x-d)m_1g+\delta(x-d)F(t) \tag{23}$$

where $F(t)$ is a stationary Gaussian random process with zero mean; m_1g is the vehicle gravity force (a constant force); the total vehicle load on the bridge $P(t)$ is a stationary Gaussian random process with a mean value of m_1g.

Let the external load $p(x, t)$ be a non-stationary process with mean (deterministic) value m_1g and with a centred (random) value $F(t)$. It is assumed that $F(t)$ is independent of the mean deterministic deflection of the bridge, i.e., the inertial forces in the vehicle due to the mean deterministic deflections of the bridge are disregarded. Eq. (22) may be rewritten as two equations:

$$\overline{m}\frac{\partial^2 y_{b1}}{\partial t^2}+c\frac{\partial y_{b1}}{\partial t}+EI\frac{\partial^4 y_{b1}}{\partial x^4}=\delta(x-d)m_1g \tag{24}$$

$$\overline{m}\frac{\partial^2 y_b}{\partial t^2}+c\frac{\partial y_b}{\partial t}+EI\frac{\partial^4 y_b}{\partial x^4}=\delta(x-d)F(t) \tag{25}$$

In other words, the total deflection $\tilde{y}_b(x, t)$ of the bridge due to the moving vehicle force is expressed as the sum of the deflection y_{b1} of the bridge due to the moving constant force m_1g and the deflection y_b (as shown in Fig. 1(c)) due to the moving dynamic vehicle load $F(t)$. Note that $\tilde{y}_b=y_{b1}+y_b$, y_{b1} is a deterministic function, and y_b is a random function.

The first, Eq. (24), is valid for mean deterministic values of random function $y_b(x, t)$, $p(x, t)$, and $Y_j(t)$ while the second, Eq. (25), is valid for their centred "random" components.

2. Response of Bridge to Static Vehicle Load

The statistical characteristics of the first order (mean value of $\tilde{y}_b$) can be obtained from Eq. (24). The solution of the equation may be calculated as a response of the bridge to a moving constant load m_1g. For the case of a simply-supported bridge, the solution of Eq. (24) can be expressed in the following form (Fryba, 1999)

$$y_{b1}(x,t)=v_0\sum_{j=1}^{\infty}\frac{\sin\frac{j\pi x}{L}}{j^2(j^2(j^2-\alpha^2)^2+4\alpha^2\beta^2)}$$

$$\cdot\{j^2(j^2-\alpha^2)\sin\frac{j\pi Vt}{L}$$

$$-\frac{j\alpha(j^2(j^2-\alpha^2)-2\beta^2)}{(j^4-\beta^2)^{1/2}}e^{-\omega_b t}\sin\omega_j' t$$

$$-2j\alpha\beta(\cos\frac{j\pi Vt}{L}-e^{-\omega_b t}\cos\omega_j' t)\} \tag{26}$$

Here the following notation has been introduced:

$$v_0=\frac{(m_1g)L^3}{48EI} \tag{27}$$

is the static deflection of the bridge at the middle span under a constant load m_1g at the same point,

$$\alpha=\frac{V}{2f_1L} \tag{28}$$

is the dimensionless speed parameter where is the first-mode natural frequency, and

$$\beta=\frac{\omega_b}{\omega_1}=\frac{\vartheta}{2\pi} \tag{29}$$

is the dimensionless damping parameter where $\omega_b = c/2\overline{m}$ is the circular frequency of damping of the bridge, $\omega_1 = 2\pi f_1$ and ϑ is the logarithmic decrement of damping.

3. Response of Bridge to Dynamic Vehicle Load

The statistical characteristics of the second order (variation of $\tilde{y}_b$) can be obtained from Eq. (25). One form of solution of Eq. (25) can be obtained by separation of variables, assuming that the solution has the form

$$y_b(x,t) = \sum_{j=1}^{\infty} \psi_j(x) Y_j(t) \tag{30}$$

In other words, it is assumed that the free-vibration motions consist of a series of constant shape $\psi_j(x)$ and the amplitude of which is varying with time according to $Y_j(t)$. For the undamped free vibration analysis considering the boundary conditions at the ends of the bridge segment, the undamped angular frequencies ω_j and the mode shapes $\psi_j(x)$ of the bridge can readily be evaluated.

For a simply-supported bridge, the undamped angular frequencies ω_j and the modes $\psi_j(x)$ of the bridge can be given by

$$\omega_j = \left(j\frac{\pi}{L}\right)^2 \sqrt{\frac{EI}{\overline{m}}} \tag{31}$$

and

$$\psi_j(x) = \sqrt{2}\sin\left(j\frac{\pi x}{L}\right) \tag{32}$$

respectively. The modes $\psi_j(x)$ satisfy the orthogonal conditions

$$\int_0^L \psi_j(x)\psi_k(x)dx = L\delta_{jk} \tag{33}$$

where δ_{jk} is the Kronecker delta function.

Substituting Eq. (30) into Eq. (25), multiplying through by $\psi_j(x)$, integrating over x, and using the orthogonal conditions, leads to the uncoupled equation of motion for $Y_j(t)$

$$\ddot{Y}_j + \beta_j \dot{Y}_j + \omega_j^2 Y_j = G_j(t) \tag{34}$$

where

$$\beta_j = \frac{c}{\overline{m}} \tag{35}$$

and

$$G_j(t) = \frac{1}{\overline{m}L}\int_0^L \psi_j(x)\delta(x-d)F(t)dx \tag{36}$$

The formal solution to Eq. (34) is given by the convolution integral

$$Y_j(t) = \int_0^t G_j(t-\theta)h_j(\theta)d\theta \tag{37}$$

where the impulse response function is

$$h_j(t) = \frac{e^{-0.5\beta_j t}}{\omega_j\sqrt{1-\frac{\beta_j^2}{4\omega_j^2}}}\sin\left(\omega_j\sqrt{1-\frac{\beta_j^2}{4\omega_j^2}}\,t\right) \qquad t \ge 0 \tag{38}$$

Substituting Eq. (36) into Eq. (37), the modal amplitude $Y_j(t)$ may then be written as

$$Y_j(t) = \int_0^t \frac{1}{\overline{m}L}\int_0^L \psi_j(x)\delta(x-d)F(t-\theta)dx\; h_j(\theta)d\theta$$

$$= \frac{\psi_j(d)}{\overline{m}L}\int_0^L F(t-\theta)h_j(\theta)d\theta \tag{39}$$

Thus, $y_b(x, t)$ can be obtained in the form

$$y_b(x,t) = \sum_{j=1}^{\infty}\psi_j(x)Y_j(t)$$

$$= \sum_{j=1}^{\infty}\frac{\psi_j(x)\psi_j(d)}{\overline{m}L}\int_0^t F(t-\theta)h_j(\theta)d\theta \tag{40}$$

Based on random vibration theory, the spectral density function of $y_b(x, t)$ can then be given by

$$S_{y_by_b}(x,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{y_by_b}(x,\tau)e^{-i\omega\tau}d\tau$$

$$= S_{FF}(\omega)\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{\psi_j(x)\psi_j(d)\psi_k(x)\psi_k(d)}{(\overline{m}L)^2}H_j(\omega)H_k(-\omega) \tag{41}$$

where $S_{FF}(\omega)$ is the power spectral density function of $F(t)$;

$$R_{y_by_b}(x,\tau) = E[y_b(x,t)y_b(x,t+\tau)] \tag{42}$$

$$H_j(\omega) = \frac{1}{(\omega_j^2-\omega^2)+i\beta_j\omega} \tag{43}$$

Let

$$S_{y_by_b}(x,\omega) = S_{FF}(\omega)B(x,\omega) \tag{44}$$

$$B(x,\omega) = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{\psi_j(x)\psi_j(d)\psi_k(x)\psi_k(d)}{(\overline{m}L)^2}\cdot H_j(\omega)H_k(-\omega) \tag{45}$$

VIII. DYNAMIC VEHICLE LOAD SPECTRUM

The dynamic vehicle load can be experimentally determined for a particular stretch of pavement. As the process of pavement roughness is a stationary Gaussian random process with zero mean, the load is essentially random in nature and is assumed to have properties of a stationary Gaussian random process with zero mean. A comprehensive description of the dynamic vehicle load can be obtained using power spectral density function, called dynamic vehicle load spectrum.

Since the dynamic vehicle load $F(t)$ is defined by

$$F(t)=c_0(\dot{Z}-\dot{y})+k(Z-y)=f(t)-m_1\ddot{Z} \tag{46}$$

the spectral density function of $F(t)$ can then be approximately given by

$$S_{FF}(\omega)=S_{ff}(\omega)+m_1^2\,\omega^4 S_{zz}(d,\ \omega) \tag{47}$$

Substituting Eqs. (13) and (21) into Eq. (10) with introducing Eqs. (19), (45), and $x=d$, gives

$$S_{ZZ}(d,\ \omega)=\frac{(\left|H_y(\omega)\right|^2 B(d,\ \omega)+\left|H_f(\omega)\right|^2)S_{ff}(\omega)+\left|H_y(\omega)\right|^2 S_{y_r y_r}(\omega)}{1-m_1^2\omega^4\left|H_y(\omega)\right|^2 B(d,\ \omega)} \tag{48}$$

Substituting Eq. (48) into Eq. (47), the one-sided dynamic vehicle load spectrum, $S_{FF}(\omega)$, is then given by

$$S_{FF}(\omega)=T_f(\omega)S_{ff}(\omega)+T_r(\omega)S_{y_r y_r}(\omega) \tag{49}$$

or in matrix form

$$S_{FF}(\omega)=\left\langle T_f(\omega)\ \ T_r(\omega)\right\rangle\begin{Bmatrix} S_{ff}(\omega) \\ S_{y_r y_r}(\omega)\end{Bmatrix} \tag{50}$$

where

$$T_f(\omega)=1+m_1^2\omega^4\frac{\left|H_y(\omega)\right|^2 B(d,\ \omega)+\left|H_f(\omega)\right|^2}{1-m_1^2\omega^4\left|H_y(\omega)\right|^2 B(d,\ \omega)} \tag{51}$$

and

$$T_r(\omega)=m_1^2\omega^4\frac{\left|H_y(\omega)\right|^2}{1-m_1^2\omega^4\left|H_y(\omega)\right|^2 B(d,\ \omega)} \tag{52}$$

Note that as the constant flexural rigidity, EI, of the bridge approaches infinite, Eqs. (51) and (52) can be further simplified. The simplified versions of Eqs. (51) and (52) have been developed by the authors (Lin and Weng, 2001) for dynamic vehicle loads on "rigid" pavement, due to the passage of the vehicle moving along a rough road surface and can be respectively expressed as

$$T_f(\omega)=1+m_1^2\omega^4\left|H_f(\omega)\right|^2 \tag{53}$$

and

$$T_r(\omega)=m_1^2\omega^4\left|H_y(\omega)\right|^2 \tag{54}$$

The mean square of $F(t)$ is related to $S_{FF}(\omega)$ by the equation

$$\sigma_F^2=\int_0^\infty S_{FF}(\omega)d\omega \tag{55}$$

If the $S_{FF}(\omega)$ is integrable, the closed-form solution of σ_F^2 can be evaluated by the formal integration of Eq. (55). Unfortunately, it is limited in practical cases. Thus, the values of σ_F^2 must be evaluated by numerical processes and a cut of frequency, $\tilde{\omega}$, must be applied on this numerical integration. For convenience of numerical calculation, the $S_{FF}(\omega)$, in this study, has been evaluated at equal frequency increments $\Delta\omega$, successive values of the function being identified by appropriate subscripts. The value of the integral can then be obtained approximately by summing these ordinates multiplied by appropriate weighting factors.

IX. MAGNITUDE OF DYNAMIC VEHICLE LOAD

The magnitude of dynamic vehicle load depends on the characteristics of vibrations of the bridge, the pavement roughness, the vehicle speed, and the suspension system of the vehicle. A parameter used to characterize the magnitude of the dynamic vehicle load is the 'Dynamic Load Coefficient' (DLC), which is defined as

$$DLC=\frac{\text{RMS dynamic vehicle load}}{\text{Static vehicle load}}=\frac{\sigma_F}{m_1 g} \tag{56}$$

Under normal operating conditions, DLC's of

0.1-0.3 are typical for the case of vehicle-road pavement interaction (Ervin *et al.*, 1983; Hu, 1988; Magnusson *et al.*, 1984; Sweatman, 1980; 1983). Note that the DLC is an important parameter in design and evaluation. The variation of dynamic vehicle load causes subtle problems and contributes to fatigue, surface wear, and cracking of concrete that leads to corrosion. Sweatman (1983) measured values up to 0.4 for particularly poor tandem suspensions. According to Hahn, measured peak dynamic vehicle loads usually exceed the root mean square (RMS) levels by a factor of about 3. This is consistent with a Gaussian probability distribution.

X. NUMERICAL EXAMPLES

The proposed solutions are useful in evaluating the variation of dynamic vehicle load and the DLC for a specified vehicle speed and pavement roughness. Numerical examples of a simply-supported bridge are presented as follows.

1. Parameters Considered in Analysis

In analyses of numerical examples, the parameters of vehicle weight, suspension stiffness, and suspension damping were, respectively, taken as 294 kN, 3220 kN/m, and 9 percent of the critical damping. The vibration frequency of the vehicle system was equal to 10.4 rad/s (1.65 Hz). Three different vehicle speeds were considered, 60, 100, and 140 km/hr.

It is assumed that a road profile is a realization of a random process that can be described by a power spectral density function. Four different classes of pavement roughness (including: very good, good, average, and poor pavements) for principal roads were used in the analyses. In the parametric study, the road spectra suggested by LaBarre *et al.* were used to model the road pavement roughness. The parameters n_0, ω_1, and ω_2 in Eq. (19) were taken as $1/2\pi$ (cycle/m), 2.05, and 1.44, respectively. The effect of engine motion on vehicle vibration was disregarded in numerical analyses.

The bridge was modeled as a simply-supported bridge. The mass per unit length $\overline{m}$ and flexural rigidity EI of the bridge were taken as 11000kg and 120×10^6 kN m^2, respectively. The modal damping ratios were assumed to be 0.02. Span length of 40 m was considered in this study. The first three modal frequencies of bridge vibration were 3.3, 12.9, and 29.1 Hz.

2. Numerical Results

The effects of vehicle speed and pavement roughness on the standard deviation of dynamic vehicle load are shown in Fig. 2. As shown in this figure, the standard deviation of dynamic vehicle load increases with the increases in vehicle speed and pavement roughness. In the figure, Zones 1 to 4 correspond to very good, good, average, and poor pavements, respectively. Fig. 2 also shows that, for a specified vehicle speed, as the value of pavement roughness coefficient $S(n_0)$ increases four times (e.g. the value of $S(n_0)$ changes from 2×10^{-6} m^3/cycle to 8×10^{-6} m^3/cycle or from 8×10^{-6} m^3/cycle to 32×10^{-6} m^3/cycle), the standard deviation of dynamic vehicle load increases two times. This observation indicates that, for a specified vehicle speed, the standard deviation of dynamic vehicle load is proportional to the square root of the pavement roughness coefficient $S(n_0)$. This conclusion can be further demonstrated from Eq. (55) with the help of Eqs. (49) and (19). As shown in Eq. (49), if the effect of engine motion on vehicle vibration is disregarded, the spectrum $S_{FF}(\omega)$ is proportional to the pavement roughness coefficient $S(n_0)$. Then, the variance σ_F^2 of dynamic vehicle load is also proportional to the pavement roughness coefficient $S(n_0)$. In other words, the standard deviation of dynamic vehicle load is proportional to the square root of the pavement roughness coefficient, $S(n_0)$, for a specified vehicle speed. It is noted that, in general, the effect of engine motion on dynamic vehicle load is small due to significant difference between the frequency of engine motion and of vehicle vibration.

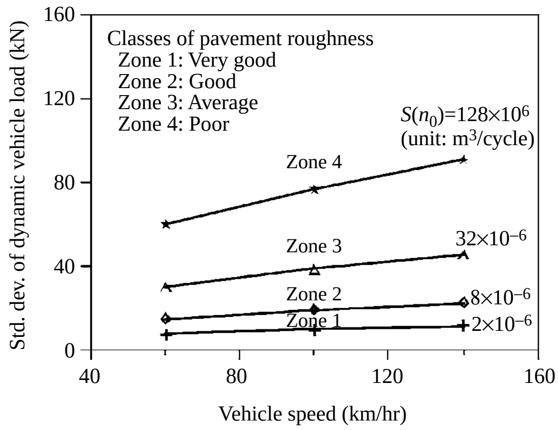


Fig. 2 Standard deviation of dynamic vehicle load versus vehicle speed for four different classes of pavement roughness

Figure 3 shows the relation between the DLC and vehicle speed for four different classes of pavement roughness from very good to poor conditions. As shown in Fig. 3, the DLC depends on vehicle speed and pavement roughness. The DLC increases with the increases in vehicle speed and pavement roughness. If the effect of engine motions on vehicle vibrations is disregarded, the DLC is also proportional

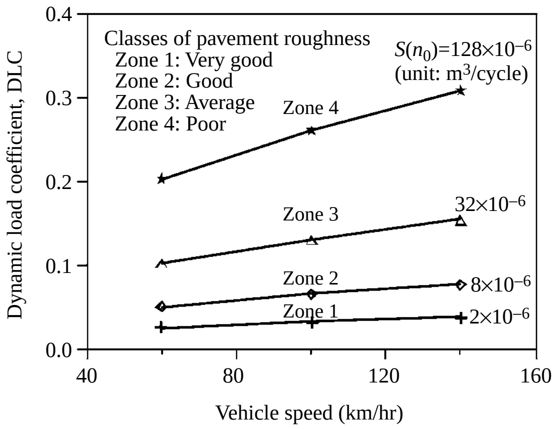


Fig. 3 Dynamic load coefficient versus vehicle speed for four different classes of pavement roughness

to the square root of the pavement roughness coefficient $S(n_0)$ for a specified vehicle speed. Note that the DLC is proportional to the standard deviation of dynamic vehicle load. Fig. 3 also shows that, for the conditions of good and average pavements (Zone 2 and 3) with vehicle speeds in the range of 60 to 100 km/hr, the values of DLC are in the range from 0.05 to 0.26. Note that, under normal operating conditions, DLC's of 0.1-0.3 are typical (Ervin *et al*., 1983; Hu, 1988; Magnusson *et al*., 1984; Sweatman, 1980; 1983).

XI. CONCLUDING REMARKS

This study develops a spectral approach for evaluating the dynamic vehicle load due to the passage of a vehicle moving at constant speed along a rough bridge surface. Based on the assumptions of linear elastic and stationary Gaussian random responses, a simple closed-form solution for predicting the variation of dynamic vehicle load on bridge decks is proposed. It is concluded that if the effect of engine motion on vehicle vibration is disregarded, both the Dynamic Load Coefficient (DLC) and standard deviation of dynamic vehicle load on bridge decks are proportional to the square root of the pavement roughness coefficient $S(n_0)$ for a specified vehicle speed. The dynamic vehicle loads vary significantly with vehicle speed and pavement roughness. However, there is no specific consideration for vehicle speed and pavement roughness in the related specifications of AASHTO.

It is noted that a real vehicle is much more complex than the simplified model adopted in this study, and the use of the calculated results according to the proposed solutions is subject to errors resulting from the simplification of the analytical model. However, the procedure developed herein can be extended to more complex situations because complete descriptions of the motion of both the vehicle and the bridge are maintained in the solution process.

NOMENCLATURE

A_0	amplitude of engine force
c_0	damping coefficient of vehicle suspension system
f	engine-induced force
F	dynamic vehicle load
H	frequency response function or transfer function
H^*	complex conjugate of H
k	spring constant of the vehicle suspension system
m_1	vehicle mass
n	spatial frequency
R	autocorrelation function
S	power spectral density function
$S(n_0)$	pavement roughness coefficient
V	vehicle speed
Y	modal amplitude of bridge vibration
y_b	bridge deflection due to dynamic vehicle load F
y_{b1}	bridge deflection due to vehicle gravity force m_1g
y_r	pavement surface elevation
Z	vehicle displacement due to dynamic vehicle load F
Z_1	vehicle displacement due to vehicle gravity force m_1g
δ	delta function
θ	random phase angle
σ_F^2	ensemble mean square of F
ψ	mode shape of bridge vibration
ω_0	circular frequency of engine force

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