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Chaos synchronization and parameter identification for loudspeaker systems

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Abstract

The identical two-degrees-of-freedom loudspeaker systems are discussed for synchronization of chaos in this paper. Two methods are used to synchronize two identical chaotic systems with different initial condition: the adaptive control and the Gerschgorin's theorem. Finally we research the parameter identification for two identical two-degrees-of-freedom loudspeaker systems by adaptive control and random optimization method.

1. Introduction

Lorenz studied the strange changes in the atmosphere which is the first example to study chaos in 1963. In the past four decades, a large number of studies have shown that chaotic phenomena are observed in many physical systems that possess nonlinearity [1,2]. It was also reported that the chaotic motion occurred in many nonlinear control systems [3].

In this paper, chaos synchronization of a two-degrees-of-freedom loudspeaker system is researched by many methods. In Section 2, a two-degrees-of-freedom loudspeaker system model and Lagrange's equations of motion for it are introduced. Next, the bifurcation diagram and the Lyapunov exponent are expressed by numerical analysis.

The identical systems are discussed for synchronization of chaos in Section 3. Two methods are presented to achieve the synchronization: the adaptive control, the Gerschgorin's theorem.

We research the parameter identification for identical two-degrees-of-freedom loudspeaker systems in Section 4. Two methods are presented to achieve the synchronization: the adaptive control and the random optimization method.

Finally, the conclusion of the whole paper is briefly stated.

2. Equations of motion

The loudspeaker system considered here is depicted in Fig. 1. It is a loudspeaker system having two-degrees-of-freedom, where one is the electric charge on the capacitor plate and the other is displacement of the parallel plate capacitor.

The state equations of loudspeaker system are described by [4]

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -a_{21}x_1 - a_{22}x_2 + a_{23}x_3 + a_{24}x_3^2 + a_{25}\sin\left(\frac{\omega}{\Omega}\right)\tau, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = a_{41}x_1 + a_{42}x_1x_3 - a_{43}x_3 - a_{44}x_4, \end{cases} \tag{2.1}$$

where $a_{21} = 1$, $a_{22} = 0.05$, $a_{23} = 2$, $a_{24} = 0.0847$, $a_{25} = \frac{A}{mx_0\Omega^2}$, $a_{41} = 0.0694$, $a_{42} = 0.0694$, $a_{43} = 1.27$, $a_{44} = 0.5$.

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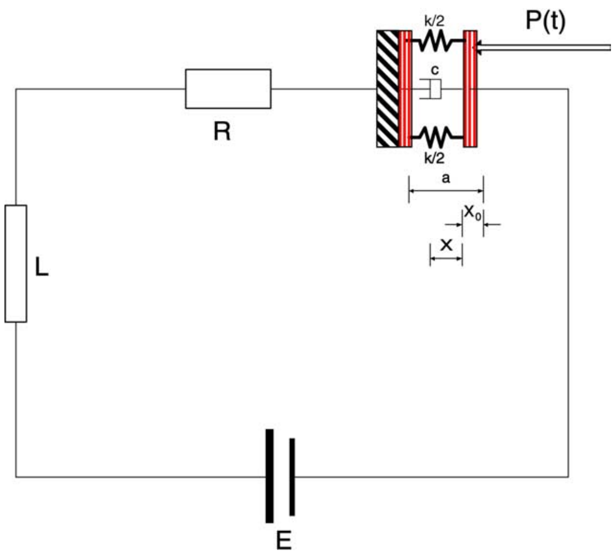

Fig. 1. A schematic diagram of loudspeaker system.

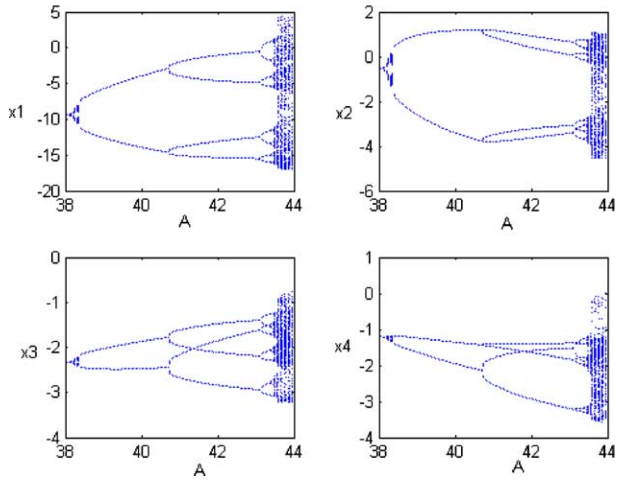

Fig. 2. Bifurcation diagram for A between 38 and 44.

The bifurcation diagram of the loudspeaker system is depicted in Fig. 2. The range of A is $[38, 44]$ with the incremental value is 0.01. Lyapunov exponents of loudspeaker system are plotted in Fig. 3.

3. Synchronization of chaos for identical systems

Two identical systems are discussed for synchronization of chaos in this paper. Two methods are presented to achieve synchronization: the adaptive control [5] and the Gerschgorin's theorem [6].

3.1. Synchronization by adaptive control

We study two identical two-degrees-of-freedom loudspeaker systems in this section. Both systems have the same form and two parameters are unknown. Two identical systems begin with two different initial conditions that will be synchronized by following methods.

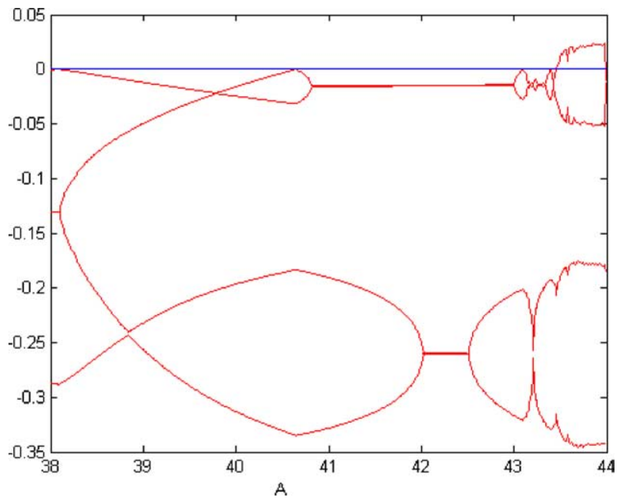

Fig. 3. The Lyapunov exponent for A between 38 and 44.

The drive system is described as Eq. (2.1).

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -a_{21}x_1 - a_{22}x_2 + a_{23}x_3 + a_{24}x_3^2 + a_{25}\sin\left(\frac{\omega}{\Omega}\right)\tau, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = a_{41}x_1 + a_{42}x_1x_3 - a_{43}x_3 - a_{44}x_4. \end{cases} \tag{3.1.1}$$

The response system is described by

$$\begin{cases} \dot{y}_1 = y_2, \\ \dot{y}_2 = -a_{21}y_1 - a_{22}y_2 + a_{23}y_3 + a_{24}y_3^2 + a_{25}\sin\left(\frac{\omega}{\Omega}\right)\tau, \\ \dot{y}_3 = y_4, \\ \dot{y}_4 = a_{41}y_1 + a_{42}y_1y_3 - a_{43}y_3 - a_{44}y_4. \end{cases} \tag{3.1.2}$$

The values of parameters are $a_{21} = 1$, $a_{23} = 2$, $a_{24} = 0.0847$, $a_{25} = 5.5652$, $a_{41} = 0.0694$, $a_{42} = 0.0694$, $a_{43} = 1.27$. The true values of "unknown" parameters are $a_{22} = 0.05$, $a_{44} = 0.5$ in numerical simulation.

The initial conditions of the drive and the response systems are $(x_1(0), x_2(0), x_3(0), x_4(0)) = (1, 0, 1, 0)$, $(y_1(0), y_2(0), y_3(0), y_4(0)) = (1.1, 0.1, 1.1, 0.1)$, respectively. The initial values of estimate for "unknown" parameters are $\hat{a}_{22}(0) = 0.1$, $\hat{a}_{44}(0) = 0.1$.

For synchronizing two two-degrees-of-freedom loudspeaker systems, we add four controllers, u_1, u_2, u_3, u_4 on the first, second, third, and fourth equations of Eq. (3.1.2), respectively.

$$\begin{cases} \dot{y}_1 = y_2 + u_1, \\ \dot{y}_2 = -a_{21}y_1 - a_{22}y_2 + a_{23}y_3 + a_{24}y_3^2 + a_{25}\sin(\frac{\omega}{\Omega})\tau + u_2, \\ \dot{y}_3 = y_4 + u_3, \\ \dot{y}_4 = a_{41}y_1 + a_{42}y_1y_3 - a_{43}y_3 - a_{44}y_4 + u_4. \end{cases} \tag{3.1.3}$$

First, subtracting Eq. (3.1.1) from Eq. (3.1.3), we can obtain the error dynamics

$$\begin{aligned} \dot{e}_1 &= e_2 + u_1, \\ \dot{e}_2 &= -a_{21}e_1 - a_{22}e_2 + a_{23}e_3 + a_{24}(y_3^2 - x_3^2) + u_2, \\ \dot{e}_3 &= e_4 + u_3, \\ \dot{e}_4 &= a_{41}e_1 + a_{42}(y_1y_3 - x_1x_3) - a_{43}e_3 - a_{44}e_4 + u_4, \end{aligned} \tag{3.1.4}$$

where $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$, $e_4 = y_4 - x_4$.

Next, a Lyapunov function is selected as

$$V(e,\tilde{a}_{22},\tilde{a}_{44}) = \tfrac{1}{2}e^{\mathrm{T}}e + \tfrac{1}{2}(\tilde{a}_{22}^2 + \tilde{a}_{44}^2), \tag{3.1.5}$$

where $\tilde{a}_{22} = a_{22} - \hat{a}_{22}$, $\tilde{a}_{44} = a_{44} - \hat{a}_{44}$ and $\hat{a}_{22}$, $\hat{a}_{44}$ are estimate values of the unknown parameters a_{22} and a_{44}, respectively.

Its derivative along the solution of Eq. (3.1.4) is

$$\begin{aligned}\frac{\mathrm{d}V(e,\tilde{a}_{22},\tilde{a}_{44})}{\mathrm{d}t} &= e^{\mathrm{T}}\dot{e} + \tilde{a}_{22}\dot{\tilde{a}}_{22} + \tilde{a}_{44}\dot{\tilde{a}}_{44} \\ &= e_1(e_2+u_1) + e_2[-a_{21}e_1 - a_{22}e_2 + a_{23}e_3 + a_{24}(y_3^2 - x_3^2) + u_2] + e_3(e_4+u_3) + e_4[a_{41}e_1 \\ &\quad + a_{42}(y_1y_3 - x_1x_3) - a_{43}e_3 - a_{44}e_4 + u_4] + \tilde{a}_{22}(-\dot{\hat{a}}_{22}) + \tilde{a}_{44}(-\dot{\hat{a}}_{44}).\end{aligned} \tag{3.1.6}$$

Select u_1, u_2, u_3, u_4, $\dot{\hat{a}}_{22}$, $\dot{\hat{a}}_{44}$ as

$$\begin{aligned} u_1 &= -e_1 - e_2, \\ u_2 &= a_{21}e_1 + \hat{a}_{22}e_2 - a_{23}e_3 - a_{24}(y_3^2 - x_3^2) - e_2, \\ u_3 &= -e_3 - e_4, \\ u_4 &= -a_{41}e_1 - a_{42}(y_1y_3 - x_1x_3) + a_{43}e_3 + \hat{a}_{44}e_4 - e_4, \\ \dot{\hat{a}}_{22} &= -e_2^2, \\ \dot{\hat{a}}_{44} &= -e_4^2. \end{aligned}$$

Then, Eq. (3.1.6) becomes

$$\frac{\mathrm{d}V(e)}{\mathrm{d}t} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 < 0. \tag{3.1.7}$$

This means that the synchronization of two identical two-degrees-of-freedom loudspeaker systems can be accomplished. The results are shown in Figs. 4–11.

3.2. Synchronization by Gerschgorin's theorem

We continue to study two identical two-degrees-of-freedom loudspeaker systems in this section. Two identical systems begin with two different initial conditions which will be synchronized by following methods. The drive system is described as Eq. (3.1.1).

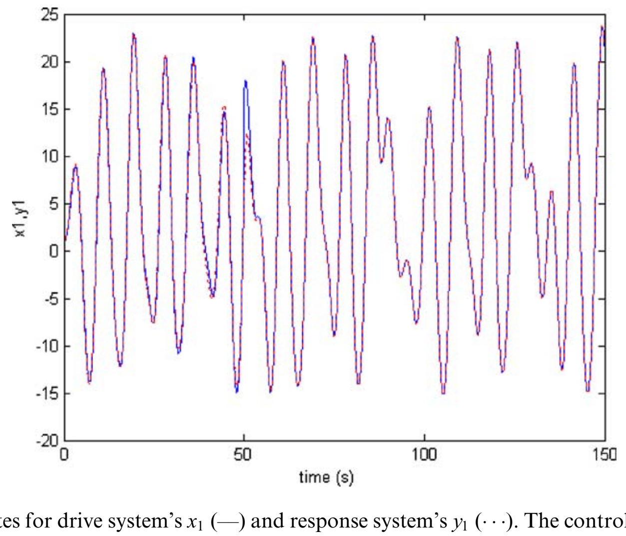

Fig. 4. The time response of states for drive system's x_1 (—) and response system's y_1 (···). The controller is acceded to response system at $t = 50$ s.

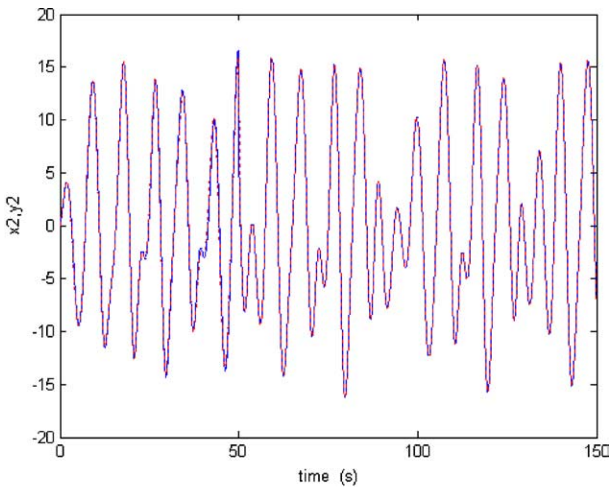

Fig. 5. The time response of states for drive system's x_2 (—) and response system's y_2 ($\cdots$). The controller is acceded to response system at $t = 50$ s.

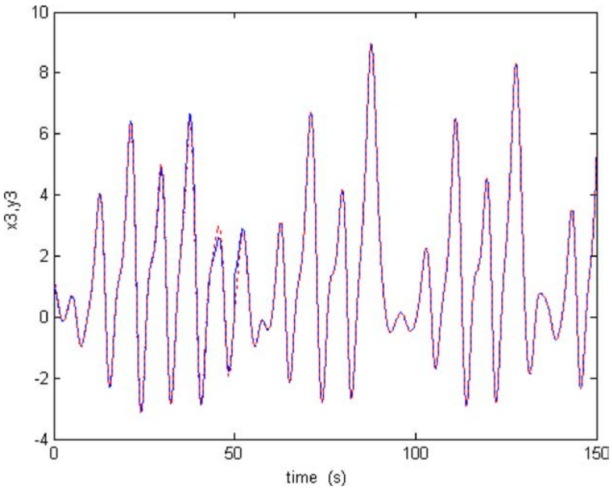

Fig. 6. The time response of states for drive system's x_3 (—) and response system's y_3 ($\cdots$). The controller is acceded to response system at $t = 50$ s.

The following response system is constructed for Eq. (3.1.2) with linear unidirectional couplings:

$$\begin{cases} \dot{y}_1 = y_2 + k_1(x_1 - y_1), \\ \dot{y}_2 = -a_{21}y_1 - a_{22}y_2 + a_{23}y_3 + a_{24}y_3^2 + a_{25}\sin\left(\frac{\omega}{\Omega}\right)\tau + k_2(x_2 - y_2), \\ \dot{y}_3 = y_4 + k_3(x_3 - y_3), \\ \dot{y}_4 = a_{41}y_1 + a_{42}y_1y_3 - a_{43}y_3 - a_{44}y_4 + k_4(x_4 - y_4). \end{cases} \tag{3.2.1}$$

The parameters of drive and response systems are chosen as $a_{21} = 1$, $a_{22} = 0.05$, $a_{23} = 2$, $a_{24} = 0.0847$, $a_{25} = 5.5652$, $a_{41} = 0.0694$, $a_{42} = 0.0694$, $a_{43} = 1.27$, $a_{44} = 0.5$.

The drive and the response systems can be written as

$$\begin{aligned} \dot{x} &= Ax + g(x) + u, \\ \dot{y} &= Ay + g(y) + u + K(x - y), \end{aligned} \tag{3.2.2}$$

where $A \in R^{n\times n}$ is a constant matrix, $g(x)$ is a nonlinear function, and u is the external input vector. K is a diagonal matrix to be designed later.

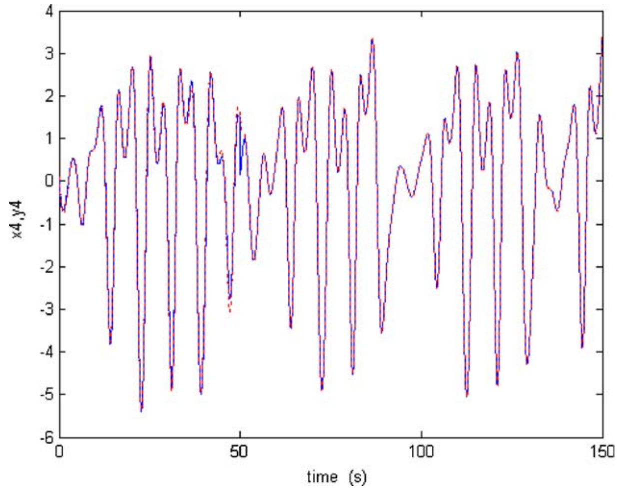

Fig. 7. The time response of states for drive system's x_4 (—) and response system's y_4 ($\cdots$). The controller is acceded to response system at $t = 50$ s.

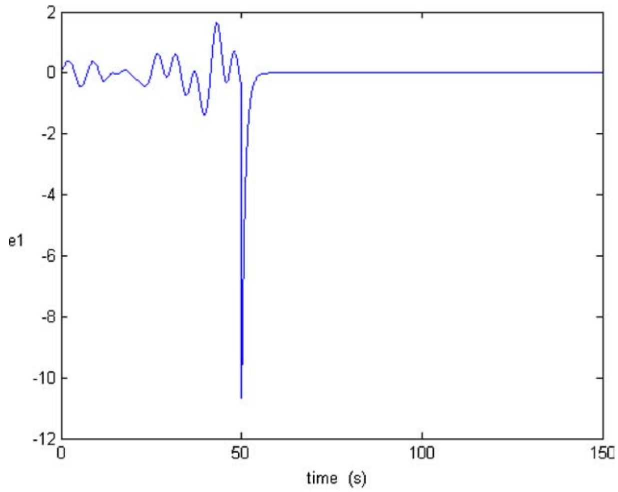

Fig. 8. Time history of error for e_1.

Assuming that

$$g(x) - g(y) = M_{x,y}(x - y). \tag{3.2.3}$$

The elements are dependent on x and y in $M_{x,y}$.

From Eq. (3.2.2), we can obtain the error dynamics

$$\dot{e} = (A - K + M_{x,y})e, \tag{3.2.4}$$

where $e = x - y$.

If $\mathbf{P}$ is a positive definite symmetric constant matrix, and all the eigenvalues of $(A - K + M_{x,y})^{\mathrm{T}}\mathbf{P} + \mathbf{P}(A - K + M_{x,y})$ are negative, the error dynamics would be asymptotically stable about $(0, 0, 0)$ [6]. In the other words, the two-degrees-of -freedom loudspeaker systems would be synchronized.

Let $\mathbf{Q} = (A - K + M_{x,y})^{\mathrm{T}}\mathbf{P} + \mathbf{P}(A - K + M_{x,y})$. The eigenvalues of $\mathbf{Q}$ are λ_i, and $\lambda_i \leqslant \mu < 0$, where μ is a negative constant:

$$\mathbf{Q} = (A - K + M_{x,y})^{\mathrm{T}}\mathbf{P} + \mathbf{P}(A - K + M_{x,y}) = [\mathbf{P}(A + M_{x,y}) + (A + M_{x,y})^{\mathrm{T}}\mathbf{P}] - [\mathbf{P}K + K^{\mathrm{T}}\mathbf{P}] = [\bar{a}_{ij}] - [b_{ij}]. \tag{3.2.5}$$

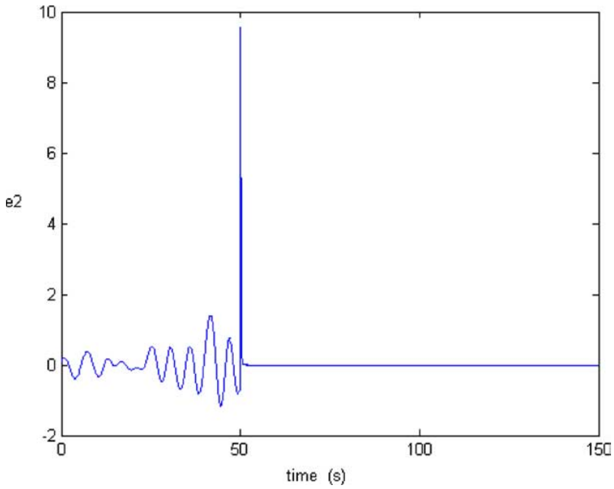

Fig. 9. Time history of error for e_2.

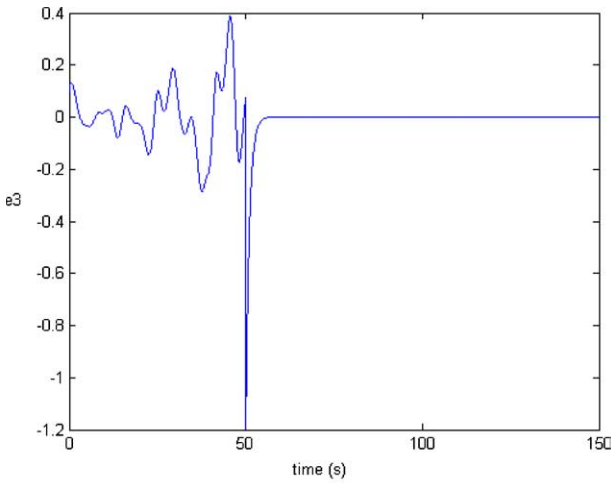

Fig. 10. Time history of error for e_3.

Let C_i be the Gerschgorin's circles of $\mathbf{Q}$, then the centers of circles are $\bar{a}_{ii} - 2k_ip_i$, the radiuses are r_i and $r_i = \sum_{j=1,j\neq i}^{n} |\bar{a}_{ij}|$. Gerschgorin's theorem guarantees that each eigenvalue of $\mathbf{Q}$, when plotted in the complex plane, must lie on or within the circle C_i. Then, we can get that $\bar{a}_{ii} - 2k_ip_i + r_i \leqslant \mu$, and the range of k_i can be obtained:

$$k_i \geqslant \frac{1}{2p_i}(\bar{a}_{ii} + r_i - \mu), \quad i = 1, 2, \ldots, n. \tag{3.2.6}$$

For convenience, we choose $\mathbf{P} = \mathbf{I}$, and Eq. (3.2.6) can be written as

$$k_i \geqslant \frac{1}{2}(\bar{a}_{ii} + r_i - \mu), \quad i = 1, 2, \ldots, n. \tag{3.2.7}$$

Considering the two-degrees-of-freedom loudspeaker systems investigated in this section, we can obtain

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -a_{21} & -a_{22} & a_{23} & 0 \\ 0 & 0 & 0 & 1 \\ a_{41} & 0 & -a_{43} & -a_{44} \end{bmatrix}, \tag{3.2.8}$$

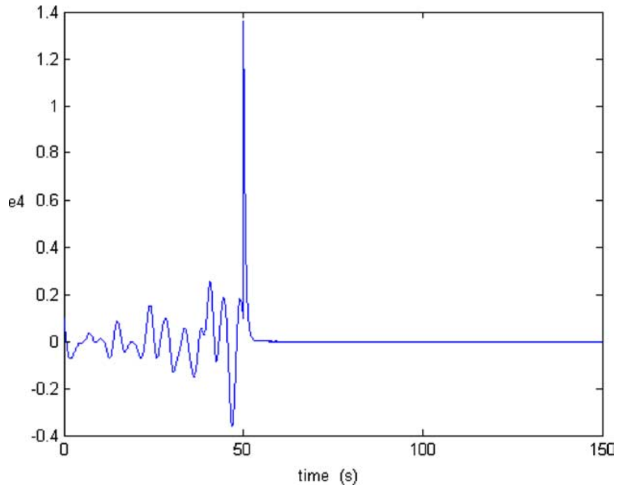

Fig. 11. Time history of error for e_4.

$$K = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix} \tag{3.2.9}$$

and

$$g(x) = \begin{bmatrix} 0 \\ a_{24}x_3^2 \\ 0 \\ a_{42}x_1x_3 \end{bmatrix}, \quad g(y) = \begin{bmatrix} 0 \\ a_{24}y_3^2 \\ 0 \\ a_{42}y_1y_3 \end{bmatrix} \tag{3.2.10}$$

then

$$\begin{aligned} g(x) - g(y) &= \begin{bmatrix} 0 \\ a_{24}(x_3^2 - y_3^2) \\ 0 \\ a_{42}(x_1x_3 - y_1y_3) \end{bmatrix} = \begin{bmatrix} 0 \\ a_{24}(x_3 + y_3)e_3 \\ 0 \\ a_{42}[(2x_3 - y_3)e_1 + x_1e_3] \end{bmatrix} = \mathbf{M}_{x,y}\mathbf{e} \\ &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & a_{24}(x_3 + y_3) & 0 \\ 0 & 0 & 0 & 0 \\ a_{42}(2x_3 - y_3) & 0 & a_{42}x_1 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}, \end{aligned} \tag{3.2.11}$$

$$M_{x,\tilde{x}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & a_{24}(x_3 + y_3) & 0 \\ 0 & 0 & 0 & 0 \\ a_{42}(2x_3 - y_3) & 0 & a_{42}x_1 & 0 \end{bmatrix}. \tag{3.2.12}$$

The error dynamics is

$$\begin{aligned} \dot{e}_1 &= e_2 - k_1e_1, \\ \dot{e}_2 &= -a_{21}e_1 - a_{22}e_2 + a_{23}e_3 + a_{24}(x_3^2 - y_3^2) - k_2e_2, \\ \dot{e}_3 &= e_4 - k_3e_3, \\ \dot{e}_4 &= a_{41}e_1 + a_{42}(x_1x_3 - y_1y_3) - a_{43}e_3 - a_{44}e_4 - k_4e_4, \end{aligned} \tag{3.2.13}$$

where

$$e_1 = x_1 - y_1, \quad e_2 = x_2 - y_2, \quad e_3 = x_3 - y_3, \quad e_4 = x_4 - y_4.$$

It follows from Eqs. (3.2.8) and (3.2.10) that

$$(A+M_{x,y})+(A+M_{x,y})^{\mathrm{T}}=\begin{bmatrix} 0 & 1-a_{21} & 0 & a_{41}+a_{42}(2x_3-y_3) \\ 1-a_{21} & -2a_{22} & a_{23}+a_{24}(x_3+y_3) & 0 \\ 0 & a_{23}+a_{24}(x_3+y_3) & 0 & 1-a_{43}+a_{42}x_1 \\ a_{41}+a_{42}(2x_3-y_3) & 0 & 1-a_{43}+a_{42}x_1 & -2a_{44} \end{bmatrix}. \tag{3.2.14}$$

In terms of Eq. (3.2.7), we can obtain the inequalities as

$$\begin{aligned} k_1 &\geqslant \tfrac{1}{2}(|1-a_{21}|+|a_{41}+a_{42}(e_3+x_3)|-\mu), \\ k_2 &\geqslant \tfrac{1}{2}(|1-a_{21}|-2a_{22}+|a_{23}+a_{24}(x_3+y_3)|-\mu), \\ k_3 &\geqslant \tfrac{1}{2}(|a_{23}+a_{24}(x_3+y_3)|+|1-a_{43}+a_{42}x_1|-\mu), \\ k_4 &\geqslant \tfrac{1}{2}(|a_{41}+a_{42}(e_3+x_3)|+|1-a_{43}+a_{42}x_1|-2a_{44}-\mu). \end{aligned} \tag{3.2.15}$$

By choosing $\mu=-0.7$, we can obtain the coupling strengths as $k_1=1.1$, $k_2=2$, $k_3=2.8$, $k_4=1.3$. The eigenvalues of $(A-K+M_{x,y})+(A-K+M_{x,y})^{\mathrm{T}}$ are −0.7, −1.6, −4.3, −8.3. The selection of μ value is to satisfy with $\lambda_i \leqslant \mu < 0$. The results are shown in Figs. 12–15, and the synchronization is accomplished.

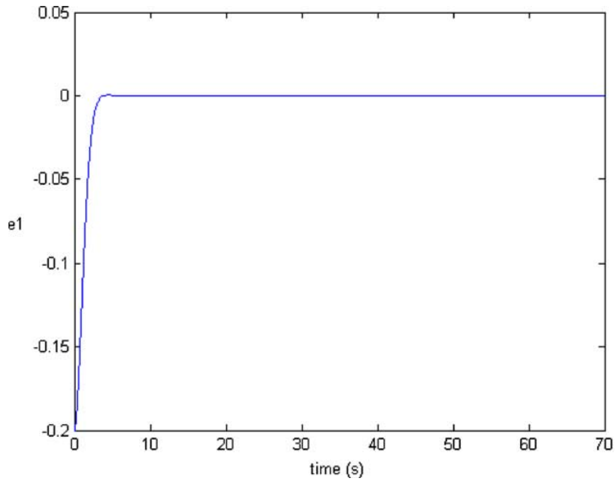

Fig. 12. Time history of error for e_1.

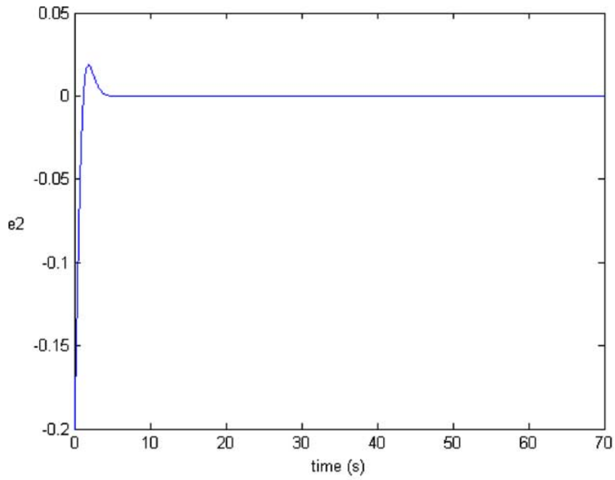

Fig. 13. Time history of error for e_2.

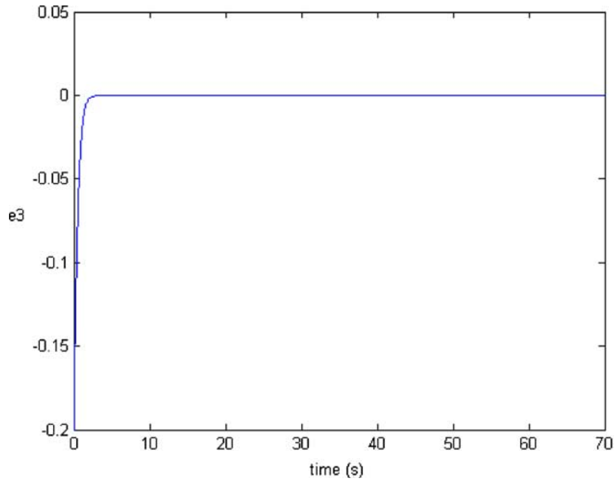

Fig. 14. Time history of error for e_3.

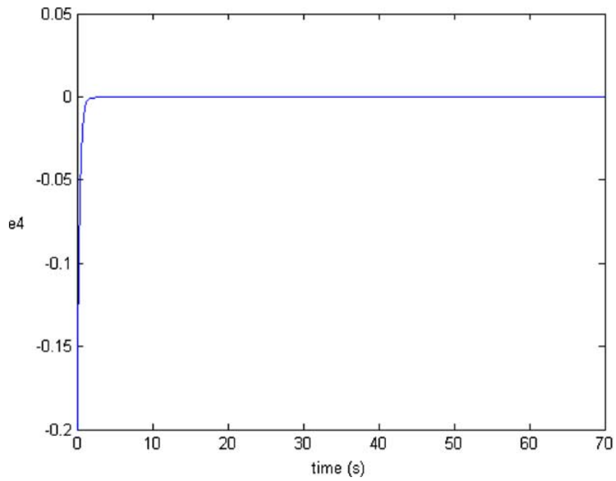

Fig. 15. Time history of error for e_4.

4. Parameter identification by adaptive control

In this paper, both parameter identification and synchronization of chaos are proposed. Two methods are presented to achieve parameter identification: the adaptive control method [7] and the random optimization [8]. In addition, we also keep on discussing identical two-degrees-of-freedom loudspeaker systems in this part.

4.1. Parameters identification by adaptive control

In this section, both parameters identification and synchronization of chaos are proposed by adaptive control method. Two parameters are uncertain in the response system.

The drive system is described by Eq. (2.1).

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -a_{21}x_1 - a_{22}x_2 + a_{23}x_3 + a_{24}x_3^2 + a_{25}\sin\left(\frac{\omega}{\Omega}\right)\tau, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = a_{41}x_1 + a_{42}x_1x_3 - a_{43}x_3 - a_{44}x_4. \end{cases} \tag{4.1.1}$$

The response system is described as

$$\begin{cases} \dot{y}_1 = y_2, \\ \dot{y}_2 = -a_{21}y_1 - \alpha_{22}(t)y_2 + a_{23}y_3 + a_{24}y_3^2 + a_{25}\sin\left(\frac{\omega}{\Omega}\right)\tau, \\ \dot{y}_3 = y_4, \\ \dot{y}_4 = a_{41}y_1 + a_{42}y_1y_3 - a_{43}y_3 - \alpha_{44}(t)y_4, \end{cases} \tag{4.1.2}$$

where α_{22} and α_{44} are two parameters of uncertainty. The initial conditions of the drive and the response systems are $(x_1(0), x_2(0), x_3(0), x_4(0)) = (1, 0, 1, 0)$, $(y_1(0), y_2(0), y_3(0), y_4(0)) = (1.2, 0.2, 1.2, 0.2)$, respectively.

The drive and the response systems can be written as

$$\begin{aligned} \dot{x} &= f(x) + (F_1(x)a_{22} + F_2(x)a_{44}), \\ \dot{y} &= f(y) + (F_1(y)\alpha_{22} + F_2(y)\alpha_{44}), \end{aligned} \tag{4.1.3}$$

where

$$F_1(x)^{\mathrm{T}} = (\,0 \quad x_2 \quad 0 \quad 0\,), \quad F_2(x)^{\mathrm{T}} = (\,0 \quad 0 \quad 0 \quad x_4\,).$$

For synchronizing two identical two-degrees-of-freedom loudspeaker systems, we add four controllers, u_1, u_2, u_3, u_4 on the first, second, third, fourth equations of Eq. (4.1.2), respectively.

$$\begin{cases} \dot{y}_1 = y_2 + u_1, \\ \dot{y}_2 = -a_{21}y_1 - \alpha_{22}(t)y_2 + a_{23}y_3 + a_{24}y_3^2 + a_{25}\sin\left(\frac{\omega}{\Omega}\right)\tau + u_2, \\ \dot{y}_3 = y_4 + u_3, \\ \dot{y}_4 = a_{41}y_1 + a_{42}y_1y_3 - a_{43}y_3 - \alpha_{44}(t)y_4 + u_4. \end{cases} \tag{4.1.4}$$

We have some work to do first before solving our problem. To consider the special case when the drive and the response systems have the same parameters which are time-invariant. In other words, α_{22} and α_{44} are rewritten as a_{22} and a_{44} in Eq. (4.1.4). Subtracting it from Eq. (4.1.1), we can obtain the error dynamics:

$$\begin{aligned} \dot{e}_1 &= e_2 + u_1, \\ \dot{e}_2 &= -a_{21}e_1 - a_{22}e_2 + a_{23}e_3 + a_{24}(y_3^2 - x_3^2) + u_2, \\ \dot{e}_3 &= e_4 + u_3, \\ \dot{e}_4 &= a_{41}e_1 + a_{42}(y_1y_3 - x_1x_3) - a_{43}e_3 - a_{44}e_4 + u_4, \end{aligned} \tag{4.1.5}$$

where

$$e_1 = y_1 - x_1, \quad e_2 = y_2 - x_2, \quad e_3 = y_3 - x_3, \quad e_4 = y_4 - x_4.$$

The Lyapunov function is selected as

$$V(e) = \tfrac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2). \tag{4.1.6}$$

Then its derivative along the solution of Eq. (4.1.5) is

$$\begin{aligned} \frac{\mathrm{d}V(e)}{\mathrm{d}t} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 \\ &= e_1(e_2 + u_1) + e_2[-a_{21}e_1 - a_{22}e_2 + a_{23}e_3 + a_{24}(y_3^2 - x_3^2) + u_2] + e_3(e_4 + u_3) + e_4[a_{41}e_1 + a_{42}(y_1y_3 - x_1x_3) \\ &\qquad - a_{43}e_3 - a_{44}e_4 + u_4]. \end{aligned} \tag{4.1.7}$$

The controllers are chosen as

$$\begin{aligned} u_1 &= -e_1 - e_2, \\ u_2 &= a_{21}e_1 + a_{22}e_2 - a_{23}e_3 - a_{24}\left(y_3^2 - x_3^2\right) - e_2, \\ u_3 &= -e_3 - e_4, \\ u_4 &= -a_{41}e_1 - a_{42}(y_1y_3 - x_1x_3) + a_{43}e_3 + a_{44}e_4 - e_4. \end{aligned} \tag{4.1.8}$$

Then, Eq. (4.1.7) can be rewritten as

$$\frac{dV}{dt} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 < 0. \tag{4.1.9}$$

This means that synchronization of the drive–response system.

Here, we use the results of this special case to solve our problem. The drive system is described as Eq. (4.1.1). The response system is described as Eq. (4.1.4). Let

$$\begin{aligned} \dot{\alpha}_{22}(t) &= -F_1^{\mathrm{T}}(x)(\operatorname{grad} V(e))^{\mathrm{T}} = x_2 e_2, \\ \dot{\alpha}_{44}(t) &= -F_2^{\mathrm{T}}(x)(\operatorname{grad} V(e))^{\mathrm{T}} = x_4 e_4. \end{aligned} \tag{4.1.10}$$

The controllers are chosen as

$$\begin{aligned} u_1 &= -e_1 - e_2, \\ u_2 &= a_{21}e_1 + \alpha_{22}(t)e_2 - a_{23}e_3 - a_{24}(y_3^2 - x_3^2) - e_2, \\ u_3 &= -e_3 - e_4, \\ u_4 &= -a_{41}e_1 - a_{42}(y_1y_3 - x_1x_3) + a_{43}e_3 + \alpha_{44}e_4 - e_4. \end{aligned} \tag{4.1.11}$$

According to the drive system Eq. (4.1.1) and the controlled response system Eq. (4.1.4), we have the following error dynamical system:

$$\dot{e} = f(y) - f(x) + F(y)\alpha - F(x)a + U, \tag{4.1.12}$$

where

$$U^{\mathrm{T}} = [u_1 \quad u_2 \quad u_3 \quad u_4].$$

The Lyapunov function is selected as

$$\overline{V}(e, \alpha) = V(e) + \tfrac{1}{2}(\alpha - a)^{\mathrm{T}}(\alpha - a). \tag{4.1.13}$$

Its derivative along the solution of system Eq. (6.1.11) satisfies [7]

$$\begin{aligned} \frac{d\overline{V}}{dt} &= (\operatorname{grad} V(e), f(y) - f(x) + F(y)\alpha - F(x)a + U) + \dot{\alpha}^{\mathrm{T}}(\alpha - a) \\ &= (\operatorname{grad} V(e), f(y) - f(x) + F(y)\alpha - F(x)\alpha + U) + [\operatorname{grad} V(e)F(x)(\alpha - a)] + \dot{\alpha}^{\mathrm{T}}(\alpha - a) \\ &= -e_1^2 - e_2^2 - e_3^2 - e_4^2 < 0. \end{aligned} \tag{4.1.14}$$

This means that synchronization of the drive–response system. The results are shown in Figs. 16–21, parameters identification and synchronization of chaos are accomplished.

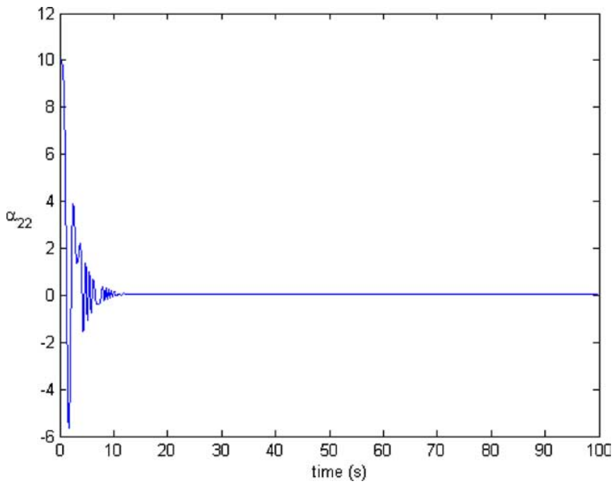

Fig. 16. Graph of the parameter identification result.

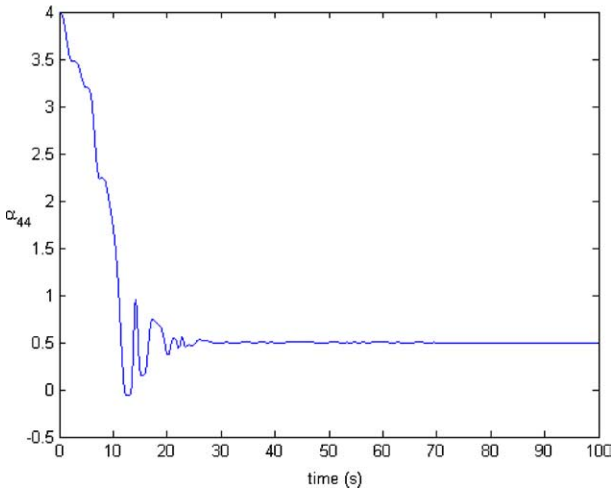

Fig. 17. Graph of the parameter identification result.

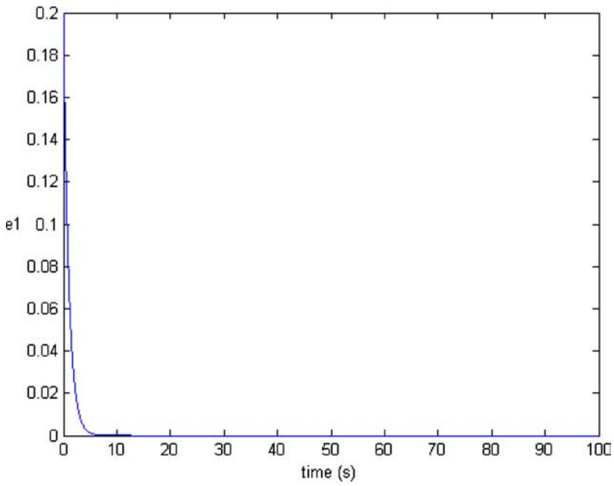

Fig. 18. Time history of error for e_1.

4.2. Parameter identification by random optimization

We investigate two identical loudspeaker systems in this section. Both systems have the same parameters, but a parameter of the response system is unknown. Our work is to identify the unknown parameter. In this section, parameter identification is proposed by random optimization.

The drive system is described as Eq. (2.1).

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -a_{21}x_1 - a_{22}x_2 + a_{23}x_3 + a_{24}x_3^2 + a_{25}\sin\left(\frac{\omega}{\Omega}\right)\tau, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = a_{41}x_1 + a_{42}x_1x_3 - a_{43}x_3 - a_{44}x_4. \end{cases} \tag{4.2.1}$$

The response system is described as

$$\begin{cases} \dot{y}_1 = y_2, \\ \dot{y}_2 = -a_{21}y_1 - \alpha_{22}y_2 + a_{23}y_3 + a_{24}y_3^2 + a_{25}\sin\left(\frac{\omega}{\Omega}\right)\tau, \\ \dot{y}_3 = y_4, \\ \dot{y}_4 = a_{41}y_1 + a_{42}y_1y_3 - a_{43}y_3 - a_{44}y_4. \end{cases} \tag{4.2.2}$$

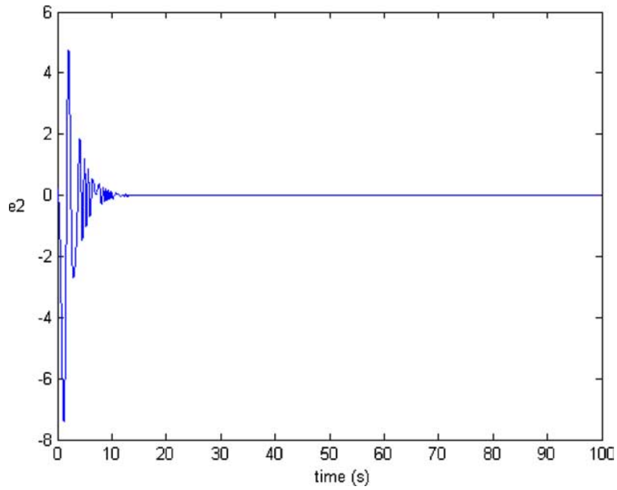

Fig. 19. Time history of error for e_2.

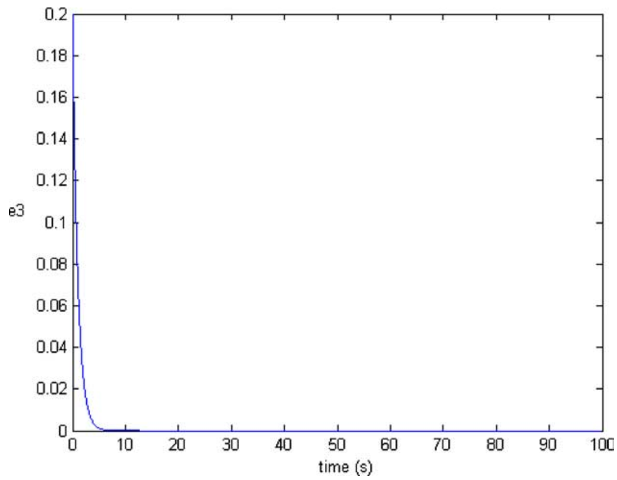

Fig. 20. Time history of error for e_3.

The parameter α_{22} is unknown in the response system. The initial conditions of the drive and the response systems are $(x_1(0), x_2(0), x_3(0), x_4(0)) = (1, 0, 1, 0)$, $(y_1(0), y_2(0), y_3(0), y_4(0)) = (1.2, 0.2, 1.2, 0.2)$, respectively.

To synchronize two identical loudspeaker systems, we add one coupling term, $k(x_1 - y_1)$, on the first equation of Eq. (4.2.2).

$$\begin{cases} \dot{y}_1 = y_2 + k(x_1 - y_1), \\ \dot{y}_2 = -a_{21}y_1 - \alpha_{22}y_2 + a_{23}y_3 + a_{24}y_3^2 + a_{25}\sin\left(\frac{\omega}{\Omega}\right)\tau, \\ \dot{y}_3 = y_4, \\ \dot{y}_4 = a_{41}y_1 + a_{42}y_1y_3 - a_{43}y_3 - a_{44}y_4. \end{cases} \tag{4.2.3}$$

Define the difference by

$$U = \int_{0.9T}^{T} |x_1 - y_1|^2 \, \mathrm{d}t, \tag{4.2.4}$$

where T is the simulation time, chosen as 100 s.

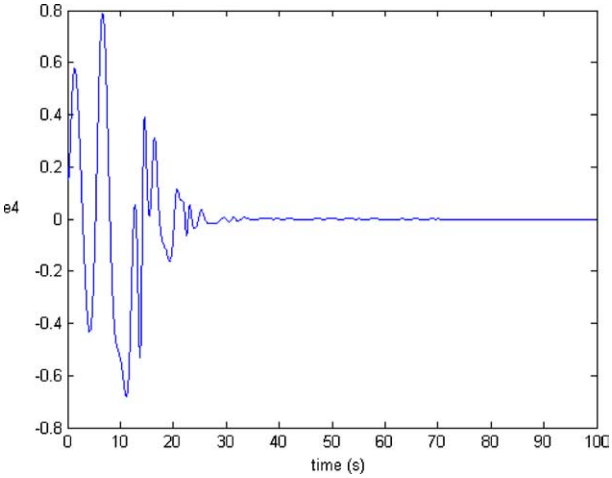

Fig. 21. Time history of error for e_4.

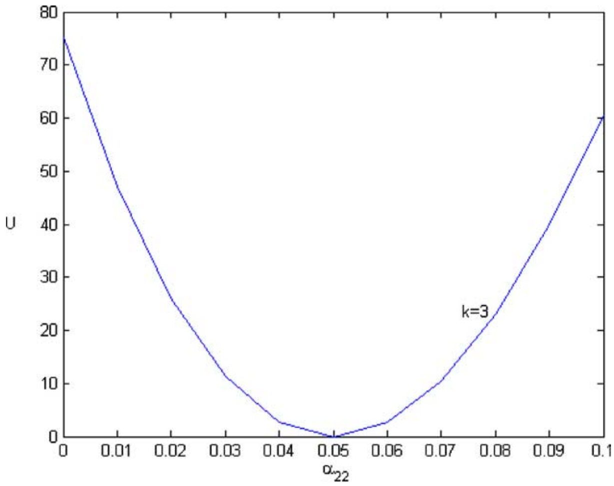

Fig. 22. Difference with respect to the parameter α_{22} for $k = 3$.

The difference U can be considered as a function of α_{22} and k. If k is sufficiently large and α_{22} is close to a_{22}, the difference U would tend to zero. In the other word, with sufficiently large value of k, if U is small, α_{22} would be close to a_{22}. The result is shown in Fig. 22.

To identify the unknown parameter of the response system, we use the random optimization method. The algorithm is as follows.

First, choose a sufficiently large value of k. In our case, we choose $k = 3$. By estimating initial value of α_{22}, we can calculate the difference U.

The parameter α_{22} is randomly modified as

$$\alpha_{22_m} = \alpha_{22} + r, \tag{4.2.5}$$

where r is a random number which obeys the Gaussian distribution with variance $\sigma = 0.001$.

Substituting the modified parameter α_{22_m} into Eq. (4.2.3), we can obtain y_1'. The difference between two systems is

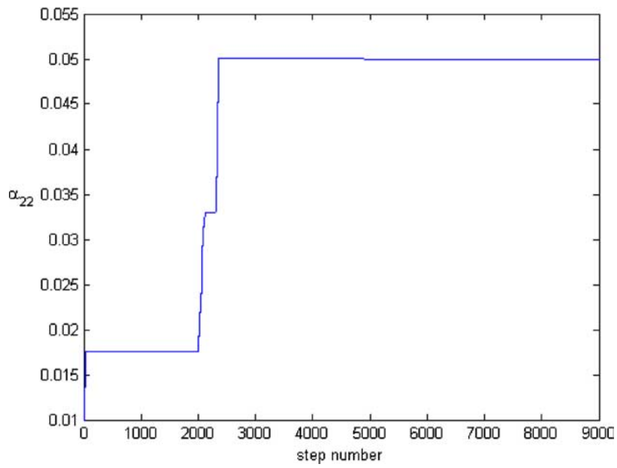

Fig. 23. Time evolution of α_{22} by random optimization process.

$$U' = \int_{0.9T}^{T} |x_1 - y'_1|^2 \, dt. \tag{4.2.6}$$

If the difference U' is smaller than U, the parameter is changed from α_{22} to α_{22_m}. On the other hand, if the difference U' is larger than U, the parameter set is unchanged and kept to be α_{22}. The processes are repeated until the difference U tends to zero.

The parameter identification can be achieved. The result is shown in Fig. 23.

5. Conclusions

In this paper, synchronization of a two-degrees-of-freedom loudspeaker system is studied. In Section 2, a two-degrees-of-freedom loudspeaker system model and its states equations of motion are introduced. Next, the bifurcation diagram and the Lyapunov exponent are expressed by numerical analysis. Then the chaos synchronization of identical systems is achieved in Section 3. Two methods are presented to achieve the synchronization: the adaptive control and the Gerschgorin's theorem. The values of state error approach zero, as display in the plots of time history of error. In other words, synchronization of chaos is realized for identical two-degrees-of-freedom loudspeaker systems.

Finally, we succeed to research the parameter identification for identical two-degrees-of-freedom loudspeaker systems by adaptive control and random optimization method. The results are demonstrated by applying various numerical results.

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