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Generalized On-Shell Ward Identities in String Theory

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It is demonstrated that an infinite set of string-tree level on-shell Ward identities, which are valid to all σ-model loop orders, can be systematically constructed without referring to the string field theory. As examples, bosonic massive scattering amplitudes are calculated explicitly up to the second massive excited states. Ward identities satisfied by these amplitudes are derived by using zero-norm states in the spectrum. In particular, the inter-particle Ward identity generated by the $D_2 \otimes D_{2'}$ zero-norm state at the second massive level is demonstrated. The four physical propagating states of this mass level are then shown to form a large gauge multiplet. This result justifies our previous consideration on higher inter-spin symmetry from the generalized worldsheet σ-model point of view.

Since Veneziano[1] derived the massless gravitational and Yang-Mills Ward identities for hidden symmetries of string theories from the canonical transformations of the string phase-space path integral, there has been an attempt by Kubota[2] to generate new Ward identities corresponding to higher massive string states. Recently, the target space-time w_∞-symmetries of $2D$ string theory, first proposed by Avan and Jevicki[3] in the context of the collective field representation of $c=1$ matrix model,[4] were suggested to associate with higher-level string states.[5] Subsequently, the corresponding on-shell Ward identities were discussed by Klebanov and many authors.[6]

The proposal of Kubota and the encouraging results of the toy 2D string theory as discussed above suggest that there exist an infinite set of Ward identities in the higher dimensional critical string theories. These Ward identities are expected to be associated with stringy symmetries proposed in Ref. 7) where symmetries are directly related to the zero-norm states in the spectrum, and may even be related to the works of Gross[8] and of Atick and Witten,[9] which claim infinite symmetry structures of string gets restored at very high energy. However, the approach of Kubota uses the vertex operator proposed by Ichinose and Sakita.[10] The vertex operator of Ref. 10) contains a string field in it, hence is analogous to the unpleasant second-quantized string field theory formalism. In addition, there are other drawbacks in the above approach, e.g., the off-shell ambiguities remain and there seems to have no underlying principle to choose appropriate gauge functions which generate the proposed higher spin Ward identities. These drawbacks make it difficult to make any physical interpretations of the identities (see Ref. 7)). A general principle which allows one to explicitly write down these Ward identities seems necessary.

In this paper, we will derive the explicit form of massive on-shell Ward identities by using zero-norm states in the bosonic string spectrum. The advantages of our approach are that off-shell ambiguities are avoided, and we can easily write down an infinite set of string-tree level on-shell Ward identities after a systematic construction of zero-norm states in the spectrum. These Ward identities, which are valid to *all* σ-model loop (α') orders, indicate that quantum string theories do possess an infinite number of high energy symmetry structures as was conjectured in Refs. 7) ~9). We will first calculate the massive string-tree level scattering amplitudes up to the second massive states in the most general gauge choice, then use the previously calculated[7] zero-norm states to derive the corresponding Ward identities. Of particular interest, the Ward identity corresponding to the $D_2 \otimes D_2'$ zero-norm state[11] explicitly relates amplitudes of *four different spin states* at the second massive level. The corresponding symmetry transformation law (in the first order weak field approximation) of the four background fields can then be constructed. This result justifies our previous consideration on higher inter-spin symmetries from the generalized σ-model point of view, and is a general feature for higher massive levels. Although the decoupling of zero-norm states from the string amplitudes has been proved for a long time by the so-called "*canceled propagator argument*" in the context of "old fashioned" operator method,[12] its implication on the physical amplitudes was always ignored and not clear so far. On the other hand, our approach will be based on Polyakov functional integral method. Moreover, through the uses of explicit form of zero-norm states which can be calculated to any higher massive level, one can easily write down infinite relations among string scattering amplitudes. Our results can be generalized to the superstring massive states. One can easily write down the symmetry transformation laws of the massive background fields after explicitly deriving the Ward identities. These transformation laws turn out to be too messy to obtain from the worldsheet σ-model approach.[13]

For simplicity we will consider one excited state whose decay process going into three tachyons. The open string massless vector amplitude has been calculated[14] to be (we use the notation of Ref. 14) and assume product of fields at the same point to be normal ordered throughout the text)

$$A=\int\prod_{i=1}^{4}dx_i\langle e^{ik_1\cdot X(x_1)}\varepsilon\cdot\partial X(x_2)e^{ik_2\cdot X(x_2)}e^{ik_3\cdot X(x_3)}e^{ik_4\cdot X(x_4)}\rangle=\varepsilon_\mu T_0{}^\mu \tag{1}$$

with

$$T_0{}^\mu=\frac{\Gamma\left(-\frac{s}{2}-1\right)\Gamma\left(-\frac{t}{2}-1\right)}{\Gamma\left(\frac{u}{2}+2\right)}[k_3{}^\mu(s/2+1)-k_1{}^\mu(t/2+1)]\,, \tag{2}$$

where ε_μ is the vector polarization and s, t and u are the usual Mandelstam variables,

$$s=-(k_1+k_2)^2\,,\quad t=-(k_2+k_3)^2\,,\quad u=-(k_1+k_3)^2\,,\quad s+t+u=\Sigma(\text{mass})^2\,. \tag{3}$$

There is only one singlet massless zero-norm state[7] in the spectrum, which is

$$k\cdot\alpha_{-1}|0,k\rangle\,,\quad -k^2=m^2=0\,. \tag{4}$$

The corresponding Ward identity is easily checked to be

$$k_{2\mu}T_0{}^\mu=[(k_2\cdot k_3)(s/2+1)-(k_2\cdot k_1)(t/2+1)]=0\,. \tag{5}$$

This is consistent with the result obtained by Veneziano through the canonical transformation of the string variables. Note that the identity in Eq. (5) is valid to all σ-model loop (α') orders.

We now turn to the first massive level state. There is only one positive-norm physical propagating mode at this level. The most general form of polarization is given by

$$(\varepsilon_{\mu\nu}\alpha_{-1}{}^{\mu}\alpha_{-1}{}^{\nu}+\varepsilon_{\mu}\alpha_{-2}{}^{\mu})|0, k\rangle\,, \quad \varepsilon_{\mu\nu}\equiv\varepsilon_{\nu\mu} \tag{6}$$

with gauge conditions

$$\varepsilon_{\nu}=-k^{\mu}\varepsilon_{\mu\nu}\,, \quad \varepsilon_{\mu}{}^{\mu}+2k^{\mu}k^{\nu}\varepsilon_{\mu\nu}=0\,, \quad k^2=-2\,. \tag{7}$$

The amplitude is defined to be

$$T_1{}^{\mu\nu}=\int\prod_{i=1}^{4}dx_i\langle e^{ik_1\cdot X}\partial X^{\mu}(x_2)\partial X^{\nu}(x_2)e^{ik_2\cdot X}e^{ik_3\cdot X}e^{ik_4\cdot X}\rangle\,, \tag{8}$$

$$T_1{}^{\mu}=\int\prod_{i=1}^{4}dx_i\langle e^{ik_1\cdot X}\partial^2 X^{\mu}(x_2)e^{ik_2\cdot X}e^{ik_3\cdot X}e^{ik_4\cdot X}\rangle\,. \tag{9}$$

The polarization doublet ($\varepsilon_{\mu\nu}$, ε_{μ}) and the amplitude doublet($T_1{}^{\mu\nu}$, $T_1{}^{\mu}$) as given above describe the dynamics of the first massive state. We will use the method suggested in Ref. 14) to calculate Eqs. (8) and (9). To calculate Eq. (8), we first absorb the kinematic factor to the exponent,

$$\varepsilon_{\mu\nu}\partial X^{\mu}\partial X^{\nu}e^{ikX}\rightarrow\exp[ik\cdot X+i\varepsilon^{(1)}\cdot\partial X+i\varepsilon^{(2)}\cdot\partial X]\,, \tag{10}$$

then the correlation function can be obtained by using the following formula:[11)]

$$\langle:e^{A}::e^{A}:\cdots:e^{A}:\rangle=\exp[\sum_{i<j}\langle A_iA_j\rangle]\,. \tag{11}$$

It is understood that only terms multilinear in $\varepsilon^{(1)}$ and $\varepsilon^{(2)}$ are picked up after evaluating the correlation function, and the factors $\varepsilon_{\mu}{}^{(1)}\varepsilon_{\nu}{}^{(2)}$ should be replaced by $\varepsilon_{\mu\nu}$. This method can be generalized to any higher rank polarization with arbitrary higher derivative $\partial^n X^{\mu}$ in the vertex operator. To make the $SL(2, R)$ gauge fixing which corresponds to the well-known Möbius transformation, we choose $x_1=0$, $0\leq x_2\leq 1$, $x_3=1$, $x_4=\infty$. After some calculation, we get

$$T_1{}^{\mu\nu}=\frac{\Gamma\left(-\frac{s}{2}-1\right)\Gamma\left(-\frac{t}{2}-1\right)}{\Gamma\left(\frac{u}{2}+2\right)}$$

$$\times[s/2(s/2+1)k_3{}^{\mu}k_3{}^{\nu}+t/2(t/2+1)k_1{}^{\mu}k_1{}^{\nu}-2(s/2+1)(t/2+1)k_1{}^{\mu}k_3{}^{\nu}]\,, \tag{12}$$

$$T_1{}^{\mu}=\frac{\Gamma\left(-\frac{s}{2}-1\right)\Gamma\left(-\frac{t}{2}-1\right)}{\Gamma\left(\frac{u}{2}+2\right)}[-k_3{}^{\mu}s/2(s/2+1)-k_1{}^{\mu}t/2(t/2+1)]\,. \tag{13}$$

There are two types of zero-norm states in the bosonic open string spectrum.[7)] They can be obtained either by standard spectrum analysis or generated in the following way,

Type I

$$L_{-1}|\chi\rangle \quad \text{where } L_m|\chi\rangle=0\,, \quad m\geq 1\,, \quad L_0|\chi\rangle=0\,, \tag{14}$$

Type II

$$(L_{-2}+3/2L^2_{-1})|\tilde{\chi}\rangle\,, \quad \text{where } L_m|\tilde{\chi}\rangle=0\,, \quad m\geq 1\,, \quad (L_0+1)|\tilde{\chi}\rangle=0\,. \tag{15}$$

Type I states have zero norm at any space time dimension, whereas type II states have zero norm only at $D=26$. A systematic construction of these zero-norm states has been given in Ref. 7). At the first massive level, we have a vector type I zero-norm state

$$[(\theta\cdot\alpha_{-1})(k\cdot\alpha_{-1})+\theta\cdot\alpha_{-2}]|0,k\rangle\,, \quad \theta\cdot k=0,\ -k^2=m^2=2\,, \tag{16}$$

and a singlet type II zero-norm state

$$[1/2\alpha_{-1}\cdot\alpha_{-1}+3/2(k\cdot\alpha_{-1})^2+5/2k\cdot\alpha_{-2}]|0,k\rangle\,. \tag{17}$$

The corresponding Ward identities are

$$k_{(\mu}\theta_{\nu)}T_1^{\mu\nu}+\theta_\mu T_1^{\mu}=\frac{\Gamma\left(-\frac{s}{2}-1\right)\Gamma\left(-\frac{t}{2}-1\right)}{\Gamma\left(\frac{u}{2}+2\right)}P(s,t)=0 \tag{18}$$

and

$$(3/2k_\mu k_\nu+1/2\eta_{\mu\nu})T_1^{\mu\nu}+5/2k_\mu T_1^{\mu}=\frac{\Gamma\left(-\frac{s}{2}-1\right)\Gamma\left(-\frac{t}{2}-1\right)}{\Gamma\left(\frac{u}{2}+2\right)}P'(s,t)=0\,, \tag{19}$$

where we have used the on-shell condition of k_i. The factors P and P' are polynomials of s and t, and can be explicitly verified to be zero after some calculation. Equations (18) and (19) are Ward identities associated with the stringy symmetries derived from worldsheet σ-model point of view in Ref. 7).

We now come to deriving the interesting inter-particle Ward identity which begins to show up at the second massive states. We will first discuss the open string case. Before doing this, let us first classify the physical degrees of freedom at this mass level. There are two positive-norm physical propagating states at this mass level.[15] The *most general form* of the first state Ψ_1 is given by

$$\{\varepsilon_{\mu\nu\lambda}\alpha_{-1}{}^{\mu}\alpha_{-1}{}^{\nu}\alpha_{-1}{}^{\lambda}+\varepsilon_{(\mu\nu)}\alpha_{-1}{}^{(\mu}\alpha_{-2}{}^{\nu)}+\varepsilon_\mu\alpha_{-3}{}^{\mu}\}|0,k\rangle\,, \quad \varepsilon_{\mu\nu\lambda}=\varepsilon_{(\mu\nu\lambda)} \tag{20}$$

with gauge conditions

$$\varepsilon_{(\mu\nu)}=-3/2k^\lambda\varepsilon_{\mu\nu\lambda}\,, \quad \varepsilon_\mu=1/4k^\nu k^\lambda\varepsilon_{\mu\nu\lambda}\,, \quad -k^2=4\,,$$

$$2\varepsilon^{\mu}{}_{\mu\lambda}+k^\mu k^\nu\varepsilon_{\mu\nu\lambda}=0\,. \tag{21}$$

Note that the form of Eqs. (20) and (21) differs from those of Ref. 15) by a zero-norm

states. One might call Ψ_1 the totally symmetric "spin" three state, and the remaining polarization $\varepsilon_{(\mu\nu)}$ and ε_μ are mere gauge artifacts.
The amplitudes are given by

$$T_2^{\mu\nu\lambda}=\int\prod_{i=1}^{4}dx_i\langle e^{ik_1\cdot X}\partial X^\mu\partial X^\nu\partial X^\lambda e^{ik_2\cdot X}e^{ik_3\cdot X}e^{ik_4\cdot X}\rangle\,, \tag{22}$$

$$T_2^{(\mu\nu)}=\int\prod_{i=1}^{4}dx_i\langle e^{ik_1\cdot X}\partial^2 X^{(\mu}\partial X^{\nu)}e^{ik_2\cdot X}e^{ik_3\cdot X}e^{ik_4\cdot X}\rangle\,, \tag{23}$$

$$T_2^{\mu}=1/2\int\prod_{i=1}^{4}dx_i\langle e^{ik_1\cdot X}\partial^3 X^\mu e^{ik_2\cdot X}e^{ik_3\cdot X}e^{ik_4\cdot X}\rangle\,. \tag{24}$$

The polarization triplet $\{\varepsilon_{(\mu\nu\lambda)},\ \varepsilon_{(\mu\nu)},\ \varepsilon_\mu\}$ and the amplitude triplet $\{T^{\mu\nu\lambda},\ T^{(\mu\nu)},\ T^\mu\}$ as given above describe the Ψ_1 state. The second state Ψ_2 is given by

$$\varepsilon_{[\mu\nu]}\alpha_{-1}{}^{[\mu}\alpha_{-2}{}^{\nu]}|0,k\rangle \tag{25}$$

with gauge conditions

$$k^\mu\varepsilon_{[\mu\nu]}=0\,,\quad -k^2=4\,. \tag{26}$$

The amplitude is given by

$$T_2^{[\mu\nu]}=\int\prod_{i=1}^{4}dx_i\langle e^{ik_1\cdot X}\partial^2 X^{[\mu}\partial X^{\nu]}e^{ik_2\cdot X}e^{ik_3\cdot X}e^{ik_4\cdot X}\rangle\,. \tag{27}$$

Equations (22)~(24) and (27) can be calculated by using the same technique presented in Eq. (10) to be

$$T_2^{\mu\nu\lambda}=\frac{\Gamma\left(-\frac{s}{2}-1\right)\Gamma\left(-\frac{t}{2}-1\right)}{\Gamma\left(\frac{u}{2}+2\right)}\times\{-t/2(t^2/4-1)k_1{}^\mu k_1{}^\nu k_1{}^\lambda+3(s/2+1)t/2(t/2+1)k_1{}^{(\mu}k_1{}^\nu k_3{}^{\lambda)}$$
$$-3s/2(s/2+1)(t/2+1)k_1{}^{(\mu}k_3{}^\nu k_3{}^{\lambda)}+s/2(s^2/4-1)k_3{}^\mu k_3{}^\nu k_3{}^\lambda\}\,, \tag{28}$$

$$T_2^{(\mu\nu)}=\frac{\Gamma\left(-\frac{s}{2}-1\right)\Gamma\left(-\frac{t}{2}-1\right)}{\Gamma\left(\frac{u}{2}+2\right)}\{t/2(t^2/4-1)k_1{}^\mu k_1{}^\nu-(s/2+1)t/2(t/2+1)k_1{}^{(\mu}k_3{}^{\nu)}$$
$$+s/2(s/2+1)(t/2+1)k_3{}^{(\mu}k_1{}^{\nu)}-s/2(s^2/4-1)k_3{}^\mu k_3{}^\nu\}\,, \tag{29}$$

$$T_2^{\mu}=\frac{\Gamma\left(-\frac{s}{2}-1\right)\Gamma\left(-\frac{t}{2}-1\right)}{\Gamma\left(\frac{u}{2}+2\right)}\{s/2(s^2/4-1)k_3{}^\mu-t/2(t^2/4-1)k_1{}^\mu\}\,, \tag{30}$$

$$T_2^{[\mu\nu]}=\frac{\Gamma\left(-\frac{s}{2}-1\right)\Gamma\left(-\frac{t}{2}-1\right)}{\Gamma\left(\frac{u}{2}+2\right)}\{s/2(s/2+1)(t/2+1)$$
$$+(s/2+1)t/2(t/2+1)\}k_3{}^{[\mu}k_1{}^{\nu]}\,. \tag{31}$$

The zero-norm states of this mass level can be calculated to be

$$C, \quad [k_\lambda \theta'_{\mu\nu}\alpha_{-1}{}^\lambda \alpha_{-1}{}^\mu \alpha_{-1}{}^\nu + \theta'_{(\mu\nu)}\alpha_{-1}{}^\mu \alpha_{-2}{}^\nu]|0, k\rangle,$$

$$\theta'_{\mu\nu} = \theta'_{\nu\mu}, \quad k^\mu \theta'_{\mu\nu} = \eta^{\mu\nu}\theta'_{\mu\nu} = 0; \tag{32}$$

$$D_1, \quad \{(5/2 k_\mu k_\nu \theta'_\lambda + \eta_{\mu\nu}\theta'_\lambda)\alpha_{-1}{}^\nu \alpha_{-1}{}^\mu \alpha_{-1}{}^\lambda + 9k_{(\mu}\theta'_{\nu)}\alpha_{-2}{}^\mu \alpha_{-1}{}^\nu + 3\theta'_\mu \alpha_{-3}{}^\mu\}|0, k\rangle,$$

$$k\cdot\theta' = 0; \tag{33}$$

$$D_2, \quad \{(1/2 k_\mu k_\nu \theta_\lambda + 2\eta_{\mu\nu}\theta_\lambda)\alpha_{-1}{}^\nu \alpha_{-1}{}^\mu \alpha_{-1}{}^\lambda + 9k_{[\mu}\theta_{\nu]}\alpha_{-2}{}^{[\mu}\alpha_{-1}{}^{\nu]} + 12\theta_\mu \alpha_{-3}{}^\mu\}|0, k\rangle,$$

$$k\cdot\theta = 0; \tag{34}$$

$$E, \quad \{(3/5 k_\mu k_\nu k_\lambda + 1/5\eta_{\mu\nu}k_\lambda)\alpha_{-1}{}^\nu \alpha_{-1}{}^\mu \alpha_{-1}{}^\lambda$$

$$+ (2/5\eta_{\mu\nu} + 11/5 k_\mu k_\nu)\alpha_{-1}{}^\mu \alpha_{-2}{}^\nu - 2k_\mu \alpha_{-3}{}^\mu\}|0, k\rangle, \tag{35}$$

where D_1 and D_2 states are obtained by symmetrizing and antisymmetrizing those terms which involve $\alpha_{-1}{}^\mu \alpha_{-2}{}^\nu$ in the original type I and type II vector zero-norm states. Ward identities can now be easily written down

$$k_\lambda \theta'_{\mu\nu} T_2{}^{(\mu\nu\lambda)} + \theta'_{\mu\nu} T_2{}^{(\mu\nu)} = 0, \tag{36}$$

$$(5/2 k_\mu k_\nu \theta'_\lambda + \eta_{\mu\nu}\theta'_\lambda) T_2{}^{(\mu\nu\lambda)} + 9k_\mu \theta'_\nu T_2{}^{(\mu\nu)} + 3\theta'_\mu T_2{}^\mu = 0, \tag{37}$$

$$(1/2 k_\mu k_\nu \theta_\lambda + 2\eta_{\mu\nu}\theta_\lambda) T_2{}^{(\mu\nu\lambda)} + 9k_\mu \theta_\nu T_2{}^{[\mu\nu]} + 12\theta_\mu T_2{}^\mu = 0, \tag{38}$$

$$(3/5 k_\mu k_\nu k_\lambda + 1/5\eta_{\mu\nu}k_\lambda) T_2{}^{(\mu\nu\lambda)} + (2/5\eta_{\mu\nu} + 11/5 k_\mu k_\nu) T_2{}^{(\mu\nu)} - 2k_\mu T_2{}^\mu = 0. \tag{39}$$

Equations (36) ~ (39) can be explicitly verified after some lengthy algebra. One has to check the vanishing of a polynomial of sixth degree in s, t and θ in each case. It is now easy to see the identities (36), (37) and (39) correspond to Ψ_1 state, whereas identity (38) generated by D_2 zero-norm state relates amplitudes for Ψ_1 and Ψ_2 states. Equation (38) implies that Ψ_1 and Ψ_2 indeed form a gauge multiplet. This is consistent with the result obtained in Ref. 11) from the worldsheet σ-model point of view. Note that one has to consider the *amplitude triplet* of Ψ_1, $\{T_2{}^{(\mu\nu\lambda)}, T_2{}^{(\mu\nu)}, T_2{}^\mu\}$, in order to see apparently the inter-particle symmetry. The usual "gauge choice" for Ψ_1, $\{\varepsilon_{(\mu\nu\lambda)}, T_2{}^{(\mu\nu\lambda)}\}$ with traceless and transverse gauge conditions, will *hide* this interesting "hidden stringy symmetry".

We can now derive the closed string Ward identities by using the relationship between closed and open string amplitudes[14)]

$$A_{\text{closed}} = -\pi\kappa^2 \sin(\pi k_2\cdot k_3) A_{\text{open}}(s, t)\bar{A}_{\text{open}}(t, u). \tag{40}$$

For example, the Ward identity corresponding to $D_2 \otimes D'_2$ zero-norm state is

$$(1/4 k_\mu k_\nu k_\alpha k_\beta \theta_{\lambda\gamma} + \eta_{\mu\nu}k_\alpha k_\beta \theta_{\lambda\gamma} + \eta_{\alpha\beta}k_\mu k_\nu \theta_{\lambda\gamma} + 4\eta_{\mu\nu}\eta_{\alpha\beta}\theta_{\lambda\gamma}) T_2{}^{(\mu\nu\lambda)(\alpha\beta\gamma)}$$

$$+ (9/2 k_\mu k_\nu k_\alpha \theta_{\lambda\beta} + 18\eta_{\mu\nu}k_\alpha \theta_{\lambda\beta}) T_2{}^{(\mu\nu\lambda)[\alpha\beta]}$$

$$+ (9/2 k_\alpha k_\beta k_\mu \theta_{\gamma\nu} + 18\eta_{\alpha\beta}k_\mu \theta_{\gamma\nu}) T_2{}^{[\mu\nu](\alpha\beta\gamma)}$$

$$+ 81 k_\mu k_\alpha \theta_{\nu\beta} T_2{}^{[\mu\nu][\alpha\beta]} + (6k_\mu k_\nu \theta_{\lambda\alpha} + 24\eta_{\mu\nu}\theta_{\lambda\alpha}) T_2{}^{(\mu\nu\lambda)\alpha}$$

$$+(6k_\alpha k_\beta\theta_{\gamma\lambda}+24\eta_{\alpha\beta}\theta_{\gamma\lambda})T_2^{\lambda(\alpha\beta\gamma)}+108k_\mu\theta_{\nu\alpha}T_2^{[\mu\nu]\alpha}$$
$$+108k_\alpha\theta_{\beta\mu}T_2^{\mu[\alpha\beta]}+144\theta_{\mu\lambda}T_2^{\mu\lambda}=0\,, \tag{41}$$

where $\theta_{\mu\nu}$ is a constant tensor with $k^\mu\theta_{\mu\nu}=k^\nu\theta_{\mu\nu}=0$. In Eq. (41) the definition of, e.g., $T_2^{(\mu\nu\lambda)[\alpha\beta]}$ is

$$T_2^{(\mu\nu\lambda)[\alpha\beta]}=\int\prod_{i=1}^{4}d^2z_i\langle e^{ik_1\cdot X}\partial X(^\mu\partial X^\nu\partial X^\lambda)\bar\partial^2X[^\alpha\bar\partial X^\beta]e^{ik_2\cdot X}e^{ik_3\cdot X}e^{ik_4\cdot X}\rangle\,, \tag{42}$$

others can be similarly written down. One can also write down the symmetry transformation in the first order weak field approximation for Eq. (41). Let $M_{\mu\nu\lambda,\alpha\beta\gamma}(X)$, $E_{\mu\nu,\alpha\beta\gamma}(X)$, $G_{\mu\nu\lambda,\alpha\beta}(X)$ and $A_{\mu\nu,\alpha\beta}(X)$ be the four physical propagating background fields of the second massive level ($\mu\nu\lambda=(\mu\nu\lambda)$ and $\mu\nu=[\mu\nu]$), the inter-particle gauge transformation is (replace k_μ by ∂_μ, and $\eta_{\mu\nu}$ by $-\eta_{\mu\nu}$ in Eq. (41) according to our convention in Ref. 7))

$$\delta M_{\mu\nu\lambda,\alpha\beta\gamma}=1/4\,\partial_{(\mu}\partial_\nu\partial_{(\alpha}\partial_\beta\theta_{\lambda)\gamma)}-\eta_{(\mu\nu}\partial_{(\alpha}\partial_\beta\theta_{\lambda)\gamma)}-\eta_{(\alpha\beta}\partial_{(\mu}\partial_\nu\theta_{\lambda)\gamma}+4\eta_{(\mu\nu}\eta_{(\alpha\beta}\theta_{\lambda)\gamma)}\,,$$
$$\delta E_{\mu\nu,\alpha\beta\gamma}=9/2\,\partial_{(\alpha}\partial_\beta\partial_{[\mu}\theta_{\gamma)\nu]}-18\eta_{(\alpha\beta}\partial_{[\mu}\theta_{\gamma)\nu]}\,,$$
$$\delta G_{\mu\nu\lambda,\alpha\beta}=9/2\,\partial_{(\mu}\partial_\nu\partial_{[\alpha}\theta_{\lambda)\beta]}-18\eta_{(\mu\nu}\partial_{[\alpha}\theta_{\lambda)\beta]}\,,$$
$$\delta A_{\mu\nu,\alpha\beta}=81\,\partial_{[\mu}\partial_{[\alpha}\theta_{\nu]\beta]}\,, \tag{43}$$

where $\partial^\mu\theta_{\mu\nu}(X)=\partial^\nu\theta_{\mu\nu}(X)=0$ and $(\Box-4)\,\theta_{\mu\nu}(X)=0$, the other five background fields are just gauge artifacts according to Eq. (21). Equation (43) is consistent with the result we obtained from generalized σ-model point of view in Ref 11). However, the method used in this paper can be easily generalized to superstring massive cases, which turn out to be too messy to obtain from σ-model approach.[13)]

We have considered only four-point amplitudes which contain only one non-tachyon particle. In general, we can consider n-point amplitudes consisting of any kind of massive states. This will give us all kinds of Ward identities including those which are associated with inter-mass level symmetries.[16)] Moreover, our construction has interesting implication on low dimensional string theory. Works in this direction is currently being progressed.

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