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Comments on "Sensitivity of Failure Detection Using Generalized Observers"*

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Key Words—Failure detection; observers; sensitivity; Bezoút identity.

Abstract—By considering the sensitivity-robustness problem in analytical redundancy-based fault detection schemes, Manquez, H. J. and C. P. Diduch (1992). Sensitivity of failure detection using generalized observers. *Automatica*, **28**, 837–840, proposed a new General Observer to overcome the limits on achievable sensitivity imposed by the structure of classical full-order observers and partial state observers. It is shown here that this General Observer can be found to be equivalent to the partial state observer using the generalized Bezout identity. Two corrected representations of equations (20) and (23) in Manquez and Diduch's paper are also provided.

CONSIDER THE GENERAL OBSERVER in Fig. 1 (Manquez and Diduch, 1992), in which $P(s)$ is the physical plant, $K(s)$ is the controller, $P_0(s)$ is the nominal plant model, $G(s)$ is the generalized observer gain, and $y_e(s)$ is the estimation of plant output. The plant output estimate is given by

$$y_e(s) = [I + P_0(s)G(s)]^{-1}P_0(s)u(s) + [I + P_0(s)G(s)]^{-1}P_0(s)G(s)y_m(s). \quad (1)$$

Note that an observer system is a data processor, because it processes both the plant input signal $u(t)$ and measured output signal $y_m(t)$ to generate the estimated signal, e.g. $y_e(s)$ in the General Observer. Thus, according to (1) an equivalent implementation of the General Observer can be given in another form, as shown in Fig. 2.

A double coprime factorization of $P_0(s)$ is written as

$$P_0(s) = N_r(s)D_r^{-1}(s) = N_l^{-1}N_l(s), \quad (2)$$

where $N_r(s)$, $D_r(s)$ and $N_l(s)$, $D_l(s)$ are right and left coprime $\mathbf{RH}_\infty$-matrices, respectively. For this double coprime factorization, there exist $\mathbf{RH}_\infty$-matrices $Y_r(s)$, $X_r(s)$, $Y_l(s)$ and $X_l(s)$ satisfying the following:

$$\begin{bmatrix} Y_r(s) & X_r(s) \\ -N_l(s) & D_l(s) \end{bmatrix}\begin{bmatrix} D_r(s) & -X_l(s) \\ N_r(s) & Y_l(s) \end{bmatrix} = \begin{bmatrix} \mathrm{I} & 0 \\ 0 & \mathrm{I} \end{bmatrix} \quad (3a)$$

(forward Bezout identity),

$$\begin{bmatrix} D_r(s) & X_l(s) \\ -N_r(_s) & Y_l(s) \end{bmatrix}\begin{bmatrix} Y_r(s) & -X_r(s) \\ N_l(s) & D_l(s) \end{bmatrix} = \begin{bmatrix} \mathrm{I} & 0 \\ 0 & \mathrm{I} \end{bmatrix} \quad (3b)$$

(reverse Bezout identity),

which are called the generalized Bezout identity. The results derived by Manquez and Diduch show that the set of all compensators that stabilizes the General Observer is given by

$$G(s) = \{[Y_r(s) - R(s)N_l(s)]^{-1}[X_r(s) + R(s)D_l(s)]: |Y_r(s) - R(s)N_l(s)| \neq 0\} =: Y^{-1}(s)X(s) \quad (4)$$

or

$$G(s) = \{[X_l(s) + D_r(s)Q(s)][Y_l(s) - N_r(s)Q(s)]^{-1}: |Y_l(s) - N_r(s)Q(s)| \neq 0\} =: \hat{X}(s)\hat{Y}^{-1}(s), \quad (5)$$

where $R(s)$, $Q(s)$ are $\mathbf{RH}_\infty$-matrices. Using equations (2), (3) and (5), the observer sensitivity, $S_0(s)$, can be derived here as follows:

$$S_0(s) = [I + P_0(s)G(s)]^{-1} = [I + D_l^{-1}N_l\hat{X}\hat{Y}^{-1}]^{-1} = \hat{Y}[D_l\hat{Y} + N_l\hat{X}]^{-1}D_l = \hat{Y}D_l = [Y_l(s) - N_r(s)Q(s)]D_l(s). \quad (6)$$

We note here that equation (6) gives the corrected form for equation (20) in Manquez and Diduch (1992). Moreover, a corrected representation of equation (23) in their paper is also given by the following:

$$\|W(j\omega)[Y_l(j\omega) - N_r(j\omega)Q(j\omega)]D_l(j\omega)\|_\infty < \alpha.$$

Now, by applying equation (6), we rewrite equation (1) in factorization form as:

$$\begin{aligned} y_e(s) &= S_0(s)P_0(s)u(s) + S_0(s)P_0(s)G(s)y_m(s) \\ &= \hat{Y}D_lD_l^{-1}N_lu(s) + \hat{Y}D_lD_l^{-1}N_l\hat{X}\hat{Y}^{-1}y_m(s) \\ &= \hat{Y}(s)N_l(s)u(s) + \hat{Y}(s)N_l(s)\hat{X}(s)\hat{Y}^{-1}(s)y_m(s). \end{aligned} \quad (7)$$

From equation (3) we have $\hat{Y}N_l = N_rY$ and $\hat{X}\hat{Y}^{-1} = Y^{-1}X$; equation (7) then becomes

$$y_e(s) = N_r(s)Y(s)u(s) + N_r(s)X(s)y_m(s). \quad (8)$$

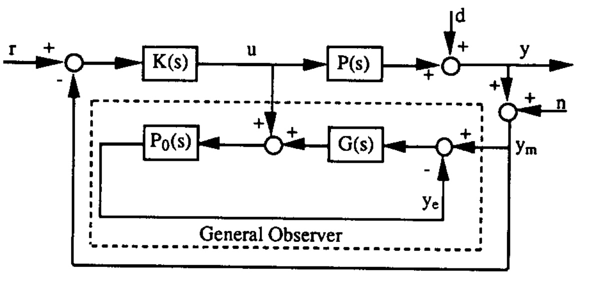

FIG. 1. The General Observer (Manquez and Diduch, 1992).

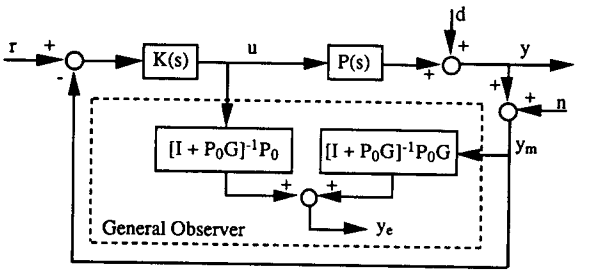

FIG. 2. An equivalent implementation of the General Observer based on equation (1).

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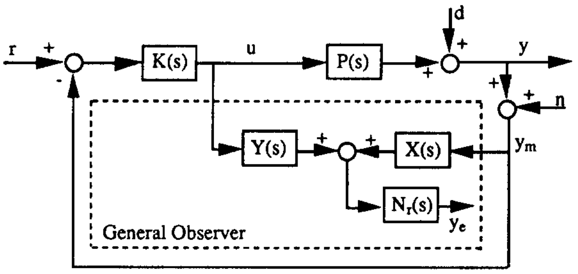

FIG. 3. An equivalent implementation of the General Observer based on equation (8).

Based on equation (8), an equivalent implementation of the General Observer can be obtained in the form shown in Fig. 3, which is the same as the partial state observer (see also Fig. 3 in Manquez and Diduch, 1992). On the basis of the above derivation, we do show that a partial state observer can indeed be constructed that results in the 'same equation' as the General Observer, but with a much simpler structure. However, when there exists uncertainty in the system, these two systems may not be equivalent in real implementation. The partial state observer can certainly generate y_c, but it may not generate a robust estimate of y_c due to the presence of the feedforward term $N_r(s)$. The General Observer can generate a robust estimate of y_c which is a feature essential to any observer-based failure detection scheme. The partial state observer in the open-loop form may find the applications in cases where the observer sensitivity with respect to $N_r(s)$ is not an issue.

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REFERENCE

Manquez, H. J. and C. P. Diduch (1992). Sensitivity of failure detection using Generalized Observers. *Automatica*, **28**, 837–840.