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Improved method for measuring small optical rotation angle of chiral medium

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Abstract

Based on the principle of Feng et al., an improved optical method for measuring small rotational angle in chiral medium is proposed. When a quarter-wave plate and two analyzers with proper azimuth angles are arranged in the two outputs of a Mach-Zehnder interferometer (with sample been inserted in one of the light passages), the final phase difference of the two interference signals used to determine the rotational angle is greatly enhanced, which is found to be a about 15× the result obtained with the Feng et al.'s method. The feasibility of the measuring method was demonstrated by our experimental results. This method should bear the merits of high accuracy, short sample medium length, and simpler operational endeavor.

Keywords: Optical rotation angle; Chiral medium; Heterodyne interferometry

1. Introduction

When linearly polarized light passes through a chiral medium, its plane of polarization rotates [1,2]. The amount of angular rotation per unit thickness is known as the rotatory power of the chiral medium, which can be measured with a polarimeter [3] constructed on the base of Malus's law. Several methods such as the high accuracy universal polarimetry (HAUP) [4,5] and the quasi-heterodyne/heterodyne interferometry [6–10] were proposed to improve the rotatory power measurement. The HAUP method measures the output intensities at several different azimuthal angles of the polarizer and the analyzer by a conventional polarimetry setup. The exact relation between the output intensity and the angular positions of the polarizer and the analyzer are derived with fitting procedure. The desired rotational angle is obtained by removing systematic errors with either the reference crystal method or the method of Meekes et al. [5]. In the quasi-heterodyne interferometry [6–8], the rotational angle is determined by an intensity variation of an associated light beam. In the heterodyne interferometry [9,10], on the other hand, the rotational angle is inferred from the phase difference between two interference signals. All these methods did result in better

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resolution and higher accuracy. However, they require thicker chiral media to achieve their derived results. Our method utilized Feng et al.'s [10] consideration enhances the phase difference of interference signals to improve resolution. Experimentally, the sample medium is situated in one arm of a Mach-Zehnder interferometer and the two outputs from the interferometer separately pass through polarization components. When the azimuth angles of these components are chosen properly, the phase difference determined with heterodyne interferometric technique of the interference signal is greatly enhanced to result in accurate rotational angle. A solution of 6×10^{-5} deg from our experimental method is 15 times better than that obtained with Feng et al.

2. Principle

The experimental setup pertinent to this measurement is shown in Fig. 1. Laser light passing through a half-wave plate H has the Jones vector

$$E_i = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \tag{1}$$

where θ measures the fast axis at $\theta/2$ from the horizontal x-axis. The Jones vector after emerging from an electro-optic modulator (EO) driven with angular frequency ω then becomes [11]

$$E_i' = \begin{pmatrix} e^{i\omega t/2} & 0 \\ 0 & e^{-i\omega t/2} \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \begin{pmatrix} \cos\theta \cdot e^{i\omega t/2} \\ \sin\theta \cdot e^{-i\omega t/2} \end{pmatrix}. \tag{2}$$

Light in Mach-Zehnder interferometer (Fig. 1) after been split by PBS travels in two paths: (a) PBS $\rightarrow$ M_a $\rightarrow$ S $\rightarrow$ BS and (b) PBS $\rightarrow$ M_b $\rightarrow$ BS, with the sample in the (a) path. The transmitted p- polarized light and reflected s- polarized light at BS superimposed to produce the amplitude E_t which is to end at D_t:

$$E_t = E_{p1} + E_{s1} = \cos\theta \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} e^{i[(\omega t/2) - \phi_{Ma}]} + \sin\theta \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-i[(\omega t/2) + kd + \phi_{Mb} + (\phi_{BS}/2)]}, \tag{3}$$

whereas, the transmitted s-polarized light and the reflected p-polarized light also summed to bear the amplitude E_r which is to end at D_r:

$$E_r = E_{p2} + E_{s2} = \cos\theta \begin{pmatrix} \cos\alpha \cdot e^{i\phi_{BS}/2} \\ \sin\alpha \cdot e^{-i\phi_{BS}/2} \end{pmatrix} e^{i[(\omega t/2) - \phi_{Ma}]} + \sin\theta \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-i[(\omega t/2) + kd + \phi_{Mb}]}, \tag{4}$$

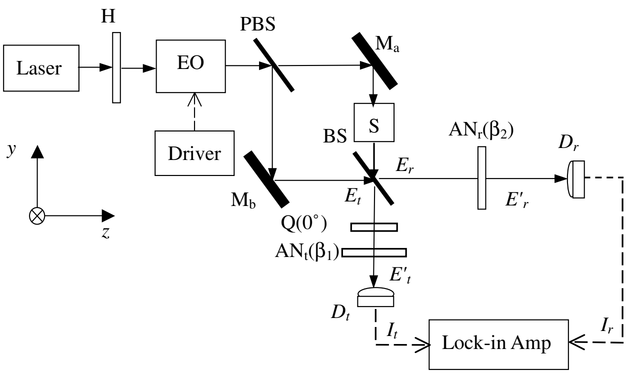

Fig. 1. Schematic diagram for measuring the optical rotation angle of a chiral medium, H: half-wave plate; EO: electro-optic modulator; PBS: polarizing beam-splitter; M_a and M_b: mirrors; S: sample medium; BS: beam-splitter; Q: quarter-wave plate; AN: analyzer; D: photodetector.

where subscripts p and s denote p- and s-polarizations, 1 and 2 mark the outputs from BS and α is the rotational angle of the incident light inflicted by the chiral medium. Let d be the optical path difference between these two paths, ϕ_{BS} be the phase difference between p- and s-polarizations of the reflection at BS, and ϕ_{Ma} and ϕ_{Mb} be that at M_a and M_b, respectively.

After passing through a quarter-wave plate Q (fast axis lying at the x-axis), and an analyzer AN_t (with the transmission axis being at β_1 to the x-axis), E_t becomes E_t' and is detected by D_t:

$$\begin{aligned} E_t' &= \mathrm{AN}_t(\beta_1)\cdot \mathrm{Q}(0°)\cdot E_t \\ &= \begin{pmatrix} \cos^2\beta_1 & \sin\beta_1\cos\beta_1 \\ \sin\beta_1\cos\beta_1 & \sin^2\beta_1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & \mathrm{i} \end{pmatrix} \\ &\quad\times\left\{\cos\theta\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}\mathrm{e}^{\mathrm{i}[(\omega t/2)-\phi_{\mathrm{Ma}}]}+\sin\theta\begin{pmatrix}0\\1\end{pmatrix}\mathrm{e}^{-\mathrm{i}[(\omega t/2)+kd+\phi_{\mathrm{Mb}}+(\phi_{\mathrm{BS}}/2)]}\right\} \\ &= \left[A_1\cos\theta\cdot\mathrm{e}^{\mathrm{i}[(\omega t/2)+\phi_t-\phi_{\mathrm{Ma}}]}+A_2\sin\theta\cdot\mathrm{e}^{-\mathrm{i}[(\omega t/2)+kd-(\pi/2)+\phi_{\mathrm{Mb}}+(\phi_{\mathrm{BS}}/2)]}\right]\begin{pmatrix}\cos\beta_1\\ \sin\beta_1\end{pmatrix}. \end{aligned} \tag{5}$$

The intensity of which hence is

$$I_t = \left|E_t'\right|^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\omega t+\psi_t), \tag{6}$$

where

$$\psi_t = \phi_t + kd + (\phi_{\mathrm{BS}}/2) + (\phi_{\mathrm{Mb}} - \phi_{\mathrm{Ma}}) - (\pi/2), \tag{7}$$

$$\phi_t = \tan^{-1}(\tan\beta_1\tan\alpha), \tag{8}$$

$$A_1 = \cos\theta\sqrt{(\cos\beta_1\cos\alpha)^2 + (\sin\beta_1\sin\alpha)^2}, \tag{9}$$

and

$$A_2 = \sin\theta\sin\beta_1. \tag{10}$$

On the other hand, E_r passing through an analyzer AN_r (with the transmission axis being at β_2 to the x-axis) becomes E_r' and is detected by photodetector D_r with

$$\begin{aligned} E_r' &= \mathrm{AN}_r(\beta_2)\cdot E_r \\ &= \begin{pmatrix} \cos^2\beta_2 & \sin\beta_2\cos\beta_2 \\ \sin\beta_2\cos\beta_2 & \sin^2\beta_2 \end{pmatrix} \\ &\quad\times\left\{\cos\theta\begin{pmatrix}\cos\alpha\cdot\mathrm{e}^{\mathrm{i}\phi_{\mathrm{BS}}/2}\\ \sin\alpha\cdot\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{BS}}/2}\end{pmatrix}\mathrm{e}^{\mathrm{i}[(\omega t/2)-\phi_{\mathrm{Ma}}]}+\sin\theta\begin{pmatrix}0\\1\end{pmatrix}\mathrm{e}^{-\mathrm{i}[(\omega t/2)+kd+\phi_{\mathrm{Mb}}]}\right\} \\ &= \left[B_1\cos\theta\cdot\mathrm{e}^{\mathrm{i}[(\omega t/2)+\phi_t-\phi_{\mathrm{Ma}}]}+B_2\sin\theta\cdot\mathrm{e}^{-\mathrm{i}[(\omega t/2)+kd+\phi_{\mathrm{Mb}}]}\right]\begin{pmatrix}\cos\beta_2\\ \sin\beta_2\end{pmatrix}. \end{aligned} \tag{11}$$

The intensity of which is

$$I_r = \left|E_r'\right|^2 = B_1^2 + B_2^2 + 2B_1B_2\cos(\omega t+\psi_r), \tag{12}$$

where

$$\psi_r = kd + (\phi_{\mathrm{Mb}} - \phi_{\mathrm{Ma}}) + \phi_r, \tag{13}$$

$$\phi_r = \tan^{-1}\left[\frac{\cos(\beta_2+\alpha)}{\cos(\beta_2-\alpha)}\tan(\phi_{\mathrm{BS}}/2)\right], \tag{14}$$

$$B_1 = \cos\theta\sqrt{\cos^2\beta_2\cos^2\alpha + \sin^2\beta_2\sin^2\alpha + \frac{1}{2}\sin 2\alpha\sin 2\beta_2\cos\phi_{\mathrm{BS}}}, \tag{15}$$

and

$$B_2 = \sin\theta\sin\beta_2. \tag{16}$$

The intensities I_t and I_r, are sent to a lock-in amplifier for phase analysis. To obtain the final phase difference $\phi(=\phi_t - \phi_r)$, we calculate $\Delta\psi(=\psi_t - \psi_r)$ as $\Delta\psi$ can be determined in advance. The phase difference

$$\Delta\psi = \psi_t - \psi_r = (\phi_t - \phi_r) - (\pi/2) + (\phi_{\mathrm{BS}}/2) = \phi - (\pi/2) + (\phi_{\mathrm{BS}}/2), \tag{17}$$

where $\phi = \Delta\psi + (\pi/2) - (\phi_{\mathrm{BS}}/2)$ is obtained from Eq. (17) as ϕ_{BS} there can be estimated with Chiu's method [11].

Finally, the rotational angle α is determined with Eqs. (8) and (14). Therefore,

$$\alpha = -\tan^{-1}\left\{\frac{[C(1-EF) - D(E-F)] - \sqrt{[C(1-EF) - D(E-F)]^2 + 4CD(1+EF)(E+F)}}{2CD(1+EF)}\right\}, \tag{18}$$

where $C = \tan\beta_1$, $D = \tan\beta_2$, $E = \tan\phi$, and $F = \tan(\phi_{\mathrm{BS}}/2)$.

3. Experiments and results

We measured the rotational angles introduced by a half-wave plate with different azimuth angles, and six glucose solutions in different weight percent, namely 0.1%, 0.5%, 1%, 10%, 15%, and 20%. Each solution was held in a quartz cell of 10 mm long. A He–Ne laser 632.8 nm line modulated by an electro-optic modulator (Model 4002, New Focus) was served as the heterodyne light source. The heterodyne generates a frequency difference of 1 kHz for p- and s-polarized light. In order to have better contrast, the conditions of $\theta = 3.5°$, $\beta_1 = 88.0°$, and $\beta_2 = 85.0°$ were chosen, while $\phi_{\mathrm{BS}} = 25.5°$ was measured in advance. At first, we inserted a half-wave plate into the interferometer to replace the sample cell. The results measured at some

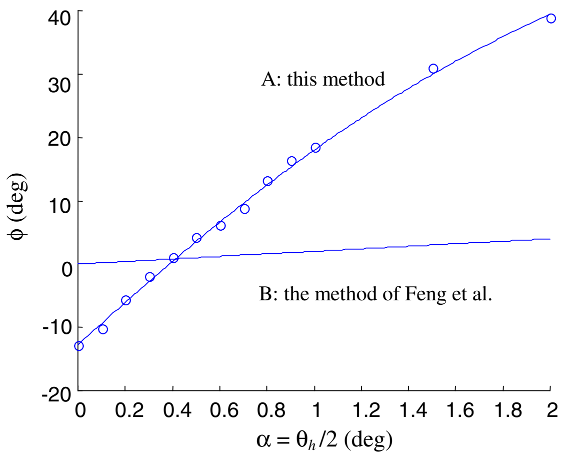

Fig. 2. Measurement results and theoretical curves of ϕ versus α.

Table 1
Experimental results and the reference data

Solutions	ϕ	α	α_{ref}
Glucose ($w = 0.1\%$)	−12.880	−0.00389	−0.00448
Glucose ($w = 0.5\%$)	−13.563	−0.02415	−0.02240
Glucose ($w = 1\%$)	−14.322	−0.04677	−0.04480
Glucose ($w = 10\%$)	−28.660	−0.47499	−0.46502
Glucose ($w = 15\%$)	−37.080	−0.73489	−0.71187
Glucose ($w = 20\%$)	−44.790	−0.98510	−0.96858

ϕ (degree): final phase difference; α (degree): optical rotation angle; α_{ref} (degree): calculated optical rotation angle from [9,12,13]. $[\alpha_s]$: specific rotation: $44.8^\circ\,(\mathrm{g/cc})^{-1}\,\mathrm{dm}^{-1}$ for glucose at 632.8 nm.

chosen angles are marked as “o” in Fig. 2 and that of theoretical calculations in smooth curve, where the ordinate represents ϕ, and abscissa represents the rotational angle α expressed in a half of the azimuth angle θ_h of the fast axis of the half-wave plate. For comparison, the theoretical curves of ϕ versus α calculated by this method (curve A) and that by the method of Feng et al. (curve B) are also presented where, in small angle region, the slope of curve A is seen to be almost $15\times$ that of curve B. It is thus seen that the sensitivity of our method is more than an order of magnitude greater. The measured ϕ and their associated α values of the samples are listed in Table 1, where the specific rotations $[\alpha_s]$ are also included. The specific rotation quantity is defined by $[\alpha_s] = \alpha/C \cdot L$, with C being the concentration of the chiral medium, and L the length of the cell holding chiral medium. The data of $[\alpha_s]$ in each solution are obtainable from [9,12,13]. By substituting the information into this equation, the associated reference values of rotational angle α_{ref} were calculated and also presented in Table 1 for comparison.

4. Measurement resolution

To find the resolution, $\Delta\alpha$, we calculate $\tan\phi$ and its differential deviation. From Eqs. (8) and (14), we arrive at

$$\tan\phi = \left(\frac{\tan\beta_1 \cdot \tan\alpha \cdot \cos(\beta_2 - \alpha) - \tan(\phi_{BS}/2) \cdot \cos(\beta_2 + \alpha)}{\cos(\beta_2 - \alpha) + \tan\beta_1 \cdot \tan(\phi_{BS}/2) \cdot \tan\alpha \cdot \cos(\beta_2 + \alpha)}\right). \tag{19}$$

As α is small quantity ($\alpha < 1^\circ$), Eq. (19) can be cast into

$$\tan\phi \cong \left(\frac{\tan\beta_1 \cdot \tan\alpha - \tan(\phi_{BS}/2)}{1 + \tan\beta_1 \cdot \tan(\phi_{BS}/2) \cdot \tan\alpha}\right), \tag{20}$$

and, thus, the errors $\Delta\alpha$ and $\Delta\phi$ can be related as follows:

$$\Delta\alpha \cong \frac{1}{\sec^2\alpha}\left[\frac{\sec^2(\phi_{BS}/2)\sec^2\phi}{(1 - \tan\phi_{BS}\tan\phi)\tan\beta_1}\right]\Delta\phi. \tag{21}$$

When the second harmonic uncertainties and the polarization-mixing uncertainties [14] were considered, the net phase difference uncertainties $\Delta\phi$ reduced to 0.0014°. Substituting our experimental conditions $\beta_1 = 88.0^\circ$, $\beta_2 = 85.0^\circ$, $\phi_{BS} = 25.5^\circ$, and $\Delta\phi = 0.0014^\circ$ into Eq. (21), we have $\Delta\alpha = 6 \times 10^{-5}$ deg.

According to the definition of rotatory power, $\alpha L = 2\pi g/\lambda$ [15], we have $\Delta\alpha = (2\pi\Delta g L)/\lambda$, with g and Δg as the chiral parameter [16] and its error. Substituting our experimental conditions $L = 10$ mm, $\lambda = 632.8$ nm, and $\Delta\alpha = 6\times10^{-5}$ deg. into this equation, we obtain $\Delta g = 1.1\times10^{-11}$. As we had reported $\Delta g = 2 \times 10^{-8}$ in our previous paper [17], we also increased the resolution in the present method.

5. Conclusion

Based on the principle of Feng et al., an improved method for measuring small optical rotation angle in chiral medium is proposed. Two groups of output beams from a Mach-Zehender interferometer are arranged to pass through several polarization components separately and recombine for interference. The phase difference was measured with the heterodyne interferometric technique and the associated optical rotation angle was estimated. When azimuth angles of polarization components were chosen appropriately, the final phase difference ϕ between these two interference signals increased greatly. Hence, the required optical length of the sample medium can be decreased dramatically. It takes only 1/15 of the length required by the method of Feng et al. The feasibility of this method was demonstrated by its measurement resolution of 6×10^{-5} deg.

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