# A simplification of entanglement purification

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**Abstract**

This study proposes a modification of existing entanglement purification protocols for pairs of qubits. The proposed protocol restores a desired pure state by standard purification local operations and classical communications, without preliminarily estimating the entangled state to be purified. The proposed protocol is demonstrated to outperform the previously proposed IBM and Oxford protocols.
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Methods of processing quantum information such as quantum teleportation [1], quantum data compression [2,3], and quantum cryptography [4] rely on the transmission of maximally entangled qubit pairs over quantum channels between a sender (Alice) and a receiver (Bob). The quantum channel is always noisy, so the pairs shared by Alice and Bob are not the pure pairs that were intended at the beginning of the quantum processing. The quantum resource in the noisy channel then can be viewed as a mixed state, or equivalently, an ensemble of pure states associated with definite random probabilities. The probabilities associated with the pure states in the ensemble are random and so should be unknown to Alice and Bob before a quantum process is performed. Accordingly, Alice and Bob must perform entanglement purification to regain, at least asymptotically, the desired maximally entangled pure state if the mixed state is distillable. This aim can be achieved using consecutive local operations and classical communications (LOCC).

Two typical recurrence methods of entanglement purification should be mentioned. Bennett et al. [5,6] presented the first entanglement purification protocol (the IBM protocol) for faithful quantum teleportation. Later, Deutsch et al. [7] proposed an improved protocol called "Quantum Privacy Amplification" (QPA, or the Oxford protocol) with reference to the security of quantum cryptography over noisy channels. Both IBM and Oxford protocols can purify a desired

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maximally entangled pure state from every distillable mixed state whose components are initially unknown by Alice and Bob. By using the IBM protocol, Alice and Bob can asymptotically regain the desired pure state, but must perform more operations than required by the Oxford protocol to twirl the state into a Werner state [8], of which the fidelity relative to the desired pure state always exceeds $1/2$, during the purification LOCC operations. The Oxford protocol can provide a higher output yield (the purified pair per impure input pair) than the IBM protocol, especially when the initial fidelity with respect to the desired pure state of the input state is close to $1/2$. In particular, the Oxford protocol can purify any state whose average fidelity with respect to at least one maximally entangled pure state exceeds $1/2$ and can be directly applied to purify states that are not necessarily of the Werner form. However, the Oxford protocol occasionally purifies a pure state other than the desired one, and so can yield two possible pure states, depending on the initial mixed state. Therefore, as well as performing the purification LOCC operations, Alice and Bob must then transform the pure state with the greatest component ($> 1/2$) in the input mixed state into the desired state. Such an action increases the operating time, by adding local unitary operations and classical communications to identify the mixed state, so some pairs are consumed before the standard purification LOCC operations can be performed. The output yields induced by the IBM and Oxford protocols are rather poor, but can be increased to some extent if both protocols are combined with hashing protocols, as described in Refs. [5,6]. Modified protocols dedicated to increasing the yield of an entanglement purification procedure have already been proposed, as in Refs. [9–11]. Although the modified methods can increase yields, they require more simultaneous local unitary operations and classical communications in the reordering schemes and hashing protocols [5,6] that are combined with the standard purification protocols.

Usually, a protocol is said to outperform another protocol if either the yield of the former exceeds that of the latter given the same operation times, or the former requires less time than the latter to output equal yields. Instead of trying to increase the yield, this work proposes the idea of establishing entanglement purification protocols that require fewer operations than the standard IBM and Oxford protocols. These pro-
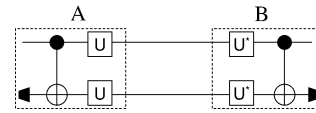


Fig. 1. The standard purification LOCC operations includes the local controlled-NOT operation, single qubit measurement, and local unitary operation in each party. Note that the classical communication is not shown in this figure.

tocols can purify a desired pure state using the standard LOCC operations alone. When these protocols are used, the mixed state to be purified does not need to be transformed into the Werner state nor to be reordered so its fidelity with respect to the desired pure state is the largest. Moreover, one of the protocols proposed in this work in fact can provide a higher yield than the Oxford protocol.

Fig. 1 depicts the standard purification LOCC operation considered in this work. In each purification LOCC operation, Alice and Bob first performs local operations by applying operators $U$ and $U^*$, respectively, which are defined below. Then Alice or Bob individually performs a quantum control-not operation and measures the target qubits in the computational basis. If the outcomes that are communicated via a classical channel are the same, then the control pair is maintained for the next step and the target pair is discarded. If the outcomes differ, then both pairs are discarded. The state to be purified in the purification LOCC operation does not have to be of Werner form. The mixed state can be expressed in the Bell basis $\{|\Phi^+\rangle, |\Psi^-\rangle, |\Psi^+\rangle, |\Phi^-\rangle\}$:

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle \pm |11\rangle\big),$$

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}\big(|01\rangle \pm |10\rangle\big), \tag{1}$$

where $|0\rangle$ and $|1\rangle$ form the computational basis of the two-dimensional space belonging to the EPR pairs. Let $\{a_0, b_0, c_0, d_0\}$ be the average initial diagonal elements of the density operator representing the mixed state before the protocol begins, and let $\{a_r, b_r, c_r, d_r\}$ be the average diagonal elements of the surviving state after the $r$th step. A purification LOCC operation can be shown relative to a nonlinear map, where the diagonal entries of the surviving state after the LOCC operation are nonlinear functions of those before the operation. Therefore, the purification protocol considered

herein is composed of consecutive nonlinear maps of
the Bell-diagonal elements used to transform an ini-
tial state asymptotically to a desired pure state. Sup-
pose that state $|\Phi^+\rangle\langle\Phi^+|$ is to be purified, a distillable
initial state $\{a_0, b_0, c_0, d_0\}$ is then mapped stepwise to
converge to the desired attractor $\{1, 0, 0, 0\}$ when the
step number $r$ is sufficiently large. However, the in-
trinsic properties of the nonlinear map are such that
the desired attractor is not the only one, as noted by
Macchiavello [12], who investigated analytical con-
vergence in the recurrence scheme of the QPA pro-
tocol. The interesting nonlinear behavior of the recur-
rence scheme in a distillation protocol is dominated
by the local unitary operators $U$ and $U^*$ applied by
Alice and Bob during the purification LOCC opera-
tion. A generalized expression for $U$, controlled by
two phases $\theta$ and $\phi$, is

$$U(\theta, \phi) = \begin{bmatrix} \cos(\frac{\theta}{2}) & -e^{-i\phi}\sin(\frac{\theta}{2}) \\ e^{i\phi}\sin(\frac{\theta}{2}) & \cos(\frac{\theta}{2}) \end{bmatrix}. \tag{2}$$

Different choices of $\theta$ and $\phi$ lead to different
destinations of the protocol. For example, in using the
original QPA protocol, Alice and Bob choose $\theta = \phi = \pi/2$, and so apply the operator

$$U\left(\frac{\pi}{2}, \frac{\pi}{2}\right) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}. \tag{3}$$

In this case, a map $\{a_{r-1}, b_{r-1}, c_{r-1}, d_{r-1}\} \rightarrow \{a_r, b_r, c_r, d_r\}$ is obtained, according to the following rela-
tions:

$$a_r = \frac{a_{r-1}^2 + b_{r-1}^2}{p_{r-1}}, \qquad b_r = \frac{2c_{r-1}d_{r-1}}{p_{r-1}},$$
$$c_r = \frac{c_{r-1}^2 + d_{r-1}^2}{p_{r-1}}, \qquad d_r = \frac{2a_{r-1}b_{r-1}}{p_{r-1}},$$
$$\text{for } \theta = \phi = \pi/2, \tag{4}$$

where $p_{r-1} = (a_{r-1} + b_{r-1})^2 + (c_{r-1} + d_{r-1})^2$ is the
probability in the $r$th step that Alice and Bob obtain
coinciding outcomes in measurements of the target
pairs (so only $p_{r-1}/2$ of the pairs before the $r$th step
survive after the step ). Define the domains

$$\mathcal{D}_a = \{a \in (0.5, 1]; a + b + c + d = 1\},$$
$$\mathcal{D}_b = \{b \in (0.5, 1]; a + b + c + d = 1\},$$
$$\mathcal{D}_c = \{c \in (0.5, 1]; a + b + c + d = 1\},$$
$$\mathcal{D}_d = \{d \in (0.5, 1]; a + b + c + d = 1\},$$

$$\mathcal{D}_{ab} = \mathcal{D}_a \cup \mathcal{D}_b,$$
$$\mathcal{D}_{cd} = \mathcal{D}_c \cup \mathcal{D}_d,$$
$$\mathcal{D}_{abcd} = \mathcal{D}_a \cup \mathcal{D}_b \cup \mathcal{D}_c \cup \mathcal{D}_d. \tag{5}$$

The case in which an initial mixed state is to be pu-
rified is in the applicable domain $\mathcal{D}_{abcd}$ is consid-
ered below, because any state $\rho \in \mathcal{D}_{abcd}$ is distillable.
For the Oxford protocol, an initial state in the domain
$\mathcal{D}_{ab}$ has been proven to eventually able to mapped
to converge to the attractor $\{1, 0, 0, 0\}$ which repre-
sents the desired pure state $|\Phi^+\rangle\langle\Phi^+|$. However, if
the initial state is in the domain $\mathcal{D}_{cd}$, then it will be
mapped to approach another attractor $\{0, 0, 1, 0\}$, the
pure state $|\Psi^+\rangle\langle\Psi^+|$. Finally, according to Ref. [7],
Alice and Bob will regain the desired pure state from
any state $\rho \in \mathcal{D}_{abcd}$ using the QPA protocol, provided
that they first make additional efforts besides the stan-
dard purification LOCC operations to transform the
pure state $|\Psi^+\rangle\langle\Psi^+|$ or $|\Phi^-\rangle\langle\Phi^-|$ into the desired
state $|\Phi^+\rangle\langle\Phi^+|$ if the input state is in the domain $\mathcal{D}_{cd}$.
Such efforts are also meaningful if the QPA is consid-
ered to be combined with the hashing protocol [5,6]
to improve its output yield. These tedious transforma-
tions cannot be avoided even when the input state is al-
ready in the domain $\mathcal{D}_{ab}$, because Alice and Bob have
no idea about whether the input state is in the domain
$\mathcal{D}_{ab}$ or $\mathcal{D}_{cd}$. For instance, if the input state has the el-
ement $c_0 = 0.7$, then Alice and Bob should transform
the state $|\Psi^+\rangle\langle\Psi^+|$ into $|\Phi^+\rangle\langle\Phi^+|$ before the purifi-
cation procedure so the mixed state will in turn have
the element $a_0 = 0.7$.

However, if Alice and Bob choose $\theta = \pi/2$ and
$\phi = 0$, then they have the operator

$$U(\pi/2, 0) = \mathbf{XH} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \tag{6}$$

where $\mathbf{X}$ refers to the quantum NOT gate and $\mathbf{H}$
is the Hadamard transformation. Accordingly, the
recurrence scheme in this case is described by

$$a_r = \frac{a_{r-1}^2 + c_{r-1}^2}{p_{r-1}}, \qquad b_r = \frac{2b_{r-1}d_{r-1}}{p_{r-1}},$$
$$c_r = \frac{b_{r-1}^2 + d_{r-1}^2}{p_{r-1}}, \qquad d_r = \frac{2a_{r-1}c_{r-1}}{p_{r-1}},$$
$$\text{for } \theta = \pi/2, \phi = 0, \tag{7}$$

where $p_{r-1} = (a_{r-1} + c_{r-1})^2 + (b_{r-1} + d_{r-1})^2$. No-
tably, the relations (7) can also be obtained from

the utility of the Hadamard transformation only, such that $U = \mathbf{H}$, but this transformation does not belong to the SU(2) operator defined in (2). Although the analytical convergence of the recurrence scheme (7) has not yet been proven, an initial state in some domain $\mathcal{D}_u \subset \mathcal{D}_{abcd}$, which is not yet defined, will be mapped to approach the periodic attractor which represents stepwise changes between $\{0.5, 0, 0, 0.5\}$ and $\{0.5, 0, 0.5, 0\}$, whereas a state in the domain $\mathcal{D}_u^c$, where $\mathcal{D}_u^c \cup \mathcal{D}_u = \mathcal{D}_{abcd}$, will be mapped to converge to the fixed attractor $\{1, 0, 0, 0\}$, as desired. For instance, it can be easily verified that the initial state $\{0.1, 0.2, 0.6, 0.1\}$ will be mapped to converge to the fixed attractor but the initial state $\{0.2, 0.1, 0.6, 0.1\}$ will be mapped to approach the mentioned periodic attractor. Consequently, a protocol that uses the operator $\mathbf{XH}$, unlike the QPA protocol, will not necessarily purify pure maximally entangled pairs.

In this work, a protocol is called a one-map protocol if Alice and Bob each uses only one single local operator in all purification LOCC operations, such as in the IBM and Oxford protocols. The above examples reveal that if only standard purification LOCC operations are implemented, all one-map protocols yield an attractor in addition to the desired one, $\{1, 0, 0, 0\}$, for a state $\rho \in \mathcal{D}_{abcd}$ to be mapped to converge to. This situation thus becomes the ultimate limitation on the one-map algorithm. Therefore, this work will present a perspective of hybrid maps for a purification protocol in which the fixed state $\{1, 0, 0, 0\}$ can be the only attractor for an initial state $\rho \in \mathcal{D}_{abcd}$ to be mapped to approach. The simple idea can be interpreted briefly as follows. If a one-map protocol, say, one controlled by $\theta_1$ and $\phi_1$, is known, in which a state $\rho$ that belongs to some defined domain $\mathcal{D}_1 (\subset \mathcal{D}_{abcd})$ can be mapped to approach the fixed attractor $\{1, 0, 0, 0\}$, then all that is required is to find another map, controlled by $\theta_0$ and $\phi_0$, in which a state $\rho \in \mathcal{D}_{abcd}$ will be mapped on to a subdomain of the defined $\mathcal{D}_1$. Such a protocol is called a two-map protocol, which can ensure that Alice and Bob regain the desired pure state $|\Phi^+\rangle\langle\Phi^+|$ using the standard purification LOCC operations alone.

With reference to the above idea, the most difficult task is to define the domain $\mathcal{D}_1$. Fortunately, Macchiavello [12] defined domain $\mathcal{D}_1$ for the QPA protocol, where $\mathcal{D}_1 = \mathcal{D}_{ab}$, as defined in Eq. (5). Therefore the QPA protocol is currently the most convenient one-map protocol to be improved by applying the proposed

idea. In contrast, no definition of the corresponding $\mathcal{D}_1$ has been proven for the one-map protocol described in Eq. (7). A concrete example of the application of the proposed idea utilizes these two one-map protocols. In this example the option $\theta_1 = \pi/2$ and $\phi_1 = \pi/2$ will be chosen and the choice $\theta_0 = \pi/2$ and $\phi_0 = 0$ follows accordingly. First, $(1 - 2a_r)$ and $(1 - 2c_r)$ are derived for $\theta_0 = \pi/2$ and $\phi_0 = 0$. From Eq. (7),

$$1 - 2a_r = \frac{(1 - 2a_{r-1})(1 - 2c_{r-1})}{p_{r-1}},$$

$$1 - 2c_r = \frac{(1 - 2b_{r-1})(1 - 2d_{r-1})}{p_{r-1}}, \tag{8}$$

for arbitrary positive integer $r$. Clearly, since $p_{r-1} > 0$, if $a_0 > 1/2$ or $c_0 > 1/2$, then after the first purification LOCC operation $a_1 > 1/2$, whereas if $b_0 > 1/2$ or $d_0 > 1/2$, then $c_1 > 1/2$, which implies $a_2 > 1/2$ after the second purification LOCC operation. Accordingly, the one-map protocol (7) can be applied to map an initial state $\rho \in \mathcal{D}_{abcd}$ in two steps on to the domain $\mathcal{D}_a$, which is exactly a subdomain of $\mathcal{D}_1 (= \mathcal{D}_{ab})$ for the standard QPA protocol. This is the two-map protocol to be proposed herein. Using this two-map protocol (TM1), Alice and Bob agree that in the first two steps of the purification procedure, they will apply the operators $U(\pi/2, 0)$ and $U^*(\pi/2, 0)$, respectively, to map a state $\rho \in \mathcal{D}_{abcd}$ on to the domain $\mathcal{D}_a = \{a \in (0.5, 1], a + b + c + d = 1\}$, they will then apply the standard QPA operators $U(\pi/2, \pi/2)$ and $U^*(\pi/2, \pi/2)$ to purify the surviving state to the desired state $|\Phi^+\rangle\langle\Phi^+|$ in the remaining purification LOCC operations. Interestingly, an alternative two-map protocol (TM2) can also be used, in which the operators $U(\pi/2, 0)$ and $U^*(\pi/2, 0)$ are applied only at the second purification LOCC operation, since after the first LOCC operation, in which the QPA operators $U(\pi/2, \pi/2)$ and $U^*(\pi/2, \pi/2)$ are used, the state has been mapped on to the domain $\mathcal{D}_{ac}$ [12]. However, as will be shown later, protocol TM1 outperforms TM2.

Apparently, the protocols TM1 and TM2 are composed of only the standard purification LOCC operations, without any additional local operations or classical communication in transforming the mixed state into a Werner state, as needed in the IBM protocol, or transforming one of the Bell states with the greatest fidelity into the desired pure state $|\Phi^+\rangle$ in advance of the Oxford operations. Therefore, the proposed purification algorithm require fewer operations
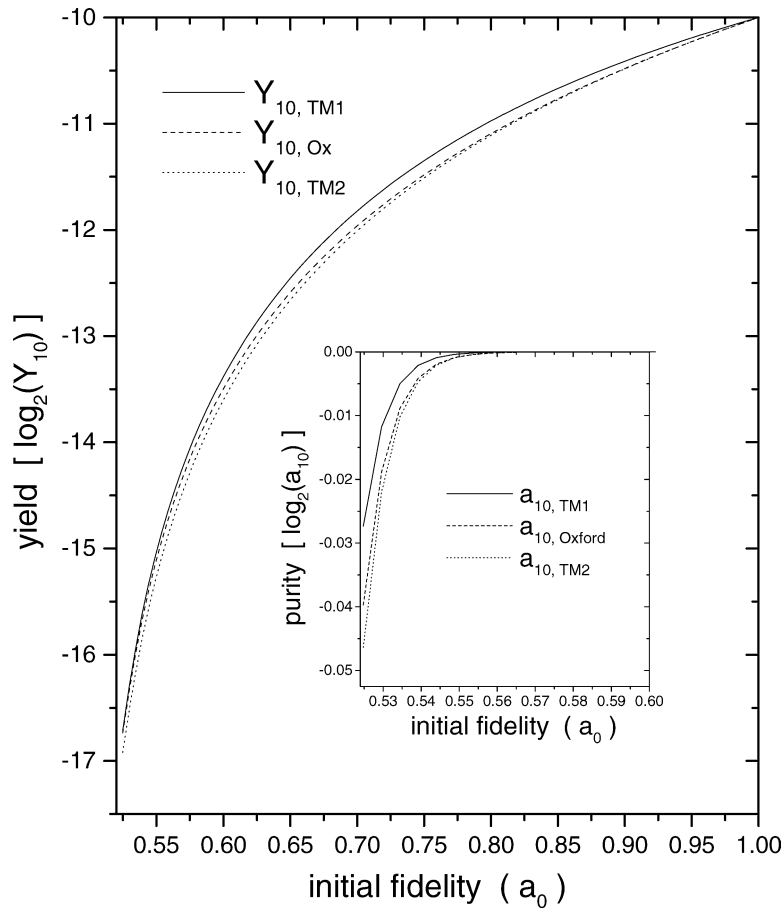
Fig. 2. The variations of the yield and purity (in the inserted diagram) after the 10th LOCC operation.

than the IBM or Oxford protocol. Moreover, protocol TM1 provides higher yields and fidelities than the Oxford protocol (which outperforms the IBM protocol), while protocol TM2 performs almost as well as the Oxford protocol. In the numerical simulations, the yield, or the fraction of surviving pairs, defined by $Y_r = p_0 p_1 \cdots p_{r-1}(2^{-r})$, was first computed for up to $r = 10$ for each input state to be purified. Fig. 2 plots the variations of the yield as functions of the initial fidelity $a_0$. In Fig. 2, and in the following figures, each yield (and each purity) was the average over ten thousand random states with the same initial fidelity. Fig. 2 also shows the corresponding purities after ten iterations. Although, after ten iterations, the resulting purities generated using the Oxford, TM1, and TM2 are high, the corresponding yields are rather poor, especially when the initial fidelity is close to $1/2$.

The yield, however, can be further improved by combining the recurrence method with the hashing protocol [5,6] as long as the purity is high enough (higher than 0.8107 for a Werner state) when the recurrence scheme is implemented in only a few iterations. Fig. 3 shows the yields $Y_5$ and the corresponding purities $a_5$ produced by the Oxford and the TM1 protocols after five iterations, respectively. This figure demonstrates that when the initial fidelities exceed some specific values near $1/2$ for both cases the hashing protocols can then be applied after five iterations in the recurrence schemes. (The specific fidelities can be lowered as the number of iterations increases.) Fig. 3 reveals that after five iterations, the surviving fraction $Y_{5,TM1}$ and the corresponding purity $a_{5,TM1}$, produced by the TM1 protocol slightly exceed the surviving fraction $Y_{5,Ox}$ and the purity $a_{5,Ox}$, which are obtained using
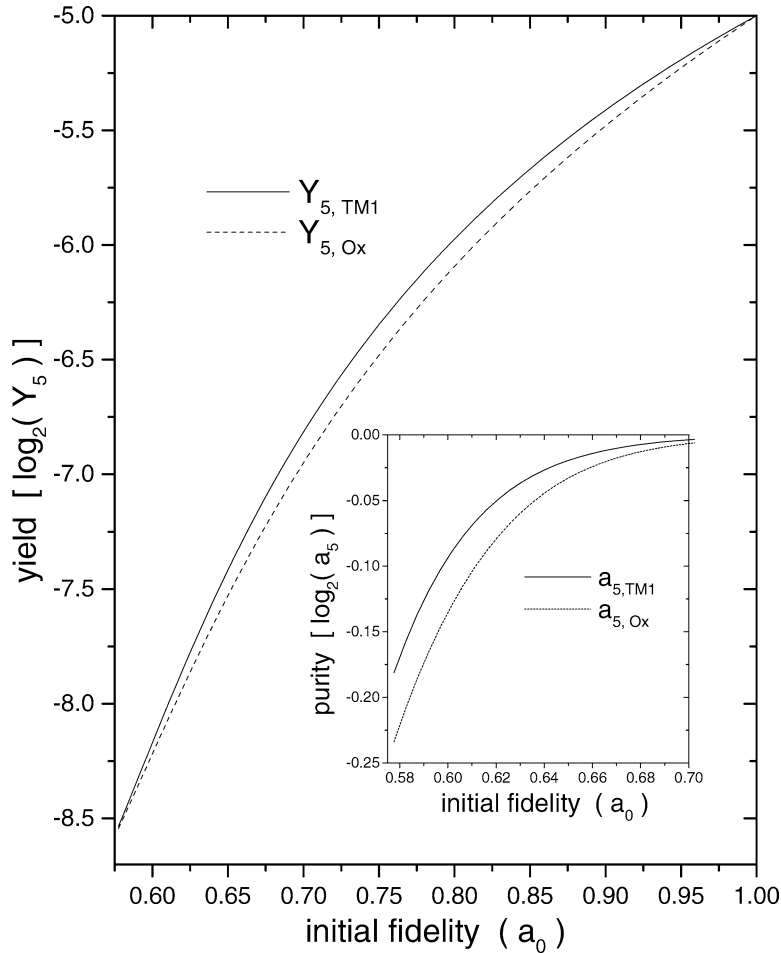
Fig. 3. The variations of the yield and purity (in the inserted diagram) after the 5th LOCC operation.

the Oxford protocol. The slight differences between $Y_5$ and $a_5$ can, however, create a significant difference between the improved yields when the hashing protocol is switched on after the five iterations. Fig. 4 presents the evidence of this claim, showing both the improved yields $Y'_{5,\text{TM1}}$ and $Y'_{5,\text{Ox}}$ and the ratio of the improved yields $(Y'_{5,\text{TM1}}/Y'_{5,\text{Ox}})$ as functions of the initial fidelity; the improved yield is defined by $Y'_r = Y_r(1 - S(\rho_r))$, where $S(\rho_r)$ is the von Neumann entropy of the surviving mixed state $\rho_r$. Fig. 4 clearly shows that the ratio $Y'_{5,\text{TM1}}/Y'_{5,\text{Ox}}$, which always exceeds unity, increases as the initial fidelity becomes closer to $1/2$.

In summary, in the recurrence scheme of a one-map entanglement purification protocol, the nonlinear be-

havior of the four Bell-diagonal elements of the density matrix representing the mixed state to be purified reveals that an attractor other than the desired fixed attractor always exists, indicating that not all distillable input states can be purified to the desired maximally entangled pure state by performing standard purification LOCC operations in a one-map protocol. Therefore, typical IBM and Oxford protocols require some tedious efforts to be made to purify a desired pure state from any distillable state beyond the purification LOCC operations. The proposed two-map purification protocols TM1 and TM2, in contrast, can ensure that all the distillable input states can be purified to the desired pure state by applying standard purification LOCC operations. That an entanglement can be
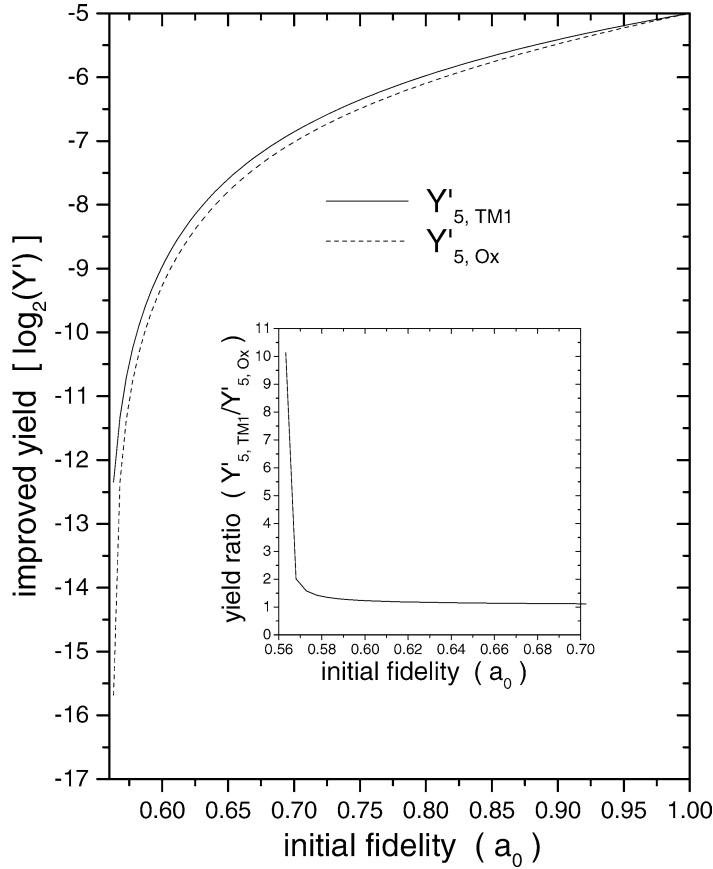
Fig. 4. The variations of the improved yields $Y'_{5,\text{TM1}}$ and $Y'_{5,\text{Ox}}$ and the comparing ratio $(Y'_{5,\text{TM1}}/Y'_{5,\text{Ox}})$ (in the inserted diagram).

purified by standard purification LOCC operations is crucially important in substantially improving the purification process. Such an improvement eliminates the need to identify the mixed state and therefore the consumption of any pair of qubits before the purification LOCC operations are performed. The proposed two-map protocols outperform the one-map IBM and Oxford protocols in the sense that they require the shortest operation times in yielding a given amount of useful EPR pairs. Moreover, the protocol TM1 is found to be able to generate higher yields and purities than the Oxford protocol. This fact is crucial when the hashing protocol is combined with the recurrence algorithm to improve the output yield. The proposed two-map protocols, however, like the standard IBM and Oxford protocols, should be implemented if the initial state possesses a fidelity that is very close to $1/2$ only after the fidelity of the state has been enhanced. For

instance, only inseparable two-qubit state with "free" entanglement, however small, has been shown [13] to be able to be distilled to a pure form using local filtering [14,15] to enhance the fidelity of the state first. An interaction with the environment [16] can even be allowed to enhance the fidelity of a quantum teleportation. The fidelity enhancement, however, is beyond the scope of this work.

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