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Decision Aiding

Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS

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Abstract

The multiple criteria decision making (MCDM) methods VIKOR and TOPSIS are based on an aggregating function representing "closeness to the ideal", which originated in the compromise programming method. In VIKOR linear normalization and in TOPSIS vector normalization is used to eliminate the units of criterion functions. The VIKOR method of compromise ranking determines a compromise solution, providing a maximum "group utility" for the "majority" and a minimum of an individual regret for the "opponent". The TOPSIS method determines a solution with the shortest distance to the ideal solution and the greatest distance from the negative-ideal solution, but it does not consider the relative importance of these distances. A comparative analysis of these two methods is illustrated with a numerical example, showing their similarity and some differences.

Keywords: Multiple criteria analysis; Compromise; VIKOR; TOPSIS; Comparison

1. Introduction

Many papers have proposed analytical models as aids in conflict management situations. Among the numerous approaches available for conflict management, one of the most prevalent is multicriteria decision making. Multicriteria decision making (MCDM) may be considered as a complex and dynamic process including one managerial level and one engineering level (Duckstein and Opricovic, 1980). The managerial level defines the goals, and chooses the final "optimal" alternative. The multicriteria nature of decisions is emphasized at this managerial level, at which public officials called "decision makers" have the power to accept or reject the solution proposed by the engineering level. These decision makers, who provide the preference structure, are "off line" from the optimization procedure done at the engineering level. Very often, the preference structure is based on political rather than only technical criteria. In such cases, a system analyst can aid the decision making process by making a comprehensive analysis and by listing the important properties of noninferior

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and/or compromise solutions (Yu, 1973). The engineering level of the MCDM process defines alternatives and points out the consequences of choosing any one of them from the standpoint of various criteria. This level also performs the multicriteria ranking of alternatives.

The main steps of multicriteria decision making are the following:

(a) Establishing system evaluation criteria that relate system capabilities to goals;
(b) Developing alternative systems for attaining the goals (generating alternatives);
(c) Evaluating alternatives in terms of criteria (the values of the criterion functions);
(d) Applying a normative multicriteria analysis method;
(e) Accepting one alternative as "optimal" (preferred);
(g) If the final solution is not accepted, gather new information and go into the next iteration of multicriteria optimization.

Steps (a) and (e) are performed at the upper level, where decision makers have the central role, and the other steps are mostly engineering tasks. For step (d), a decision maker should express his/her preferences in terms of the relative importance of criteria, and one approach is to introduce criteria weights. These weights in MCDM do not have a clear economic significance, but their use provides the opportunity to model the actual aspects of decision making (the preference structure). In this paper, we consider "importance weights" which represent the relative importance of criteria. Another approach is to introduce weights in a simple aggregating function (weighted sum), where weights reflect both criterion importance and measurement scale ("trade-offs" weights). Since criteria usually are expressed in different units (noncommensurable) it is difficult to determine the values of such weights. There are applications with "objective weights" determined from a performance matrix, and these have no relationship with preference of the decision maker (Deng et al., 2000).

In the engineering level, the main efforts are in generating and evaluating the alternatives (steps (b) and (c)); and these efforts are different for individual projects, since projects vary in the types of needs they meet or the problems they solve. The physical, environmental, and social settings in which planning takes place also differ from one location to another. Alternatives can be generated and their feasibility can be tested by mathematical models, physical models, and/or by experiments on the existing system or other similar systems. Constraints are seen as high-priority objectives, which must be satisfied in the process of generating alternatives. Generating alternatives can be a very complex process, since there is no general procedure or model, and no mathematical procedure could replace human creativity in generating and evaluating alternatives. However, after generating and evaluating the alternatives, an MCDM method (such as TOPSIS or VIKOR) could be applied to rank alternatives and to propose a solution to the decision maker.

Multicriteria optimization is the process of determining the best feasible solution according to the established criteria (representing different effects). Practical problems are often characterized by several noncommensurable and conflicting (competing) criteria, and there may be no solution satisfying all criteria simultaneously. Thus, the solution is a set of noninferior solutions, or a compromise solution according to the decision makers' preferences.

Most multicriteria methods require definition of quantitative weights for the criteria, in order to assess the relative importance of the different criteria. The paper by Mareschal (1988) considers the stability of the ranking results during changes of the criteria weights. The procedure for sensitivity analysis defines stability intervals for the weights. The values of the weight of one criterion within the stability interval do not alter the results obtained with the initial set of weights, since all other weights have initial ratios. Wolters and Mareschal (1995) considered new types of stability analysis for additive MCDM methods, including additive utility function and outranking methods such as PROMETHEE (Brans et al., 1984; Olson, 2001). However, the compromise ranking method (called VIKOR) does not belong to this class of methods, but rather determines the weight stability intervals, using the methodology presented in Opricovic (1998).

A compromise solution for a problem with conflicting criteria can help the decision makers to reach a final decision. The foundation for compromise solution was established by Yu (1973) and Zeleny (1982). The compromise solution is a feasible solution, which is the closest to the ideal, and a compromise means an agreement established by mutual concessions. The VIKOR method was introduced as one applicable technique to implement within MCDM (Opricovic, 1998). The TOPSIS method determines a solution with the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution, but it does not consider the relative importance of these distances (Hwang and Yoon, 1981; Yoon, 1987).

In this paper two MCDM methods, VIKOR and TOPSIS are compared, focusing on modelling aggregating function and normalization, in order to reveal and to compare the procedural basis of these two MCDM methods. A comparative analysis is illustrated with a numerical example.

2. VIKOR method

The VIKOR method was developed for multicriteria optimization of complex systems. It determines the compromise ranking-list, the compromise solution, and the weight stability intervals for preference stability of the compromise solution obtained with the initial (given) weights. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It introduces the multicriteria ranking index based on the particular measure of "closeness" to the "ideal" solution (Opricovic, 1998).

Assuming that each alternative is evaluated according to each criterion function, the compromise ranking could be performed by comparing the measure of closeness to the ideal alternative. The multicriteria measure for compromise ranking is developed from the L_p-*metric* used as an aggregating function in a compromise programming method (Yu, 1973; Zeleny, 1982). The various J alternatives are denoted as $a_1, a_2, \ldots, a_J$. For alternative a_j, the rating of the ith aspect is denoted by f_{ij}, i.e. f_{ij} is the value of ith criterion function for the alternative a_j; n is the number of criteria.

Development of the VIKOR method started with the following form of L_p-*metric*:

$$L_{p,j} = \left\{ \sum_{i=1}^{n} \left[w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-) \right]^p \right\}^{1/p},$$

$$1 \leqslant p \leqslant \infty; \quad j = 1, 2, \ldots, J.$$

Within the VIKOR method $L_{1,j}$ (as S_j in Eq. (1)) and $L_{\infty,j}$ (as R_j in Eq. (2)) are used to formulate ranking measure. The solution obtained by $\min_j S_j$ is with a maximum group utility ("majority" rule), and the solution obtained by $\min_j R_j$ is with a minimum individual regret of the "opponent".

The compromise solution F^c is a feasible solution that is the "closest" to the ideal F^*, and compromise means an agreement established by mutual concessions, as is illustrated in Fig. 1 by $\Delta f_1 = f_1^* - f_1^c$ and $\Delta f_2 = f_2^* - f_2^c$.

The compromise ranking algorithm VIKOR has the following steps:

(a) Determine the best f_i^* and the worst f_i^- values of all criterion functions, $i = 1, 2, \ldots, n$. If the ith function represents a benefit then:

$$f_i^* = \max_j f_{ij}, \qquad f_i^- = \min_j f_{ij}.$$

(b) Compute the values S_j and $R_j, j = 1, 2, \ldots, J$, by the relations

$$S_j = \sum_{i=1}^{n} w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-), \tag{1}$$

$$R_j = \max_i \left[w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-) \right], \tag{2}$$

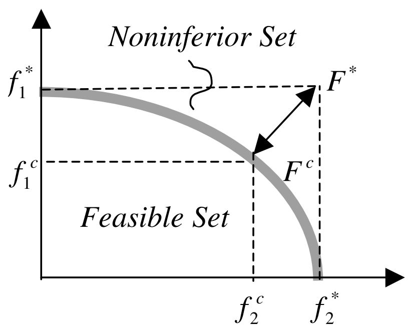

Fig. 1. Ideal and compromise solutions.

where w_i are the weights of criteria, expressing their relative importance.

(c) Compute the values Q_j, $j = 1, 2, \ldots, J$, by the relation

$$Q_j = v(S_j - S^*)/(S^- - S^*) + (1 - v)(R_j - R^*)/(R^- - R^*) \quad (3)$$

where

$$S^* = \min_j S_j, \quad S^- = \max_j S_j,$$

$$R^* = \min_j R_j, \quad R^- = \max_j R_j,$$

and v is introduced as weight of the strategy of "the majority of criteria" (or "the maximum group utility"), here $v = 0.5$.

(d) Rank the alternatives, sorting by the values S, R and Q, in decreasing order. The results are three ranking lists.

(e) Propose as a compromise solution the alternative (a') which is ranked the best by the measure Q (minimum) if the following two conditions are satisfied:

C1 ."Acceptable advantage":

$$Q(a'') - Q(a') \geqslant DQ$$

where a'' is the alternative with second position in the ranking list by Q; $DQ = 1/(J - 1)$; J is the number of alternatives.

C2 ."Acceptable stability in decision making": Alternative a' must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be: "voting by majority rule" (when $v > 0.5$ is needed), or "by consensus" $v \approx 0.5$, or "with veto" ($v < 0.5$). Here, v is the weight of the decision making strategy "the majority of criteria" (or "the maximum group utility").

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives a' and a'' if only condition C2 is not satisfied, or
- Alternatives $a', a'', \ldots, a^{(M)}$ if condition C1 is not satisfied; and $a^{(M)}$ is determined by the relation $Q(a^{(M)}) - Q(a') < DQ$ for maximum M (the positions of these alternatives are "in closeness").

The best alternative, ranked by Q, is the one with the minimum value of Q. The main ranking result is the compromise ranking list of alternatives, and the compromise solution with the "advantage rate".

Ranking by VIKOR may be performed with different values of criteria weights, analyzing the impact of criteria weights on proposed compromise solution. The VIKOR method determines the weight stability intervals, using the methodology presented in Opricovic (1998). The compromise solution obtained with initial weights (w_i, $i = 1, \ldots, n$), will be replaced if the value of a weight is not within the stability interval. The analysis of weight stability intervals for a single criterion is performed for all criterion functions, with the same (given) initial values of weights. In this way, the preference stability of an obtained compromise solution may be analyzed using the VIKOR program.

VIKOR is a helpful tool in multicriteria decision making, particularly in a situation where the decision maker is not able, or does not know to express his/her preference at the beginning of system design. The obtained compromise solution could be accepted by the decision makers because it provides a maximum "group utility" (represented by $\min S$, Eq. (1)) of the "majority", and a minimum of the individual regret (represented by $\min R$) of the "opponent". The compromise solutions could be the basis for negotiations, involving the decision makers' preference by criteria weights.

3. TOPSIS method

TOPSIS (technique for order preference by similarity to an ideal solution) method is presented in Chen and Hwang (1992), with reference to Hwang and Yoon (1981). The basic principle is that the chosen alternative should have the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution.

The TOPSIS procedure consists of the following steps:

(1) Calculate the normalized decision matrix. The normalized value r_{ij} is calculated as

$$r_{ij} = f_{ij} \Bigg/ \sqrt{\sum_{j=1}^{J} f_{ij}^2},$$

$$j = 1, \ldots, J; \;\; i = 1, \ldots, n.$$

(2) Calculate the weighted normalized decision matrix. The weighted normalized value v_{ij} is calculated as

$$v_{ij} = w_i r_{ij}, \quad j = 1, \ldots, J; \;\; i = 1, \ldots, n,$$

where w_i is the weight of the ith attribute or criterion, and $\sum_{i=1}^{n} w_i = 1$.

(3) Determine the ideal and negative-ideal solution.

$$A^* = \{v_1^*, \ldots, v_n^*\} = \{(\max_j v_{ij} | i \in I'), (\min_j v_{ij} | i \in I'')\},$$

$$A^- = \{v_1^-, \ldots, v_n^-\} = \{(\min_j v_{ij} | i \in I'), (\max_j v_{ij} | i \in I'')\},$$

where I' is associated with benefit criteria, and I'' is associated with cost criteria.

(4) Calculate the separation measures, using the n-dimensional Euclidean distance. The separation of each alternative from the ideal solution is given as

$$D_j^* = \sqrt{\sum_{i=1}^{n} (v_{ij} - v_i^*)^2}, \quad j = 1, \ldots, J. \tag{4}$$

Similarly, the separation from the negative-ideal solution is given as

$$D_j^- = \sqrt{\sum_{i=1}^{n} (v_{ij} - v_i^-)^2}, \quad j = 1, \ldots, J. \tag{5}$$

(5) Calculate the relative closeness to the ideal solution. The relative closeness of the alternative a_j with respect to A^* is defined as

$$C_j^* = D_j^- / (D_j^* + D_j^-), \quad j = 1, \ldots, J. \tag{6}$$

(6) Rank the preference order.

Eq. (6) represents the "basic principle" in the TOPSIS method (Chen and Hwang, 1992). In step 5, the sentences "*Calculate the relative closeness to the ideal solution. The relative closeness of alternative a_j with respect to A^* is defined as:*" which are taken from the book by Chen and Hwang (1992, p. 39), although simplified seem incorrect. In Triantaphyllou (2000, p. 21) states that "*The best (optimal) alternative can now be decided according to the preference rank order of C_j^*. Therefore, the best alternative is the one that has the shortest distance to the ideal solution. The previous definition can also be used to demonstrate that any alternative which has the shortest distance from the ideal solution is also guaranteed to have the longest distance from the negative-ideal solution*". This could be correct, but not always (see Section 4.1, condition 2 in Eq. (7)). In fact, the chosen alternative has the maximum value of C_j^*, defined in Eq. (6), with the intention to minimize the distance from the ideal solution and to maximize the distance from the negative-ideal solution. The previous authors did not consider the relative importance of distances D_j^* (Eq. (4)) and D_j^- (Eq. (5)) within Eq. (6), although this could be a major concern in decision making. They simply summed D_j^* and D_j^- in Eq. (6), without using any parameter that could represent the relative importance of these two distances. This issue was considered by Lai et al. (1994) introducing the "satisfactory level" for both criteria of the shortest distance from the ideal and the farthest distance from the negative ideal, and concluding "*The compromise solution will exist at the point where the satisfactory levels of both criteria are the same. In future studies, applying compensatory operators should be emphasized*". Thus, the relative importance remained an open question. Deng et al. (2000) presented a modified TOPSIS method, for which they used the ranking index formulated in Eq. (6).

4. Comparing VIKOR and TOPSIS

The MCDM methods VIKOR and TOPSIS are based on an aggregating function representing closeness to the reference point(s). Our comparative analysis points out that these two methods introduce different forms of aggregating function (L_p-*metric*) for ranking. The VIKOR method introduces Q_j function of L_1 and L_∞, whereas the TOPSIS method introduces C_j^* function of L_2.

These two MCDM methods use different kinds of normalization to eliminate the units of criterion functions: the VIKOR method uses linear normalization, and the TOPSIS method uses vector normalization.

4.1. Aggregating function

The VIKOR method is based on following aggregating function (derived from L_p-*metric*):

$$L_{p,j} = \left\{ \sum_{i=1}^{n} \left[w_i (f_i^* - f_{ij})/(f_i^* - f_i^-) \right]^p \right\}^{1/p}, \quad 1 \leqslant p \leqslant \infty; \; j = 1, \ldots, J.$$

The measure L_{pj} represents the distance of the alternative a_j to the ideal solution, as introduced by Duckstein and Opricovic (1980).

In the VIKOR method $S_j = L_{1,j}$ (Eq. (1)) and $R_j = L_{\infty,j}$ (Eq. (2)), $j = 1, \ldots, J$, are introduced (as "boundary measures"). The solution obtained by $\min_j S_j$ is with a maximum "group utility" ("majority" rule). The solution obtained by $\min_j R_j$ is with a minimum individual regret of an "opponent". According to Eqs. (1) and (2), the VIKOR result stands only for the given set of alternatives. Inclusion (or exclusion) of an alternative could affect the VIKOR ranking of new set of alternatives. This seems logical if VIKOR ranking is considered as a competition. By fixing the best f_i^* and the worst f_i^- values, this effect could be avoided, but that would mean that the decision maker could define a fixed ideal solution.

The TOPSIS method introduces an aggregating function for ranking in Eq. (6). According to the formulation of C_j^* (ranking index), alternative a_j is better than a_k if $C_j^* > C_k^*$ or $D_j^-/(D_j^* + D_j^-) > D_k^-/(D_k^* + D_k^-)$, which will hold if

1. $D_j^* < D_k^*$ and $D_j^- > D_k^-$; or

2. $D_j^* > D_k^*$ and $D_j^- > D_k^-$, but $D_j^* < D_k^* D_j^- / D_k^-$. (7)

Condition 1 shows the "regular" situation, when alternative a_j is better than a_k because it is closer to the ideal and farther from the negative-ideal. On the contrary, condition 2 in Eq. (7) shows that an alternative a_j could be better than a_k even though a_j is farther from ideal then a_k. Let a_k be the alternative with $D_k^* = D_k^-$ and $C_k^* = 0.5$. In this case, all alternatives a_j with $D_j^* > D_k^*$ and $D_j^- > D_j^*$ are better ranked than a_k, although a_k is closer to the ideal A^*. The distances considered by VIKOR and TOPSIS are illustrated in Fig. 2. An alternative a_j is better then a_k as a TOPSIS result, but a_k is better then a_j ranked by VIKOR because a_k is closer to the ideal solution. The relative importance of distances D_j^* (Eq. (4)) and D_j^- (Eq. (5)) was not considered within Eq. (6), although it could be a major concern in decision making.

4.2. Normalization effects

Normalization is used to eliminate the units of criterion functions, so that all the criteria are dimensionless. The same criterion function could be evaluated in different convertible units, for example: length f_i [m] or ϕ_i [km], and temperature $f_i[C^0]$ or $\phi_i[F^0]$. These "convertible" units are related as follows:

$$\phi_{ij} = \alpha f_{ij} + \beta, \quad \alpha > 0. \tag{8}$$

Does evaluation of the ith criterion function as f_i or ϕ_i affect the result of MCDM method? The answer should be NO, although there are normalization procedures with effects on the final MCDM result (Pavlicic, 2001).

The VIKOR method uses a normalized value as follows:

$$d_{ij}(f) = (f_i^* - f_{ij})/(f_i^* - f_i^-). \tag{9}$$

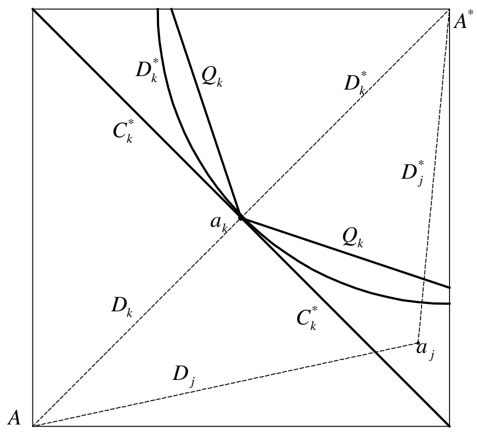

Fig. 2. VIKOR and TOPSIS distances.

The normalized value $d_{ij}(\phi)$ is the following:

$$d_{ij}(\phi) = (\phi_i^* - \phi_{ij})/(\phi_i^* - \phi^-).$$

Since in Eq. (8), $\alpha > 0$ and $\beta = \text{const.}$, we have:

$$\phi_{ij}^* = \alpha f_{ij}^* + \beta, \quad \text{and} \quad \phi_{ij}^- = \alpha f_{ij}^- + \beta$$

and the following equality holds:

$$d_{ij}(\phi) = d_{ij}(f).$$

The normalized value in the VIKOR method does not depend on the evaluation unit of a criterion function.

The normalized value r_{ij} in the TOPSIS method is calculated as

$$r_{ij}(f) = f_{ij} \Big/ \sqrt{\sum_{j=1}^{J} f_{ij}^2},$$

$$r_{ij}(\phi) = \phi_{ij} \Big/ \sqrt{\sum_{j=1}^{J} \phi_{ij}^2}$$

$$= (\alpha f_{ij} + \beta) \Big/ \sqrt{\sum_{j=1}^{J} (\alpha f_{ij} + \beta)^2}.$$

The normalized value in the TOPSIS method could depend on the evaluation unit if $\phi_{ij} = \alpha f_{ij} + \beta$, and the vector normalization leads to $r_{ij}(\phi) \neq r_{ij}(f)$. The equality $r_{ij}(\phi) = r_{ij}(f)$ holds only if $\phi_{ij} = \alpha f_{ij}$.

Linear normalization, such as that in Eq. (9), was subsequently introduced into the TOPSIS method by Lai and Hwang (1994, p. 72), as the following:

$$r_{ij} = f_{ij}/(f_i^* - f_i^-), \quad i \in I' \text{ (benefits)};$$

and

$$r_{ij} = f_{ij}/(f_i^- - f_i^*), \quad i \in I'' \text{ (costs)}. \tag{10}$$

The normalized value in Eq. (10) does not depend on the evaluation unit of a criterion function.

4.3. Essentials of VIKOR and TOPSIS

The main features of VIKOR and TOPSIS are summarized here in order to clarify the differences between these two methods.

Procedural basis. Both methods assume that there exists a performance matrix $\|f\|_{n\times J}$ obtained by the evaluation of all the alternatives in terms of each criterion. Normalization is used to eliminate the units of criterion values. An aggregating function is formulated and it is used as a ranking index. In addition to ranking, the VIKOR method proposes a compromise solution with an advantage rate.

Normalization. The difference appears in the normalization used within these two methods. The VIKOR method uses linear normalization in Eq. (9), and the normalized value does not depend on the evaluation unit of a criterion. The TOPSIS method uses vector normalization, and the normalized value could be different for different evaluation unit of a particular criterion. A later version of the TOPSIS method uses linear normalization in Eq. (10).

Aggregation. The main difference appears in the aggregation approaches. The VIKOR method introduces an aggregating function representing the distance from the ideal solution. This ranking index is an aggregation of all criteria, the relative importance of the criteria, and a balance between total and individual satisfaction. The TOPSIS method introduces the ranking index in Eq. (6), including the distances from the ideal point and from the negative-ideal (nadir) point. These distances in TOPSIS are simply summed in Eq. (6), without considering their relative importance. However, the reference point could be a major concern in decision making, and to be as close as possible to the ideal is the rationale of human choice. Being far away from a nadir point could be a goal only in a particular situation, and the relative importance remains an open question (see Section 3). The TOPSIS method uses n-dimensional Euclidean distance that by itself could represent some balance between total and individual satisfaction, but uses it in a different way than VIKOR, where weight v is introduced in Eq. (3).

Solution. Both methods provide a ranking list. The highest ranked alternative by VIKOR is the closest to the ideal solution. However, the highest ranked alternative by TOPSIS is the best in terms of the ranking index, which does not mean that it is always the closest to the ideal solution. In addition to ranking, the VIKOR method proposes a compromise solution with an advantage rate.

5. Numerical example

A mountain climber (beginner) must choose an alternative from a set of three alternatives, i.e. destinations $\{A_1, A_2, A_3\}$. The alternatives are evaluated as presented in Tables 1 and 2. Let us suppose that both criteria are equally important, i.e. the weights of criteria are $w_i = 1/2$, $i = 1, 2$.

The relationships between values of criteria f and ϕ are as follows:

$$\phi_{1j} = f_{1j} + 5 \quad \text{and} \quad \phi_{2j} = f_{2j}/1000 - 1.$$

The results obtained by the VIKOR and the TOPSIS methods are presented in Table 3.

The compromise solution obtained by VIKOR is A_2. The same solution is obtained for *problem f* and *problem* ϕ, illustrating that the "convertible" units related as $\phi_{ij} = \alpha f_{ij} + \beta$ do not affect the result of the VIKOR method.

The final rankings obtained by the TOPSIS criterion values, C_j^* $(j = 1, 2, 3)$, are different for two normatively equivalent problems. Thus, although A_1 is proclaimed as the best solution for *problem f*, alternative A_2 is suggested as the best for the *problem* ϕ (see Table 3, "TOPSIS vector normalization"). This shows that the TOPSIS results with vector normalization depend on the convertible units related as $\phi_{ij} = \alpha f_{ij} + \beta$. According to the TOPSIS results, alternative A_2 is the closest to the ideal A^* for both problems (f and ϕ), but different alternatives are denoted as the "farthest" from the "negative ideal". In *problem f*, the TOPSIS method considers A_1 as the "farthest" from the "negative ideal", while in *problem* ϕ, alternative A_3 is proclaimed the "farthest". The VIKOR method considers both A_1 and A_3 as the "farthest" from the "negative ideal".

Table 1
Problem f

Criteria	Alternatives		
	A_1	A_2	A_3
f_1—Risk, subjective evaluation, scale: 1, 2, 3, 4, 5	1	2	5
f_2—Altitude, evaluated in meters above the sea (m.a.s)	3000	3750	4500

$f_1^* = 1$, $f_2^* = 4500$.

Table 2
Problem ϕ (the same alternatives as in the Problem f)

Criteria	Alternatives		
	A_1	A_2	A_3
ϕ_1—Risk, subjective evaluation, scale: 6, 7, 8, 9, 10	6	7	10
ϕ_2—Altitude, in kilometres above the foothill (km.a.fh), fh = 1000 m.a.s	2	2.75	3.5

$\phi_1^* = 6$, $\phi_2^* = 3.5$.

Analyzing the results of TOPSIS, we find that the results from TOPSIS with vector normalization for *problem f* are interesting due to the following. According to C_j^*, the best solution is A_1, i.e. $C_1^* = 0.762$, $C_2^* = 0.722$; and it is the best according to D_j^-, i.e. $D_1^- = 0.365$, $D_2^- = 0.280$. However, A_1 is not the closest to the ideal, i.e. $D_1^* = 0.114$, $D_2^* = 0.108$. According to relation (7), in this case, A_1 is ranked best by TOPSIS, although it is not the closest to the ideal, because $D_1^* < D_2^* D_1^- / D_2^-$, i.e. $0.114 < 0.141$. This is an example of the case discussed in Section 4.1 in Eq. (7) and presented in Fig. 2.

Ranking results by TOPSIS with linear normalization by Eq. (10) are the same as the results by the VIKOR method (see Table 3, "TOPSIS linear normalization").

Figs. 3–6 illustrate the positions of the alternatives and their distances to the ideal A^* and negative-ideal A^-.Weight stability intervals for a single criterion obtained by VIKOR are as follows:

$0.415 \leqslant w_1 \leqslant 0.630$ (input $w_1 = 0.5$),
$0.370 \leqslant w_2 \leqslant 0.585$ (input $w_2 = 0.5$).

The alternative A_2 is a single compromise solution with the weights within these intervals. It is ranked as first in the set of compromise solution $\{A_2, A_1\}$ with the weights outside the above intervals, but within the following intervals:

$0.366 \leqslant w_1 \leqslant 0.746$ (input $w_1 = 0.5$),
$0.254 \leqslant w_2 \leqslant 0.634$ (input $w_2 = 0.5$).

Table 3
Results obtained by VIKOR and TOPSIS

		Alternatives			Ranking
		A_1	A_2	A_3	
Linear normalization $d(f) = d(\phi)$	d_{1j}	0	0.25	1	A_1, A_2, A_3
	d_{2j}	1	0.5	0	A_3, A_2, A_1
Vector normalization $r(f) \neq r(\phi)$	$r_{1j}(f)$	0.183	0.365	0.913	A_1, A_2, A_3
	$r_{2j}(f)$	0.456	0.570	0.684	A_3, A_2, A_1
	$r_{1j}(\phi)$	0.441	0.515	0.735	A_1, A_2, A_3
	$r_{2j}(\phi)$	0.410	0.564	0.717	A_3, A_2, A_1
VIKOR	S	0.5	0.375	0.5	$A_2, A_1 \approx A_3$
	R	0.5	0.25	0.5	$A_2, A_1 \approx A_3$
	Q	1	0	1	$A_2, A_1 \approx A_3$
TOPSIS vector normalization	$D^*(f)$	0.114	0.108	0.365	A_2, A_1, A_3
	$D^-(f)$	0.365	0.280	0.114	A_1, A_2, A_3
	$C^*(f)$	0.762	0.722	0.238	A_1, A_2, A_3
	$D^*(\phi)$	0.154	0.085	0.147	A_2, A_3, A_1
	$D^-(\phi)$	0.147	0.134	0.154	A_3, A_1, A_2
	$C^*(\phi)$	0.489	0.612	0.511	A_2, A_3, A_1
TOPSIS linear normalization	D^*	0.500	0.280	0.500	$A_2, A_1 \approx A_3$
	D^-	0.500	0.451	0.500	$A_1 \approx A_3, A_2$
	C^*	0.500	0.617	0.500	$A_2, A_1 \approx A_3$

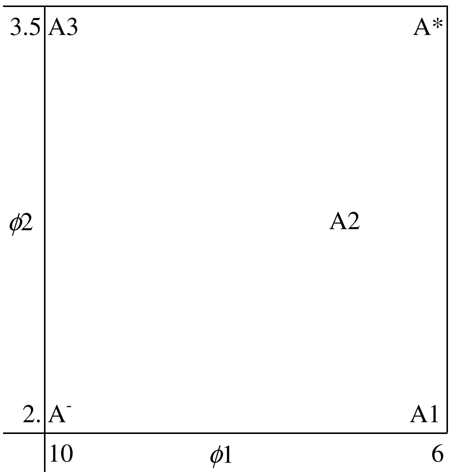

Fig. 3. Original positions.

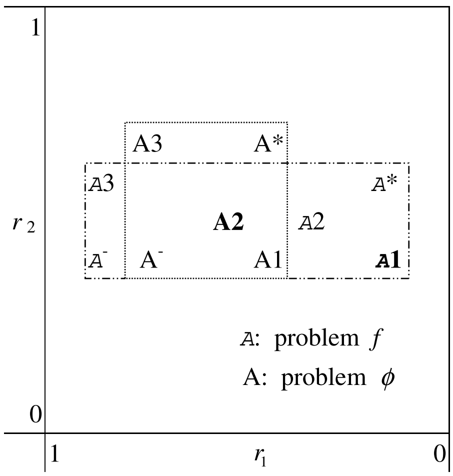

Fig. 4. Vector normalized space.

In this example, the alternative A_2 is a real compromise, as "something between extremes". The TOPSIS method with vector normalization selects A_1 as a compromise solution for *problem f*, although it is difficult to accept A_1 as a compromise being better than A_2 (see Fig. 3).

These results illustrate the difference between MCDM methods VIKOR and TOPSIS. The normalized value in the VIKOR method does not depend on the evaluation unit of a criterion function, whereas the normalized values by vector normalization in the TOPSIS method may depend on the evaluation unit (see $r(f) \neq r(\phi)$ in Table 3).

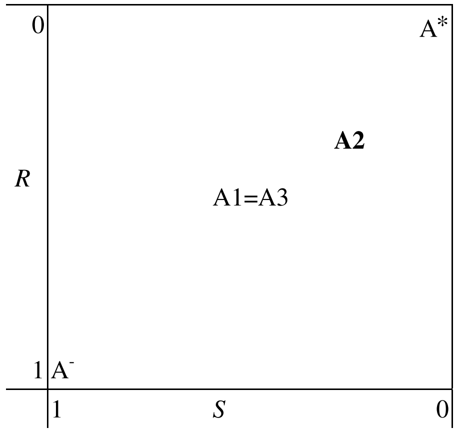

Fig. 5. Distance by VIKOR.

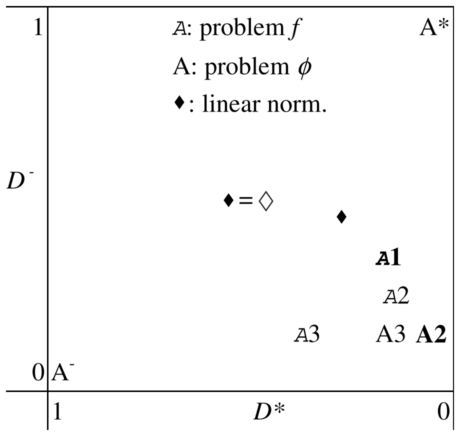

Fig. 6. Distances by TOPSIS.

6. Conclusions

The MCDM methods VIKOR and TOPSIS are both based on an aggregating function representing "closeness to the ideal". The VIKOR method introduces the ranking index based on the particular measure of "closeness" to the ideal solution. In contrast, the basic principle of the TOPSIS method is that the chosen alternative should have the "shortest distance" from the ideal solution and the "farthest distance" from the "negative-ideal" solution. The TOPSIS method introduces two "reference" points, but it does not consider the relative importance of the distances from these points.

These two MCDM methods use different kinds of normalization to eliminate the units of criterion functions, whereas the VIKOR method uses linear normalisation, the TOPSIS method uses vector normalization. The normalized value in the VIKOR method does not depend on the evaluation unit of a criterion function, whereas the normalized values by vector normalization in the TOPSIS method may depend on the evaluation unit.

A comparative analysis shows that these two methods use different normalizations and that they introduce different aggregating functions for ranking.

The paper does not consider the trade-offs involved by normalization in obtaining the aggregating function (Q or C^*), and this topic remains for further research.

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