Robust Cerebellar Model Articulation Controller Design for Unknown Nonlinear Systems

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***Abstract*—In this study, a robust cerebellar model articulation controller (RCMAC) is designed for unknown nonlinear systems. The RCMAC is comprised of a cerebellar model articulation controller (CMAC) and a robust controller. The CMAC is utilized to approximate an ideal controller, and the weights of the CMAC are on-line tuned by the derived adaptive law based on the Lyapunov sense. The robust controller is designed to guarantee a specified H^∞ robust tracking performance. In the RCMAC design, the sliding-mode control method is utilized to derive the control law, so that the developed control scheme has more robustness against the uncertainty and approximation error. Finally, the proposed RCMAC is applied to control a chaotic circuit. Simulation results demonstrate that the proposed control scheme can achieve favorable tracking performance with unknown the controlled system dynamics.**

***Index Terms*—Cerebellar model articulation controller (CMAC), chaotic system, robust control, sliding-mode control.**

I. INTRODUCTION

RECENTLY, there has been some increasing interest on the study of analysis and control of the nonlinear systems. Various control methodologies have been developed by many researchers from a point of view of dynamic system theory and traditional feedback control. However, since these design methods are based on a good understanding of the controlled system dynamics and its environment; the objection for the real-time application is unrealizable for the unknown systems. To tackle this problem, some intelligent control techniques (e.g., fuzzy control, neural network and neuro-fuzzy control) have represented a design method for control of unknown dynamic systems [2], [5]–[7]. Neural network is a model-free approach, which is generally considered suitable for controlling imprecisely defined systems. The success key element is the neural network can approximate a mapping through choosing adequate learning method. In recent years, the cerebellar model articulation controller (CMAC) has been adopted widely for the control of complex dynamical systems owing to its fast learning property, good generalization capability, and simple computation compared with the neural networks [3], [4], [8]. The application of CMAC is not only for the control problems but also for model-free function approximation. The CMAC has been already validated that it can approximate a nonlinear function over a domain of interest to any desired accuracy.

If the exact model of the controlled system is well known, there exists an ideal controller to achieve favorable control performance by possible canceling all the system uncertainties. Since the system parameters and the external disturbances may be perturbed or unknown, the ideal controller is always unobtainable. To overcome this problem, a robust design technique termed as sliding-mode control has been presented to confront these uncertainties [9]. However, to satisfy the existence condition of the sliding mode, sliding-mode control suffers from large control chattering that may excite the unmodeled high frequency response of the systems due to the discontinuous switching and imperfect implementations.

This study successfully develops a robust cerebellar model articulation controller (RCMAC) to achieve H^∞ robust tracking performance. Three important control design techniques, i.e., CMAC control approach, H^∞ tracking theory, and sliding-mode control design have been employed together to develop the robust control algorithm. The developed RCMAC is comprised of a CMAC and a robust controller. The CMAC is designed to mimic an ideal controller and the robust controller is designed to attenuate the effect of the approximation error between the CMAC and the ideal controller. Thus, the derived control scheme has more robustness against the uncertainties and approximation error. To investigate the effectiveness of the proposed control scheme, it is applied to control a chaotic circuit. The major contributions of this study are: 1) the successful investigation of the RCMAC system without using prior knowledge of the controlled plant and (2) the successful application of the RCMAC system for the accuracy control of a chaotic circuit system.

II. PROBLEM STATEMENT AND IDEAL CONTROL

Consider the nth-order nonlinear system of the form

$$x^{(n)} = f(\mathbf{x}) + u \tag{1}$$

where $\mathbf{x} = [x, \dot{x}, \ldots, x^{(n-1)}]^T \in R^n$ is the state vector of the system, which is assumed to be available for measurement, $f(\mathbf{x})$ is the nonlinear system dynamics which can be unknown, and $u \in R$ is the input of the system. The tracking control problem of the system is to find a control law so that the state trajectory x can track a reference command x_c. The tracking error is defined as

$$e = x_c - x. \tag{2}$$

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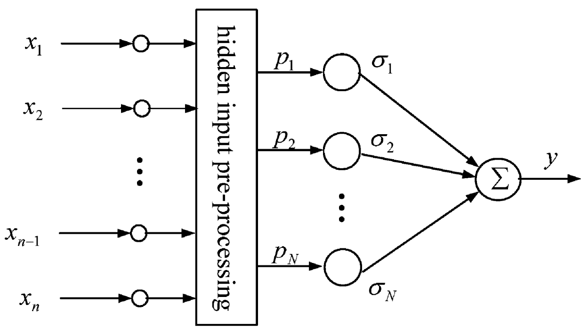

Fig. 1. Network structure of a CMAC.

Assume that the parameters of the controlled system in (1) are well known, there exits an ideal controller [9]

$$u^* = -f(\mathbf{x}) + x_c^{(n)} + \mathbf{k}^T\mathbf{e} \tag{3}$$

where $\mathbf{e} = [e, \dot{e}, \ldots, e^{(n-1)}]^T \in R^n$ is the tracking error vector and $\mathbf{k} = [k_n, \ldots, k_2, k_1]^T \in R^n$, in which k_i $(i = 1, 2, \ldots, n)$ are positive constants. Applying the ideal controller (3) to system (1) results in the following error dynamics:

$$e^{(n)} + k_1 e^{(n-1)} + \cdots + k_n e = 0. \tag{4}$$

If $k_i, i = 1, 2, \ldots, n$ are chosen such that all roots of the polynomial $h(s) \triangleq s^n + k_1 s^{n-1} + \cdots + k_n$ lie strictly in the open left half of the complex plane, then it implies that $\lim_{t\to\infty} e = 0$ for any starting initial conditions. The error dynamics (4) can be rewritten in a vector form as

$$\dot{\mathbf{e}} = \mathbf{A_m}\mathbf{e} \tag{5}$$

where

$$\mathbf{A_m} = \begin{bmatrix} 0 & 1 & 0 & \cdots 0 \\ \vdots & \ddots & \ddots & \ddots 0 \\ 0 & \cdots & 0 & 1 \\ -k_n & -k_{n-1} & \cdots & -k_1 \end{bmatrix}.$$

If the system dynamics $f(\mathbf{x})$ cannot be exactly obtained, the ideal controller is unobtainable. Thus, a model-free design method termed as the RCMAC will be developed for the unknown nonlinear systems.

III. Robust Control System Design via Sliding-Mode Control Technique

The developed RCMAC is comprised of a CMAC and a robust controller. The CMAC is utilized to approximate the ideal controller and the robust controller is designed to achieve a specified H^∞ robust tracking performance. The detail is described as follows.

A. Description of CMAC

A CMAC neural network is depicted in Fig. 1, which can be considered as "1"-layer feedforward neural network with input preprocessing element [8]. If $\sigma(\cdot)$ is a continuous discriminate function, the neural network output performs the mapping according to

$$y = \sum_{i=1}^{N} w_i \sigma_i(p_i) \tag{6}$$

where w_i are the output layer weight values, p_i are the network input to the ith neuron, $\sigma(\cdot): R \to R$ is the activation function, and N is the number of units (also called nodes and neurons) in the hidden layer. For ease of notation, the neural network equation (6) can be expressed in a compact vector form as

$$y = \mathbf{w}^T\boldsymbol{\sigma} \tag{7}$$

where $\mathbf{w} = [w_1, w_2, \ldots, w_N]^T \in R^N$ and $\boldsymbol{\sigma} = [\sigma_1, \sigma_2, \ldots, \sigma_N]^T \in R^N$. It has been proven that there exists a neural network approximator in (7) such that it can uniformly approximate any nonlinear even time-varying function $\hbar$ [4]. By the universal approximation theorem, there exists an ideal neural network approximator y^* such that

$$\hbar = y^* + \varepsilon(t) = \mathbf{w}^{*T}\boldsymbol{\sigma} + \varepsilon(t) \tag{8}$$

where $\mathbf{w}^*$ is the optimal vector of $\mathbf{w}$ and $\varepsilon(t)$ denotes the approximation error. The approximation error generally decreases as the number of the neurons N increases. In fact, the optimal vector that is needed to best approximate a given nonlinear function $\hbar$ is difficult to determine and might not even be unique. Thus, a neural network estimator is defined as

$$\hat{y} = \hat{\mathbf{w}}^T\boldsymbol{\sigma} \tag{9}$$

where $\hat{\mathbf{w}} = [\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_N]^T \in R^N$ is the estimated vector of $\mathbf{w}^*$. Define the estimated error $\tilde{y}$ as

$$\tilde{y} = \hbar - \hat{y} = y^* - \hat{y} + \varepsilon = \tilde{\mathbf{w}}^T\boldsymbol{\sigma} + \varepsilon \tag{10}$$

where $\tilde{\mathbf{w}} = \mathbf{w}^* - \hat{\mathbf{w}}$. In the following, an update law will be derived to on-line tune the estimated vector to achieve favorable estimation.

B. RCMAC Design

The developed RCMAC feedback control system is shown in Fig. 2. The RCMAC is comprised of a CMAC and a robust controller as follows:

$$u = u_{\mathrm{CMAC}} + u_R = \hat{\mathbf{w}}^T\boldsymbol{\sigma} + u_R \tag{11}$$

where the CMAC u_{CMAC} is designed to approximate the ideal controller u^*, and the robust controller u_R is designed to achieve H^∞ robust tracking performance. Substituting (11) into (1), yields

$$x^{(n)} = f(\mathbf{x}) + u_{\mathrm{CMAC}} + u_R. \tag{12}$$

Using (3) and (12), the error dynamic equation is obtained as

$$\dot{\mathbf{e}} = \mathbf{A_m}\mathbf{e} + \mathbf{b_m}(u^* - u_{\mathrm{CMAC}} - u_R) \tag{13}$$

where $\mathbf{b_m} = [0, \ldots, 0, 1]^T \in R^n$.

Define a sliding surface as [10]

$$s(t) = C(\mathbf{e}) - C(\mathbf{e}_0) - \int_0^t \frac{\partial C(\mathbf{e})}{\partial \mathbf{e}} \mathbf{A_m}\mathbf{e}\, d\tau \tag{14}$$

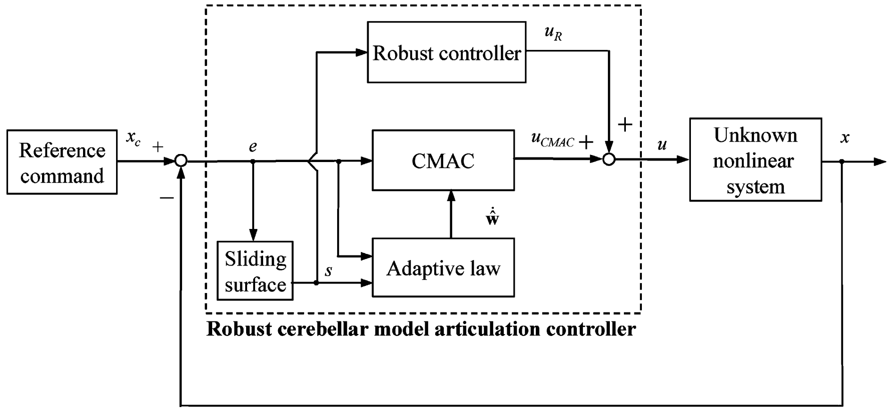

Fig. 2. RCMAC feedback control system.

where $C(\mathbf{e})$ is a function to be designed, and $\mathbf{e}_0$ is the initial state of $\mathbf{e}(t)$. Differentiating (14) with respect to time and using the error dynamic equation (15) gives

$$\dot{s} = \frac{\partial C(\mathbf{e})}{\partial \mathbf{e}}\dot{\mathbf{e}} - \frac{\partial C(\mathbf{e})}{\partial \mathbf{e}}\mathbf{A_m}\mathbf{e} = u^* - u_{\text{CMAC}} - u_R \quad (15)$$

where $C(\mathbf{e})$ satisfies $(\partial C(\mathbf{e})/\partial \mathbf{e}) = [0,\ldots,0,1]$. Since u_{CMAC} is used to approximate u^*, using the approximation error in (10), (15) can be rewritten as

$$\dot{s}(t) = \tilde{\mathbf{w}}^T\boldsymbol{\sigma} + \varepsilon(t) - u_R. \quad (16)$$

In case of the existence of $\varepsilon(t)$, consider a specified H^∞ tracking performance [2], [7]

$$\int_0^T s^2(t)\,dt \le s^2(0) + \frac{1}{\kappa}\tilde{\mathbf{w}}(0)\tilde{\mathbf{w}}(0) + \delta^2\int_0^T \varepsilon^2(t)\,dt \quad (17)$$

where κ is a positive gain and δ is a prescribed attenuation constant. If the system starts with initial conditions $s(0) = 0$ and $\tilde{\mathbf{w}}(0) = 0$, the H^∞ tracking performance in (17) can be rewritten as

$$\sup_{\varepsilon\in L_2[0,T]} \frac{\|s\|}{\|\varepsilon\|} \le \delta \quad (18)$$

where $\|s\|^2 = \int_0^T s^2(t)\,dt$ and $\|\varepsilon\|^2 = \int_0^T \varepsilon^2(t)\,dt$. If $\delta = \infty$, this is the case of minimum error tracking control [2], [7]. Therefore, the following theorem can be stated and proved.

Theorem 1: Consider the nth-order nonlinear system expressed by (1). If the RCMAC system is designed as (11), in which the adaptation law of $\hat{\mathbf{w}}$ is designed as (19), and the robust controller is designed as (20)

$$\dot{\hat{\mathbf{w}}} = -\dot{\tilde{\mathbf{w}}} = \kappa s\boldsymbol{\sigma} \quad (19)$$

$$u_R = \frac{(\delta^2+1)s}{2\delta^2} \quad (20)$$

where κ presents a learning rate, then the desired robust tracking performance in (17) can be achieved for a prescribed attenuation level δ.

Proof: Define a Lyapunov function in the following form:

$$V(s,\tilde{\mathbf{w}},t) = \frac{1}{2}s^2 + \frac{\tilde{\mathbf{w}}^T\tilde{\mathbf{w}}}{2\kappa}. \quad (21)$$

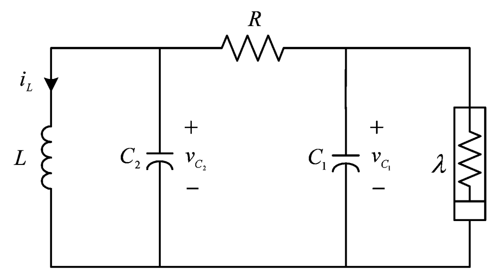

Fig. 3. Chua's chaotic circuit.

Differentiating (21) with respect to time and using (16), (19), and (20), gives

$$\begin{aligned}\dot{V}(s,\tilde{\mathbf{w}},t) &= s\dot{s} + \frac{\tilde{\mathbf{w}}^T\dot{\tilde{\mathbf{w}}}}{\kappa}\\ &= s(\tilde{\mathbf{w}}^T\boldsymbol{\sigma} + \varepsilon - u_R) + \frac{\tilde{\mathbf{w}}^T\dot{\tilde{\mathbf{w}}}}{\kappa}\\ &= \tilde{\mathbf{w}}^T\left(s\boldsymbol{\sigma} + \frac{\dot{\tilde{\mathbf{w}}}}{\kappa}\right) + s(\varepsilon - u_R)\\ &= s\varepsilon - \frac{(\delta^2+1)s^2}{2\delta^2}\\ &= -\frac{s^2}{2} - \frac{1}{2}\left(\frac{s}{\delta} - \delta\varepsilon\right)^2 + \frac{1}{2}\delta^2\varepsilon^2\\ &\le -\frac{s^2}{2} + \frac{1}{2}\delta^2\varepsilon^2. \end{aligned} \quad (22)$$

Assume $\varepsilon \in L_2[0,T), \forall T \in [0,\infty)$. Integrating the above equation from $t=0$ to $t=T$ yields

$$V(T) - V(0) \le -\frac{1}{2}\int_0^T s^2\,dt + \frac{1}{2}\delta^2\int_0^T \varepsilon^2\,dt. \quad (23)$$

Since $V(T) \ge 0$, it implies the following inequality:

$$\frac{1}{2}\int_0^T s^2\,dt \le V(0) + \frac{1}{2}\delta^2\int_0^T \varepsilon^2\,dt \quad (24)$$

Using (21), the above inequality is equivalent to the following

$$\int_0^T s^2\,dt \le s^2(0) + \frac{1}{\kappa}\tilde{\mathbf{w}}(0)^T\tilde{\mathbf{w}}(0) + \delta^2\int_0^T \varepsilon^2\,dt. \quad (25)$$

Thus, the proof is completed. □

In the following, the design algorithm of the developed RCMAC scheme is summarized as follows:

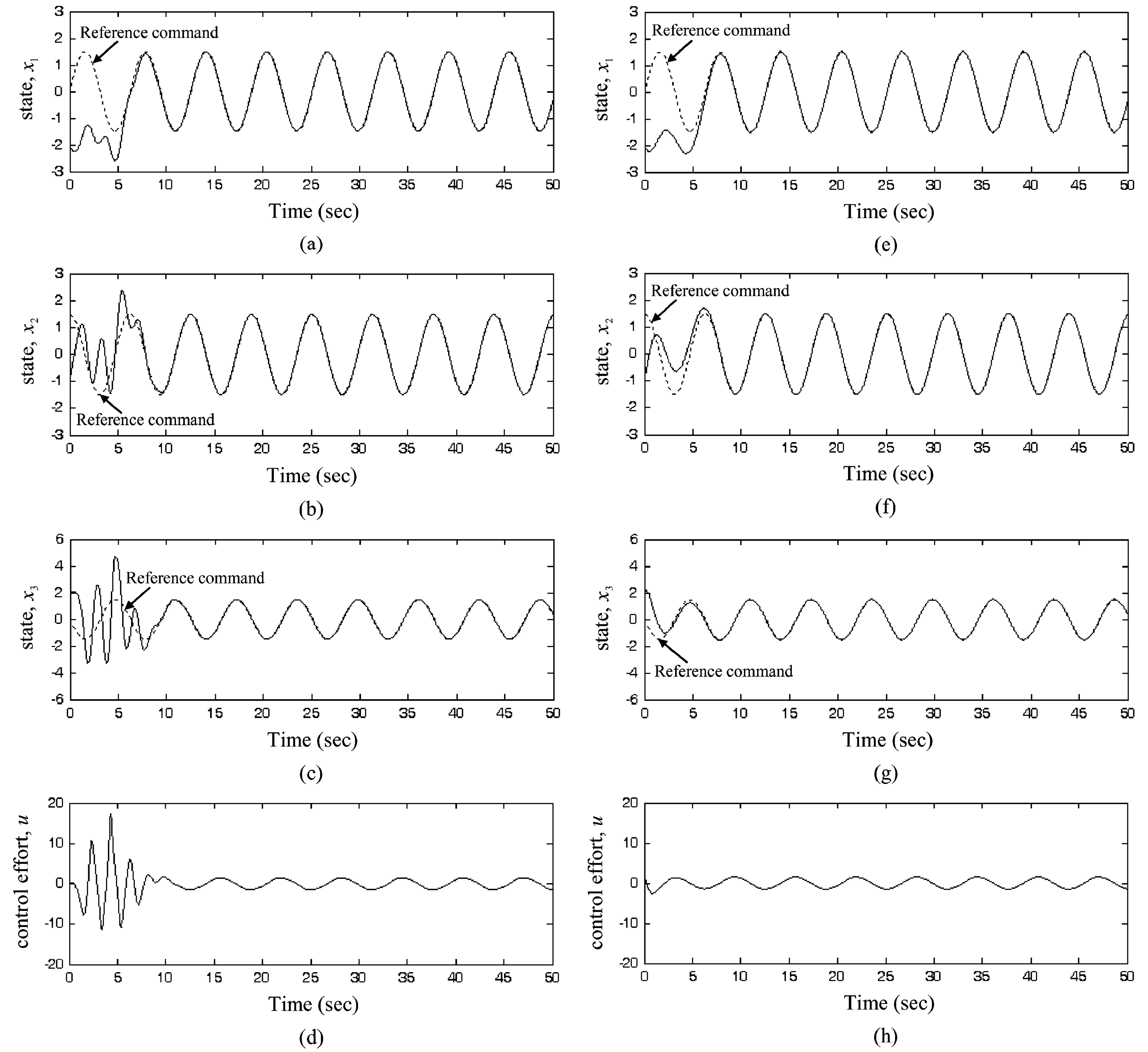

Fig. 4. Simulation results of RCMAC for the Chua's chaotic circuit system. (a) State response x_1. (b) State response x_2. (c) State response x_3. (d) Associated effort u (for $\delta = 1.0$). (e) State response x_1. (f) State response x_2. (g) State response x_3. (h) Associated effort u (for $\delta = 0.1$).

Step 1) Select control parameters $k_i, i = 1, 2, \ldots, n$ such that the roots of (4) are in the open left half plane and with desired convergent performance for e; and then define the sliding surface as (14).

Step 2) The CMAC is given as (9) with $\hat{\mathbf{w}}$ on-line tuned by (19).

Step 3) The robust controller is designed as (20) where δ is an attenuation constant specified by the designers.

Step 4) The RCMAC is given as (11).

IV. Simulation Results

In this section, the proposed design technique is applied to control a chaotic circuit system to verify its effectiveness. It should be emphasized that the development of the RCMAC does not need to know the system dynamic function of the controlled system.

A third-order Chua's chaotic circuit, as shown in Fig. 3, is a simple electronic system that consists of one linear resistor (R), two capacitors (C_1, C_2), one inductor (L), and one nonlinear resistor (λ) [1], [11]. It has been shown to own very rich nonlinear dynamics such as chaos and bifurcations. The dynamic equations of Chua's circuit are written as

$$\begin{aligned}\dot{v}_{C_1} &= \frac{1}{C_1}\left(\frac{1}{R}(v_{C_2} - v_{C_1}) - \lambda(v_{C_2})\right)\\ \dot{v}_{C_2} &= \frac{1}{C_2}\left(\frac{1}{R}(v_{C_1} - v_{C_2}) + i_L\right)\\ \dot{i}_L &= -\frac{1}{L}v_{C_1}\end{aligned} \tag{26}$$

where the voltages v_{C_1}, v_{C_2}, and current i_L are state variables, and λ denotes the nonlinear resistor, which is a function of the voltage across the two terminals of C_1. The λ is defined as a cubic function as

$$\lambda(v_{C_1}) = av_{C_1} + cv_{C_1}^3 \quad (a < 0, c > 0). \tag{27}$$

Choose the parameters as $R = 10/7, C_1 = 1, C_2 = 19/2, L = 19/14, a = -4/5$, and $c = 2/45$ [11]. Furthermore, the state equations in (26) are not in the standard canonical form in (1). Therefore, a linear transformation is needed to transform them into the form of (1). According to [11], the dynamic equations of transformed Chua's circuit can be rewritten as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= f(\mathbf{x}) + u \end{aligned} \tag{28}$$

where $f(\mathbf{x}) = (14/1805)x_1 - (168/9025)x_2 + (1/38)x_3 - (2/45)((28/361)x_1 + (7/95)x_2 + x_3)^3$ and $\mathbf{x} = [x_1, x_2, x_3]^T$ is the state vector of the system which is assumed to be available, and u is the control effort. Following the design procedure, the control parameters and sliding surface are chosen as $k_1 = 3, k_2 = 3, k_3 = 1$, and $s = \ddot{e} - \ddot{e}(0) + \int_0^t (3\ddot{e} + 3\dot{e} + e)\, dt$. The CMAC is given as (9) with $\hat{\mathbf{w}}$ on-line tuned by (19) where k is chosen as 20; and the robust controller is given as (20). Then, the RCMAC is given as (11). The simulation results of the RCMAC feedback control system are shown in Fig. 4. The state responses x_1, x_2, and x_3 are shown in Fig. 4(a), (b), and (c), respectively, and the associated effort is shown in Fig. 4(d) for $\delta = 1.0$. Meanwhile, The state responses x_1, x_2 and x_3 are shown in Fig. 4(e), (f), and (g), respectively, and the associated effort is shown in Fig. 4(h) for $\delta = 0.1$. From Fig. 4(e), (f), and (g), the state responses can achieve faster tracking performance than the control results proposed in [11]. Furthermore, the associated effort u of the RCMAC is smaller than the control effort proposed in [11]. From these simulation results, it can be seen that favorable tracking performance can be achieved for the proposed RCMAC by specifying a small attenuation constant.

V. Conclusion

This study has successfully developed a RCMAC system, which is comprised of a CMAC and a robust controller. The CMAC is utilized to approximate an ideal controller and the robust controller is utilized to attenuate the tracking error with a specified H^∞ tracking performance. The developed control scheme has the advantages that it can on-line tune the parameters of the CMAC even unknown of the system dynamics and it can achieve a specified robust tracking performance. For investigating the effectiveness of the proposed RCMAC system, it is applied to control a chaotic circuit system. Simulation results indicate that favorable tracking performance can be achieved by specifying a small attenuation constant.

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