Procedure for supplier selection based on C_{pm} applied to super twisted nematic liquid crystal display processes

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Published online: 21 Feb 2007.

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Procedure for supplier selection based on C_{pm} applied to super twisted nematic liquid crystal display processes

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The loss-based process capability index C_{pm}, sometimes called the Taguchi index, has been proposed to the manufacturing industry as a method to measure process performance. The index C_{pm} takes into account the targeting degree of the process, which essentially measures process performance based on average process loss. Based on the C_{pm} index, a mathematically complicated approximation method was developed previously for selecting a subset of processes containing the best supplier from a given set of processes. The present paper implements this method and develops a practical step-by-step procedure to aid supplier selection decisions. The accuracy of the selection method is investigated using a simulation technique. The accuracy study provides useful information about the sample size required for a designated selection power. A two-phase selection procedure is developed to select a better supplier and to examine the magnitude of the difference between the two suppliers. Also investigated is a real-world case on the super twisted nematic liquid crystal display manufacturing process, and it is applied to the selection procedure using actual data collected from the factories to reach a decision in supplier selection.

1. Introduction

The aim of process capability indices is to provide numerical measures of whether or not the reproduction ability of a manufacturing process meets a predetermined level of production tolerance. Process capability indices have received considerable research attention and increased use in process assessments and purchasing decisions in the automotive and other industries during the last decade. Examples include Pearn *et al.* (1992, 1998), Pearn and Chen (1997), Kotz and Lovelace (1998), Palmer and Tsui (1999), Kotz and Johnson (2002), Pearn and Lin (2003a, b), Pearn and Shu (2003a–c), Spiring *et al.* (2003) and many others. Those indices are effective tools for process capability analysis and quality assurance, and the formulae for those indices are easy to understand and straightforward to apply. The C_p index was developed by Kane (1986) and it considers the overall process variability relative to the manufacturing tolerance to measure process precision (product consistency). Due to simplicity of the design, C_p cannot reflect the tendency of process centring (targeting):

$$C_p = \frac{USL - LSL}{6\sigma}.$$

In order to reflect the deviations of process mean from the target value, several indices similar in nature to C_p have been proposed. Those indices attempt to take the magnitude of process variance as well as process departures from the target value into consideration. One of those indices is C_{pk} defined as:

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \quad \frac{\mu - LSL}{3\sigma}\right\},$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation and T is the target value. The index C_{pk} was developed because C_p does not adequately deal with cases where process mean μ is not centred. However, C_{pk} alone still cannot provide adequate measure of process centring. That is, a large C_{pk} does not really say anything about the location of the mean in the tolerance interval. The C_p and C_{pk} indices are appropriate measures of progress for quality improvement paradigms in which reduction of variability is the guiding principle and process yield is the primary measure of success. However, they are not related to the cost of failing to meet customers' requirement. Taguchi, on the other hand, emphasizes the loss in a product's worth when one of its characteristics departs from the customers' ideal value T.

To help account for this, Hsiang and Taguchi (1985) introduced the index C_{pm}, which was also proposed independently by Chan *et al.* (1988). The index is related to the idea of squared error loss, loss $(X) = (X - T)^2$, and this loss-based process capability index C_{pm}, sometimes called the Taguchi index. The index emphasizes on measuring the ability of the process to cluster around the target, which therefore reflects the degrees of process targeting (centring). The index C_{pm} incorporates with the variation of production items with respect to the target value and the specification limits preset in the factory. The index C_{pm} is defined as:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{USL - LSL}{6\gamma},$$

where $\gamma^2 = \sigma^2 + (\mu - T)^2 = E[(X - T)]^2$ is the major part of the denominator of C_{pm}, which incorporates two variation components: (1) variation to the process mean and (2) deviation of the process mean from the target. For on target processes, C_{pm} index reaches its maximum, implying that the process capability runs under the desired condition. On the other hand, smaller values of C_{pm} mean higher expected loss and poorer process capability. Therefore, the index C_{pm} is considered to be more sensitive than C_p and C_{pk} in reflecting the process loss. Boyles (1991) has provided a definitive analysis of C_{pm} and its usefulness in measuring process centring. He notes that both C_{pk} and C_{pm} coincide with C_p when $\mu = T$ and decrease as μ moves away from T. However, $C_{pk} < 0$ for $\mu > USL$ or $\mu < LSL$, whereas C_{pm} of process with $|\mu - T| = \Delta > 0$ is strictly bounded above by the C_p value of a process with $\sigma = \Delta$.

In the initial stage of production setting, the decision-maker usually faces the problem of selecting the best manufacturing supplier from several available manufacturing suppliers. There are many factors, such as quality, cost, service and so on, that need to be considered in selecting the best suppliers. Process yield is currently defined as the percentage of the processed product units passing the inspections.

Traditionally, the fraction of non-conformities for manufacturing processes has been calculated by counting the number of non-conforming items in the sample. With the fraction non-conforming now commonly less than 0.01%, which is often expressed in parts per million (ppm), those traditional methods for estimating the fraction non-conforming no longer work since all reasonably sized samples would contain zero defective items. Several selection rules have been proposed for selecting the means or variances in analysis of variance (see Gibbons *et al.* 1977, Gupta and Panchapakesan 1979, Gupta and Huang 1981 for more details). Process capability indices are useful management tools, particularly in the manufacturing industry, which provide common quantitative measures on manufacturing capability and production quality. In the situation of the manufacturing process being control, it is assumed that the quality characteristic X is normally distributed, USL and LSL are usually fixed and determined in advance, the larger C_p is equivalent to looking for the smallest σ^2. Tseng and Wu (1991) considered the problem of selecting the best manufacturing process from k available manufacturing processes based on the precision index C_p and a modified likelihood ratio selection rule is proposed. Chou (1994) developed three one-sided tests (C_p, C_{PU}, C_{PL}) for comparing two process capability indices in order to choose between competing processes when the sample sizes are equal. Based on the C_{pm} index, a mathematically complicated approximation method is developed by Huang and Lee (1995) for selecting a subset of processes containing the best supplier from a given set of processes.

Under the circumstance, to search the larger C_{pm}'s which are used to provide a unitless measure of the process performance is equivalent to looking for a smaller γ^2. The present paper implements this method and develops a practical step-by-step procedure for practitioners to use in making supplier selection decisions. In practice, the process mean and process variance are unknown. To calculate the index, sample data must be collected, and a great degree of uncertainty may be introduced into capability assessments due to sampling errors. Thus, the distributional properties of the estimated index C_{pm} are then introduced and the unbiased estimator of loss function, $\hat{\gamma}^2$, is considered. Accuracy of the selection method is investigated using a simulation technique. The accuracy study provides useful information about the sample size required for designated selection power. Subsequently, also investigated is a real-world case on the super twisted nematic liquid crystal display (STN-LCD) manufacturing process and the selection procedure is applied using actual data collected from the factories to reach a decision in supplier selections.

2. Distribution of the estimated C_{pm}

Since the process mean μ and the process variance σ^2 must be estimated from the sample, the estimated index $\hat{C}_{pm}$ is obtained by replacing μ and σ^2 by their estimators. Chan *et al.* (1988) and Boyles (1991) proposed two different estimators of C_{pm}, respectively, defined as the following:

$$\hat{C}_{pm(CCS)} = \frac{d}{3\sqrt{s^2 + [n/(n-1)](\bar{x}-T)^2}} \text{ and } \hat{C}_{pm(B)} = \frac{d}{3\sqrt{s_n^2 + (\bar{x}-T)^2}}.$$

where $d = (USL - LSL)/2$ is the half width of the specification interval, $\bar{x} = \sum_{i=1}^{n} x_i/n$, $s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2/(n-1)$ and $s_n^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2/n$. In fact, the two estimators, $\hat{C}_{pm(CCS)}$ and $\hat{C}_{pm(B)}$, are asymptotically equivalent. Assuming that

the process data are normally distributed and $T = M$, Chan *et al.* (1988) derived the probability density function of $\hat{C}_{pm(CCS)} = Y$ as:

$$f_Y(y) = \frac{a}{2^{n/2-1}y^3}\exp\left[-\frac{1}{2}\left(\frac{a}{y^2} + \lambda\right)\right]\sum_{j=1}^{\infty}\frac{\lambda^j(a/y^2)^{n/2+j-1}}{j!\Gamma(n/2+j)2^{2j}}, \ y > 0,$$

where $a = C_{pm}^2(1 + \lambda/n)(n - 1)$ and $\lambda = n(\mu - T)^2/\sigma^2$. Note that $\hat{C}_{pm(CCS)}$ can be shown to be functions of the inverse moments of a non-central chi-square distribution. An alternative but equivalent formula was provided by Pearn *et al.* (1992).

The distributional properties of $\hat{C}_{pm(CCS)}$ are intractable for asymmetrical specifications $((USL + LSL)/2 \neq T)$. When the case of $(USL + LSL)/2 = T$, $\hat{C}_{pm(CCS)}$ is a biased estimator of C_{pm}, but is asymptotically unbiased. For detailed descriptions and proofs of the properties of $\hat{C}_{pm(CCS)}$, see Chan *et al.* (1988). On the other hand, Boyles (1991) considered that it would be more appropriate to replace the factor $n - 1$ by n in the denominator since the term $\hat{\gamma}_{(B)} = s_n^2 + (\bar{x} - T)^2$ in the denominator of $\hat{C}_{pm(B)}$ is the uniformly minimum variance unbiased estimator (UMVUE) of the term $\sigma^2 + (\mu - T)^2$. Note that $\bar{x}$ and s_n^2 are the maximum likelihood estimators (MLEs) of μ and σ^2, respectively. Hence, the estimated $\hat{C}_{pm(B)}$ is also the MLE of C_{pm}.

The approach by simply looking at the calculated values of the estimated indices and then making a conclusion on whether the given process is capable is highly unreliable as the sampling errors have been ignored. As the use of the capability indices grows more widespread, users are becoming educated and sensitive to the impact of the estimators and their sampling distributions on constructing confidence intervals and performing hypothesis testing. Under the assumption of normality, Kotz and Johnson (1993) obtained the rth moment and calculated the first two moments, the mean and variance of $\hat{C}_{pm}$. Cheng (1994) developed a hypothesis-testing procedure where tables of the approximate p values were provided for some commonly used capability requirements, using the natural estimator of C_{pm}. The practitioners can use the obtained results to determine if their process satisfies the targeted quality condition. However, Cheng's approach requires further estimation of the distribution characteristic $(\mu - T)/\sigma$ when calculating the p values, which introduces additional sampling errors, thus making the decisions less reliable. Zimmer and Hubele (1997) provided tables of exact percentiles for the sampling distribution of the estimator $\hat{C}_{pm}$. Zimmer *et al.* (2001) proposed a graphical procedure to obtain exact confidence intervals for C_{pm}, where the parameter $(\mu - T)/\sigma$ is assumed to be a known constant. On the other hand, using the method similar to that presented in Vännman (1997), Pearn and Shu (2003a) obtained an exact form of the cumulative distribution function of $\hat{C}_{pm}$. Under the assumption of normality, the cumulative distribution function of $\hat{C}_{pm}$ can be expressed in terms of a mixture of the chi-square distribution and the normal distribution:

$$F_{\hat{C}_{pm}}(x) = 1 - \int_0^{b\sqrt{n}/(3x)} G\left(\frac{b^2 n}{9x^2} - t^2\right)\left[\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})\right]dt, \quad (1)$$

for $x > 0$, where $b = d/\sigma$, $\xi = (\mu - T)/\sigma$, $G((\cdot)$ is the cumulative distribution function of the chi-square distribution with degree of freedom $n - 1$, χ_{n-1}^2, and $\phi(\cdot)$ is the probability density function of the standard normal distribution $N(0, 1)$. Note that

one would obtain an identical equation if ξ was substituted by $-\xi$ in equation (1) for fixed values of x and n.

3. Selecting a better supplier with a smaller γ^2

Huang and Lee (1995) considered the supplier selection problem based on the index C_{pm} and developed a rather complicated method for supplier selection applications. The method essentially compares the average loss of a group of candidate processes and selects a subset of these processes with small process loss γ^2, which with a certain level of confidence contains the best process. Because the specification limits are usually fixed and determined in advance, searching the largest C_{pm} is the equivalent to looking for the smallest γ^2. The selection rule of Huang and Lee is retaining the population i in the selected subset if and only if $\hat{\gamma}_i^2 \leq c \times \min_{j \neq i}^{1 \leq j \leq k} \hat{\gamma}_j^2$, where c is determined by a function of parameters, which can be determined by calculating from collected samples. Note that the choice of c must be larger than 1 but as small as possible. The method, however, provides no indication on how one could proceed further with selecting the best population among those chosen from the subset of populations. This method is investigated for cases with two candidate processes. Let π_i be the population with mean μ_i and variance σ_i^2, $i = 1, 2$, and $x_{i1}, x_{i2}, \ldots, x_{in_i}$ are the independent random samples from π_i, $i = 1, 2$. When the populations are ranked in terms of γ_i^2, one wants to select the better process with a smaller value of γ_i^2. A correct selection is denoted as CS, and the ordered γ^2 as $\gamma_{[1]}^2 < \gamma_{[2]}^2$ is assumed.

Denote $\pi_{(i)}$ as the population associated with $\gamma_{[i]}^2$, $i = 1, 2$. The better population is then $\pi_{(1)}$. We wish to define a procedure with selection rule R such that the probability of a correct selection is no less than a pre-assigned number p^* and $0.5 < p^* < 1$, i.e. $\Pr(\text{CS} | R) \geq p^*$. This requirement is referred to as the p^* condition. The selection rule R based on the unbiased and consistent estimators $\hat{\gamma}_i^2$ of $\hat{\gamma}_i^2$, $i = 1, 2$, and $\hat{\gamma}_i^2$ is defined as follows:

$$\hat{\gamma}_i^2 = \frac{\sum_{j=1}^{n_i} (x_{ij} - T)^2}{n_i} = \frac{(n_i - 1)S_i^2 + n_i(\bar{x}_i - T)^2}{n_i},$$

$$S_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2, \ \bar{x}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} x_{ij}.$$

For cases with two candidate processes, comparing $\hat{C}_{pm1}$ and $\hat{C}_{pm2}$ is equivalent to comparing $\hat{\gamma}_1^2$ and $\hat{\gamma}_2^2$. Hence, by the result of Pearn *et al.* (1992) are:

$$\hat{\gamma}_i^2 \sim \frac{\sigma_i^2}{n_i} \chi_{n_i}^2(\lambda_i), \lambda_i = n_i \left(\frac{\mu_i - T}{\sigma_i} \right)^2,$$

where $\chi_{n_i}^2(\lambda_i)$ is the non-central chi-squared distribution with degrees of freedom and non-centrality parameter λ_i.

3.1. *Selection rule R*

Consider the problem of selecting two populations with the smaller $\hat{\gamma}^2$. The selection rule R is as follows: consider π_i as the better supplier if and only if

$\hat{\gamma}_i^2 \leq c \times \hat{\gamma}_j^2$ and $\hat{\gamma}_j^2 > c \times \hat{\gamma}_i^2$, $i = 1, 2$ and $i \neq j$. To satisfy the p^* condition, then:

$$c_1 = \exp\left\{-2A_1\sqrt{\frac{1}{\hat{v}_{[1]}}} + \left(\frac{1}{\hat{v}_{[1]}} - \frac{1}{\hat{v}_{[2]}}\right)\sqrt{\frac{\hat{v}_{[2]}}{\hat{v}_{[1]}}}\right\},$$

$$c_2 = \exp\left\{-2A_2\sqrt{\frac{1}{\hat{v}_{[1]}}} + \left(\frac{1}{\hat{v}_{[1]}} - \frac{1}{\hat{v}_{[2]}}\right)\sqrt{\frac{\hat{v}_{[2]}}{\hat{v}_{[1]}}}\right\}.$$

Choose the value of c that is larger than 1 and choose it as small as possible, so that:

$$c = \min\{c_1, c_2\}, \text{ if } c_1 > 1 \text{ and } c_2 > 1$$
$$c = c_1, \text{ if } c_1 > 1 \text{ and } c_2 \leq 1; c = c_2, \text{ if } c_2 > 1 \text{ and } c_1 \leq 1,$$

where

$$A_1 = \frac{-d_2 + \sqrt{d_2^2 - 4d_1d_3}}{2d_1}, \; A_2 = \frac{-d_2 - \sqrt{d_2^2 - 4d_1d_3}}{2d_1}$$

$$d_1 = a\left(1 + \frac{a_2}{a_1}\right) + \frac{a_2a^2}{a^2}\left(\frac{a_1 + a_2}{a_1}\right), \; d_2 = b\sqrt{1 + \frac{a_2}{a_1}} + \frac{ab}{a^*}\left(\frac{\sqrt{a_1 + a_2}}{a_1}\right)\left(\frac{a_2}{\sqrt{a_1}}\right)$$

$$d_3 = \frac{b^2a_2}{4a^*a_1} - \ln\left(2p^*\sqrt{2a^*}\right), \; a^* = 0.5 - a \times \frac{a_2}{a_1}, \; a_1 = \frac{1}{\hat{v}_{[1]}}, \; a_2 = \frac{1}{\hat{v}_{[2]}}$$

$$b = -0.513277, \; a = -0.085514$$

$$\hat{v}_1 = \frac{\left(n_1 + \hat{\lambda}_1\right)^2}{n_1 + 2\hat{\lambda}_1}, \; \hat{v}_2 = \frac{\left(n_2 + \hat{\lambda}_2\right)^2}{n_2 + 2\hat{\lambda}_2}, \; \hat{\lambda}_1 = n_1\left(\frac{\bar{x}_1 - T}{S_1}\right)^2, \; \hat{\lambda}_2 = n_2\left(\frac{\bar{x}_2 - T}{S_2}\right)^2,$$

where $\hat{v}_i$ is used to estimate v_i, $i = 1, 2$, and ordered $\hat{v}_i$ are denoted by $\hat{v}_{[1]} \leq \hat{v}_{[2]}$.

3.2. Selection procedure

The selection procedure is based on a mathematically complicated approximation method developed by Huang and Lee (1995) for selecting a problem. To make this method practical for in-plant applications, the selection procedure can be summarized and expand as follows:

Step 1. Input the original sample data of size n_i, $i = 1, 2$, set the specification limits *USL*, *LSL*, target value T, the probability p^*, and the constants $a = -0.085514$, $b = -0.513277$.

Step 2. Calculate the sample mean $\bar{x}_i$, sample standard deviation S_i, $\hat{\lambda}_i$ and $\hat{\gamma}_i^2$, $i = 1, 2$:

$$\bar{x}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} x_{ij}, S_i = \left[\frac{1}{n_i - 1}\sum_{j=1}^{n_i}(x_{ij} - \bar{x}_i)^2\right]^{1/2}, \; \hat{\lambda}_i = n_i\left(\frac{\bar{x}_i - T}{S_i}\right)^2$$

$$\hat{\gamma}_i^2 = \frac{\sum_{j=1}^{n_i}(x_{ij} - T)^2}{n_i} = \frac{(n_i - 1)S_i^2 + n_i(\bar{x}_i - T)^2}{n_i}.$$

Step 3. Calculate $\hat{v}_i, a_j (a_j = 1/\hat{v}_{[j]})$, and a^*:

$$\hat{v}_1 = \frac{\left(n_1 + \hat{\lambda}_1\right)^2}{n_1 + 2\hat{\lambda}_1}, \ \hat{v}_2 = \frac{\left(n_2 + \hat{\lambda}_2\right)^2}{n_2 + 2\hat{\lambda}_2}, \ a_1 = \frac{1}{\hat{v}_{[1]}}, \ a_2 = \frac{1}{\hat{v}_{[2]}}, \ a^* = 0.5 - \frac{a_2}{a_1}.$$

Step 4. Calculate d_1, d_2, d_3, and A_1, A_2:

$$d_1 = a\left(1 + \frac{a_2}{a_1}\right) + \frac{a_2 a^2}{a^2}\left(\frac{a_1 + a_2}{a_1}\right), \ d_2 = b\sqrt{1 + \frac{a_2}{a_1}} + \frac{ab}{a^*}\left(\frac{\sqrt{a_1 + a_2}}{a_1}\right)\left(\frac{a_2}{\sqrt{a_1}}\right)$$

$$d_3 = \frac{b^2 a_2}{4a^* a_1} - \ln\left(2p^* \sqrt{2a^*}\right), \ A_1 = \frac{-d_2 + \sqrt{d_2^2 - 4d_1 d_3}}{2d_1}, \ A_2 = \frac{-d_2 - \sqrt{d_2^2 - 4d_1 d_3}}{2d_1}.$$

Step 5. Calculate c_1 and c_2, then choose the value c from c_1 and c_2:

$$c_1 = \exp\left\{-2A_1\sqrt{\frac{1}{\hat{v}_{[1]}}} + \left(\frac{1}{\hat{v}_{[1]}} - \frac{1}{\hat{v}_{[2]}}\right)\sqrt{\frac{\hat{v}_{[2]}}{\hat{v}_{[1]}}}\right\}$$

$$c_2 = \exp\left\{-2A_2\sqrt{\frac{1}{\hat{v}_{[1]}}} + \left(\frac{1}{\hat{v}_{[1]}} - \frac{1}{\hat{v}_{[2]}}\right)\sqrt{\frac{\hat{v}_{[2]}}{\hat{v}_{[1]}}}\right\}.$$

Then choose the value c that is greater than 1 but as small as possible:

$$c = \min\{c_1, c_2\}, \text{ if } c_1 > 1 \text{ and } c_2 > 1$$
$$c = c_1, \text{ if } c_1 > 1 \text{ and } c_2 \leq 1; \ c = c_2, \text{ if } c_2 > 1 \text{ and } c_1 \leq 1.$$

Step 6. Conclude which supplier is better using the following rule R:

If $\hat{\gamma}_1^2 \leq c \times \hat{\gamma}_2^2$ and $\hat{\gamma}_2^2 > c \times \hat{\gamma}_1^2$ then we conclude that π_1 is the better supplier.
If $\hat{\gamma}_2^2 \leq c \times \hat{\gamma}_1^2$ and $\hat{\gamma}_1^2 > c \times \hat{\gamma}_2^2$ then we conclude that π_2 is the better supplier.
If $\hat{\gamma}_1^2 \leq c \times \hat{\gamma}_2^2$ and $\hat{\gamma}_2^2 \leq c \times \hat{\gamma}_1^2$, then there is not enough information to make a supplier selection.

4. Selection power analysis

Huang and Lee (1995) proposed a mathematically complicated approximation method for selecting a subset of processes containing the best supplier from a given set of processes based on the index C_{pm}. The method essentially compares the average loss of a group of candidate processes and selects a subset of these processes with small process loss γ^2, which with a certain level of confidence contains the best process. The accuracy of the selection method is investigated by using the simulation technique. The accuracy analysis provides useful information about the sample size required for the designated selection power.

4.1. *Sample size required for designated selection power*

In practice, if a new supplier II wants to compete for the orders by claiming that its capability is better than the existing supplier I, then the new supplier must furnish convincing information justifying the claim with a prescribed level of confidence. Thus, the sample size required for a designated selection power must be determined to collect actual data from the factories. The method, however, applies some approximating results and provides no indication on how one could further proceed to

select the best population among those chosen subsets of populations. This method was investigated for cases with two candidate processes. If the minimum requirement of C_{pm} values for two candidate processes, C_{pm0}, and the minimal difference $\delta = C_{pm2} - C_{pm1}$ are determined, then the sample size required needs to sample such that the suppliers must be differentiated with designated selection power. Thus, based on the proposed selection procedures, if $\hat{\gamma}_2^2 \leq c \times \hat{\gamma}_1^2$ and $\hat{\gamma}_1^2 > c \times \hat{\gamma}_2^2$, then it is concluded that the C_{pm} of π_2 is better than π_1. Otherwise, one would believe that the existing supplier I is better than the new supplier II since there is not sufficient information to reject the null hypothesis. The selection method and accuracy analysis were investigated using a simulation technique with a simulated 10 000 numbers. For users' convenience in applying the procedure in practice, the sample sizes required for various designated selection power were tabulated as 0.90, 0.95, 0.975, 0.99. The selection power calculates the probability of rejecting the null hypothesis $H_0 : C_{pm1} \geq C_{pm2}$, while actually $C_{pm1} < C_{pm2}$ is true, using the simulation technique. Tables 1–4 summarize the sample size required for various capability requirements $C_{pm} = 1.00$, 1.33, 1.50, 1.67 and difference $\delta = 0.05(0.05)1.00$ under the p^* condition $= 0.95$, respectively. For example, if the capability requirement of

C_{pm1}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
C_{pm2}	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
0.90	3408	898	414	240	165	118	90	71	59	50
0.95	4351	1120	520	347	204	151	115	91	73	63
0.975	5130	1356	640	371	250	180	137	109	91	76
0.99	6131	1631	785	451	303	220	171	135	110	93
C_{pm1}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
C_{pm2}	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00
0.90	43	37	32	29	26	24	22	22	19	18
0.95	53	48	41	37	33	31	29	28	27	26
0.975	65	57	50	45	40	37	34	30	28	27
0.99	80	70	61	56	49	45	40	38	35	33

Table 1. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under $p^* = 0.95$, with $C_{pm1} = 1.00$, $C_{pm2} = 1.05(0.05)2.00$.

C_{pm1}	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33
C_{pm1}	1.38	1.43	1.48	1.53	1.58	1.63	1.68	1.73	1.78	1.83
0.90	5900	1520	694	400	269	194	147	115	94	79
0.95	7493	1297	896	530	343	246	191	149	119	102
0.975	9014	2350	1060	622	401	301	231	178	147	120
0.99	10999	2859	1315	765	499	368	272	222	175	149
C_{pm1}	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33
C_{pm2}	1.88	1.93	1.98	2.03	2.08	2.13	2.18	2.23	2.28	2.33
0.90	67	59	52	45	41	36	33	32	29	26
0.95	85	73	65	59	52	46	43	39	35	33
0.975	103	90	78	69	64	56	51	48	43	39
0.99	127	109	95	85	76	70	64	56	52	49

Table 2. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under $p^* = 0.95$, with $C_{pm1} = 1.33$, $C_{pm2} = 1.38(0.05)2.33$.

C_{pm1}	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50
C_{pm2}	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00
0.90	7394	1941	891	513	338	245	184	145	118	96
0.95	9506	2460	1120	657	430	308	232	180	151	125
0.975	11503	3001	1338	801	515	376	283	220	180	151
0.99	13502	3540	1634	974	627	457	340	268	221	177
C_{pm1}	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50
C_{pm2}	2.05	2.10	2.15	2.20	2.25	2.30	2.35	2.40	2.45	2.50
0.90	83	71	62	55	49	45	39	38	35	32
0.95	106	91	79	71	63	56	51	48	44	40
0.975	125	109	95	85	75	69	63	57	53	50
0.99	155	134	115	103	92	83	75	71	65	60

Table 3.   Sample size required for power $= 0.90$, 0.95, 0.975, 0.99 under $p^* = 0.95$, with $C_{pm1} = 1.50$, $C_{pm2} = 1.55(0.05)2.50$.

C_{pm1}	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67
C_{pm2}	1.72	1.77	1.82	1.87	1.92	1.97	2.02	2.07	2.12	2.17
0.90	9291	2360	1091	630	408	292	223	173	141	115
0.95	12004	3034	1387	807	531	371	282	220	177	151
0.975	14297	3700	1650	970	629	448	338	260	218	180
0.99	17990	4400	2000	1163	765	544	400	325	255	220
C_{pm1}	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67
C_{pm2}	2.22	2.27	2.32	2.37	2.42	2.47	2.52	2.57	2.62	2.67
0.90	100	85	75	66	60	52	49	43	39	38
0.95	125	108	95	85	75	67	63	55	51	48
0.975	154	130	115	102	91	82	75	66	63	56
0.99	185	159	140	120	112	99	91	83	74	69

Table 4.   Sample size required for power $= 0.90$, 0.95, 0.975, 0.99 under $p^* = 0.95$, with $C_{pm1} = 1.67$, $C_{pm2} = 1.72(0.05)2.67$.

suppliers C_{pm} is set to 1.00 and $\delta = 0.30$, one would need to collect 151 samples to satisfy the designated selection power $= 0.95$.

Note that the sample size required is a function of C_{pm}, the difference δ between two suppliers and the designated selection power. Tables 1–4 show that the larger the difference δ between two suppliers, the smaller the sample size required for a fixed selection power. For fixed δ and C_{pm}, the sample size required increases as the designated selection power increases. This phenomenon can be explained easily, since the smaller the difference and the larger the designated selection power, the more the collected sample is required to account for the smaller uncertainty in the estimation.

4.2.   Phase I: Supplier selection

In most applications, the supplier selection decisions would be based solely on the hypothesis testing comparing the two C_{pm} values: $H_0 : C_{pm1} \geq C_{pm2}$ versus $H_1 : C_{pm1} < C_{pm2}$. If the test rejects the null hypothesis $H_0 : C_{pm1} \geq C_{pm2}$, then there is sufficient information to conclude that the new supplier II is superior to the original supplier I, and the decision of the replacement would be suggested.

For the Phase I of the supplier selection problem, the practitioner should input the preset minimum requirement of C_{pm} values, and the minimal difference that must

be differentiated between suppliers with designated selection power. The practitioner alternatively might check tables 1–4 for the sample size required for p^* condition $= 0.95$, with designated selection powers $= 0.90$, 0.95, 0.975, 0.99. In this case, one only needs to compare the test statistic $\hat{\gamma}_i^2$, $i = 1, 2$, with the selection value c based on the selection procedure corresponding to the preset capability requirement and required sample sizes.

4.3. *Phase II: Magnitude outperformed measurement*

In Phase I of the supplier selection problem, the supplier selection decisions would be based solely on the hypothesis testing comparing the two C_{pm} values without investigating further the magnitude of the difference between the two suppliers. In other applications, the supplier selection decisions would be based on the hypothesis test comparing the two C_{pm} values: $H_0 : C_{pm1} + h \geq C_{pm2}$ versus $H_1 : C_{pm1} + h < C_{pm2}$, where $h > 0$ is a specified constant. If the test rejects the null hypothesis $H_0 : C_{pm1} + h \geq C_{pm2}$, then there is sufficient information to conclude that supplier II is significantly better than supplier I by a magnitude of h, and the replacement would then be made due to the high cost of the supplier replacement. In this case, one would have to compare the test statistic $\hat{\gamma}_i^2$, $i = 1, 2$, with the selection value c corresponding to the preset capability requirement for a given sample and designated selection power to ensure that the magnitude of the difference between the two suppliers exceeds h. Note that C_{pm1} must be greater than the preset capability requirement, and $C_{pm2} = C_{pm1} + h$, where $h = \max\{h' |$ test rejects $C_{pm1} + h' \geq C_{pm2}\}$. The basic problem of checking whether or not the two suppliers meeting the preset capability requirement could be solved by finding the lower confidence bounds on their process capabilities.

4.4. *Comments on the classical approach*

The classical approach for estimating the fraction of defectives is to take a sample of size n and calculate the proportion D/n, where D is the number of defective items in the sample. Note that for processes with a very low fraction of defectives, the classical approach requires a large sample size for the sample to contain at least one defective item. Table 5 shows the sample sizes required for the sample to include at least one defective item with a probability of 95% for various fractions of defectives in ppm. For capability requirement $C_{pm} = 1.33$ (33.04 ppm), the classical approach requires the sample size $> 90\,000$. Therefore, the classical approach is not feasible for real applications with a lower fraction of defectives.

5. Application example STN-LCD

Liquid crystals have been used for display applications with various configurations. Most of the recently produced displays involve the use of either twisted nematic (TN) or super twisted nematic (STN) liquid crystals. The technology for the latter was introduced recently to improve the performance of LCD without using

p	100 000	10 000	1000	100	10	1	0.1
n	29	299	2996	29 957	299 572	2 995 731	29 957 322

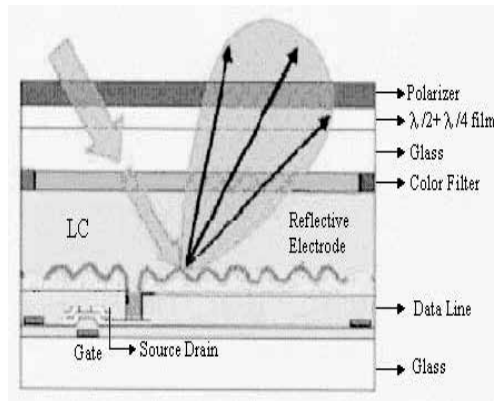Table 5.   Sample sizes required for various p (ppm).

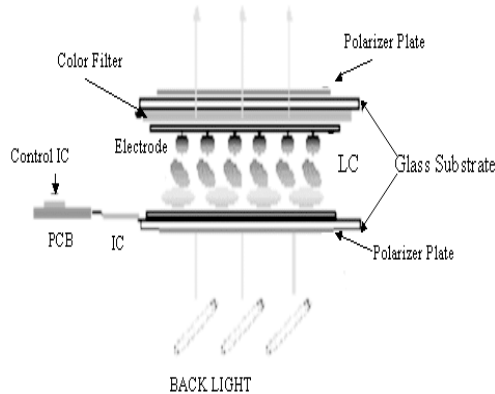Figure 1. Assembly drawing for the STN-LCD product.

Figure 2. Custom glass and modules of the STN-LCD product.

the TFT. A larger twist angle results in a significantly larger electro-optical distortion. This leads to substantial improvement in the contract and viewing angles over TN displays. The STN-LCD products are popularly used to make personal digital assistants (PDAs), notebook personal computers, word processors and other peripherals. Due to the advancement of modern manufacturing technology in making STN-LCD and relatively low production costs, STN-LCD has remained at a competitive advantage in the marketplace. A typical assembly drawing for the STN-LCD product is shown in figure 1, and the custom glass and modules of the STN-LCD product are shown in figure 2.

An increasing number of personal computers are now network-ready and multimedia capable and are equipped with CD-ROM drives. Due to advances in telecommunications' technology, simple monochromatic displays are no longer in popular demand. The next generation of telecommunication products will require displays with rich, graphic-quality images and personal interfaces. Therefore, future displays must be clearer and sharper to meet these demands. Until this point, STN-LCD have been used mainly to display still images, and because of the slow response time needed to process still images, STN-LCD have not been able to reproduce animated images at an adequate contrast level. Thus, with the growing popularity

of multimedia applications, there is a need for PCs equipped with colour STN-LCD capable of processing animated pictures instead of still images. The space between the glass substrate is filled with liquid crystal material and the thickness of the liquid crystal is kept uniform with glass fibres or plastic balls as spacers. Therefore, the STN-LCD is sensitive to the thickness of the glass substrates.

To illustrate which of the two suppliers has a better process capability, a case study on STN-LCD manufacturing processes in a science-based industrial park manufacturer in Taiwan is presented. These factories manufacture various types of LCD. For a particular model of the STN-LCD investigated, the *USL* of a glass substrate's thickness was set to 0.77 mm, the *LSL* of a glass substrate's thickness was set to 0.63 mm, and the target value was set to $T = 0.70$ mm. If the characteristic data do not fall within the tolerance (*LSL*, *USL*), the lifetime or reliability of the STN-LCD will be discounted.

5.1. *Data analysis and supplier selection*

For Phase I of the Supplier Selection problem, the practitioner should input the preset minimum requirement of C_{pm} values, and the minimal difference that must be differentiated between suppliers with a designated selection power. If the minimum requirement of an STN-LCD product is $C_{pm} = 1.00$, $\delta = 0.25$ with a selection power of 0.95. By checking table 1, the sample size required for estimation is 204. Thus, the glass substrate's thickness data taken from two LCD suppliers are shown in table 6. To confirm if the data of both suppliers are normally distributed, a Shapiro–Wilk test for normality is performed (figures 3 and 4). Because the $p > 0.05$, the null hypothesis is not rejected because the data are normally distributed. Histograms of the data are shown in figures 5 and 6.

5.2. *Phase I: Supplier selection*

The aim is to determine if supplier II has a better process capability than supplier I, i.e. hypothesis testing must be performed to compare the two C_{pm} values, $H_0: C_{pm1} \geq C_{pm2}$ versus $H_1: C_{pm1} < C_{pm2}$. First, calculate the sample means, sample standard deviations, sample estimators of $\hat{C}_{pm}$, $\hat{\gamma}^2$, and $\hat{v}$ for suppliers I and II (table 7). Based on the selection procedure, $c_1 = 1.241426$ and $c_2 = 1.478218$. Choose the value of c that is larger than 1 and as small as possible, so $c = \min\{c_1, c_2\} = 1.241426$. In this case, one only needs to compare the test statistic $\hat{\gamma}_i^2$, $i = 1, 2$, with the selection value c. Since $\hat{\gamma}_2^2 \leq c \times \hat{\gamma}_1^2$ and $\hat{\gamma}_1^2 > c \times \hat{\gamma}_2^2$, it is concluded that π_2 is a better supplier with a larger process capability C_{pm}.

5.3. *Phase II: Magnitude outperformed measurement*

To investigate further the magnitude of the capability difference between the two suppliers, the supplier selection decisions would find a magnitude of h such that $C_{pm2} = C_{pm1} + h$, where $h = \max\{h' \mid \text{test rejects } C_{pm1} + h' \geq C_{pm2}\}$. From the estimation of Phase I, the obtained selection values c and the decision based on the selection procedure for $h = 0.01$, 0.05, 0.10, 0.12(0.01)0.15 are shown in table 8. Therefore, from the analysis of magnitude outperformed detection based on sample statistics, the magnitude of the difference between the two suppliers is $h = 0.14$. That is, it is concluded that $C_{pm2} > C_{pm1} + 0.14$.

Supplier I							Supplier II				
0.688	0.719	0.666	0.698	0.707	0.709	0.709	0.698	0.695	0.693	0.692	0.683
0.725	0.706	0.679	0.731	0.697	0.715	0.697	0.716	0.690	0.697	0.708	0.695
0.711	0.701	0.706	0.696	0.699	0.685	0.690	0.698	0.719	0.750	0.693	0.695
0.712	0.702	0.697	0.679	0.698	0.700	0.708	0.717	0.729	0.693	0.749	0.729
0.698	0.717	0.683	0.688	0.691	0.706	0.730	0.718	0.706	0.717	0.712	0.741
0.687	0.699	0.730	0.709	0.708	0.710	0.741	0.727	0.713	0.698	0.724	0.698
0.712	0.702	0.695	0.716	0.679	0.677	0.715	0.717	0.699	0.713	0.710	0.718
0.679	0.694	0.700	0.695	0.700	0.708	0.730	0.697	0.678	0.719	0.733	0.710
0.707	0.723	0.711	0.693	0.670	0.723	0.694	0.728	0.709	0.708	0.705	0.721
0.691	0.713	0.680	0.719	0.691	0.680	0.696	0.747	0.707	0.739	0.721	0.688
0.686	0.684	0.727	0.705	0.685	0.670	0.711	0.730	0.715	0.696	0.715	0.709
0.714	0.695	0.685	0.696	0.733	0.710	0.702	0.735	0.728	0.728	0.735	0.688
0.679	0.673	0.715	0.680	0.691	0.706	0.726	0.709	0.727	0.678	0.737	0.707
0.684	0.691	0.708	0.716	0.679	0.718	0.723	0.690	0.705	0.710	0.710	0.721
0.705	0.704	0.729	0.698	0.716	0.689	0.726	0.711	0.729	0.722	0.704	0.730
0.709	0.711	0.719	0.678	0.669	0.711	0.729	0.727	0.685	0.684	0.692	0.704
0.684	0.713	0.691	0.731	0.691	0.710	0.713	0.710	0.710	0.734	0.691	0.723
0.688	0.708	0.670	0.693	0.696	0.703	0.715	0.711	0.713	0.726	0.704	0.714
0.676	0.685	0.728	0.713	0.685	0.697	0.709	0.690	0.694	0.694	0.698	0.718
0.693	0.699	0.710	0.699	0.711	0.681	0.715	0.682	0.703	0.713	0.701	0.748
0.696	0.698	0.691	0.693	0.700	0.720	0.742	0.697	0.702	0.735	0.662	0.711
0.677	0.669	0.690	0.724	0.690	0.685	0.639	0.698	0.712	0.705	0.691	0.764
0.704	0.712	0.690	0.716	0.693	0.714	0.717	0.721	0.706	0.700	0.723	0.725
0.736	0.721	0.679	0.713	0.728	0.730	0.720	0.736	0.699	0.722	0.686	0.698
0.707	0.683	0.700	0.683	0.715	0.723	0.722	0.705	0.740	0.691	0.709	0.716
0.676	0.711	0.702	0.714	0.701	0.702	0.693	0.720	0.704	0.716	0.696	0.704
0.675	0.697	0.685	0.695	0.740	0.697	0.712	0.715	0.684	0.714	0.692	0.733
0.705	0.691	0.699	0.716	0.701	0.681	0.691	0.705	0.724	0.704	0.744	0.716
0.687	0.714	0.688	0.706	0.702	0.695	0.695	0.717	0.711	0.680	0.696	0.685
0.682	0.685	0.727	0.686	0.712	0.717	0.702	0.680	0.680	0.711	0.725	0.734
0.688	0.728	0.694	0.701	0.715	0.687	0.712	0.712	0.741	0.696	0.687	0.742
0.702	0.713	0.677	0.731	0.708	0.677	0.723	0.724	0.714	0.703	0.708	0.718
0.692	0.669	0.710	0.708	0.704	0.686	0.702	0.681	0.713	0.720	0.713	0.672
0.688	0.713	0.687	0.715	0.670	0.697	0.715	0.710	0.699	0.706	0.716	0.715

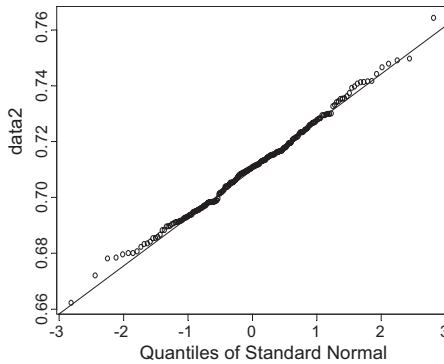Table 6.   Sample data collected from the two suppliers.

Figure 3.   Normal probability plot for thickness data of supplier I.
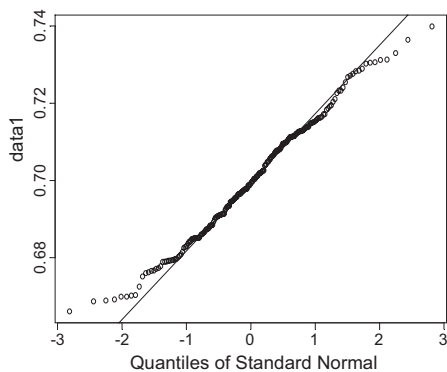
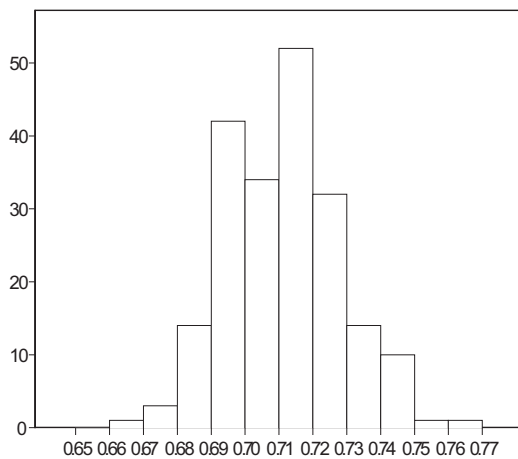Figure 4. Normal probability plot for thickness data of supplier II.
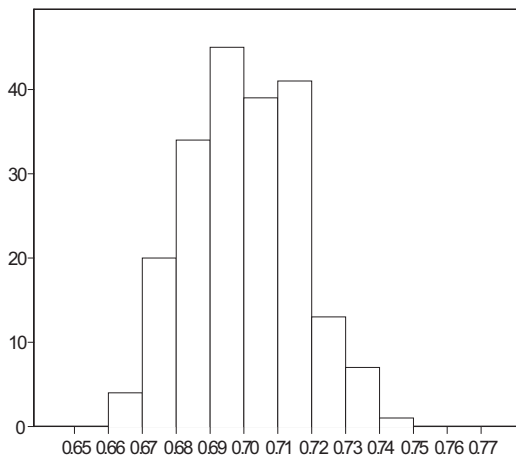

Figure 5. Histogram for supplier I.

Figure 6. Histogram for supplier II.

Population	$\bar{X}$	S	$\hat{C}_{pm}$	$\hat{\gamma}^2$	Rank $\hat{\gamma}^2$	$\hat{v}$	Rank $\hat{v}$
I	0.7106	0.01695	1.1705	3.974×10^{-3}	2	221.3295	2
II	0.6998	0.01593	1.4687	2.524×10^{-3}	1	204	1

Table 7.   Calculated sample statistics for the two suppliers.

$\hat{C}_{pm1}$	1.1805	1.2205	1.2705	1.2905	1.3005	1.3105	1.3205
$\hat{C}_{pm2}$	1.4687	1.4687	1.4687	1.4687	1.4687	1.4687	1.4687
h	0.01	0.05	0.10	0.12	0.13	0.14	0.15
c	1.241459	1.241602	1.241821	1.241922	1.241976	1.242032	1.242091
							do not
Decision	reject H_0	reject H_0	reject H_0	reject H_0	reject H_0	reject H_0	reject H_0

Table 8.   Magnitude of the outperformed detection of the selection procedure.

6.   Conclusions

In the initial stage of production setting, the decision-maker usually faces the problem of selecting the best manufacturing supplier from several available manufacturing suppliers. According to today's modern quality improvement theory, reduction of the process loss is as important as increasing the process yield. The use of loss functions in quality assurance settings has grown with the introduction of Taguchi's philosophy. The index C_{pm} incorporates with the variation of production items with respect to the target value and the specification limits preset in the factory. Huang and Lee (1995) proposed a mathematically complicated approximation method for selecting a subset of processes containing the best supplier from a given set of processes based on the index C_{pm}. The present paper implements this method and develops a practical step-by-step procedure for practitioners to use in making supplier selection decisions. Accuracy of the selection method is investigated by using a simulation technique. The accuracy analysis provides useful information about the sample size required for designated selection power. A two-phase selection procedure is developed to select a better supplier and further to examine the magnitude of the difference between the two suppliers. To make this method practical for in-plant applications, an application example of STN-LCD manufacturing processes, under a specific power, was also presented illustrating the sample size information to distinguish which supplier has better process capability.

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