Alternating Coordinates Minimization Algorithm for Estimating Parameters of Partial Erasure Plus Transition Shift Model

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***Abstract*—The identification of model parameters of a high-density recording channel is usually difficult and complicated. In this paper, we successfully apply the alternating coordinates minimization (ACM) algorithm for estimating parameters of a partial erasure plus transition shift model (PETSM). The resulting algorithm turns out to iteratively solve two least square problems and is guaranteed to converge. Furthermore, the obtained model for a nonlinear partial response channel is more accurate than conventional models such that the maximum likelihood (ML) detector has better bit error rate (BER) performance without increasing its realization complexity. Computer simulations show that the ACM algorithm can accurately estimate the model parameters and the BER for the detector is significantly improved especially when the transition shift parameter is large.**

***Index Terms*—Alternating coordinates minimization, maximum likelihood detector, partial erasure ratio, partial response channel, transition shift parameter.**

I. Introduction

NONLINEAR distortions are the primary factors to limit the detector performance in high-density magnetic storage [1], [2]; these distortions are mainly the transition shift and the partial erasure. Several models have been presented to characterize the nonlinear distortions [3], [4], including the partial erasure plus transition shift model (PETSM) and simple partial erasure model (SPEM). However, the model parameters are usually difficult to estimate or measure [5], [6]. Recently, the authors applied the expectation-maximization (EM) algorithm [7] for identifying the parameters of a SPEM, and assumed that the effect of transition shift had been precompensated. This assumption makes the EM approach difficult to estimate model parameters directly from the measurement data without proper precompensation.

In this paper, the alternating coordinates minimization (ACM) algorithm [8], [9] is successfully applied for estimating parameters of a PETSM, including both nonlinear effects of transition shift and partial erasure. The resulting algorithm turns out to iteratively solve two least square problems and is guaranteed to converge. The obtained model for a nonlinear partial response channel is more accurate than conventional models such that the maximum likelihood (ML) detector has better performance without increasing its realization complexity. The algorithm, therefore, enables us to accurately estimate parameters directly from the measurement data and to design a detector with improved performance.

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II. Channel Model

The sampled output, y_m, of the PETSM [4] is given by

$$y_m = \sum_{k=-L_1}^{L_2} r_{m-k} b_{m-k} s\left(kT - \frac{b_{m-k} b_{m-k-1}}{4}\epsilon T\right) \quad (1)$$

where L_1, L_2 express the duration of the channel, $s(t)$ is the Lorentzian function, ϵ is the transition shift parameter, r_m represents the partial erasure effect, determined as follows:

$$r_m = \begin{cases} 1, & b_{m-1} = b_{m+1} = 0 \\ \gamma_1, & |b_{m-1}| \neq |b_{m+1}| \\ \gamma_2, & |b_{m-1}| = |b_{m+1}| = 2 \end{cases} \quad (2)$$

where γ_1 and γ_2 denote the partial erasure ratios. Note that the common setting for $\gamma_2 = \gamma_1^2$ has been relaxed and thus the model flexibility is enhanced. The data b_m is obtained by the nonreturn-to-zero-inverted (NRZI) encoding of the plus and minus binary recorded data a_m. Thus $b_m = a_m - a_{m-1}$ and b_m may be of values $\{+2, 0, -2\}$. Since the product $b_{m-k} b_{m-k-1}$ for all k can only be either -4 or 0, we denote a switch function q_m by

$$q_m = \begin{cases} 1, & b_{m-1} b_m = 0 \\ 0, & b_{m-1} b_m = -4 \end{cases}. \quad (3)$$

The PETSM (1) then can be represented in a new formulation

$$y_m = \sum_{k=-L_1}^{L_2} r_{m-k} b_{m-k} [q_{m-k} g_k + (1 - q_{m-k}) f_k] \quad (4)$$

where the channel parameters $g_k = s(kT)$ and $f_k = s(kT + \epsilon T)$ for $k = -L_1, \ldots, 0, \ldots, L_2$. Therefore, the parameters of this model (4) consist of γ_1, γ_2, g_k, and f_k. The problem here is to find the model parameters for minimizing the following square output error

$$J = \frac{1}{N} \sum_{m=1}^{N} (z_m - y_m)^2 \quad (5)$$

where z_m is the sampled measurements of a magnetic recording channel and N is the number of sample data.

III. ACM Algorithm for Estimating Model Parameters

A. Formulation of Two "Linear" Equations

Divide the model parameters into two vectors $\boldsymbol{h}$ and $\boldsymbol{\lambda}$, with $\boldsymbol{h}$ representing the channel parameters and $\boldsymbol{\lambda}$ representing the partial erasure ratio parameters, as follows:

$$\boldsymbol{h} = [g_{-L_1}, f_{-L_1}, \ldots, g_0, f_0, \ldots, g_{L_2}, f_{L_2}]^T \tag{6}$$
$$\boldsymbol{\lambda} = [\gamma_1, \gamma_2]^T \tag{7}$$

where the superscript T denotes the transpose operation. Denote $p_m = r_m b_m$ and

$$\boldsymbol{u}_m = [p_{m+L_1} q_{m+L_1}, p_{m+L_1}(1 - q_{m+L_1}), \ldots, p_m q_m, p_m(1 - q_m), \ldots, p_{m-L_2} q_{m-L_2}, p_{m-L_2}(1 - q_{m-L_2})]^T \tag{8}$$

then the model output y_m is a linear form of the channel parameters $\boldsymbol{h}$

$$y_m = \boldsymbol{u}_m^T \boldsymbol{h}. \tag{9}$$

Similarly, the model output also can be formulated as a linear form of the partial erasure ratio parameters $\boldsymbol{\lambda}$

$$y_m = x_m + \boldsymbol{c}_m^T \boldsymbol{\lambda} \tag{10}$$

where $\boldsymbol{c}_m = [\alpha_m, \beta_m]^T$, and

$$x_m = \sum_{k=-L_1}^{L_2} b_{m-k} I_{m-k} [q_{m-k} g_k + (1 - q_{m-k}) f_k],$$
$$I_{m-k} = \begin{cases} 1, & \text{if } b_{m-k-1} = b_{m-k+1} = 0 \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

$$\alpha_m = \sum_{k=-L_1}^{L_2} b_{m-k} I_{m-k} [q_{m-k} g_k + (1 - q_{m-k}) f_k],$$
$$I_{m-k} = \begin{cases} 1, & \text{if } |b_{m-k-1}| \neq |b_{m-k+1}| \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

$$\beta_m = \sum_{k=-L_1}^{L_2} b_{m-k} I_{m-k} [q_{m-k} g_k + (1 - q_{m-k}) f_k],$$
$$I_{m-k} = \begin{cases} 1, & \text{if } |b_{m-k-1}| = |b_{m-k+1}| = 2 \\ 0, & \text{otherwise} \end{cases}. \tag{13}$$

Note that (9) and (10) look like linear equations but in fact they are nonlinear.

B. ACM Algorithm for Estimating Model Parameters

The ACM algorithm iteratively performs the following two major operations until convergence; the iteration number is denoted by the subscript k. The first operation is given $\boldsymbol{h} = \boldsymbol{h}^{(k-1)}$ to solve $\boldsymbol{\lambda}^{(k)}$ for minimizing J, which can be expressed as

$$J(\boldsymbol{\lambda}) = \frac{1}{N} \|\boldsymbol{z} - \boldsymbol{x} - C\boldsymbol{\lambda}\|_2 \tag{14}$$

where $\boldsymbol{z} = [z_1, \ldots, z_N]^T, \boldsymbol{x} = [x_1, \ldots, x_N]^T$, and $C = [\boldsymbol{c}_1, \ldots, \boldsymbol{c}_N]^T$. Since the vector $\boldsymbol{x}$ and matrix C are evaluated under the condition $\boldsymbol{h} = \boldsymbol{h}^{(k-1)}$, the performance J in (14) is obviously a quadratic function of $\boldsymbol{\lambda}$ and the unique solution of $\boldsymbol{\lambda}^{(k)}$ can be obtained

$$\boldsymbol{\lambda}^{(k)} = (C^T C)^{-1} C^T (\boldsymbol{z} - \boldsymbol{x}). \tag{15}$$

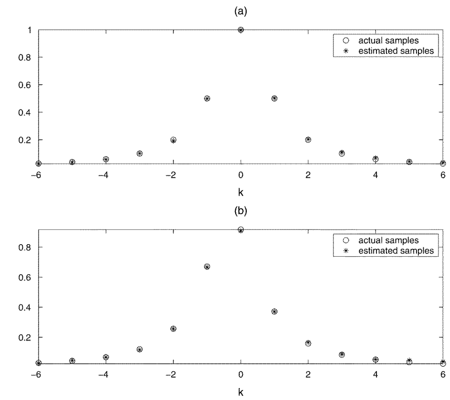

Fig. 1. (a) Samples of the channel parameters g_ks and their estimates at the 257th iteration and (b) samples of the channel parameters f_ks and their estimates at the 257th iteration.

Similarly, the second operation is given $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{(k)}$, obtained in the previous operation, to solve $\boldsymbol{h}^{(k)}$ for minimizing J, which is

$$J(\boldsymbol{h}) = \frac{1}{N} \|\boldsymbol{z} - U\boldsymbol{h}\|_2 \tag{16}$$

where $U = [\boldsymbol{u}_1, \ldots, \boldsymbol{u}_N]^T$, and each vector $\boldsymbol{u}_m$ is evaluated using (8) with $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{(k)}$. The performance function J in (16) is also quadratic and the solution is

$$\boldsymbol{h}^{(k)} = (U^T U)^{-1} U^T \boldsymbol{z}. \tag{17}$$

Each operation involves a quadratic minimization and solves a unique minimum. Thus, $J(\boldsymbol{\lambda}, \boldsymbol{h})$ is guaranteed nonincreasing. Furthermore, since $J(\boldsymbol{\lambda}, \boldsymbol{h})$ is bounded below by zero, the ACM algorithm will always converge. The algorithm here terminates when the measure, $\|\boldsymbol{h}^{(k)} - \boldsymbol{h}^{(k-1)}\|_2 + \|\boldsymbol{\lambda}^{(k)} - \boldsymbol{\lambda}^{(k-1)}\|_2$, is less than a predetermined small value δ.

C. Simulation Example

Let 1000 measurement data z_m be generated as

$$z_m = \sum_{k=-L_1}^{L_2} r_{m-k} b_{m-k} s\left(kT - \frac{b_{m-k} b_{m-k-1}}{4} \epsilon T\right) + n_m \tag{18}$$

where $s(t)$ is also a Lorentzian function with $\text{PW}_{50} = 2, \gamma_1 = 0.7, \gamma_2 = 0.49, \epsilon = 0.3$, and n_m is the additive white Gaussian noise. The noise variance is set to -22.63 dB to make the signal-to-noise ratio (SNR), defined as $10 \log (\text{E}[(z_m - n_m)^2]/\text{E}[n_m^2])$, equal 20 dB. The channel lengths are given by $L_1 = L_2 = 6$. The ACM algorithm is initialized with all model parameters set to zeros and the predetermined value $\delta = 0.005$. Here, the algorithm terminated at the 257th iteration, and the convergent average square output error is -22.51 dB. The estimated partial erasure ratios $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are respectively 0.7013 and 0.4942, and the obtained model parameters are shown in Fig. 1, which illustrates that the ACM algorithm can accurately estimate the model parameters.

IV. Nonlinear PR4 Channel: Modeling and Detector Performance

For high-density magnetic storage, the model duration is usually long; this makes the complexity to realize the detector,

TABLE I
TOBTAINED SDRs IN dB OF EACH MODEL FOR PR4 CHANNEL WITH VARIOUS TRANSITION SHIFT VALUES

ϵ	proposed model	Ryan's model	SPEM	LSM
0.1	19.6836	19.1828	18.0882	9.3072
0.2	18.9795	17.1686	14.9247	7.7017
0.3	18.4324	14.5409	11.9430	5.9520
0.4	17.9602	11.7578	9.5355	4.4162
0.5	17.7718	9.5909	8.3392	3.8054

designed for the obtained model, prohibitively high. Therefore, the magnetic channel is often equalized to a partial response model and then the ML detector is designed to improve the performance in high-density magnetic storage [10], [11]. Assume that the magnetic channel is equalized to the class-IV partial response (PR4) with minimum bandwidth [2], then its impulse response $s(t)$ is given by

$$s(t) = \frac{\sin\frac{\pi t}{T}}{\frac{\pi t}{T}} + \frac{\sin\frac{\pi(t-T)}{T}}{\frac{\pi(t-T)}{T}}. \quad (19)$$

If the nonlinear effects of partial erasure and transition shift are considered, the readback sampled signal, z_m, is obtained by

$$z_m = \sum_{k=-L_1}^{L_2} r_{m-k} b_{m-k} s\left(kT - \frac{b_{m-k}b_{m-k-1}}{4}\epsilon T\right) + n_m \quad (20)$$

where L_1 and L_2 represent the effective channel lengths of $s(t-\epsilon T)$, and n_m denotes the noise which is normally colored because of the PR4 equalizer. In this paper, the proposed model for the nonlinear PR4 channel is given by

$$y_m = r_m b_m[q_m g_0 + (1-q_m)f_0] + r_{m-1}b_{m-1}[q_{m-1}g_1 + (1-q_{m-1})f_1]. \quad (21)$$

The parameters $\gamma_1, \gamma_2, g_0, f_0, g_1, f_1$ in (21) are further estimated by the ACM algorithm.

We use the signal-to-distortion (SDR) ratio, defined as $10\log(\mathrm{E}[(z_m-n_m)^2]/\mathrm{E}[(z_m-y_m)^2])$, to measure the model capability. The measurement data, for each value of ϵ from 0.1 to 0.5, are generated using (20) with $L_1 = L_2 = 10, \gamma_1 = 0.7$, and $\gamma_2 = 0.49$. For simplicity, white noise n_m is used and the SNR is 20 dB. The resulting SDRs are listed in Table I. The linear superposition model (LSM) results in the lowest SDR because it ignores the nonlinear effects. The SPEM [1] also yields poor performance in SDR when the transition shift parameter is equal to 0.2 or larger. While Ryan's model [2], because of linearization, produces high SDRs only for small ϵ, our model (21) always results in a very high SDR even for ϵ as large as 0.5.

The trellis diagram of the proposed model can be derived from (21) and is identical to those in [1] and [2], except that the model output is modified. The Viterbi algorithm is used to realize the ML detector for the detection of b_m. Since the data $b_m = a_m - a_{m-1}, a_m$ can be recovered by the relation $a_m = a_{m-1}$ when $b_m = 0$ or $a_m = b_m/2$ when $b_m \neq 0$. Note that recovering a_m from b_m may cause error propagation; this effect, however, is minor in our simulation. The bit error rate (BER) performance of a_m for each model under various SNRs for $\epsilon = 0.3$ is shown in Fig. 2. Hence, the complexity of the ML detector is not increased and the performance is improved because of the increasing model accuracy.

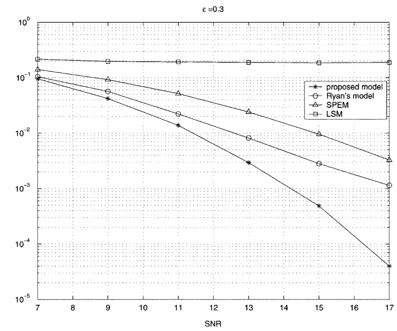

Fig. 2. Bit error rate of a nonlinear PR4 channel for $\epsilon = 0.3$.

V. CONCLUSION

We have applied the alternating coordinates minimization algorithm for estimating the parameters of a PETSM. This algorithm can also be used to identify the parameters of the nonlinear PR4-equalized channel. The obtained model greatly increases the modeling accuracy and improves the performance of the corresponding ML detector without increasing its realization complexity.

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