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ORIGINAL ARTICLE

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C_{pm} MPPAC for manufacturing quality control applied to the precision voltage reference process

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Abstract The multiprocess performance analysis chart (MPPAC) based on the process capability index C_{pm}, called C_{pm} MPPAC, is developed for analysing the manufacturing quality of a group of processes in a multiple process environment. The C_{pm} MPPAC conveys critical information of multiple processes regarding the departure of the process and process variability from one single chart. Existing research works on MPPAC are restricted to obtaining quality information from one single sample of each process ignoring sampling errors. The information provided from existing MPPAC charts, therefore, is unreliable and misleading and results in incorrect decisions. In this paper, we consider the natural estimator of C_{pm} based on multiple samples. Based on the natural estimator of C_{pm}, we consider the sampling errors by providing an explicit formula with the Matlab program to obtain the estimation accuracy of the C_{pm}. We tabulate the sampling accuracy of C_{pm} for sample size determination so that the engineers/practitioners can use it for their in-plant applications. An example of multiple precision voltage reference (PVR) processes is presented to illustrate the applicability of C_{pm} MPPAC for manufacturing quality control.

Keywords Multiprocess performance analysis chart · Maximum likelihood estimator · Process capability index · Sample size determination

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1 Introduction

Process capability indices (PCIs) have been widely used in various manufacturing industries, to provide numerical measures on process potential and process performance. The two most commonly used process capability indices are C_p and C_{pk} introduced by Kane [1]. These two indices are defined in the following equation:

$$C_p = \frac{USL-LSL}{6\sigma}$$
$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}\frac{\mu-LSL}{3\sigma}\right\},$$

where $USL$and LSL are the upper and the lower specification limits, respectively, μis the process mean and s is the process standard deviation. The index C_p measures the process variation relative to the manufacturing tolerance, with reflects only the process potential. The index C_{pk} measures process performance based on the process yield (percentage of conforming items) without considering the process loss (a new criteria for process quality championed by Hsiang and Taguchi [2]). Taking into consideration the process departure (which reflects the process loss), Chan et al. [3] developed the index C_{pm}, which measures the ability of the process to cluster around the target. The index C_{pm} is defined as:

$$C_{pm} = \frac{USL-LSL}{6\sqrt{\sigma^2+(\mu-T)^2}} = \frac{d}{3\sqrt{\sigma^2+(\mu-T)^2}},$$

where T is the target value, and $d=(USL-LSL)/2$ is half of the length of the specification interval (LSL, USL). Ruczinski [4] showed that Yield $=2F(3C_{pm})-1$, or the fraction of nonconformities $=2F(-3C_{pm})$, where $F(.)$ is the cumulative function of the standard normal distribution. Table 1 displays various values of $C_{pm}=$ 0.95(0.05) 2.00, and the corresponding nonconformities (in PPM). For example, if a process has capability with $C_{pm}=1.25$, then the manufacturing process yield would be at least 99.982%. Some common used values of C_{pm} are: 1/3 (the process is incapable), 1/2 (the process is

Table 1 Various values of $C_{pm}=0.95(0.01)2.00$ and the corresponding nonconformities (in PPM)

C_{pm}	PPM
0.95	4371.923
1.00	2699.796
1.05	1632.705
1.10	966.848
1.15	560.587
1.20	318.217
1.25	176.835
1.30	96.193
1.35	51.218
1.40	26.691
1.45	13.614
1.50	6.795
1.55	3.319
1.60	1.587
1.65	0.742
1.70	0.340
1.75	0.152
1.80	0.067
1.85	0.029
1.90	0.012
1.95	0.005
2.00	0.002

incapable), 1.00 (the process is normally called capable), 1.33 (the process is normally called satisfactory), 1.67 (the process is normally called good), and 2.00 (the process is normally called super).

Statistical process control charts have been widely used for monitoring individual factory manufacturing processes on a routine basis. Those charts are essential tools for the control and improvement of these processes. In the multiprocess environment where a group of processes need to be monitored and controlled, it could be difficult and time-consuming for factory engineers or supervisors to analyse the individual chart in order to evaluate the overall performance of factory process control activities. Singhal [5, 6] introduced the multiprocess performance analysis chart (MPPAC) using the process capability indices C_p and C_{pk} which can be implemented for illustrating and analysing the performance of a group of processes in a multiple process environment by including the departure of the process mean from the target value, process variability, capability zones and expected fallout outside specification limits on a single chart. Pearn and Chen [7] proposed a modification to MPPAC combining the more advanced process capability index, C_{pm}, to identify the problems causing the processes failing to centre around the target. Pearn et al. [8] introduced the MPPAC based on the incapability index. Chen et al. [9] presented a modification to the MPPAC.

With respect to these studies, there are some limitations and shortcomings included. First, the existing MPPACs based on the process capability indices are restricted to obtaining quality information by calculating one single sample data for each process. In practice, however, manufacturing information is often derived from multiple samples rather than one single sample, particularly when a daily-based process control plan is implemented for monitoring process stability. Second, most existing MPPACs using process capability indices simply use the estimates of the indices on the chart and then make a conclusion on whether processes meet the capability requirement and directions need to be taken for further quality improvement. Their approach is highly unreliable since sampling errors are ignored. Therefore, in this paper, we introduce a new control chart of C_{pm} MPPAC. The natural estimator of C_{pm} based on multiple samples is investigated. We also consider the sampling errors and develop a Matlab program for determining the overall number of observations and sub-samples required by an estimating accuracy of the C_{pm}. An example of precision voltage reference (PVR) processes is presented to illustrate the applicability of C_{pm} MPPAC for production quality control.

2 The C_{pm} MPPAC

Singhal [5] indicated that the MPPAC can be used to evaluate the performance of a single process as well as multiple-processes; to set the priorities among multiple processes for quality improvement, and to indicate whether reducing the variability, or the departure of the process mean should be focused upon; to provide an easy way to qualify the process improvement by comparing the locations on the chart of the processes before and after the improvement effort. Since C_{pm} simultaneous measures process variability and centreing, a C_{pm} MPPAC would provide a convenient way to identify problems in process capability after statistical control is established. Based on the definition, we first set $C_{pm}=h$, for various h values, then a set of $C_{pm}=h$ values satisfying the equation:

$$\left(\frac{\sigma}{d/3}\right)^2+\left(\frac{\mu-T}{d/3}\right)^2=\left(\frac{1}{h}\right)^2$$

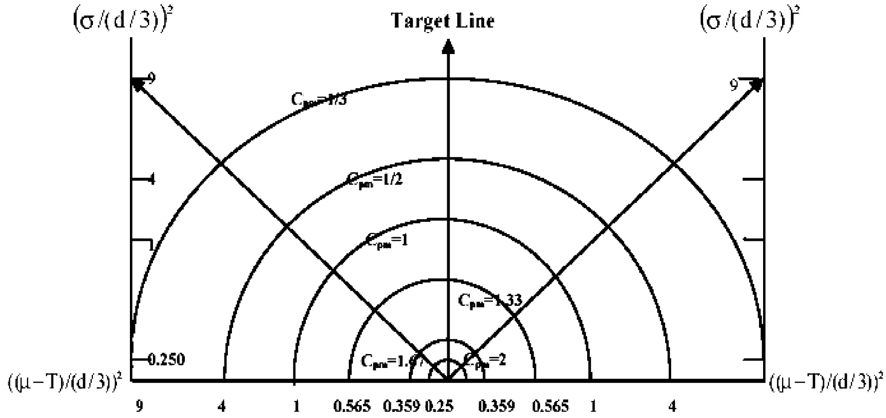
can be plotted to form a contour (indifference curve) of $C_{pm}=h$. These contours are semicircles centreed at $(T, 0)$ with radius $1/h$. The more capable the process, the smaller the semicircle is. Figure 1 plots the six contours on the C_{pm} MPPAC for the six values, $C_{pm}=1/3$, 1/2, 1, 1.33, 1.67 and 2. On the C_{pm} MPPAC, we note that:

[a] The horizontal line and vertical line through the plotted point intersect the vertical axis and horizontal axis at points represented by $(\sigma/(d/3))^2$and $((\mu-T)/(d/3))^2$, respectively.

[b] The distance between 0 and the intersection between point the vertical line through the plotted point and the horizontal axis, denoting the departure of the process mean from the target.

[c] The distance between 0 and the point, which the horizontal line through the plotted point intersecting the vertical axis, denotes the process variance.

Fig. 1 The contours of C_{pm} MPPAC



[d] For the points inside the semicircular contour (indifference curve) $C_{pm}=h$, the corresponding C_{pm}values are larger than h. For the points outside the semicircular contour $C_{pm}=h$, the corresponding C_{pm}values are smaller than h.

[e] As the point gets closer to the target, the value of the C_{pm} becomes larger, and the process performance is better.

[f] For processes with fixed values of C_{pm}, the points within the two 45° lines envelop, and the process variability is contributed mainly by the process variance.

[g] For processes with fixed values of C_{pm}, the points outside the two 45° lines envelop, the process variability is contributed mainly by the process departure.

In general, the process parameters $\mu$and σ^2 are unknown. But, in practice $\mu$and σ^2 can be estimated by sample data obtained from stable processes. In the next section, estimating C_{pm} and the estimation accuracy based on multiple samples is investigated.

3 Estimating C_{pm} based on multiple samples

Kirmani et al. [10] indicated that a common practice of the process capability estimation in the manufacturing industry is to first implement a routine-basis data collection program for monitoring/controlling the process stability, then to analyse the past "in control" data. For multiple samples of m_s groups each of with sizes n are chosen randomly from a stable process which follows a normal distribution $N(\mu,\sigma^2)$. Let $\bar{X}_i = \sum_{j=1}^{n} x_{ij}/n$ and $S_i = \left[(n)^{-1}\sum_{j=1}^{n}\left(x_{ij}-\bar{X}_i\right)^2\right]^{1/2}$ be the ith sample mean and the sample standard deviation, respectively. We consider the following natural estimator of C_{pm}:

$$\tilde{C}_{pm}^{M} = \frac{USL - LSL}{6\sqrt{S_p^2 + \left(\bar{\bar{X}} - T\right)^2}},$$

where $\bar{\bar{X}} = \sum_{i=1}^{m_s}\bar{X}_i\Big/m_s$ and $S_p^2 = \sum_{i=1}^{m_s} S_i^2\Big/m_2$. If the process follows the normal distribution $N(\mu,\sigma^2)$, then

$$\tilde{C}_{pm}^{M} = \sqrt{N}\frac{USL-LSL}{6\sigma}\left[\frac{NS_p^2}{\sigma^2} + \frac{N\left(\bar{\bar{X}}-\mu\right)^2}{\sigma^2} + \frac{N(\mu-T)^2}{\sigma^2}\right]^{1/2},$$

where $\sum_{i=1}^{m_s} n = N$.

The $NS_p^2\Big/\sigma^2$ and $N\left(\bar{\bar{X}}-\mu\right)\Big/\sigma^2$ are distributed as ordinary central Chi-square distribution with N-m_s and one degree of freedom, $\chi^2_{N-m_s}$and χ^2_1, respectively. Therefore,

$$\tilde{C}_{pm}^{M} \sim \frac{USL-LSL}{6\sigma}\sqrt{\frac{N}{\chi^2_{N-m_s+1,\lambda}}} = C_P\sqrt{\frac{N}{\chi^2_{N-m_s+1,\lambda}}},$$

where $\chi^2_{N,\lambda}$ denotes the noncentral Chi-square distribution with N degrees of freedom and noncentral parameter $\lambda = N((\mu-T)/\sigma)^2$. The rth moment (about zero) can be obtained as the following equation:

$$\begin{aligned} E\left[\tilde{C}_{Pm}^{M}\right]^r &= \left(\sqrt{N}C_P\right)^r E\left(\chi^2_{N-m_s+1,\lambda}\right)^{-r/2} \\ &= \left(\frac{\sqrt{N}C_P}{\sqrt{2}}\right)^r \exp\left(\frac{-\lambda}{2}\right) \\ &\quad\times\sum_{j=0}^{\infty}\left\{\frac{(\lambda/2)^j}{j!}\times\frac{\Gamma((2j+N-m_s+1-r)/2)}{\Gamma((2j+N-m_s+1)/2)}\right\}. \end{aligned}$$

The probability density function (PDF) of natural estimator of C_{pm} can be easily attained as the following equation:

$$\begin{aligned} f(x) &= \frac{2^{(1-N^*)/2}C'^{(N^*+1)}}{3^{(N^*+1)}x^{(N^*+2)}}\exp\left[-\frac{\lambda}{2}-\frac{C'^2}{18x^2}\right] \\ &\quad\times\sum_{j=0}^{\infty}\left\{\left[\frac{\lambda C'^2}{36x^2}\right]^j\left[j!\Gamma\left(\frac{N^*+1+2j}{2}\right)\right]^{-1}\right\}, \end{aligned}$$

where $C' = 3\sqrt{N}C_P, N - m_s = N^*, \text{and } x > 0$

Using the method similar to that presented in Vännman [11], we may obtain an exact form of the cumulative distribution function of $\tilde{C}_{Pm}^{M}$. The cumulative distribution function of $\tilde{C}_{Pm}^{M}$ can be expressed in terms of a mixture of the ordinary central Chi-square distribution and the normal distribution:

$$F_{\tilde{C}_{Pm}^{M}}(x) = 1 - \int_0^{b\sqrt{N}/(3x)} G\left(\frac{b^2N}{9x^2} - t^2\right) \times \left[\phi\left(t + \xi\sqrt{N}\right) + \phi\left(t - \xi\sqrt{N}\right)\right]dt,$$

where $b = d/\sigma$, $\xi = (\mu\text{-}T)/\sigma$, $G(\cdot)$ is the cumulative distribution function of the ordinary central Chi-square distribution $\chi^2_{N-m_s}$, and $F(\cdot)$ is the probability density function of the standard normal distribution $N(0, 1)$. We note that for cases with one single sample, $m_s = 1$, the special case of multiple samples, the statistical properties of the estimator of C_{pm} are proposed by Chan et al. [3], Boyles [12], Pearn et al. [13], Kotz and Johnson [14], Vännman and Kotz [15] and Vännman [11].

4 The estimation accuracy of C_{pm}

For processes with a target value setting to the mid-point of the specification limits ($T = (USL + LSL)/2$), the index may be rewritten as follows. We note that when $C_{pm} = C$, $b = d/\sigma$ can be expressed as $b = 3C\sqrt{1 + \xi^2}$. Thus, the index C_{pm} may be expressed as a function of the characteristic parameter ξ.

$$C_{pm} = \frac{d}{3\sigma\sqrt{1 + \xi^2}} = \frac{d/\sigma}{3\sqrt{1 + \xi^2}},$$

where$\xi = (\mu - T)/\sigma$. Hence, given the total number of observations N, the number of sub-samples m_s with the confidence level γ, the parameter ξ, and the estimating accuracy R_{pm} can be obtained using numerical integration technique with iterations, to solve the following Eq. 11. It should be noted, particularly, that Eq. 1 is an even function of ξ. Thus, for both $\xi = \xi_0$ and $\xi = -\xi_0$ we may obtain the same total observations N.

$$\int_0^{R_{pm}\sqrt{N(1+\xi^2)}} G\left(R_{pm}^2 N(1 + \xi^2) - t^2\right) \times \left[\phi\left(t + \xi\sqrt{N}\right) + \phi\left(t - \xi\sqrt{N}\right)\right]dt = 1 - \gamma. \quad (1)$$

4.1 The estimation accuracy R_{pm} and parameter ξ

Since the process parameters $\mu$and σ are unknown, then the distribution characteristic parameter, $\xi = (\mu - T)/\sigma$ is also unknown, which has to be estimated in real applications, naturally by substituting μ and σ by the sample mean $\bar{\bar{X}}$and the sample standard deviation S_p. Such approach introduces additional sampling errors from estimating ξ in finding the sample accuracy, and certainly would make our approach (and of course including all the existing methods) less reliable. Consequently, any decisions made would result in less production yield assurance to the factories, and provide less quality protection to the customers. To eliminate the need for further estimating the distribution characteristic parameter $\xi = (\mu - T)/\sigma$, we examine the behaviour of the sample accuracy R_{pm} against the parameter $\xi = (\mu - T)/\sigma$.

We perform extensive calculations to obtain the R_{pm} for $\xi = 0(0.1)3.00$, the total number of observations $N = 200$, $m_s = 1$, 10, 20, 40, 50 and 100 with confidence level$\gamma = 0.90$, 0.95, 0.975 and 0.99. The results indicate that (i) the sample precision is increasing as ξ increases, and is decreasing as m_s increases, (ii) the sample precision R_{pm} obtains its minimum at $\xi = 0$ in all cases. Hence, for practical purpose we may solve Eq. 1 with $\xi = 0$ to obtain the required sample accuracy for given N, m_s and γ, without having to further estimate the parameter ξ. Thus, the level of confidence γ can be ensured, and the decisions made based on such approach are indeed more reliable. Figure 2 plots the curves of the sample accuracy R_{pm} versus the parameter ξ for $N = 200$ and $m_s = 1$, 10, 20, 40, 50 and 100 with confidence level $\gamma = 0.95$. For bottom curve 1, $m_s = 100$; for bottom curve 2, $m_s = 50$; for bottom curve 3, $m_s = 40$; for top curve 3, $m_s = 20$; for top curve 2, $m_s = 10$; for top curve 1, $m_s = 1$.

5 The sample size determination for C_{pm} MPPAC

Using Eq. 1, we may compute the estimation accuracy R_{pm}. Three auxiliary functions for evaluating R_{pm} are included here: (a) the cumulative distribution function of the chi-square $\chi^2_{N-m_s}$, $G(\cdot)$, (b) the probability density function of the standard normal distribution $F(\cdot)$, and (c) the function of the numerical evaluate integration using the recursive adaptive Simpson quadrature—”quad”. The program sets $(\mu - T)/\sigma = 0$, reads the number of

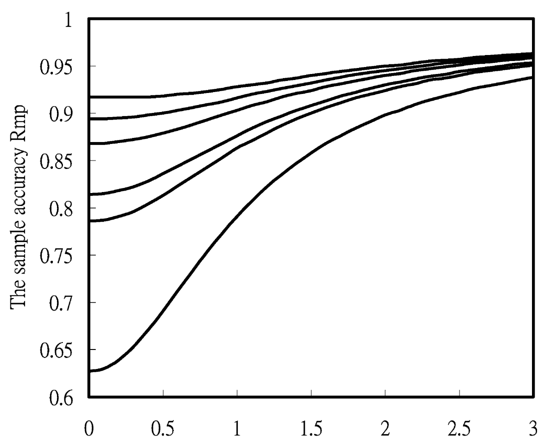


Fig. 2 Plots of R_{mp} vs$|\xi|$ for $N = 200$, $m_s = 1$, 10, 20, 40, 50, 100 (from top to bottom), $\gamma = 0.95$

sub-samples m_s, the total number of observations N, and the confidence level γ, then outputs with the estimating precision R_{pm}. The program, actual executed inputs and outputs are listed below.

Matlab Program for Computing the Accuracy

```
%-----------------------------------
% This program calculates the sample
% accuracy of Cpm for given sample
% observations, the number of samples
% and confidence level.
%-----------------------------------
clear global
[N1 ms1 r1]=read('Enter the total
observations, the number of subsamples,
and confidence level:');
global b N epsilon eCpm ms
N=N1;
r=r1;
ms=ms1;
epsilon=0;
eCpm=1.0;
b=0;d=0;
c=0.2:0.025:3;
for i=1:1:113
b=0;d=0;y=0;
b=3*c(i)*sqrt(1+epsilon^2);
d=b*sqrt(N)/(3*eCpm);
y=quad('Cpm',0,d);
if (y-(1-r))>0
break
end
end
c=0.2+0.025*(i-1):-0.001:0.2;
for k=1:(0.025*(i-1)*1000)+1
b=0;d=0;y=0;
b=3*c(k)*sqrt(1+epsilon^2);
d=b*sqrt(N)/(3*eCpm);
y=quad('Cpm',0,d);
if ((1-r)-y)>0.0001
break
end
end
fprintf('The Estimating Accuracy is
%g\n',c(k)/eCpm)
%-----------------------------------
% read.m file.
%-----------------------------------
function [a1, a2, a3]=read(labl)
if nargin==0,
labl='?';
end
n=nargout;
str=input(labl,'s');
str=['[',str,']'];
v=eval(str);
L=length(v);
if L>=n, v=v(1:n);
else, v=[v,zeros(1,n-L)];
end
for j=1:nargout
eval(['a', int2str(j),'=v(j);']);
end
%-----------------------------------
% Cpm.m file.
%-----------------------------------
function Q1=Cpm(t)
global N b epsilon eCpm ms
Q1=chi2cdf(((b^2*N/(9*eCpm^2))-
t.^2),
N-ms).*(normpdf((t+epsilon*sqrt(N)))+
normpdf((t-epsilon*sqrt(N))));
%-----------------------------------
% The end.
%-----------------------------------
```

Input Enter the total observations, the number of subsamples, and confidence level: 100,20,0.95

Output The estimating accuracy is 0.782

The sample size determination is essential to most factory applications, particularly for those implementing a routine-basis data collection plan for monitoring and controlling process quality. It directly relates to the sampling cost of a data collection plan. Table 2a and 2b displays the sample size N and number of samples m_s required and the corresponding precision of the estimation R_{pm}. For example, $\gamma=0.95$, $N=150$, $m_s=30$ gives $R_{pm}=0.802$. Thus, the true value of C_{pm}, is no less than $\tilde{C}_{pm}^{M}\times 0.802$. On the other hand, if $R_{pm}=0.802$ is chosen, then we may determine $N=102$ with $m_s=17$ (each sample with six observations). Similarly, if $R_{pm}=0.85$ is chosen, then we may determine $N=190$ with $m_s=10$ and $\gamma=0.975$, $N=198$ with $m_s=6$ and $\gamma=0.90$, or $N=256$ with $m_s=32$ and $\gamma=0.975$, depending on which sampling plan is more appropriate to the application.

6 Manufacturing quality control for multiple PVR processes

In the following discussion, we investigate the production quality of a group of multiple processes for manufacturing the precision voltage reference devices. Voltage references are essential to the accuracy and performance of analog systems. They are used in many types of analog circuitry for signal processing, such as analog to digital (AD) or digital to analog (DA) converters and smart sensors. Precision voltage references can be used in constructing an accuracy regulated supply that could have better characteristics than some regulator chips, which sometimes can dissipate too much power. Another application of the voltage references is creating a precision constant current supply. In addition, voltage references are needed in the test equipment, which must be accurate, such as voltmeters, ohmmeters and ammeters.

Table 2 The total number of sample observations, $nm_s = N$, number of samples, m_s and the precision of estimation with $\gamma = 0.90, 0.95, 0.975, 0.99$

n	4				5				6				8				10				12			
$m_{s\gamma}$	0.90	0.95	0.975	0.99	0.9	0.95	0.975	0.99	0.90	0.95	0.975	0.99	0.90	0.95	0.975	0.99	0.9	0.95	0.975	0.99	0.90	0.95	0.975	0.99
5	0.682	0.630	0.587	0.538	0.727	0.680	0.641	0.596	0.759	0.715	0.679	0.637	0.800	0.762	0.730	0.693	0.827	0.792	0.763	0.729	0.845	0.814	0.787	0.756
6	0.692	0.649	0.608	0.563	0.740	0.697	0.661	0.619	0.771	0.731	0.697	0.659	0.811	0.776	0.746	0.712	0.837	0.805	0.778	0.747	0.855	0.826	0.801	0.772
7	0.708	0.663	0.626	0.583	0.751	0.711	0.676	0.637	0.781	0.744	0.712	0.676	0.820	0.787	0.759	0.728	0.845	0.815	0.790	0.761	0.862	0.835	0.812	0.785
8	0.717	0.675	0.640	0.599	0.760	0.722	0.689	0.652	0.789	0.754	0.724	0.690	0.827	0.796	0.770	0.740	0.851	0.823	0.800	0.773	0.868	0.843	0.821	0.796
9	0.725	0.685	0.651	0.613	0.767	0.731	0.700	0.665	0.795	0.762	0.734	0.702	0.833	0.804	0.779	0.751	0.856	0.830	0.808	0.782	0.873	0.849	0.828	0.805
10	0.731	0.694	0.661	0.625	0.773	0.739	0.709	0.676	0.801	0.770	0.743	0.712	0.838	0.810	0.787	0.760	0.861	0.836	0.815	0.790	0.877	0.854	0.835	0.812
11	0.737	0.701	0.670	0.635	0.778	0.745	0.718	0.685	0.806	0.776	0.750	0.721	0.842	0.816	0.793	0.767	0.865	0.841	0.821	0.797	0.880	0.859	0.840	0.819
12	0.742	0.708	0.678	0.644	0.783	0.751	0.725	0.694	0.810	0.781	0.757	0.728	0.846	0.821	0.799	0.774	0.868	0.846	0.826	0.804	0.884	0.863	0.845	0.824
13	0.747	0.713	0.685	0.652	0.787	0.757	0.731	0.701	0.814	0.786	0.763	0.735	0.849	0.825	0.804	0.778	0.871	0.849	0.831	0.809	0.886	0.866	0.849	0.829
14	0.751	0.719	0.691	0.659	0.791	0.761	0.736	0.708	0.818	0.791	0.768	0.741	0.852	0.829	0.809	0.785	0.874	0.853	0.835	0.814	0.889	0.870	0.853	0.834
15	0.755	0.723	0.696	0.666	0.794	0.766	0.741	0.714	0.821	0.795	0.772	0.747	0.855	0.832	0.813	0.790	0.876	0.856	0.838	0.818	0.891	0.873	0.856	0.838
16	0.758	0.728	0.702	0.672	0.797	0.770	0.746	0.719	0.823	0.798	0.777	0.752	0.857	0.835	0.817	0.795	0.879	0.859	0.842	0.822	0.893	0.875	0.860	0.842
17	0.761	0.731	0.706	0.677	0.800	0.773	0.750	0.724	0.826	0.802	0.781	0.756	0.860	0.838	0.820	0.799	0.881	0.861	0.845	0.826	0.895	0.878	0.862	0.845
18	0.764	0.735	0.710	0.682	0.802	0.776	0.754	0.728	0.828	0.805	0.784	0.760	0.862	0.841	0.823	0.802	0.882	0.864	0.848	0.829	0.897	0.880	0.865	0.848
19	0.766	0.738	0.714	0.687	0.805	0.779	0.758	0.733	0.830	0.807	0.787	0.764	0.864	0.843	0.826	0.806	0.884	0.866	0.850	0.832	0.898	0.882	0.867	0.851
20	0.769	0.741	0.718	0.691	0.807	0.782	0.761	0.737	0.832	0.810	0.790	0.768	0.865	0.846	0.829	0.809	0.886	0.868	0.853	0.835	0.890	0.884	0.870	0.853
21	0.771	0.744	0.721	0.695	0.809	0.785	0.764	0.740	0.834	0.812	0.793	0.771	0.867	0.848	0.831	0.812	0.887	0.870	0.855	0.838	0.901	0.885	0.872	0.856
22	0.773	0.747	0.724	0.699	0.811	0.787	0.767	0.744	0.836	0.814	0.796	0.774	0.868	0.850	0.833	0.815	0.889	0.872	0.857	0.840	0.902	0.887	0.874	0.858
23	0.775	0.749	0.727	0.702	0.812	0.789	0.770	0.747	0.838	0.817	0.798	0.777	0.870	0.851	0.836	0.817	0.890	0.873	0.859	0.842	0.904	0.888	0.875	0.860
24	0.777	0.752	0.730	0.705	0.814	0.791	0.772	0.750	0.839	0.818	0.801	0.778	0.871	0.853	0.838	0.820	0.891	0.875	0.861	0.845	0.905	0.890	0.877	0.862
25	0.778	0.754	0.733	0.708	0.816	0.793	0.774	0.752	0.841	0.820	0.803	0.783	0.872	0.855	0.840	0.822	0.892	0.876	0.863	0.847	0.906	0.891	0.879	0.864
26	0.780	0.756	0.735	0.711	0.817	0.795	0.777	0.755	0.842	0.822	0.805	0.785	0.874	0.856	0.841	0.824	0.893	0.878	0.864	0.849	0.907	0.892	0.880	0.866
27	0.782	0.758	0.738	0.714	0.818	0.797	0.779	0.758	0.843	0.824	0.807	0.787	0.875	0.858	0.843	0.826	0.894	0.879	0.866	0.850	0.908	0.894	0.881	0.867
28	0.783	0.760	0.740	0.717	0.820	0.799	0.781	0.760	0.844	0.825	0.809	0.789	0.876	0.859	0.845	0.828	0.895	0.880	0.867	0.852	0.908	0.895	0.883	0.869
29	0.784	0.762	0.742	0.719	0.821	0.800	0.783	0.762	0.846	0.827	0.810	0.792	0.877	0.860	0.846	0.830	0.896	0.881	0.869	0.854	0.909	0.896	0.884	0.871
30	0.786	0.763	0.744	0.721	0.822	0.802	0.784	0.764	0.847	0.828	0.812	0.793	0.878	0.862	0.848	0.831	0.897	0.882	0.870	0.855	0.910	0.897	0.885	0.872
31	0.787	0.765	0.746	0.724	0.823	0.803	0.786	0.766	0.848	0.829	0.814	0.795	0.879	0.863	0.849	0.833	0.898	0.883	0.871	0.857	0.911	0.898	0.886	0.873
32	0.788	0.766	0.748	0.726	0.824	0.805	0.788	0.768	0.849	0.831	0.815	0.797	0.880	0.864	0.850	0.835	0.899	0.884	0.872	0.858	0.912	0.899	0.887	0.875
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35	0.791	0.771	0.752	0.732	0.827	0.809	0.792	0.774	0.851	0.834	0.819	0.802	0.882	0.867	0.854	0.839	0.901	0.887	0.876	0.862	0.913	0.901	0.890	0.878
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37	0.793	0.773	0.755	0.735	0.829	0.811	0.795	0.777	0.853	0.836	0.822	0.805	0.883	0.869	0.856	0.841	0.902	0.889	0.877	0.864	0.915	0.903	0.892	0.880
38	0.794	0.774	0.757	0.737	0.830	0.812	0.796	0.778	0.854	0.837	0.823	0.806	0.884	0.870	0.857	0.843	0.903	0.890	0.878	0.865	0.915	0.903	0.893	0.881
39	0.795	0.775	0.758	0.738	0.831	0.813	0.797	0.780	0.855	0.838	0.824	0.808	0.885	0.870	0.858	0.844	0.903	0.890	0.879	0.866	0.916	0.904	0.894	0.882
40	0.796	0.776	0.760	0.740	0.832	0.814	0.799	0.781	0.855	0.839	0.825	0.809	0.885	0.871	0.859	0.845	0.904	0.891	0.880	0.867	0.916	0.905	0.895	0.883

Table 3 The precision voltage reference specifications

Code	V	Precision	*LSL*	*USL*
A	5	±0.2%	4.99	5.01
B	10	±0.025%	9.9975	10.0025
C	15	±0.1%	14.985	15.015
D	20	±0.05%	19.99	20.01
E	1	±0.025%	0.99975	1.00025
F	0.5	±0.002%	0.49999	0.50001
G	3	±0.01%	2.9997	3.0003
H	12	±0.05%	8.994	9.006
I	9	±0.2%	8.982	9.018
J	6	±0.2%	5.988	6.012
K	3	±0.05%	2.9985	3.0015
L	18	±0.05%	17.991	18.009

Table 4 The calculated statistics of the ten processes

Process	$\bar{\bar{X}}$	S_P	$\left[\left(\bar{\bar{X}}-T\right)/(d/3)\right]^2$	$[S_P/(d/3)]^2$
A	4.999529	0.001491	0.02	0.2
B	10.00111	0.000667	1.78	0.64
C	14.99325	0.004796	1.82	0.92
D	19.99795	0.002728	0.38	0.67
E	1.00003	0.00015	0.13	3.24
F	0.499996	1.49E−06	1.44	0.2
G	2.999946	7.87E−05	0.29	0.62
H	11.99864	0.002272	0.46	1.29
I	9.004948	0.005333	0.68	0.79
J	6.00337	0.0032	0.71	0.64
K	3.000087	0.000296	0.03	0.35
L	17.99944	0.002057	0.035	0.47

Consider the following case taken from a microelectronics manufacturing factory making precision voltage reference devices. Twelve specific types of precision voltage reference devices extensively used on PC-based instrumentation and test equipment with different precision voltage references specifications are selected in this study. Their precision voltage reference specifications are displayed in Table 3. A sample data collection plan is implemented in the factory on a daily basis to monitor/control production quality. The factory resource and sampling schedule allow the data collection plan be implemented with $N=150$ with $m_s=15$ (each sample with 10 observations). Checking Table 2a and

Table 5 The $\tilde{C}_{pm}^{M}$ minimum true capability C_{pm}, and the maximum nonconformities (in PPM)

Process	$\tilde{C}_{pm}^{M}$	C_{pm}	PPM
A	2.132	1.825	0.0438
B	0.643	0.550	98943
C	0.604	0.517	120900
D	0.976	0.835	8439
E	0.545	0.467	161210
F	0.781	0.669	44750
G	1.048	0.897	4331
H	0.756	0.647	52258
I	0.825	0.706	34175
J	0.861	0.737	27036
K	1.622	1.389	30.86
L	1.407	1.205	300.35

2b, we obtain the estimation accuracy $R_{pm}=0.856$, with confidence $\gamma=0.95$.

The calculated overall sample mean, the pooled sample standard deviation, the estimate $\tilde{C}_{pm}^{M}$, the minimum true value and the maximum nonconformities are displayed in Table 4 and Table 5. Figure 3plots the C_{pm} MPPAC for the 12 processes using the data summarised in Table 4. We analyse these process points in Fig. 3 and obtain the following critical summary information of the quality condition for all processes.

[a] The plotted point E is outside the contour of $C_{pm}=1/2$. It indicates that the process has a very low capability. Since the point E is close to the target line, the process mean is close to the target value, and the poor capability is mainly contributed by the significant process variation. Thus, immediate quality improvement actions must be taken for reducing the process variance.

[b] The plotted points H, D, and G lies outside of the contour of $C_{pm}=1$. It indicates that the capability C_{pm} is less than 1. Since the point lies inside the two 45° lines envelope range, it indicates that the process variation measure, $(\sigma/(d/3))^2$, is more significant than the departure measure, $((\mu-T)/(d/3))^2$. Thus, reducing the process variance should be set to higher priority than that of reducing the process departure.

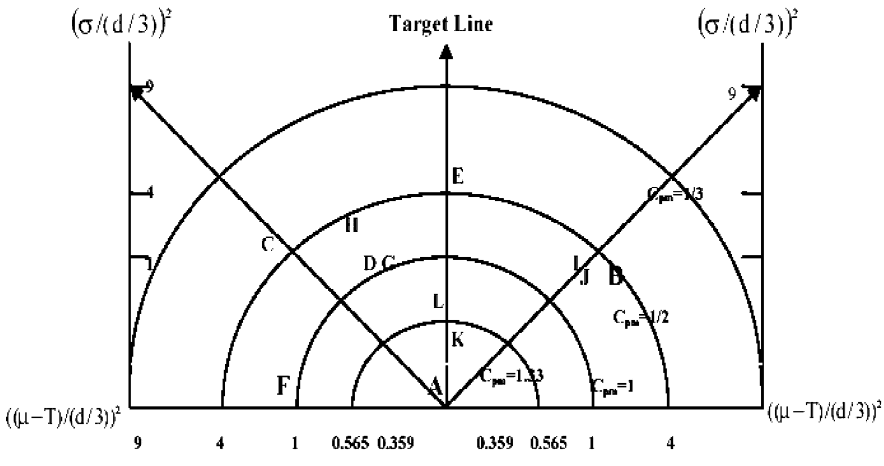

Fig. 3 The C_{pm} MPPAC for the application

[c] The plotted points C, F and B lie outside the contour of $C_{pm}=1$. Since these points also lie outside the two 45° lines envelope range, it indicates that the departure measure, $((\mu-T)/(d/3))^2$ is higher than process variation measure, $(\sigma/(d/3))^2$. Thus, quality improvement effort for these processes should be first focused on reducing their process departure from the target value T, then only the reduction of the process variance can be considered.

[d] The plotted points I and J are very close to the two 45° lines, and are outside the contour of $C_{pm}=1$. It indicates that the poor capability of both processes is contributed to equally significantly by the process mean departure and process variance.

[e] The plotted points K and L lie inside the contours of $C_{pm}=1.33$ and $C_{pm}=1$, respectively. It means that both process capability values C_{pm} are greater than 1. Capabilities of both processes are consider satisfactory. They have lower priorities in allocating quality improvement efforts than other processes.

[f] Process A is near to 0 and its variation measure is small. Therefore, process A is considered performing well. No immediate improvement activities need to be taken.

7 Conclusions

Conventional investigations on manufacturing quality control are restricted to obtaining quality information based on one single sample for each process ignoring sampling errors. The proposed C_{pm} MPPAC using process capability index C_{pm} is useful for manufacturing quality control of a group of processes in a multiple process environment. In this paper, we introduced a new control chart called C_{pm} MPPAC using the natural estimator of C_{pm} based on multiple samples. We investigated the accuracy of the estimation as a function of the process characteristic parameter $\xi=(\mu-T)/\sigma$, given a group of multiple control chart samples. Information regarding the true capability values and the maximum nonconformities (in PPM) is provided for production quality control. Appropriate sample sizes are then recommended to the proposed C_{pm} MPPAC for multiple processes production quality control. This approach ensures that the critical information conveyed from the C_{pm} MPPAC based on multiple control chart samples, is more reliable than all other existing methods. We developed a Matlab computer program to calculate the estimating accuracy and provided convenient tables for practitioners to use in determining appropriate sample sizes needed for their factory applications. An example of PVR manufacturing processes is given to illustrate the applicability of the proposed C_{pm} MPPAC.

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