Diversity-Multiplexing Tradeoff in MIMO Gaussian Interference Channels

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***Abstract*—In this paper we analyze the generalized degrees of freedom (GDOF) and the DMT performances of both the symmetric and asymmetric MIMO Gaussian interference fading channel with a fixed-power-split HK scheme for two transmit-receive pairs. Exact characterizations of both the GDOF and DMT performance measures are given. It is shown in the SIMO case that when the number of receive antennas is at least two, full region of GDOF can be achieved, as if the non-intending transmitter does not exist. The same conclusion is applied to the DMT measure as well. In particular, if the channel is symmetric, both receivers are able to achieve the single-user performance regime.**

I. Introduction

The Gaussian interference fading channel (GIFC) models the situation when two or more transmit-receive pairs try to communicate via common communication channel. In the GIFC model, it is assumed that there is no cooperation between any of the transmitters and receivers. For example, in the case of two transmit-receive pairs, where each transmitter is equipped with n_t transmit antennas, and each receiver has n_r receive antennas, the MIMO-GIFC can be described by the following channel input-output relations:

$$\begin{cases} \underline{y}_1 &= \sqrt{\mathrm{SNR}_{11}} H_{11} \underline{x}_1 + \sqrt{\mathrm{INR}_{21}} H_{21} \underline{x}_2 + \underline{z}_1 \\ \underline{y}_2 &= \sqrt{\mathrm{INR}_{12}} H_{11} \underline{x}_1 + \sqrt{\mathrm{SNR}_{22}} H_{22} \underline{x}_2 + \underline{z}_2 \end{cases} \tag{1}$$

where $\underline{x}_i$ is the signal vector sent out by the transmitter TX_i, $\underline{y}_j$ is the signal vector received by the receiver RX_j, H_{ij} is the $(n_r \times n_t)$ channel matrix between TX_i and RX_j, and $\underline{z}_j$ is the additive noise vector at RX_j. Entries of H_{ij} and $\underline{z}_j$ are modeled as i.i.d. complex Gaussian random variables with zero mean and unit variance, i.e., $\mathbb{CN}(0,1)$. SNR_{ii} is the signal-to-noise ratio between TX_i and its intending receiver RX_i and INR_{ij} models the interference-to-noise ration between TX_i and its nonintending receiver RX_j. The transmitter signal vector $\underline{x}_i$ is assumed to satisfy the average power constraint $\mathbb{E}\left\|\underline{x}_i\right\|_F^2 \leq 1$, where by $\left\|\underline{x}_i\right\|_F$ we mean the Frobenius norm of the vector $\underline{x}_i$.

Determining the capacity region of GIFC has been a very difficult problem. Most of earlier works focus on the case when $n_t = n_r = 1$ (henceforth will be termed *SISO-GIFC* in this paper) and when the channel state information (CSI) that includes the channel coefficients $H_{ij} = h_{ij}$, SNR_{ii} and INR_{ij}, are known completely to all transmitters and receivers. Depending on the various relations between $\mathrm{INR}_{ij}\left|h_{ij}\right|^2$ and $\mathrm{SNR}_{ii}\left|h_{ii}\right|^2$ for all $i \neq j$, the GIFC can be classified into three different regimes, namely, the *very strong interference regime*, the *strong interference regime*, and the *weak interference regime*.

Carleil [1] showed that the capacity region in very strong regime can be easily determined by treating the interference signal as noise. For the strong interference regime, the capacity region has been completely determined by Sato [2]. Recently, Etkin *et al.* [3] provided several capacity outer bounds for the weak interference regime, and then applied these bounds to show that a fixed Han-Kobayashi (HK) [4] scheme with a specific power-split between the private and common messages achieves the capacity region of the weak regime to within one-bit. Capacity regions of the *mixed interference regime* [3], i.e., one transmit-receive pair in the weak regime and the other in the strong regime, are also characterized to within one-bit.

Let SNR denote the base-line SNR, and set $\mathrm{SNR}_{ii} = \mathrm{SNR}^{\beta_{ii}}$ and $\mathrm{INR}_{ij} = \mathrm{SNR}^{\beta_{ij}}$. The only interesting case is the one when $\beta_{ii} > 0$ for $i = 1, 2$. Values of β_{ij}, $i \neq j$ vary in different interference regimes. Furthermore, let $R_i = r_i \log \mathrm{SNR}$ be the sum-rate of the private and common messages sent by TX_i, where r_i is the generalized degree of freedom (GDOF) defined by Etkin *et al.* [3]. The concept of GDOF is analogous to that of multiplexing gain in diversity-multiplexing tradeoff (DMT) proposed by Zheng and Tse [5]. With the above, we say the MIMO-GIFC is *symmetric* if $\beta_{11} = \beta_{22}$, $\beta_{12} = \beta_{21}$, and $r_1 = r_2$; otherwise the channel is said to be *asymmetric*.

If full CSI is available at both TX_i and RX_j for all i, j, in [6] Akuiyibo and Lévêque studied the DMT performance in symmetric SISO-GIFC based on the capacity outer bounds proposed by Etkin *et al.* [3]. When CSI is available only at the receivers, in [7], [8] the authors analyzed the DMT in symmetric SISO-GIFC when a fixed-power-split HK scheme is used. Weng and Tuninetti [9] focused on the case of asymmetric SISO-GIFC and analyzed the DMT performances of the fixed-power-split HK scheme as well as several known capacity outer bounds. For MIMO-GIFC, Akuiyibo *et al.* [10] presented upper bounds of the DMT in symmetric MIMO-GIFCs when $\beta_{ii} = \beta_{ij}$ for all i, j and when $n_t = n_r$.

In this paper we will analyze the GDOF and the DMT performances of both the symmetric and asymmetric MIMO GIFCs with a fixed-power-split HK scheme. We will assume that full CSI is available only at the receivers, and the

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transmitters are only aware of the β_{ij}s. The regions of GDOF of these channels will be characterized at high SNR regime completely in Section III. DMT analyses of MISO- and SIMO-GIFCs are presented in Section IV.

II. HK Region of General MIMO-GIFCs

Assume the senders are aware of the β_{ij}s and transmit using a fixed-power-split HK scheme where the transmitted signal vector $\underline{x}_i$ consists of two independent parts, the private signal $\underline{u}_i$ and the common signal $\underline{w}_i$. Following [3], we shall set the power level of the private messages such that they are received at, or below, the level of the noise. Thus, we have

$$\underline{x}_i = \sqrt{\frac{1}{1+\mathrm{INR}_{ij}}}\underline{u}_i + \sqrt{\frac{\mathrm{INR}_{ij}}{1+\mathrm{INR}_{ij}}}\underline{w}_i \tag{2}$$

for $i \neq j \in \{1,2\}$, where $\mathbb{E}\,\|\underline{u}_i\|_F^2, \mathbb{E}\,\|\underline{w}_i\|_F^2 \leq 1$. With such fixed-power-split, the resulting rate region is completely characterized by the constraints given in [4], or equivalently by the simplified constraints given by Chong *et al.* [11]. For example, one of the simplified constraints asserts

$$R_1 \leq \mathbb{E}\Bigg\{\log\left|I + \mathrm{SNR}_{11}H_{11}H_{11}^\dagger + \frac{\mathrm{INR}_{21}}{1+\mathrm{INR}_{21}}H_{21}H_{21}^\dagger\right| - \log\left|I + \frac{\mathrm{INR}_{21}}{1+\mathrm{INR}_{21}}H_{21}H_{21}^\dagger\right|\Bigg\},$$

where I is the identity matrix of appropriate size, and where by $|A|$ we mean the determinant of matrix A. In high SNR regime, we can further simplify the above constraint by showing $\mathbb{E}\log\left|I + \mathrm{SNR}_{11}H_{11}H_{11}^\dagger + \frac{\mathrm{INR}_{21}}{1+\mathrm{INR}_{21}}H_{21}H_{21}^\dagger\right| = \mathbb{E}\left|I + \mathrm{SNR}_{11}H_{11}H_{11}^\dagger\right| + O(1)$ and $\mathbb{E}\left|I + \frac{\mathrm{INR}_{21}}{1+\mathrm{INR}_{21}}H_{21}H_{21}^\dagger\right| = O(1)$, independent of the values of INR_{21}.

A. Some Useful Techniques

To establish our claim, we present the following theorem which is actually more powerful than what is needed at the moment.

Theorem 1: Let $\mathrm{SNR}_1 = \mathrm{SNR}^{\beta_1}$ and $\mathrm{SNR}_2 = \mathrm{SNR}^{\beta_2}$ with $\beta_1 > 0$ and $\beta_2 \leq 0$, and let H_1 and H_2 be two i.i.d. $(n_r \times n_t)$ random matrices with i.i.d. $\mathbb{CN}(0,1)$ entries; then

$$\Pr\left\{\log\left|I + \mathrm{SNR}_1H_1H_1^\dagger + \mathrm{SNR}_2H_2H_2^\dagger\right| \leq R\right\} \doteq \mathrm{SNR}^{-d(r)},$$

where $R = r\log\mathrm{SNR}$, and

$$d(r) \;=\; \beta_1\, d^*_{n_t,n_r}\left(\frac{r}{\beta_1}\right). \tag{3}$$

$\doteq$ denotes the exponential equality defined in [5]. $d^*_{n_t,n_r}(\ell)$ is the single-block point-to-point DMT for n_t transmit antennas and n_r receive antennas given in [5], i.e., $d^*_{n_t,n_r}(\ell)$ is the piecewise linear function obtained by connecting the points $(\ell,(n_t-\ell)(n_r-\ell))$ for $\ell = 0,1,\cdots,\min\{n_t,n_r\}$. ■

Remark 2: From Theorem 1, the maximal multiplexing gain for nonzero $d(r)$ equals $\beta_1\min\{n_t,n_r\}$, and it depends only on the value of SNR_1 whenever $\mathrm{SNR}_2 \dot{\leq} 1$. It then follows that $\mathbb{E}\log\left|I + \mathrm{SNR}_1H_1H_1^\dagger + \mathrm{SNR}_2H_2H_2^\dagger\right| = \mathbb{E}\log\left|I + \mathrm{SNR}_1H_1H_1^\dagger\right| + O(1)$. This proves our earlier claim. ■

To prove Theorem 1 we call for the following theorem due to Weyl [12] for the perturbation of Hermitian matrices.

Theorem 3 (Weyl): Let A and E be Hermitian matrices of size $(n \times n)$. Let $\lambda_1 \leq \cdots \leq \lambda_n$ be the ordered eigenvalues of A and let $\hat{\lambda}_1 \leq \cdots \leq \hat{\lambda}_n$ be the ordered eigenvalues of $A+E$. Then

$$\left|\hat{\lambda}_i - \lambda_i\right| \leq \|E\|_2\,, \quad i = 1,2,\cdots,n \tag{4}$$

where $\|E\|_2 := \max\left\{\sqrt{\ell}:\ \ell \text{ an eigenvalue of } EE^\dagger\right\}$ is the spectral norm of E. ■

With the above theorem, note that

$$\mathrm{SNR}_1H_1H_1^\dagger + \mathrm{SNR}_2H_2H_2^\dagger = \mathrm{SNR}_1\left(H_1H_1^\dagger + \mathrm{SNR}^\delta H_2H_2^\dagger\right),$$

where $\delta := \beta_2 - \beta_1 \leq -\beta_1$ by assumption. Let $\lambda_1 \leq \cdots \leq \lambda_K$ be the ordered nonzero eigenvalues of $H_1H_1^\dagger$, and let $\hat{\lambda}_1 \leq \cdots \leq \hat{\lambda}_K$ be the ordered nonzero eigenvalues of $\left(H_1H_1^\dagger + \mathrm{SNR}^\delta H_2H_2^\dagger\right)$, where $K = \min\{n_t,n_r\}$. By Theorem 3 we have

$$\left|\hat{\lambda}_i - \lambda_i\right| \leq \left\|\mathrm{SNR}^\delta H_2H_2^\dagger\right\|_2 \leq \left\|\mathrm{SNR}^\delta H_2H_2^\dagger\right\|_F \;\dot{\leq}\; \mathrm{SNR}^\delta$$

with probability 1, where the last exponential inequality follows from $\left\|H_2H_2^\dagger\right\|_F \dot{>} \mathrm{SNR}^0$ has probability zero. Set $\alpha_i := -\log_{\mathrm{SNR}}\lambda_i$; then we have

$$\left(\mathrm{SNR}^{-\alpha_i} - \mathrm{SNR}^\delta\right)^+ \leq \hat{\lambda}_i \leq \mathrm{SNR}^{-\alpha_i} + \mathrm{SNR}^\delta$$

where $(x)^+ := \max\{x,0\}$. Thus with $0 \leq \alpha_i \leq \beta_1$, we always have

$$\left(\mathrm{SNR}^{\beta_1-\alpha_i} \pm \mathrm{SNR}^{\beta_1+\delta}\right)^+ \;\doteq\; \mathrm{SNR}^{\beta_1-\alpha_i},$$

as $\beta_1 + \delta = \beta_2 \leq 0$ by hypothesis. Finally, the proof is complete after noting that

$$\Pr\left\{\log\left|I + \mathrm{SNR}_1H_1H_1^\dagger + \mathrm{SNR}_2H_2H_2^\dagger\right| \leq R\right\}$$
$$= \Pr\left\{\sum_{i=1}^K(\beta_1-\alpha_i)^+ \leq r\right\} = \Pr\left\{\sum_{i=1}^K\left(1-\frac{\alpha_i}{\beta_1}\right)^+ \leq \frac{r}{\beta_1}\right\}$$

and after invoking the point-to-point DMT by Zheng and Tse [5, Theorem 4].

B. Simplified HK Region at High SNR Regime

Armed with Theorem 1, we now present the simplified HK region of the fixed-power-split HK scheme at high SNR regime. Using the result given by Chong *et al.* [11], our simplified HK constraints are the following:

$$R_1 \leq \mathbb{E}\log\left|I + \mathrm{SNR}_{11}H_{11}H_{11}^\dagger\right| \tag{5}$$

$$R_2 \leq \mathbb{E}\log\left|I + \mathrm{SNR}_{22}H_{22}H_{22}^\dagger\right| \tag{6}$$

$$R_1 + R_2 \leq \mathbb{E}\log\left|I + \mathrm{SNR}_{11}H_{11}H_{11}^\dagger + \mathrm{INR}_{21}H_{21}H_{21}^\dagger\right| + \mathbb{E}\log\left|I + \frac{\mathrm{SNR}_{22}}{1+\mathrm{INR}_{21}}H_{22}H_{22}^\dagger\right| \tag{7}$$

$$R_1 + R_2 \le \mathbb{E}\log\left|I + \mathrm{SNR}_{22}H_{22}H_{22}^{\dagger} + \mathrm{INR}_{12}H_{12}H_{12}^{\dagger}\right| + \mathbb{E}\log\left|I + \frac{\mathrm{SNR}_{11}}{1+\mathrm{INR}_{12}}H_{11}H_{11}^{\dagger}\right| \quad (8)$$

$$R_1 + R_2 \le \mathbb{E}\log\left|I + \frac{\mathrm{SNR}_{11}}{1+\mathrm{INR}_{12}}H_{11}H_{11}^{\dagger} + \mathrm{INR}_{21}H_{21}H_{21}^{\dagger}\right| + \mathbb{E}\log\left|I + \frac{\mathrm{SNR}_{22}}{1+\mathrm{INR}_{21}}H_{22}H_{22}^{\dagger} + \mathrm{INR}_{12}H_{12}H_{12}^{\dagger}\right| \quad (9)$$

$$2R_1 + R_2 \le \mathbb{E}\log\left|I + \mathrm{SNR}_{11}H_{11}H_{11}^{\dagger} + \mathrm{INR}_{21}H_{21}H_{21}^{\dagger}\right| + \mathbb{E}\log\left|I + \frac{\mathrm{SNR}_{11}}{1+\mathrm{INR}_{12}}H_{11}H_{11}^{\dagger}\right| + \mathbb{E}\log\left|I + \frac{\mathrm{SNR}_{22}}{1+\mathrm{INR}_{21}}H_{22}H_{22}^{\dagger} + \mathrm{INR}_{12}H_{12}H_{12}^{\dagger}\right| \quad (10)$$

$$R_1 + 2R_2 \le \mathbb{E}\log\left|I + \mathrm{SNR}_{22}H_{22}H_{22}^{\dagger} + \mathrm{INR}_{12}H_{12}H_{12}^{\dagger}\right| + \mathbb{E}\log\left|I + \frac{\mathrm{SNR}_{22}}{1+\mathrm{INR}_{21}}H_{22}H_{22}^{\dagger}\right| + \mathbb{E}\log\left|I + \frac{\mathrm{SNR}_{11}}{1+\mathrm{INR}_{12}}H_{11}H_{11}^{\dagger} + \mathrm{INR}_{21}H_{21}H_{21}^{\dagger}\right|. \quad (11)$$

III. GDOF of General MIMO-GIFCs

In Section II-B, we give the simplified rate constraints (5)-(11) for the fixed-power-split HK scheme in MIMO-GIFC. To relate them to the GDOF, we recall from [3], [5] that the GDOF is defined as the region of (r_1, r_2) such that (R_1, R_2) with $R_i = r_i \log \mathrm{SNR}$ is within the HK region by letting $\mathrm{SNR} \to \infty$ while keeping $\beta_{i,j}$ constant for all i, j.

A quick examination of constraints (5)-(11) shows the determination of GDOF in general hinges on the finding of asymptotic expression of $\mathbb{E}\log\left|I + \mathrm{SNR}_1 H_1 H_1^{\dagger} + \mathrm{SNR}_2 H_2 H_2^{\dagger}\right|$ with $\mathrm{SNR}_1 = \mathrm{SNR}^{\beta_1}$ and $\mathrm{SNR}_2 = \mathrm{SNR}^{\beta_2}$ for all possible values of β_1 and β_2. It should be noted that such expression corresponds to the GDOF of the general MIMO multiple-access (MAC) channels where the users transmit at different asymptotic SNR levels. The case of either $\beta_1 \le 0$ or $\beta_2 \le 0$ is already seen in Remark 2. For the other cases, we have the following theorem.

Theorem 4 (GDOF in general MIMO-MAC channel): Let SNR_1, SNR_2, β_1 and β_2 be defined as above, and let H_1 and H_2 be $(n_r \times n_t)$ random matrices with i.i.d. $\mathbb{CN}(0,1)$ entries. Assume $\beta_1 \ge \beta_2 \ge 0$; then

$$\mathbb{E}\log\left|I + \mathrm{SNR}_1 H_1 H_1^{\dagger} + \mathrm{SNR}_2 H_2 H_2^{\dagger}\right| \doteq \mathfrak{C}(\beta_1, \beta_2)\log \mathrm{SNR}$$

where the function $\mathfrak{C}(\beta_1, \beta_2)$ is given by

$$\mathfrak{C}(\beta_1, \beta_2) = \begin{cases} \beta_1 n_r & \text{if } n_r \le n_t, \\ \beta_1 n_t + \beta_2(n_r - n_t) & \text{if } n_t \le n_r \le 2n_t, \\ (\beta_1 + \beta_2)n_t & \text{if } 2n_t \le n_r. \end{cases} \quad (12)$$

■

Applying Theorem 4 to constraints (5)-(11), we immediately obtain the region of GDOF of general MIMO-GIFCs for both symmetric and asymmetric GIFCs. Below we only present the result when $\beta_{11}, \beta_{22} \ge 0$. The other non-interesting cases can be easily obtained in a similar manner.

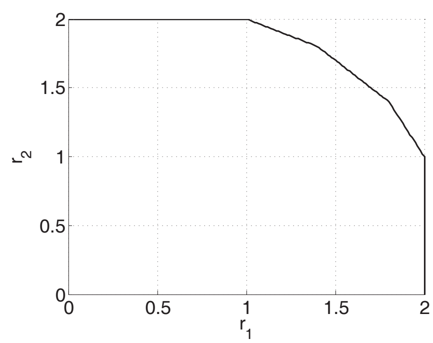

Fig. 1. Region of GDOF when $n_t = 2$, $n_r = 3$, $\beta_{11} = \beta_{22} = 1$, $\beta_{12} = \frac{2}{3}$, and $\beta_{21} = 0.8$.

Theorem 5 (GDOF in general MIMO-GIFC): Let β_{11}, β_{12}, β_{21}, and β_{22} be defined as before. Set $K := \max\{n_t, n_r\}$. Assume $\beta_{11}, \beta_{22} \ge 0$. Then the GDOF region of the fixed-power-split HK scheme is given by all $r_1, r_2 \ge 0$ that satisfy the following constraints:

$$\begin{cases} r_1 & \le & K\beta_{11}, \\ r_2 & \le & K\beta_{22}, \\ r_1 + r_2 & \le & \mathfrak{C}(\beta_{11}, (\beta_{21})^+) + K(\beta_{22} - \beta_{21})^+, \\ r_1 + r_2 & \le & \mathfrak{C}(\beta_{22}, (\beta_{12})^+) + K(\beta_{11} - \beta_{12})^+, \\ r_1 + r_2 & \le & \mathfrak{C}((\beta_{11} - \beta_{12})^+, (\beta_{21})^+) \\ & & +\mathfrak{C}((\beta_{22} - \beta_{21})^+, (\beta_{12})^+), \\ 2r_1 + r_2 & \le & \mathfrak{C}(\beta_{11}, (\beta_{21})^+) + K(\beta_{11} - \beta_{12}) \\ & & +\mathfrak{C}((\beta_{22} - \beta_{21})^+, (\beta_{12})^+), \\ r_1 + 2r_2 & \le & \mathfrak{C}(\beta_{22}, (\beta_{12})^+) + K(\beta_{22} - \beta_{21}) \\ & & +\mathfrak{C}((\beta_{11} - \beta_{12})^+, (\beta_{21})^+). \end{cases}$$

■

Here we do not attempt to derive a unified expression for the region of GDOF as such region can be easily computed numerically by Theorem 5. The region can be much complex than those of symmetric SISO-GIFCs. For example, in Fig. 1 we demonstrate the region of GDOF when $n_t = 2$, $n_r = 3$, $\beta_{11} = \beta_{22} = 1$, $\beta_{12} = \frac{2}{3}$, and $\beta_{21} = 0.8$. It is seen that such region is shaped by five straight lines with different slopes. Nevertheless, the region does have a very simple description whenever $n_r \ge 2n_t$. We provide without proof the following corollary.

Corollary 6 (Full GDOF): Let β_{11}, β_{12}, β_{21}, and β_{22} be defined as before. Assume $\beta_{11}, \beta_{22} \ge 0$. If $n_r \ge 2n_t$, then the region of GDOF is given by

$$\{(r_1, r_2) : 0 \le r_1 \le \beta_{11}n_t, 0 \le r_2 \le \beta_{22}n_t\}. \quad (13)$$

■

In other words, when $n_r \ge 2n_t$, the additional $(n_r - n_t) \ge n_t$ receive antennas can be used to resolve the interference caused by the other non-intending transmitter, and the full rectangle region of GDOF can be restored, independent of the power levels of the interference, i.e., it is achieved in all weak, mixed, strong, or very strong interference regimes. One such example is shown in Fig. 2 for an illustration.

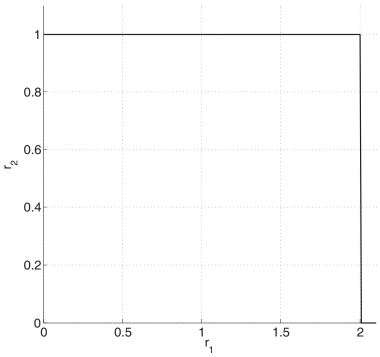

Fig. 2. Region of GDOF when $n_t = 2$, $n_r = 4$, $\beta_{11} = 1$, $\beta_{22} = 0.5$, $\beta_{12} = 0.3$, and $\beta_{21} = 0.2$.

A. *Proof-Sketch of Theorem 4*

Due to space limit, here we give only the proof of the case when $n_r \leq n_t$. We will briefly comment on the proofs of the other two cases later. First, it is easy to see

$$\left|I + \mathrm{SNR}_1 H_1 H_1^\dagger + \mathrm{SNR}_2 H_2 H_2^\dagger\right| \geq \left|I + \mathrm{SNR}_i H_i H_i^\dagger\right|$$

for $i = 1, 2$, hence taking expectation at both sides shows $\mathfrak{C}(\beta_1, \beta_2) \geq \max\{\beta_1, \beta_2\}K$. To show the converse when $n_r \leq n_t$, note simply that the function $\log|\cdot|$ is strictly convex, and

$$\begin{aligned}
&\mathbb{E}\log\left|I + \mathrm{SNR}_1 H_1 H_1^\dagger + \mathrm{SNR}_2 H_2 H_2^\dagger\right| \\
&< \log\left|\mathbb{E}\left(I + \mathrm{SNR}_1 H_1 H_1^\dagger + \mathrm{SNR}_2 H_2 H_2^\dagger\right)\right| \\
&= \log\left|I + n_t \mathrm{SNR}_1 I + n_t \mathrm{SNR}_2 I\right| \\
&\doteq \max\{\beta_1, \beta_2\} n_r \log \mathrm{SNR}.
\end{aligned}$$

Thus the case of $n_r \leq n_t$ is proven. Note that the above upper bound is loose when $n_r > n_t$. Proofs of the other two cases are much more difficult and lengthy. Let it suffice to say that our proof technique is based on partitioning the matrix $[\sqrt{\mathrm{SNR}_1} H_1 \sqrt{\mathrm{SNR}_2} H_2]$ in various ways, depending on the relation between n_r and n_t. Nevertheless, finding a general upper bound when $n_r > n_t$ is easy. To see this, let A and B be any $(m \times n)$ complex matrices. Fischer inequality [12] can be applied to establish the following inequality

$$\log\left|I + AA^\dagger + BB^\dagger\right| \;\leq\; \log\left|I + AA^\dagger\right| + \log\left|I + BB^\dagger\right|.$$

From here it leads to $\mathfrak{C}(\beta_1, \beta_2) \leq (\beta_1 + \beta_2)K$. Theorem 4 asserts that the equality holds whenever $n_r \geq 2n_t$.

IV. DMT IN MIMO-GIFCS

In Section II-B we have characterized the rate region of the fixed-power-split HK scheme using the inequalities (5)-(11) for ergodic channels where the mutual information is used as the performance measure for various systems. On the other hand, when channel is slow fading or block fading, a proper measurement of performance is the outage probability, which is defined as the probability of channel realizations $(H_{11}, \cdots, H_{22})$ such that the target rate-pair (R_1, R_2) cannot be met. For example, the outage event $\mathcal{O}_1$ characterized by the first inequality (5) is given by

$$\mathcal{O}_1 := \left\{(H_{11}, \cdots, H_{22}) : \log\left|I + \mathrm{SNR}_{11} H_{11} H_{11}^\dagger\right| \leq R_1\right\}.$$

Outage events $\mathcal{O}_i$ of the ith inequality, $i = 2, 3, \cdots, 7$, can be described in a similar fashion. Furthermore, we remark that as indicated by the proof of Theorem 1, the residual amount of information given by the term $(\frac{\mathrm{INR}_{ij}}{1+\mathrm{INR}_{ij}} H_{ij} H_{ij}^\dagger)$ can be safely ignored at high SNR regime since its contribution has order of $o(\log \mathrm{SNR})$ only.

Thus, the outage probability $P_{\text{out}}(r_1, r_2)$ is defined as

$$P_{\text{out}}(r_1, r_2) \;:=\; \Pr\{\mathcal{O}_1 \cup \cdots \cup \mathcal{O}_7\} \doteq \mathrm{SNR}^{-d_{\text{out}}(r_1, r_2)}$$

where $d_{\text{out}}(r_1, r_2)$ is the diversity, or the high-SNR exponent of the outage probability. Clearly, we have

$$\max_{1 \leq i \leq 7} \Pr\{\mathcal{O}_i\} \leq P_{\text{out}}(r_1, r_2) \leq \sum_{i=1}^{7} \Pr\{\mathcal{O}_i\}. \tag{14}$$

Let $\Pr\{\mathcal{O}_i\} \;\doteq\; \mathrm{SNR}^{-d_i(r_1, r_2)}$. It is immediate from (14) that

$$d_{\text{out}}(r_1, r_2) \;=\; \min_{i=1,\cdots,7} d_i(r_1, r_2). \tag{15}$$

Therefore, we do not have to be worried about the dependence between the inequalities (5)-(11).

A. *DMT in General MISO-GIFCs*

We begin with the DMT analysis of the case of $n_t \geq 1$ and $n_r = 1$ (hence called multiple-input single-output (MISO) channel) where the derivation is easier than the others. Again, we require the following theorem which characterizes the DMT of the basic building-block of the seven inequalities.

Theorem 7 (DMT in general MISO-MAC): Let $\mathrm{SNR}_1 = \mathrm{SNR}^{\beta_1}$ and $\mathrm{SNR}_2 = \mathrm{SNR}^{\beta_2}$, and let $\underline{h}_1$ and $\underline{h}_2$ be two i.i.d. $(1 \times n_t)$ random vectors with i.i.d. $\mathbb{C}\mathcal{N}(0, 1)$ entries; then the outage probability

$$\Pr\left\{\log\left|I + \mathrm{SNR}_1 \underline{h}_1 \underline{h}_1^\dagger + \mathrm{SNR}_2 \underline{h}_2 \underline{h}_2^\dagger\right| \leq r \log \mathrm{SNR}\right\}$$

has diversity

$$\begin{aligned}
\mu(\beta_1, \beta_2, r) &:= (\beta_1)^+ d^*_{n_t,1}\left(\frac{r}{(\beta_1)^+}\right) + (\beta_2)^+ d^*_{n_t,1}\left(\frac{r}{(\beta_2)^+}\right) \\
&= n_t\left[(\beta_1 - r)^+ + (\beta_2 - r)^+\right].
\end{aligned} \tag{16}$$

■

Armed with the above description of general DMT, characterizing the DMT in general MISO-GIFC is immediate. To compact our description, we define the following convolution operation of two DMT functions.

Definition 8: Let $f(r)$ and $g(r)$ be two DMT functions; then the convolution of $f(r)$ and $g(r)$ is defined as

$$f(r) \star g(r) := \inf\{f(x) + g(y) : x + y \leq r\}. \tag{17}$$

■

Now, the DMT in general MISO-GIFC is the following.

Corollary 9 (DMT in general MISO-GIFC): Let β_{11}, β_{12}, β_{21}, and β_{22} be defined as before. Assume $\beta_{11}, \beta_{22} \geq 0$.

Then the DMT of the fixed-power-split HK scheme for all $r_1, r_2 \geq 0$ in general MISO-GIFC is given by $\min\{d_i(r_1, r_2) : i = 1, 2, \cdots, 7\}$, where

$$\begin{cases} d_1(r_1, r_2) = n_t(\beta_{11} - r_1)^+, \\ d_2(r_1, r_2) = n_t(\beta_{22} - r_2)^+, \\ d_3(r_1, r_2) = \mu(\beta_{11}, \beta_{21}, r) \star \mu(\beta_{2d}, 0, r)|_{r=r_1+r_2}, \\ d_4(r_1, r_2) = \mu(\beta_{22}, \beta_{12}, r) \star \mu(\beta_{1d}, 0, r)|_{r=r_1+r_2}, \\ d_5(r_1, r_2) = \mu(\beta_{1d}, \beta_{21}, r) \star \mu(\beta_{2d}, \beta_{12}, r)|_{r=r_1+r_2}, \\ d_6(r_1, r_2) = \xi(\beta_{11}, \beta_{21}, \beta_{1d}, r) \star \mu(\beta_{2d}, \beta_{12}, r)|_{r=2r_1+r_2}, \\ d_7(r_1, r_2) = \xi(\beta_{22}, \beta_{12}, \beta_{2d}, r) \star \mu(\beta_{1d}, \beta_{21}, r)|_{r=r_1+2r_2}. \end{cases}$$

and where $\beta_{1d} := (\beta_{11} - \beta_{12})^+$, $\beta_{2d} := (\beta_{22} - \beta_{21})^+$, and $\xi(\beta_1, \beta_2, \beta_3, r) := n_t \cdot \inf_{p+q \leq r} \max\{(\beta_1 - p)^+, (\beta_3 - q)^+\} + (\beta_2 - p)^+\}$. ■

We remark that the need of function $\xi(\beta_1, \beta_2, \beta_3, r)$ is due to the dependence of H_{11} in (10) and H_{22} in (11). Again, here we do not attempt to find a universal expression of DMT in general MISO-GIFCs as the result in Corollary 9 can be fairly easily computed.

B. DMT in General SIMO-GIFCs

The case of $n_r \geq n_t = 1$ is referred to as the single-input multiple-output (SIMO) GIFCs. The DMT in general SIMO-GIFCs is actually much more interesting than its MISO counterpart. In Corollary 6 we have already seen that when $n_r \geq 2$, both RX_1 and RX_2 are able to achieve full GDOF from their intending transmitter, as if the non-intending transmitter does not exist. Thus, it is expected that the overall diversity takes the following very simple form:

$$d_{\text{out}}(r_1, r_2) \;=\; \min\left\{\beta_{11} d^*_{1,n_r}\left(\frac{r_1}{\beta_{11}}\right), \beta_{22} d^*_{1,n_r}\left(\frac{r_2}{\beta_{22}}\right)\right\}.$$

To see the above result, we again begin with characterizing the DMT of the basic building-block of (5)-(11).

Theorem 10 (DMT in general SIMO-MAC): Let $\text{SNR}_1 = \text{SNR}^{\beta_1}$ and $\text{SNR}_2 = \text{SNR}^{\beta_2}$, and let $\underline{h}_1$ and $\underline{h}_2$ be two i.i.d. $(n_r \times 1)$ random vectors with i.i.d. $\mathbb{C}\mathcal{N}(0, 1)$ entries; then the outage probability

$$\Pr\left\{\log\left|I + \text{SNR}_1 \underline{h}_1 \underline{h}_1^\dagger + \text{SNR}_2 \underline{h}_2 \underline{h}_2^\dagger\right| \leq r \log \text{SNR}\right\}$$

has diversity $\sigma(\beta_1, \beta_2, r) := n_r(\beta_1 - r)^+ + (n_r - 1)[\beta_2 - (r - \beta_1)^+]^+ + (\beta_2 - r)^+$. ■

Thus, the DMT in general SIMO-GIFC is the following.

Corollary 11 (DMT in general SIMO-GIFC): Let β_{11}, β_{12}, β_{21}, and β_{22} be defined as before. Assume $\beta_{11}, \beta_{22} \geq 0$. Then the DMT of the fixed-power-split HK scheme for all $r_1, r_2 \geq 0$ in general SIMO-GIFC is given by $\min\{d_i(r_1, r_2) : i = 1, 2, \cdots, 7\}$, where

$$\begin{cases} d_1(r_1, r_2) = n_r(\beta_{11} - r_1)^+, \\ d_2(r_1, r_2) = n_r(\beta_{22} - r_2)^+, \\ d_3(r_1, r_2) = \sigma(\beta_{11}, \beta_{21}, r) \star \sigma(\beta_{2d}, 0, r)|_{r=r_1+r_2}, \\ d_4(r_1, r_2) = \sigma(\beta_{22}, \beta_{12}, r) \star \sigma(\beta_{1d}, 0, r)|_{r=r_1+r_2}, \\ d_5(r_1, r_2) = \sigma(\beta_{1d}, \beta_{21}, r) \star \sigma(\beta_{2d}, \beta_{12}, r)|_{r=r_1+r_2}, \\ d_6(r_1, r_2) = \chi(\beta_{11}, \beta_{21}, \beta_{1d}, r) \star \sigma(\beta_{2d}, \beta_{12}, r)|_{r=2r_1+r_2}, \\ d_7(r_1, r_2) = \chi(\beta_{22}, \beta_{12}, \beta_{2d}, r) \star \sigma(\beta_{1d}, \beta_{21}, r)|_{r=r_1+2r_2}. \end{cases}$$

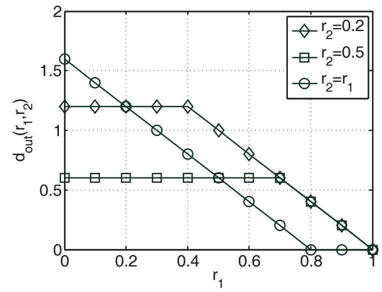

Fig. 3. DMTs in SIMO-GIFC when $n_r = 2$, $\beta_{11} = 1$, $\beta_{22} = 0.8$, $\beta_{12} = 0.3$, and $\beta_{21} = 0.6$.

and where $\beta_{1d} := (\beta_{11} - \beta_{12})^+$, $\beta_{2d} := (\beta_{22} - \beta_{21})^+$, and $\chi(\beta_1, \beta_2, \beta_3, r) := \inf\{\max\{n_r(\beta_1 - x)^+, n_r(\beta_3 - z)^+\} + (n_r - 1)(\beta_2 - y)^+ + (\beta_2 - x - y)^+ : x + y + z \leq r\}$. Moreover, when $n_r > 1$, it can be deduced that for any $\beta_{12}, \beta_{21} \in \mathbb{R}$

$$d_{\text{out}}(r_1, r_2) \;=\; n_r \cdot \min\left\{(\beta_{11} - r_1)^+, (\beta_{22} - r_2)^+\right\}.$$

■

In Fig. 3 we provide the DMT for $r_2 = 0.2$, $r_2 = 0.5$ and $r_2 = r_1$ when $n_r = 2$, $\beta_{11} = 1$, $\beta_{22} = 0.8$, $\beta_{12} = 0.3$, and $\beta_{21} = 0.6$. As $\beta_{22} < \beta_{11}$, it is seen that the DMT performance of such SIMO-GIFC is always limited by TX_2 whenever $r_2 > r_1$. On the other hand, if the channel is partially symmetric, i.e., $\beta_{11} = \beta_{22}$ and $r_1 = r_2$, then the single-user performance can always be achieved.

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