Phenomenology of a TeV right-handed neutrino and the dark matter model

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In a model of a TeV right-handed (RH) neutrino by Krauss, Nasri, and Trodden, the sub-eV scale neutrino masses are generated via a three-loop diagram with the vanishing seesaw mass forbidden by a discrete symmetry, and the TeV mass RH neutrino is simultaneously a novel candidate for cold dark matter. However, we show that with a single RH neutrino it is not possible to generate two mass-square differences as required by the oscillation data. We extend the model by introducing one more TeV RH neutrino and show that it is possible to satisfy the oscillation pattern within the modified model. After studying in detail the constraints coming from the dark matter, lepton flavor violation, the muon anomalous magnetic moment, and the neutrinoless double beta decay, we explore the parameter space and derive predictions of the model. Finally, we study the production and decay signatures of the TeV RH neutrinos at TeV $e^+e^-/\mu^+\mu^-$ colliders.

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I. INTRODUCTION

One of the most natural ways to generate a small neutrino mass is via the seesaw mechanism [1]. There are very heavy right-handed neutrinos, which are gauge singlets of the standard model (SM), and so they could have a large Majorana mass M_R. After electroweak symmetry breaking, a Dirac mass term M_D between the right-handed and the left-handed neutrinos can be developed. Therefore, after diagonalizing the neutrino mass matrix, a small Majorana mass $\sim m_D^2/M_R$ for the left-handed neutrino is obtained. This is a very natural mechanism, provided that $M_R \sim 10^{11}$–10^{13} GeV. One drawback of this scheme is that these right-handed neutrinos are too heavy to be produced in any terrestrial experiments. Therefore, phenomenologically there are not many channels to test the mechanism. Although it could be possible to get some hints from the neutrino masses and mixing, it is rather difficult to reconstruct the parameters of the right-handed neutrinos using the low energy data [2].

Another natural way to generate a small neutrino mass is via higher loop processes, e.g., the Zee model [3], with some lepton number violating couplings. However, these lepton number violating couplings are also subject to experimental constraints, e.g., $\mu \to e\gamma$, $\tau \to e\gamma$. In the Zee model, there are also extra scalars whose masses are of electroweak scale, and so can be observed at colliders [4].

On the other hand, recent cosmological observations have established the concordance cosmological model where the present energy density consists of about 73% of cosmological constant (dark energy), 23% (nonbaryonic) cold dark matter, and just 4% of baryons. To clarify the identity of the dark matter remains a prime open problem in cosmology and particle physics. Although quite a number of promising candidates have been proposed and investigated in detail, other possibilities can never be neglected.

Recently, Krauss, Narsi, and Trodden [5] considered an extension to the SM, similar to the Zee model, with two additional charged scalar singlets and a TeV right-handed neutrino. They showed that with an additional discrete symmetry the Dirac mass term between the left-handed and right-handed neutrinos are forbidden and thus avoiding the seesaw mass. Furthermore, the neutrino mass can only be generated at three-loop level, and sub-eV neutrino masses can be obtained with the masses of the charged scalars and the right-handed neutrino of order of TeV. Phenomenologically, this model is interesting because the TeV right-handed neutrino can be produced at colliders and could be a dark matter candidate.

In this work, we explore in detail the phenomenology of the TeV right-handed (RH) neutrinos. We shall extend the analysis to three families of left-handed neutrinos and explore the region of the parameters that can accommodate the present oscillation data. In the course of our study, we found that the model in Ref. [5] with a single RH neutrino cannot explain the oscillation data, because it only gives one mass-square difference. We extend the model by adding another TeV RH neutrino, which is slightly heavier than the first one. We demonstrate that it is possible to accommodate the oscillation pattern. We also obtain the relic density of the RH neutrino, and discuss the possibility of detecting them if they form a substantial fraction of the dark matter. We also study the lepton number violating processes, the muon anomalous magnetic moment, and production at leptonic colliders. In particular, the pair production of N_1N_2, N_2N_2 at $e^+e^-/\mu^+\mu^-$ colliders gives rise to very interesting signatures. The N_2 so produced will decay into N_1 plus a pair of charged leptons inside the detector. Thus, the signature would be either one or two pairs of charged leptons plus a large missing energy.

The organization is as follows. We describe the model in the next section. In Sec. III, we explore all the phenomenology associated with the TeV RH neutrino. In Sec. IV, we discuss the signatures in collider experiments. Section V is devoted to a conclusion.

II. REVIEW OF THE MODEL

The model considered in Ref. [5] has two extra charged scalar singlets S_1, S_2 and a right-handed neutrino N_R. A discrete Z_2 symmetry is imposed on the particles, such that all SM particles and S_1 are even under Z_2 but S_2, N_R are odd under Z_2. Therefore, the Dirac mass term $\overline{L}\phi N_R$ is forbidden, where ϕ is the SM Higgs boson. The seesaw mass is avoided.

In the present work, we extend the model a bit further by adding the second TeV right-handed neutrino, which also has odd Z_2 parity. The reason for this is because with only 1 TeV RH neutrino, it is impossible to obtain two mass-square differences, as required by the oscillation data. However, with two TeV RH neutrinos it is possible to accommodate two mass-square differences with the corresponding large mixing angles. We will explicitly show this result in the next section. The most general form for the interaction Lagrangian is[1]

$$\mathcal{L} = f_{\alpha\beta} L_\alpha^T C i \tau_2 L_\beta S_1^+ + g_{1\alpha} N_1 S_2^+ \ell_{\alpha R} + g_{2\alpha} N_2 S_2^+ \ell_{\alpha R} + V(S_1, S_2) + \text{H.c.} + M_{N_1} N_1^T C N_1 + M_{N_2} N_2^T C N_2, \quad (1)$$

where α, β denote the family indices, C is the charge-conjugation operator, and $V(S_1, S_2)$ contains a term $\lambda_s (S_1 S_2^*)^2$. Note that $f_{\alpha\beta}$ is antisymmetric under interchange of the family indices. Note that even with the presence of the first term in the Lagrangian it cannot give rise to the one-loop Zee diagrams for neutrino mass generation, because there is no mixing term between the Zee charged scalar S_1^+ and the standard model Higgs doublet that can generate the charged lepton mass.

If the masses of N_1, N_2, S_1, S_2 are arranged such that $M_{N_1} < M_{N_2} < M_{S_1} < M_{S_2}$, N_1 would be stable if the Z_2 parity is maintained. The N_1 could be a dark matter candidate provided that its interaction is weak enough. Also, N_1, N_2 must be pair produced or produced associated with S_2 because of the Z_2 parity. The N_2 so produced would decay into N_1 and a pair of charged leptons. The decay time may be long enough to produce a displaced vertex in the central detector. The S_2, if produced, would also decay into N_1, N_2, and a charged lepton. We will discuss the phenomenology in details in the next section.

III. PHENOMENOLOGY

A. Neutrino masses and mixings

The goal here is to find the parameter space of the model in Eq. (1) such that the neutrino mass matrix so obtained can accommodate the maximal mixing for the atmospheric neutrino, the large mixing angle for the solar neutrino, and the small mixing angle for θ_{13} [6]:

$$\Delta m_{\rm atm} \approx 2.7 \times 10^{-3} \ \text{eV}^2, \quad \sin^2 2\theta_{\rm atm} = 1.0,$$
$$\Delta m_{\rm sol} \approx 7.1 \times 10^{-5} \ \text{eV}^2, \quad \tan^2 \theta_{\rm sol} = 0.45,$$
$$\sin^2 2\theta_{13} \lesssim 0.1. \quad (2)$$

The three-loop Feynman diagram that contributes to the neutrino mass matrix has been given in Ref. [5]. The neutrino mass matrix $(M_\nu)_{\alpha\beta}$ is given by

$$(M_\nu)_{\alpha\beta} \sim \frac{1}{(4\pi^2)^3} \frac{1}{M_{S_2}} \lambda_s f_{\alpha\rho} m_{\ell_\rho} g_\rho g_\sigma m_{\ell_\sigma} f_{\sigma\beta}, \quad (3)$$

where α, β denote the flavor of the neutrino. Note that in the Zee model, the neutrino mass matrix entries are proportional to $f_{\alpha\beta}$ such that only off-diagonal matrix elements are nonzero. It is well known that the Zee model gives bimaximal mixings, which have some difficulties with the large-mixing angle solution of the solar neutrino [6]. Here in Eq. (3) we do not have the second Higgs doublet to give a mixing between the SM Higgs doublet and S_1^+, and therefore the one-loop Zee-type diagrams are not possible. However, the mass matrix in Eq. (3) allows for nonzero diagonal elements, which may allow the departure from the bimaximal mixings.

The mixing matrix between flavor eigenstates and mass eigenstates is given as

$$U_{\alpha i} = \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12} & -s_{23}c_{12} - s_{13}s_{12}c_{23} & c_{23}c_{13} \end{pmatrix}, \quad (4)$$

where we have ignored the phases. The mass eigenvalues are given by

$$U^T M U = M_{\rm diag} = \text{diag}(m_1, m_2, m_3). \quad (5)$$

The mass-square differences and mixing angles are related to oscillation data by

$$\Delta m^2_{sol} \equiv \Delta m^2_{21} = m_2^2 - m_1^2$$
$$\Delta m^2_{atm} \equiv \Delta m^2_{32} = m_3^2 - m_2^2$$
$$\theta_{sol} \equiv \theta_{12}$$
$$\theta_{atm} \equiv \theta_{23}. \quad (6)$$

From Eq. (3) the neutrino mass matrix is rewritten as

[1] In principle, there are terms sush as $N_1 N_2 \phi$ and $M N_1 N_2$. The latter explicitly gives a mixing between the two RH neutrinos, while the former also gives the mixing after the Higgs field develops a vacuum expectation value (VEV). However, the mixing term can be rotated away by redefining the N_1 and N_2 fields. Effectively, the Lagrangian has the form given in Eq. (1).

$$(M_\nu)_{\alpha\beta} \sim -\frac{\lambda_s}{(4\pi^2)^3 M_{S_2}} \begin{pmatrix} (fmg)_e^2 & (fmg)_e(fmg)_\mu & (fmg)_e(fmg)_\tau \\ (fmg)_e(fmg)_\mu & (fmg)_\mu^2 & (fmg)_\mu(fmg)_\tau \\ (fmg)_e(fmg)_\tau & (fmg)_\mu(fmg)_\tau & (fmg)_\tau^2 \end{pmatrix}, \tag{7}$$

where $(fmg)_\alpha = \Sigma_\rho f_{\alpha\rho} m_{\ell_\rho} g_\rho$, the mass eigenvalues are given by

$$m_1 = m_2 = 0, \tag{8}$$

$$m_3 \sim -\frac{\lambda_s}{(4\pi^2)^3 M_{S_2}} [(fmg)_e^2 + (fmg)_\mu^2 + (fmg)_\tau^2]. \tag{9}$$

This model obviously cannot explain the neutrino oscillation data because of the vanishing Δm_{21}^2.

Hereafter we would like to discuss a possibility to improve this shortcoming. The reason that this model predicts two vanishing mass eigenvalues is the proportionality relation in the mass matrix (7). Therefore it is necessary to break the proportionality relation. Although one way to improve the mass matrix might be to add small perturbations to the original mass matrix, we, however, found that this approach cannot resolve the difficulty. Instead, we consider a modification of the right-handed neutrino sector. As mentioned before, we employ two TeV RH neutrinos, the mass matrix (7) is replaced by

$$(M_\nu)_{\alpha\beta} \sim \frac{1}{(4\pi^2)^3} \frac{1}{M_{S_2}} \lambda_s \sum_{I=1,2} (fmg_I)_\alpha (g_I mf)_\beta, \tag{10}$$

where I denotes the two RH neutrinos.

If we assume $(fmg_2)_\mu \ll (fmg_1)_e$, Eq. (10) is rewritten as

$$(M_\nu)_{\alpha\beta} \sim -\frac{\lambda_s (fmg_1)_e^2}{(4\pi^2)^3 M_{S_2}} \begin{pmatrix} 1+c^2 & w & t+cd \\ w & w^2 & wt \\ t+cd & wt & t^2+d^2 \end{pmatrix}, \tag{11}$$

$$w = (fmg_1)_\mu/(fmg_1)_e,$$
$$t = (fmg_1)_\tau/(fmg_1)_e,$$
$$c = (fmg_2)_e/(fmg_1)_e, \tag{12}$$
$$d = (fmg_2)_\tau/(fmg_1)_e,$$

and has one zero and two nonzero eigenvalues:

$$m_\pm \sim -\frac{\lambda_s (fmg_1)_e^2}{(4\pi^2)^3 M_{S_2}} \lambda_\pm, \tag{13}$$

where

$$2\lambda_\pm = 1 + w^2 + t^2 + c^2 + d^2 \pm \sqrt{(1+w^2+t^2+c^2+d^2)^2 - 4(d^2 + c^2 w^2 + d^2 w^2 - 2cdt + c^2 t^2)}, \tag{14}$$

and each of the mixing angles is given by

$$t_{23} = \frac{w(\lambda_+ - c^2 - d^2)}{t(\lambda_+ - c^2) + cd}, \tag{15}$$

$$s_{13} = \frac{\lambda_+ - d^2 - tcd}{\sqrt{(\lambda_+ - d^2 - tcd)^2 + (1+t_{23}^2) w^2 (\lambda_+ - c^2 - d^2)^2}}, \tag{16}$$

$$c_{12} = \frac{1}{c_{13}} \frac{dw}{\sqrt{(c^2+d^2)w^2 + (ct-d)^2}}, \tag{17}$$

where we adopt the normal mass hierarchy. Indeed, we found that the correct mixing angles could not be realized if we assumed the inverted mass hierarchy here. Here $t_{23} \simeq 1$, $s_{13}^2 \ll 1$ imply $w \simeq t$, $\lambda_+ \gg c^2, d^2$ and $w^2 \gg 1$. This means $t^2 \simeq w^2 \gg 1, c^2, d^2$. Definitely, from

$$\sin^2 2\theta_{13} \simeq \frac{2}{w^2}\left(1 - \frac{tcd}{\lambda_+}\right)^2 \left(\frac{2}{1+t_{23}^2}\right) \lesssim 0.1, \tag{18}$$

we obtain $w^2 \gtrsim 20$. Since Eq. (17) is rewritten as

$$t_{12}^2 \simeq \frac{c^2 w^2 + (cw-d)^2}{d^2 w^2}, \tag{19}$$

where $c_{13} \simeq 1$ and $w \simeq t$ are used, we obtain

$$\frac{c^2}{d^2} \sim \frac{1}{4}, \tag{20}$$

by comparing with Eq. (2). From the mass-square differences,

$$\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} \simeq \left(\frac{\lambda_-}{\lambda_+}\right)^2 \simeq \left(\frac{2c^2w^2+d^2w^2-2cdw}{4w^4}\right)^2 \simeq \left(\frac{3c^2}{2w^2}\right)^2 \sim 10^{-2}, \tag{21}$$

we find

$$c^2 \sim \pm\frac{4}{3}\left(\frac{w^2}{20}\right). \tag{22}$$

Finally, $\Delta m_{atm}^2 \simeq m_3^2 = m_+^2$ is rewritten as

$$2.7\times 10^{-3}\ \ \mathrm{eV}^2 \simeq \left(-\frac{40\lambda_s(fmg_1)_e^2}{(4\pi^2)^3 M_{S_2}}\right)^2\left(\frac{w^2}{20}\right)^2, \tag{23}$$

where we used $\lambda_+ \simeq 2w^2$. In Sec. III E, we find some parameter space that leads to correct mixing angles and mass-square differences, after considering also the constraints from the dark matter relic density and lepton flavor violation.

B. Neutrinoless double beta decay

A novel feature of the Majorana neutrino is the existence of neutrinoless double beta decay, which essentially requires a nonzero entry $(M_\nu)_{ee}$ of the neutrino mass matrix. Its nonobservation has put an upper bound on the size of $(M_\nu)_{ee} \lesssim 1$ eV [7].

In the model with two RH neutrinos, $(M_\nu)_{ee}$ is estimated to be

$$(M_\nu)_{ee} \sim -\frac{\lambda_s}{(4\pi^2)^3 M_{S_2}}[(fmg_1)_e^2+(fmg_2)_e^2] \sim 3\times 10^{-3}\left(\frac{1\pm\frac{3}{4}(20/w^2)}{2}\right)\ \ \mathrm{eV} \tag{24}$$

by using Eqs. (22) and (23). Thus, we find that this model is consistent with the current experimental bound. Such a small $(M_\nu)_{ee}$ may still be within reach of the GENIUS neutrinoless double beta decay experiment [8].

C. Dark matter: Density and detection

The lightest RH neutrino is stable because of the assumed discrete symmetry. Here we consider the relic density of the lightest RH neutrino, and the relic density must be less than the critical density of the Universe. First of all, we verify that the second lightest RH neutrino is of no relevance here because of the short decay time. The heavier RH neutrino will decay into the lighter one and two right-handed charged leptons, $N_2 \to N_1 \ell_\alpha^- \ell_\beta^+$ (α,β denote flavors), and its decay width is given by

$$\Gamma_{N_2} = \frac{M_{N_2}}{512\pi^3}|g_{1\beta}g_{2\alpha}|^2\frac{1}{2\mu_s^2}\Big[2(1-\mu_s)(\mu_1-\mu_s)(\mu_1+\mu_s+\mu_1\mu_s-3\mu_s^2)\log\left(\frac{\mu_s-\mu_1}{\mu_s-1}\right)+(1-\mu_1)\mu_s(2\mu_1-5\mu_s-5\mu_1\mu_s+6\mu_s^2)-2\mu_1^2\log\mu_1\Big], \tag{25}$$

where $\mu_1 = M_{N_1}^2/M_{N_2}^2, \mu_s = M_{S_2}^2/M_{N_2}^2$. In the worst case when M_{N_2} is very close to M_{N_1}, say, they are both of order 1 TeV but differ by 1 GeV only, and we set $g_i \sim 0.1$. In this case, the decay width is then of order $10^4 - 10^5\ \mathrm{s}^{-1}$, i.e., the decay time is still many orders smaller than the age of the present Universe. Therefore, the presence of N_2 will not affect the relic density of N_1.

The relevant interactions for the annihilation is $N_1N_1 \to \ell_{\alpha R}^+ \ell_{\beta R}^-$ through charged scalar S_2^+ exchange. The corresponding invariant matrix element is given by

$$|\mathcal{M}|^2 = \frac{|g_{1\alpha}g_{1\beta}|^2}{4}\left[\frac{(2q_1\cdot p_1)2q_2\cdot p_2}{(t-M_{S_2}^2)^2}+\frac{(2q_2\cdot p_1)2q_1\cdot p_2}{(u-M_{S_2}^2)^2}-\frac{2M_{N_1}^2 2p_1\cdot p_2}{(t-M_{S_2}^2)(u-M_{S_2}^2)}\right], \tag{26}$$

where q_i and p_i are four-momenta of the incoming N_1 particles and the outgoing leptons, respectively. Then, we obtain

$$2q_1^0 2q_2^0 \sigma v = \frac{d^3p_1}{(2\pi)^2 2p_1^0}\frac{d^3p_2}{(2\pi)^2 2p_2^0}(2\pi)^2|\mathcal{M}|^2\delta^{(4)}(q_1+q_2-p_1-p_2) \tag{27}$$

$$= \frac{1}{8\pi}\frac{|g_{1\alpha}g_{1\beta}|^2}{(M_{S_2}^2+s/2-M_{N_1}^2)^2}\left[\frac{m_{l\alpha}^2+m_{l\beta}^2}{2}\left(\frac{s}{2}-M_{N_1}^2\right)+\frac{8}{3}\frac{(M_{S_2}^2-M_{N_1}^2)^2+(s/2)(M_{S_2}^2-M_{N_1}^2)+s^2/8}{(M_{S_2}^2+s/2-M_{N_1}^2)^2}\frac{s}{4}\left(\frac{s}{4}-M_{N_1}^2\right)\right], \tag{28}$$

where $m_{l\alpha}$ is the lepton mass. We expanded $|\mathcal{M}|^2$ in powers of the three-momenta of these particles and integrated over the scattering angle in the second line. Following Ref. [9], the thermal averaged annihilation rate is estimated to be

$$\langle\sigma v\rangle=\left[\frac{M_{N_1}^2 T}{2\pi^2}K_2\left(\frac{M_{N_1}}{T}\right)\right]^{-2}\frac{T}{4(2\pi)^4}\int_{4M_{N_1}^2}^{\infty}ds \times\sqrt{s-4M_{N_1}^2}K_1(\sqrt{s}/T)(2q_1^0 2q_2^0\sigma v)$$
$$\simeq\sum_f\frac{|g_{1\alpha}g_{1\beta}|^2}{32\pi}\frac{M_{S_2}^4+M_{N_1}^4}{(M_{S_2}^2+M_{N_1}^2)^4}4M_{N_1}^2\left(\frac{T}{M_{N_1}}\right)$$
$$\equiv\sigma_0\left(\frac{T}{M_{N_1}}\right), \tag{29}$$

where Σ_f denotes the summation over lepton flavors, and we have omitted the contributions from the S-wave annihilation terms, which are suppressed by the masses of the final state leptons. The relic mass density is given by

$$\Omega_{N_1}h^2=1.1\times10^9\frac{2(M_{N_1}/T)}{\sqrt{g_*}M_p\langle\sigma v\rangle}\bigg|_{T_d}\quad\mathrm{GeV}^{-1}, \tag{30}$$

where T_d is the decoupling temperature, which is determined as

$$\frac{M_{N_1}}{T_d}\simeq\ln\left[\frac{0.152}{\sqrt{g_*(T_d)}}M_p\sigma_0 M_{N_1}\right]-\frac{3}{2}\ln\ln\left[\frac{0.152}{\sqrt{g_*(T_d)}}M_p\sigma_0 M_{N_1}\right], \tag{31}$$

and g_* is the total number of relativistic degrees of freedom in the thermal bath [10].

By comparing with the recent data from the Wilkinson Microwave Anisotropy Probe (WMAP) [11], we find

$$\Omega_{DM}h^2=0.113=2.2\times10^{12}\left(\frac{M_{N_1}}{10^3\ \mathrm{GeV}}\right)\frac{(M_{N_1}/T_d)^2}{\sqrt{g_*}M_p\sigma_0 M_{N_1}}. \tag{32}$$

We can calculate σ_0 from Eqs. (31) and (32), and we obtain

$$\sigma_0\simeq1.4\times10^{-7}\left(\frac{10^2}{g_*(T_d)}\right)^{1/2}\left\{1+0.07\ln\left[\left(\frac{M_{N_1}}{10^3\ \mathrm{GeV}}\right)\times\left(\frac{10^2}{g_*(T_d)}\right)\right]\right\}\quad\mathrm{GeV}^{-2}, \tag{33}$$

if we ignore the second term in Eq. (31). Indeed, we can confirm the validity of this assumption within about 10% error by using Eq. (33). Actually, Eq. (31) is evaluated to be

$$\frac{M_{N_1}}{T_d}\simeq\ln(2.5\times10^{13})-\frac{3}{2}\ln\ln(2.5\times10^{13})=31-5.1=26. \tag{34}$$

Our result of $\langle\sigma v\rangle$ is consistent with a previous estimation [12]. Equations (29) and (33) read

$$\sum_f|g_{1\alpha}g_{1\beta}|^2\simeq1\left(\frac{M_{N_1}}{1.3\times10^2\ \mathrm{GeV}}\right)^2\left(\frac{1+M_{S_2}^2/M_{N_1}^2}{1+2}\right)^4\left(\frac{1+2^2}{1+M_{S_2}^4/M_{N_1}^4}\right). \tag{35}$$

It is obvious that the RH neutrino must be as light as $\sim10^2$ GeV and at least one of $g_{1\alpha}$ should be of order of unity, such that the relic density is consistent with the dark matter measurement.[2] As the mass difference between M_{S_2} and N_1 becomes larger, the upper bound on M_{N_1} becomes smaller provided that we keep $g\lesssim1$.

The detection of the RH neutrinos as a dark matter candidate depends on its annihilation cross section and its scattering cross section with nucleons. Conventional search of dark matter employs an elastic scattering signal of the dark matter with the nucleons. We do not expect that the N_R dark matter would be easily identified by this method, given its very mild interaction. In addition, because of the Majorana nature the annihilation into a pair charged lepton at the present velocity ($v_{\mathrm{rel}}\sim0$) is also highly suppressed by the small lepton mass, even in the case of the tau lepton. However, one possibility was pointed out by Baltz and Bergstrom [12] that the annihilation $N_1N_1\to\ell^+\ell^-\gamma$ would not suffer from helicity suppression. The rate of this process is approximately α/π times the annihilation rate at the freeze-out. As will be indicated later, the dominant mode would be $\mu^+\mu^-\gamma$. There is a slight chance to observe the excess in positron, but, however, the energy spectrum is softened because of the cascade from the muon decay. However, the chance of observing the photon spectrum is somewhat better [12].

D. Lepton flavor changing processes and $g-2$

There are two sources of lepton flavor violation in Eq. (1). The first one is from the interaction $f_{\alpha\beta}L_\alpha^T Ci\tau_2 L_\beta S_1^+$. This one is similar to the Zee model. (However, the present model would not give rise to neutrino mass terms in one loop because of the absence of the S_1^+-ϕ mixing.) The flavor violating amplitude of $\ell_\alpha\to\ell_\rho$ via an intermediate ν_β would be proportional to $|f_{\alpha\beta}f_{\beta\rho}|$. The second source is from the term $g_{I\alpha}N_I S_2^+\ell_{\alpha R}$ in the Lagrangian (1). The flavor violating am-

[2]Krauss *et al.* [5] claimed that $M_{N_R}\sim1$ TeV and $g^2\sim0.1$ is consistent with the dark matter constraint, but in their rough estimation a numerical factor of $(T_D/M_N)/8\sim200$ is missing from the equation of $\langle\sigma v\rangle$.

plitude of $\ell_\alpha \to \ell_\beta$ via an intermediate N_I would be proportional to $|g_{I\alpha} g_{I\beta}|$. We apply these two sources to the radiative decays of $\ell_\alpha \to \ell_\beta \gamma$ and the muon anomalous magnetic moment.

The new contribution to the muon anomalous magnetic moment can be expressed as

$$\Delta a_\mu = \frac{m_\mu^2}{96\pi^2}\left(\frac{|f_{\mu\tau}|^2 + |f_{\mu e}|^2}{M_{S_1}^2} + \frac{6|g_{1\mu}|^2}{M_{S_2}^2} F_2(M_{N_1}^2/M_{S_2}^2) + \frac{6|g_{2\mu}|^2}{M_{S_2}^2} F_2(M_{N_2}^2/M_{S_2}^2)\right), \tag{36}$$

where $F_2(x) = (1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x)/6(1-x)^4$. The function $F_2(x) \to 1/6$ for $x \to 0$, and $F_2(0.25) \approx 0.125$. We naively set $F_2(x) = 1/6$ for a simple estimate. Therefore, we obtain

$$\Delta a_\mu = 3\times 10^{-10}\left[(|f_{\mu\tau}|^2 + |f_{\mu e}|^2)\left(\frac{2\times 10^2 \ \text{GeV}}{M_{S_1}}\right)^2 + (|g_{1\mu}|^2 + |g_{2\mu}|^2)\left(\frac{2\times 10^2 \ \text{GeV}}{M_{S_2}}\right)^2\right] \lesssim 10^{-9}, \tag{37}$$

which implies that $f_{23}, f_{21}, g_{1\mu}, g_{2\mu}$ can be as large as $O(1)$ for $O(200 \text{ GeV})$ S_1^+, S_2^+ without contributing in a significant level to Δa_μ.

Among the radiative decays $\mu \to e\gamma$ is the most constrained experimentally, $B(\mu \to e\gamma) < 1.2\times 10^{-11}$ [13]. The contribution of the our model is

$$B(\mu \to e\gamma) = \frac{\alpha v^4}{384\pi}\left[\frac{|f_{\mu\tau} f_{\tau e}|^2}{M_{S_1}^4} + \frac{36|g_{1e} g_{1\mu}|^2}{M_{S_2}^4} F_2^2(M_{N_1}^2/M_{S_2}^2) + \frac{36|g_{2e} g_{2\mu}|^2}{M_{S_2}^4} F_2^2(M_{N_2}^2/M_{S_2}^2)\right], \tag{38}$$

where $v = 246$ GeV. Again we take $F_2(x) = 1/6$ and $O(200 \text{ GeV})$ mass for S_1^+, S_2^+ for a simple estimate:

$$B(\mu \to e\gamma) = 1.4\times 10^{-5}\left[(|f_{\mu\tau} f_{\tau e}|^2)\left(\frac{2\times 10^2 \ \text{GeV}}{M_{S_1}}\right)^4 + |g_{1e} g_{1\mu}|^2\left(\frac{2\times 10^2 \ \text{GeV}}{M_{S_2}}\right)^4 + |g_{2e} g_{2\mu}|^2\left(\frac{2\times 10^2 \ \text{GeV}}{M_{S_2}}\right)^4\right] < 1.2\times 10^{-11}, \tag{39}$$

which implies that

$$|f_{e\tau} f_{\tau\mu}| < 1\times 10^{-3},$$
$$|g_{1e} g_{1\mu}| < 1\times 10^{-3},$$
$$|g_{2e} g_{2\mu}| < 1\times 10^{-3}. \tag{40}$$

This is in contrast to a work by Dicus *et al.* [14]. In their model, the couplings g_i's are much larger than f_{ij}'s.

E. An example of consistent model parameters

Here we summarize the constraints from previous subsections, and illustrate some allowed parameter space. The prime constraints come from neutrino oscillations. The maximal mixing and the mass-square difference required in the atmospheric neutrino and the small θ_{13} read

$$f_{\tau\mu} m_\mu g_{1\mu} \simeq f_{\mu\tau} m_\tau g_{1\tau} \gg f_{e\mu} m_\mu g_{1\mu} + f_{e\tau} m_\tau g_{1\tau} \sim \sqrt{\frac{1}{\lambda_s}\left(\frac{M_{S_2}}{10^2 \ \text{GeV}}\right)} \ \text{MeV}, \tag{41}$$

where the terms $f_{\tau e} m_e g_{1e}$ and $f_{\mu e} m_e g_{1e}$ have been omitted because these terms are suppressed by electron mass. The large mixing angle and the mass-square difference required in the solar neutrino are given by

$$f_{\tau e} m_e g_{2e} + f_{\tau\mu} m_\mu g_{2\mu} \simeq 2(f_{e\mu} m_\mu g_{2\mu} + f_{e\tau} m_\tau g_{2\tau}) \gg f_{\mu e} m_e g_{2e} + f_{\mu\tau} m_\tau g_{2\tau}, \tag{42}$$

$$\left(\frac{f_{e\mu} m_\mu g_{2\mu} + f_{e\tau} m_\tau g_{2\tau}}{f_{\tau\mu} m_\mu g_{1\mu}}\right)^2 \simeq \frac{2}{3}\times 10^{-1}. \tag{43}$$

On the other hand, the dark matter constraint requires at least one of the $g_{1e}, g_{1\mu}, g_{1\tau}$ to be of order of unity. While the muon anomalous magnetic moment does not impose any strong constraints, lepton flavor violating processes, especially $B(\mu \to e\gamma)$, give the following strong constraints:

$$|f_{\mu\tau} f_{\tau e}| \lesssim 1\times 10^{-3}, \tag{44}$$

$$|g_{1e} g_{1\mu}|, |g_{2e} g_{2\mu}| \lesssim 1\times 10^{-3}. \tag{45}$$

Now, let us look for an example of consistent parameters. From Eq. (41), we obtain $|m_\mu g_{1\mu}| \simeq |m_\tau g_{1\tau}|$, in other words $|g_{1\mu}| \gg |g_{1\tau}|$, and

$$f_{\tau\mu} \gg f_{e\mu} + f_{\tau e}. \tag{46}$$

Since either $g_{1\mu}$ or g_{1e} must be of order of unity from the dark matter constraint, we take $g_{1\mu} \simeq 1$. From Eqs. (42) and (43) with $g_{2\tau} \simeq 0$, we obtain

$$f_{\tau\mu} \simeq 2 f_{e\mu}, \ |m_\mu g_{2\mu}| \gg |m_e g_{2e}| \tag{47}$$

and

$$g_{2\mu}^2 \simeq 8/3\times 10^{-1} g_{1\mu}^2 \simeq 0.27(g_{1\mu}/1)^2. \tag{48}$$

Equations (46) and (47) can be rewritten as

$$1 \gg \frac{1}{2} + \frac{f_{\tau e}}{f_{\tau\mu}}, \tag{49}$$

where we find that a mild cancellation between $f_{e\mu}$ and $f_{\tau e}$ is necessary. For instance, $f_{\tau e}/f_{\tau\mu}=-1/3$. The strong cancellation corresponds to the small θ_{13}. However, a cancellation with too high accuracy would require a λ_s, which is too big by Eq. (41). Therefore, one can say that this model predicts a relatively large mixing in θ_{13}. Now we obtain an example set of parameters that makes this model workable and it is

$$|g_{1e}|\lesssim 1\times 10^{-3},\quad |g_{1\mu}|\simeq 1,\quad |g_{1\tau}|\simeq 0.06,$$
$$|g_{2e}|\lesssim 2\times 10^{-3},\quad |g_{2\mu}|\simeq 0.5,\quad |g_{2\tau}|< 10^{-2},\tag{50}$$
$$f_{e\mu}\simeq 1\times 10^{-2},\quad f_{\tau\mu}\simeq 2\times 10^{-2},\quad f_{e\tau}\sim -f_{e\mu}.$$

IV. PRODUCTION AT $e^+e^-,\mu^+\mu^-$ COLLIDERS

The decay of N_2 may have an interesting signature, a displaced vertex, in colliders. Depending on the parameters, N_2 could be able to travel a typical distance, e.g., mm, in the detector without depositing any kinetic energy, and suddenly decay into N_1 and two charged leptons. The signature is very striking.

The N_1N_1, N_2N_2, and N_1N_2 pairs can be directly produced at e^+e^- colliders. The differential cross section for $e^+e^-\to N_IN_I$, $I=1,2$, is given by

$$\frac{d\sigma}{d\cos\theta}(e^+e^-\to N_IN_I)=\frac{g_{Ie}^4}{256\pi}\frac{\beta_I}{s}\left[\frac{(t-M_{N_I}^2)^2}{(t-M_{S_2}^2)^2}+\frac{(u-M_{N_I}^2)^2}{(u-M_{S_2}^2)^2}-\frac{2M_{N_I}^2 s}{(t-M_{S_2}^2)(u-M_{S_2}^2)}\right],\tag{51}$$

where $\beta_I=\sqrt{1-4M_{N_I}^2/s}$, $t=M_{N_I}^2-(s/2)(1-\beta_I\cos\theta)$, $u=M_{N_I}^2-(s/2)(1+\beta_I\cos\theta)$. The total cross section is obtained by integrating over the angle θ:

$$\sigma(e^+e^-\to N_IN_I)=\frac{g_{Ie}^4}{64\pi s}\frac{2(x_I-x_s)^2+x_s}{-2x_I^3+x_I^2(6x_s+1)-2x_Ix_s(3x_s+2)+x_s(1+x_s)(1+2x_s)}\bigg[\beta_I(-2x_I+2x_s+1)$$
$$+2((x_I-x_s)^2+x_s)\log\left(\frac{2x_I-2x_s+\beta_I-1}{2x_I-2x_s-\beta_I-1}\right)\bigg],\tag{52}$$

where $x_I=M_{N_I}^2/s$ and $x_s=M_{S_2}^2/s$. For N_1N_2 production the differential cross section is given by

$$\frac{d\sigma}{d\cos\theta}(e^+e^-\to N_1N_2)=\frac{|g_{1e}g_{2e}|^2}{128\pi}\frac{\beta_{12}}{s}\left[\frac{(t-M_{N_1}^2)(t-M_{N_2}^2)}{(t-M_{S_2}^2)^2}+\frac{(u-M_{N_1}^2)(u-M_{N_2}^2)}{(u-M_{S_2}^2)^2}-\frac{2M_{N_1}M_{N_2}s}{(t-M_{S_2}^2)(u-M_{S_2}^2)}\right],\tag{53}$$

and the integrated cross section is

$$\sigma(e^+e^-\to N_1N_2)=\frac{|g_{1e}g_{2e}|^2}{128\pi}\frac{\beta_{12}}{s}\frac{4}{\beta_{12}s(-1+x_1+x_2-2x_s)(-1+\beta_{12}+x_1+x_2-2x_s)(1+\beta_{12}-x_1-x_2+2x_s)}$$
$$\times\Big\{\beta_{12}s(-1+x_1+x_2-2x_s)(-1+\beta_{12}^2+2x_1-x_1^2+2x_2-6x_1x_2-x_2^2-4x_s+8x_1x_s+8x_2x_s-8x_s^2)$$
$$+s[2\sqrt{x_1x_2}+(x_1+x_2)(x_1+x_2-4x_s-1)+2x_s(2x_s+1)][\beta_{12}^2-(-1+x_1+x_2-2x_s)^2]$$
$$\times\log\left(\frac{-1-\beta_{12}+x_1+x_2-2x_s}{-1+\beta_{12}+x_1+x_2-2x_s}\right)\Big\},\tag{54}$$

where $\beta_{12}=\sqrt{(1-x_1-x_2)^2-4x_1x_2}$. The above cross section formulas are equally valid for $\mu^+\mu^-$ collisions. Since the constraints from the last section restrict g_{1e} and g_{2e} to be hopelessly small, we shall concentrate on using $g_{1\mu}$ and $g_{2\mu}$.

The production cross sections for the N_2N_2 and N_1N_2 pairs are given in Figs. 1(a) and 1(b), respectively, for $\sqrt{s}=0.5,1,1.5$ TeV and for M_{N_2} from 150 to 800 GeV, and we have set $g_{1\mu}=1$, $g_{2\mu}=0.5$ [see Eq. (50)]. In the curve for N_1N_2, we set $M_{N_1}=M_{N_2}-50$ GeV. We are particularly in-

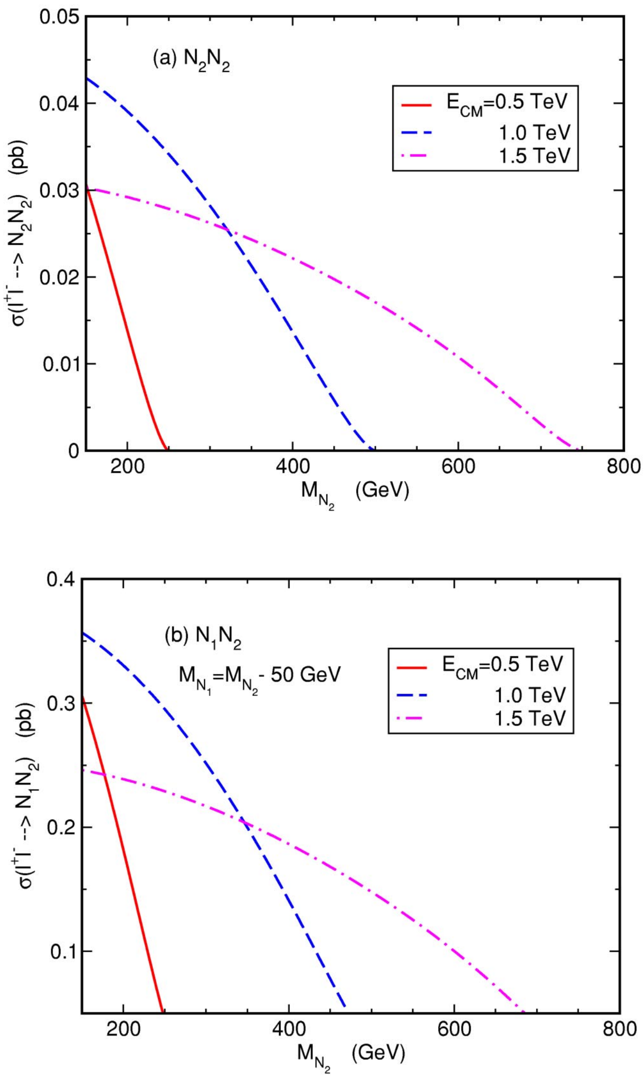

FIG. 1. Production cross sections for (a) N_2N_2 and (b) N_1N_2 pairs for $\sqrt{s}=0.5,1.0,1.5$ TeV at l^+l^- collisions. We have set $g_{1\mu}=1$, $g_{2\mu}=0.5$, as suggested by Eq. (50), $M_{S_2}=500$ GeV, and $M_{N_1}=M_{N_2}-50$ GeV.

terested in the N_1N_2, N_2N_2 production, because of its interesting signature.

As we have calculated the decay width of N_2 in Eq. (25), the N_2 can decay into N_1 plus two charged leptons, either promptly or after traveling a visible distance from the interaction point. It depends on the parameters involved, mainly the largest of $g_{1\beta}g_{2\alpha}$. As seen in Eq. (50) the largest is $|g_{1\mu}g_{2\mu}|\sim 0.5$, and so the decay of N_2 is prompt. Therefore, in the case of N_1N_2 production, the signature would be a pair of charged leptons plus missing energies, because the N_1's would escape the detection. The charged lepton pair is likely to be on one side of the event. In case of N_2N_2 production, the signature would be two pairs of charged leptons with a large missing energy. Note that in the case of N_1N_1 production, there is nothing in the final state that can be detected. From Fig. 1 the production cross sections are of order $O(10-100\ \text{fb})$, which implies plenty of events with $O(100\ \text{fb}^{-1})$ luminosity.

One may also consider $S_2^+S_2^-$ pair production. The S_2 so produced will decay into $S_2^\pm \to N_1\ell^\pm_{\alpha R}$ or $N_2\ell^\pm_{\alpha R}$, where $\ell_\alpha = e,\mu,\tau$. However, the constraints on the parameter space require the mass of M_{S_2} substantially heavier than N_1 and N_2, and therefore the $S_2^+S_2^-$ pair production cross section is relatively much smaller.

V. CONCLUSIONS

In this paper, we have discussed a model that explains the small neutrino mass and dark matter in the Universe at the same time. Such a model was proposed by Krauss *et al.* as a modification of Zee model. However, our study revealed that their original model is unfortunately not capable of explaining the neutrino oscillation pattern.

We have extended the model by introducing another right-handed neutrino. We succeed in showing that such an extension is possible to achieve the correct neutrino mixing pattern. A prediction of this model is the normal mass hierarchy. In addition, the undiscovered mixing angle θ_{13} is relatively large, because of the requirement of a mild cancellation between the parameters for a small θ_{13} and a sensible coupling of the charged scalar, λ_s.

The relic density of the lightest right-handed neutrino has also been revisited. Under the constraint by WMAP we found that the mass of the right-handed neutrino cannot be as large as TeV but only of order 1×10^2 GeV, after a careful treatment of the calculation. In addition, other constraints including the muon anomalous magnetic moment, radiative decay of muon, and neutrinoless double beta decay have also been studied. With all the constraints we are still able to find a sensible region of parameter space.

Finally, our improved model has an interesting signature at leptonic colliders via pair production of right-handed neutrinos, in particular N_1N_2 and N_2N_2. The N_2 so produced will decay into N_1 plus two charged leptons. Thus, the signature is either one or two pairs of charged leptons with a large missing energy. Hence, this model can be tested not only by neutrino experiments but also by collider experiments.

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