

Repacking on Demand for Two-Tier Wireless Local Loop

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Abstract—This paper proposes a radio channel assignment scheme called *repacking on demand (RoD)* for two-tier wireless local loop (WLL) networks. A two-tier WLL overlays a macrocell with several microcells. When a new call arrives at a two-tier WLL with RoD, if no idle channel is available in both the microcell and the macrocell, repacking is performed (i.e., a call in the macrocell is moved to its corresponding microcell), and then the reclaimed macrocell channel is used to serve the new call. An analytic model is proposed to compute the call blocking probability of the two-tier WLL with repacking. This analytic model is validated against simulation experiments. We prove that the blocking probability is not affected by the call holding time distributions, but is only dependent on the mean of the call holding times. Compared with some previous proposed schemes, RoD has low blocking probability and significantly reduces repacking rate.

Index Terms—Channel assignment, channel repacking, wireless local loop (WLL).

I. INTRODUCTION

WIRELESS local loop (WLL) provides wireless transmission paths between a *local exchange (LE)* and *customer premise equipment (CPE)*. Compared with the wireline local loop, the WLL offers advantages such as ease of installation, deployment, and concentration of resources [8]. Thus, WLL has been considered as a potential alternative for stationary subscribers to access the telephony services. Fig. 1 shows a typical WLL architecture, which consists of *subscriber terminal (ST)*, *base station (BS)*, and *base station controller (BSC)*. The ST collocated with the CPE is responsible for converting and delivering speech and control signals between the CPE (through the subscriber telephone line) and the corresponding BS (through the air interface). The BS provides radio channels for the STs in its radio coverage (i.e., cell). The BSC controls the BSs and STs to perform call setup and release between the LE and CPE.

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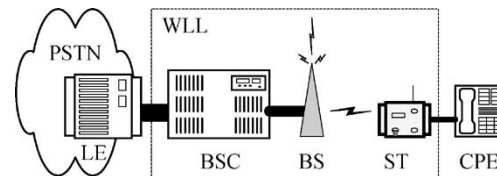


Fig. 1. Typical WLL architecture.

Traditional WLLs are “single-tier” where the BSs are populated such that the cells may be partially overlaid, but no cell is fully overlaid with another cell. In single-tier WLL, if all channels in a cell are busy, the next incoming calls are blocked even if other (nonoverlaid) cells have idle channels. Channels can be utilized more efficiently by two-tier WLL configuration consisting of low-tier BSs and high-tier BSs (see Fig. 2). In this configuration, a low-tier BS with low power transceiver provides small radio coverage (referred to as microcell), and a high-tier BS with high-power transceiver provides large radio coverage (referred to as macrocell). Every macrocell is overlaid with several microcells. Several STs are covered in each microcell. All STs are covered by the macrocell. If all channels in a microcell are busy (i.e., the microcell is *blocked*), the radio channels in the macrocell are allocated to serve the incoming calls of the blocked microcell. Thus, the call blocking effect can be reduced.

Several channel assignment approaches have been proposed for two-tier WLL [7]. A basic scheme or the so-called “no repacking” (NR) scheme was described in [11]. In this scheme, when a call (either incoming or outgoing) for an ST arrives, the WLL first tries to allocate a channel in the microcell of the ST. If no channel is available in that microcell, then the call overflows to the macrocell. If the macrocell has no idle channel, the call is blocked. Call blocking of this basic scheme can be improved by repacking techniques described as follows [16]. Consider a new call arrival $call_n$ to an ST in the i th microcell. Suppose that the i th microcell is blocked and $call_n$ is served by the macrocell. If radio channels are available in the i th microcell later, $call_n$ can be transferred back to that microcell again. The call handoff from the macrocell to the microcell is called “*repacking*.” This action increases the number of idle channels in the macrocell, and more macrocell channels can be shared by blocked microcells. Depending on the time when repacking is exercised, several schemes have been proposed. In always repacking (AR) [1], [10], [13], the WLL always moves a call (if exists) in the macrocell to the corresponding microcell as soon as a call is completed at that microcell. AR keeps maximum number of idle channels in the macrocell at the cost of high handoff rate [7]. Several studies [2]–[4], [15] have focused on radio resource management for two-tier

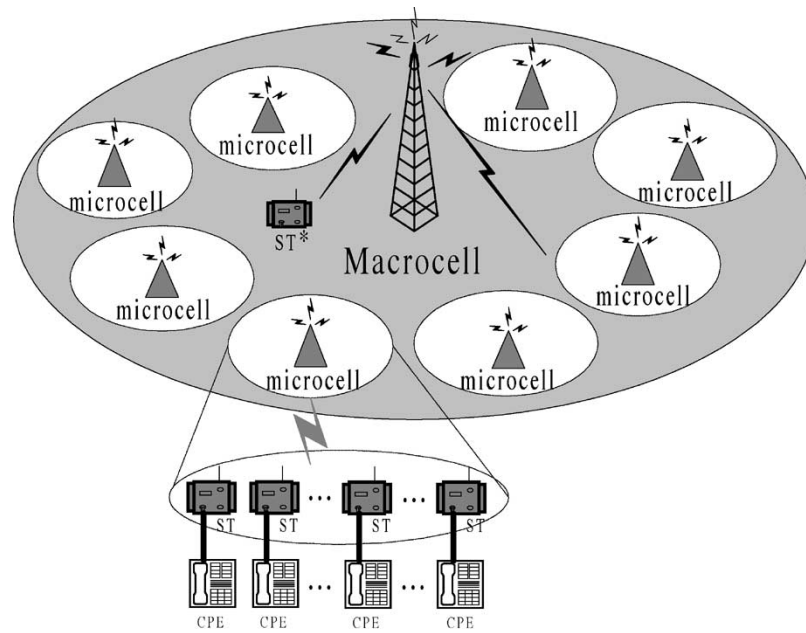


Fig. 2. Two-tier WLL configuration.

mobile systems. However, they do not consider repacking. Some packing approaches studied in [12] and [17] dealt with dynamic channel allocation for single-tier mobile systems. Some repacking schemes [5], [14] for mobile networks perform repacking based on the moving speeds of users. These schemes do not apply to two-tier WLL, and will not be elaborated further.

This paper proposes a new two-tier WLL scheme called *repacking on demand* (RoD). Unlike AR, RoD does not immediately perform repacking when a call in a microcell is completed. Instead, repacking is exercised only when it is necessary. RoD channel assignment is similar to NR, except that when a new call $call_n$ arrives, if both the microcell (of $call_n$) and the macrocell are blocked, the system attempts to move a call $call_r$ in the macrocell to the corresponding microcell (of $call_r$) that has idle channels. Then the reclaimed macrocell channel is used to serve $call_n$.

We develop an analytic model to investigate the call blocking performance of repacking schemes for two-tier WLL. This analytic model is validated against simulation experiments. We show that call blocking performance is the same for both RoD and AR. We also prove that call blocking is not affected by the call holding time distributions and is only dependent on the mean of the call holding times. We then compare the handoff performance between AR and RoD. Our study indicates that AR generates much more handoffs than RoD does.

II. RoD

This section describes the channel assignment and repacking procedures for RoD. We say that a new call attempt $call_n$ (incoming or outgoing) is generated from the i th microcell if $call_n$ is for an ST covered in the i th microcell. The WLL first assigns a channel in the i th microcell to $call_n$. If no idle channel is available in the i th microcell, then $call_n$ overflows to the macrocell.

If the macrocell has no idle channel, RoD is exercised to identify *repacking candidates*. Every repacking candidate is a call that satisfies the following criteria:

Criterion 1: the call occupies a macrocell channel.

Criterion 2: the microcell of this call has an idle channel.

RoD selects one or more repacking candidates to perform handoff from the macrocell to the corresponding microcells. Then a reclaimed macrocell channel is used to serve $call_n$. If RoD cannot find any repacking candidate, then $call_n$ is blocked.

There are several alternatives to handle the repacking candidates in RoD. In BSC-based RoD, repacking candidate selection and handoff decision are made by the BSC. Two policies can be used in BSC-based RoD. In RoD-R, the BSC randomly selects a repacking candidate for handoff. In RoD-L, the BSC selects the repacking candidate whose microcell has the least traffic loading. Both RoD-R and RoD-L can be adopted by the two-tier WLL that utilizes radio systems such as GSM/PCS1900 [9] or wideband code-division multiple-access [8], where the handoff decision is made by the network.

In ST-based RoD (referred to as RoD-ST), repacking candidate selection and handoff decision are made by the STs. RoD-ST can be adopted by the two-tier WLL that utilizes radio systems such as DECT, Personal Access Communication System (PACS), and Personal Handy-Phone System (PHS) [8]. RoD-ST is exercised when a new call attempt overflows to the macrocell and the macrocell has no idle channel. The RoD-ST message flow consists of the following steps:

- Step 1) The macrocell BS broadcasts the system information to all STs in the macrocell. The system information indicates that repacking is required.
- Step 2) Upon receipt of the system information broadcast in the macrocell, every ST that has a call connection through the macrocell (i.e., it occupies a macrocell channel) begins to listen to the system information broadcast by its microcell BS.

- Step 3) If the system information indicates that the microcell BS has idle channels, the ST executes the repacking step described next.
- Step 4) The ST sends a handoff request to the microcell BS.
- Step 5) The microcell BS accepts the request and replies an acceptance message to the ST.
- Step 6) A channel release message is sent to the macrocell BS to release the macrocell channel used by the ST.

Note that more than one call may be handed off from the macrocell to the microcells in RoD-ST. In this case, one of the released macrocell channels is chosen to serve the new call attempt.

III. ANALYTIC MODEL

This section proposes an analytic model to evaluate the call blocking performance of the repacking techniques. We assume that a macrocell is overlaid with M microcells. The macrocell has C radio channels and the i th microcell has c_i radio channels, where $1 \leq i \leq M$. We assume that the call arrivals to STs in the i th microcell (for both incoming and outgoing calls) are a Poisson stream with rate λ_i . The call holding time has a general distribution with mean $1/\mu$. Thus, the i th microcell experiences the traffic intensity $\rho_i \equiv \lambda_i/\mu$ Erlangs.

We consider two output measures. Let P_b be the blocking probability that all radio channels are busy when a call arrives. Let P_h be the handoff probability that a call is handed off from the macrocell to the microcell.

We first compute P_b for AR, and then prove that P_b for RoD is the same as that for AR. To compute P_b , we design a stochastic process for AR. A state of the process is defined by a vector $\mathbf{x} = [x_1, x_2, \dots, x_M]$, where x_i represents the number of outstanding calls generated from the i th microcell. Consider an unrestricted two-tier WLL, where the macrocell has unlimited number of channels (i.e., $C \rightarrow \infty$). The state space Γ_∞ for this unrestricted system is

$$\Gamma_\infty = \{\mathbf{x} | x_i \geq 0, 1 \leq i \leq M\}. \quad (1)$$

Let $\mathbf{X} = [X_1, X_2, \dots, X_M]$ be the random vector denoting the state of the unrestricted system in equilibrium, i.e., X_i is the number of outstanding calls from the i th microcell in equilibrium. According to the $M/G/\infty$ model [6], the equilibrium probability $P(X_i = x_i)$ that the system has x_i outstanding calls from the i th microcell is given by

$$P(X_i = x_i) = \frac{\rho_i^{x_i} e^{-\rho_i}}{x_i!}. \quad (2)$$

Since X_1, X_2, \dots, X_M are independent in the unrestricted system, the equilibrium probability $P(\mathbf{X} = \mathbf{x})$ is given by

$$P(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^M P(X_i = x_i) = \prod_{i=1}^M \left(\frac{\rho_i^{x_i} e^{-\rho_i}}{x_i!} \right). \quad (3)$$

Let $\mathbf{Y} = [Y_1, Y_2, \dots, Y_M]$ be the equilibrium random vector, where $Y_i = \max(X_i - c_i, 0)$ for $1 \leq i \leq M$. That is, in the AR scheme, Y_i represents the number of outstanding calls from the

i th microcell such that they are served by the macrocell. Then the equilibrium probability for Y_i is given by

$$P(Y_i = k) = \begin{cases} \sum_{j=0}^{c_i} P(X_i = j), & \text{if } k = 0 \\ P(X_i = c_i + k), & \text{if } k > 0. \end{cases} \quad (4)$$

For $1 \leq i \leq M$, let $S_i(\mathbf{X}) = \sum_{j=1}^i Y_j = \sum_{j=1}^i \max(X_j - c_j, 0)$. From (4), $P(S_i(\mathbf{X}) = k)$ can be computed recursively as follows:

$$P(S_i(\mathbf{X}) = k) = \begin{cases} P(Y_1 = k), & \text{if } i = 1 \\ \sum_{j=0}^k [P(S_{i-1}(\mathbf{X}) = j) \times P(Y_i = k - j)], & \text{if } i > 1. \end{cases} \quad (5)$$

In AR, if $i = M$, $S_M(\mathbf{X}) = \sum_{i=1}^M Y_i = \sum_{i=1}^M \max(X_i - c_i, 0)$ represents the total number of outstanding calls in the macrocell.

For $1 \leq i \leq j \leq M$ and $j > 1$, let

$$S_{j,-i}(\mathbf{X}) = \sum_{l=1, l \neq i}^j Y_l. \quad (6)$$

From (4), (5), and (6), $P(S_{j,-i}(\mathbf{X}) = k)$ can be computed recursively as

$$P(S_{j,-i}(\mathbf{X}) = k) = \begin{cases} P(S_{j-1}(\mathbf{X}) = k), & \text{if } j = i \\ \sum_{l=0}^k [P(S_{j-1,-i}(\mathbf{X}) = l) \times P(Y_j = k - l)], & \text{if } j > i. \end{cases} \quad (7)$$

To compute P_b in AR, we consider the following events (i.e., the subsets of sample space Γ_∞). Let event A be

$$A = \left\{ \mathbf{x} \left| 0 \leq \sum_{i=1}^M \max(x_i - c_i, 0) \leq C \right. \right\}. \quad (8)$$

From (5), the probability $P(A)$ of A is

$$P(A) = \sum_{k=0}^C P(S_M(\mathbf{X}) = k). \quad (9)$$

For $1 \leq i \leq M$, let $B_i = \{\mathbf{x} | 0 \leq x_i \leq c_i - 1\}$ represent the event that the number X_i of outstanding calls in the i th microcell is less than c_i (i.e., the i th microcell is not blocked under AR). Then from (2), the probability of B_i is

$$P(B_i) = \sum_{k=0}^{c_i-1} P(X_i = k) = \sum_{k=0}^{c_i-1} \frac{\rho_i^k e^{-\rho_i}}{k!}. \quad (10)$$

Let event $D = \{\mathbf{x} | \sum_{i=1}^M \max(x_i - c_i, 0) = C\}$. From (5), the probability of D is

$$P(D) = P(S_M(\mathbf{X}) = C). \quad (11)$$

From (6), $S_{M,-i}(\mathbf{X})$ (for $1 \leq i \leq M$) is the number of outstanding calls in the macrocell excluding the calls generated from the i th microcell. Let event $E_i = \{\mathbf{x} | \sum_{l=1, l \neq i}^M \max(x_l - c_l, 0) = C\}$. From (7), the probability of E_i is

$$P(E_i) = P(S_{M,-i}(\mathbf{X}) = C). \quad (12)$$

Now consider the stochastic process for the real two-tier WLL, where the macrocell has limited number of radio channels (i.e., $C < \infty$). The set of all possible states is the set A defined in (8). In Appendix A, we prove that the probability measure in real two-tier WLL can be expressed by the probability measure in the unrestricted two-tier WLL. That is, we have the following:

Theorem 1: Consider a two-tier WLL with $C < \infty$. Suppose that the call arrivals are a Poisson stream and the call holding times have a general distribution. Then the equilibrium probability $P_A(\mathbf{X} = \mathbf{x})$ for state $\mathbf{x} \in A$ is

$$P_A(\mathbf{X} = \mathbf{x}) = \frac{P(\mathbf{X} = \mathbf{x})}{P(A)}.$$

Theorem 1 implies that the equilibrium probability is independent of the distribution of the call holding times, but is only affected by the mean of the call holding times.

In AR, a call attempt from the i th microcell is blocked if the i th microcell is blocked (i.e., event \overline{B}_i occurs where \overline{B}_i is the complement of event B_i) and the macrocell is blocked (i.e., event D occurs). That is, the call is blocked when $\overline{B}_i \cap D$ occurs. Then the blocking probability $P_b(i)$ of the call attempt from the i th microcell is derived as

$$P_b(i) = P_A(\overline{B}_i \cap D) = P_A(D) - P_A(B_i \cap D). \quad (13)$$

Note that event B_i implies $Y_i = \max(X_i - c_i, 0) = 0$. According to the definitions of D and E_i , we have $B_i \cap D = B_i \cap E_i$. Therefore, (13) can be rewritten as

$$P_b(i) = P_A(D) - P_A(B_i \cap E_i). \quad (14)$$

Based on Theorem 1, (14) can be rewritten as

$$P_b(i) = \frac{P(D) - P(B_i \cap E_i)}{P(A)}.$$

Because events B_i and E_i are independent in the unrestricted two-tier WLL, we have

$$P_b(i) = \frac{P(D) - P(B_i)P(E_i)}{P(A)} \quad (15)$$

where $P(A)$, $P(B_i)$, $P(D)$, and $P(E_i)$ are expressed in (9)–(12), respectively. From (15), the system blocking probability P_b is given by

$$P_b = \frac{\sum_{i=1}^M \lambda_i P_b(i)}{\sum_{i=1}^M \lambda_i} = \frac{P(D)}{P(A)} - \frac{\sum_{i=1}^M \lambda_i P(B_i)P(E_i)}{P(A) \left[\sum_{i=1}^M \lambda_i \right]}. \quad (16)$$

In the next section, we will show that (16) is validated by a simulation model described in Appendix C.

Consider the homogeneous two-tier WLL, where $c_i = c_j$ and $\lambda_i = \lambda_j$ for $1 \leq i, j \leq M$. We have $B = B_i$ and $E = E_i$ for $1 \leq i \leq M$. Then (16) can be simplified as

$$P_b = \frac{P(D) - P(B)P(E)}{P(A)}.$$

To compute P_b for RoD, we prove the following theorem in Appendix B.

Theorem 2: In a two-tier WLL with $C < \infty$, both AR and RoD have the same blocking probability.

Based on Theorem 2, P_b for RoD can also be computed by using (16).

IV. RESULTS AND DISCUSSIONS

Based on the analytic model developed in the previous section and the simulation model in Appendix C, we compare NR, AR, and RoD in terms of the blocking probability P_b and the handoff probability P_h . In our numerical examples, the radio channel number $c = 8$ and the traffic intensity $\rho = 7$ Erlangs for every microcell. Similar conclusions can be drawn for various channel numbers and traffic intensities, and will not be presented in this paper. In each simulation run, $10^6 - 10^7$ call arrival events per microcell are executed to ensure that simulation results are stable. We consider various call holding time distributions in simulation experiments. The simulation results indicate that P_b is not affected by call holding time distributions, and is only affected by the mean of the distributions. This result validates Theorem 1. Simulation results also validate Theorem 2 by showing that both AR and RoD have the same P_b . Figs. 3(a) and 4(a) show that the analytic results (the “o” curves) are consistent with the simulation results (the “*” curves). For the discussion purpose, we only present the simulation results in the remainder of this section.

A. Effects of the Microcell Number M

Fig. 3(a) and (b) plots P_b and P_h as functions of M . In these figures, the call holding times are exponentially distributed with mean $1/\mu = 3$ min, $C = 12$, $c = 8$, $\rho = 7$ Erlangs, and M ranges from 4 to 52. Fig. 3(a) shows an intuitive result that for repacking and nonrepacking approaches, P_b increases as M increases (where C is fixed). Fig. 3(b) shows that P_h is a decreasing function of M for AR, which is explained as follows. Increasing overflow traffic (i.e., increasing M) causes the increase of call blocking in the macrocell and microcells. In this case, the overflow calls have less opportunities to become repacking candidates for handoff. When M is small, almost all overflow call attempts are accepted at the macrocell and are very likely to hand off to the microcells in AR, and P_h is large. For RoD-R, RoD-L, and RoD-ST, P_h increases and then decreases as M increases. This nontrivial phenomenon is explained as follows. When M is small (i.e., the overflow traffic is low), call blocking seldom occurs in the macrocell and few on-demand handoffs are exercised. Note that the traffic to every individual microcell is fixed in our experiments. If M increases (i.e., the overflow traffic increases), blocking is more likely to occur in the macrocell. In this case, the on-demand handoffs are performed frequently. When M is very large (i.e., the overflow traffic is very heavy), the number of handoff attempts from the macrocell to a microcell significantly increases. In this case, the capacity of a microcell is consumed quickly, and only a small portion of handoff attempts find repacking candidates. Therefore, repacking is not likely to be exercised, and the new calls

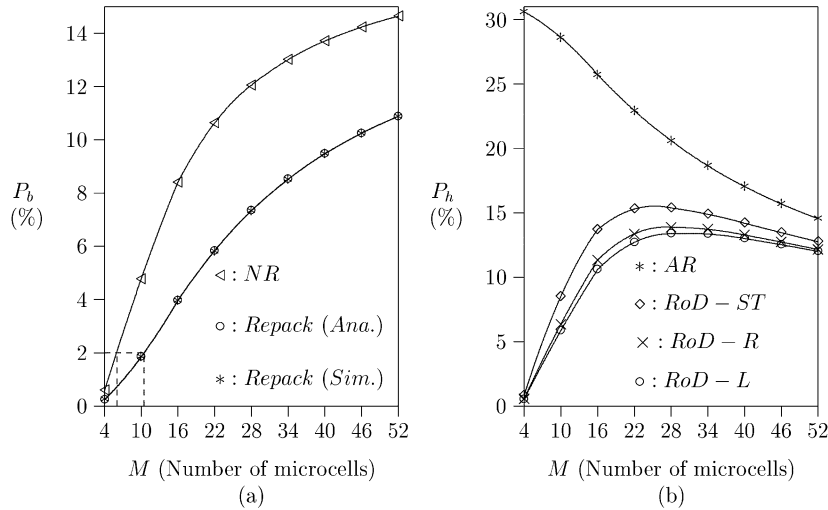


Fig. 3. Effects of M on P_b and P_h ($C = 12, c = 8, 1/\mu = 3$ min, $\rho = 7$ Erlangs). (a) Blocking probability. (b) Handoff probability.

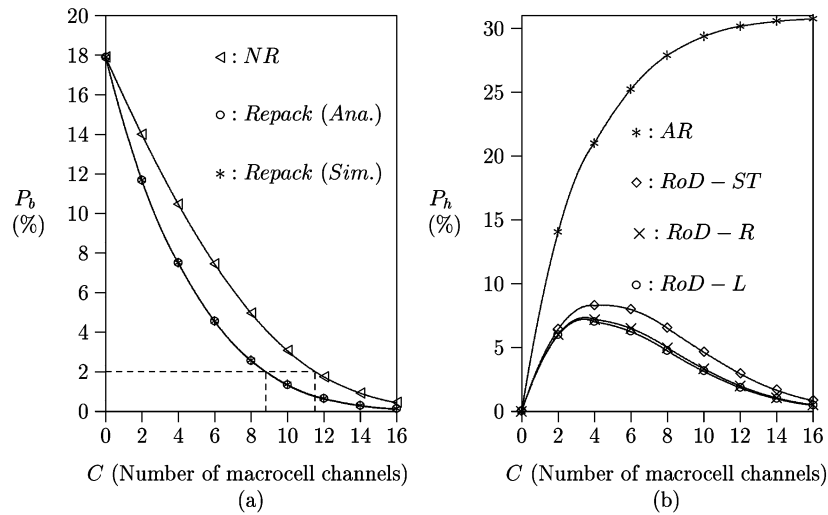


Fig. 4. Effects of C on P_b and P_h ($M = 6, c = 8, 1/\mu = 3$ min, $\rho = 7$ Erlangs). (a) Blocking probability. (b) Handoff probability.

are blocked. Specifically, for $M > 28$, P_h decreases as M increases in our experiments.

B. Effects of the Macrocell Channel Number C

Fig. 4(a) and (b) plots P_b and P_h as functions of C . In these figures, the call holding times are exponentially distributed with mean $1/\mu = 3$ min, $M = 6, c = 8, \rho = 7$ Erlangs, and C ranges from 0 to 16. Fig. 4(a) shows that P_b decreases as C increases for all approaches. When C is small, macrocell channels are the bottleneck resources. Increasing C significantly improves P_b . When C is large [$C > 14$ in Fig. 4(a)], the macrocell channels are no longer the bottleneck (i.e., the bottleneck shifts to the microcell capacities), adding extra macrocell channels only insignificantly improves P_b . Fig. 4(b) illustrates that P_h increases as C increases for AR. Increasing macrocell channels results in the increase of overflow calls in the macrocell, which causes more handoffs under AR. For RoD-R, RoD-L, and RoD-ST, P_h increases and then decreases as C increases. The reason is similar to that for the curves in Fig. 3(b).

C. Comparison of NR, AR, and RoD

In Figs. 3(a) and 4(a), repacking approaches have lower P_b than NR. If the WLL is engineered at $P_b = 2\%$ [see the horizontal dashed line in Fig. 3(a)], repacking approaches accommodate 4.5 more microcells than (or 75% improvement over) NR. Similarly, if the WLL is engineered at $P_b = 2\%$, Fig. 4(a) indicates that for fixed M , repacking approaches require 2.7 less radio channels than (or 23% improvement over) NR. In Figs. 3(b) and 4(b), we observe that $P_{h,AR} \gg P_{h,RoD-ST} > P_{h,RoD-R} > P_{h,RoD-L}$, where $P_{h,T}$ is the handoff probability for the repacking approach T . Because RoD is exercised only when repacking is necessary, $P_{h,RoD} \ll P_{h,AR}$. This effect becomes significant when the traffic load of macrocell is low (i.e., M is small or C is large). Furthermore, when the WLL is engineered at the 2% blocking probability, RoD reduces 68%–86% of handoffs over AR, as shown in Figs. 3(b) and 4(b).

D. Effects of the Variance V for the Call Holding Time

Fig. 5 plots P_h as functions of the variance V for gamma call holding time distribution. In this figure, the mean call holding time is $1/\mu = 3$ min, $M = 6, C = 8, c = 8$, and $\rho = 7$ Erlangs.

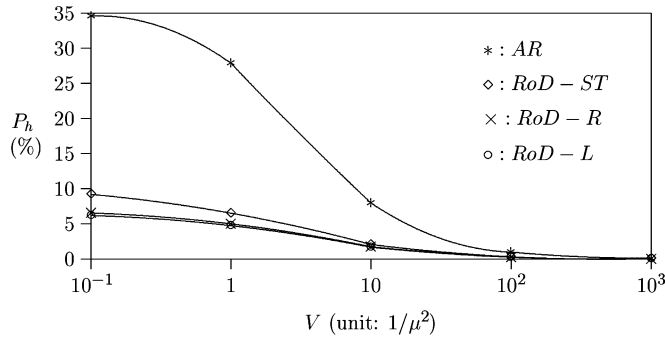


Fig. 5. Effects of variance (V) for the gamma call holding time distribution on P_h ($M = 6$, $C = 8$, $c = 8$, $1/\mu = 3$ min, and $\rho = 7$ Erlangs).

This figure indicates that P_h decreases as V increases. As previously mentioned, P_b is not affected by the variance V of the call holding times. Unlike P_b , P_h is sensitive to V . This result is explained as follows. As V increases, more large and small call holding times are observed. More short call holding times imply that more calls are completed before the next call attempts arrive. Thus, the number of handoffs decreases.

V. CONCLUSION

In this paper, we have proposed the RoD scheme for radio channel assignment in two-tier WLL networks. We developed analytic and simulation models to investigate the blocking probability P_b of two-tier WLL with repacking techniques. We proved that P_b is not affected by the call holding time distributions, and is only dependent on the mean of the call holding time. Furthermore, we showed that RoD has the same P_b as AR. Compared with the NR scheme, both AR and RoD reduce P_b at the cost of handoffs. Our study indicated that RoD has much lower handoff probability P_h than AR when the WLL is engineered at $P_b = 2\%$. In RoD, three policies (i.e., RoD-R, RoD-L, and RoD-ST) were proposed to select repacking candidates for handoff. Among these policies, RoD-L has the lowest P_h .

APPENDIX I

PROOF FOR THEOREM 1

This appendix proves Theorem 1. Consider a two-tier WLL with C radio channels in the macrocell and c_i channels in the i th microcell, where $1 \leq i \leq M$. We assume Poisson call arrival rate λ_i from the i th microcell. Let the call holding times have a general distribution $G(t)$ with density $g(t)$ and mean $1/\mu$. Then the hazard function $h(t)$ of $G(t)$ is defined by

$$h(t) = \frac{g(t)}{\bar{G}(t)}, \quad \text{where } \bar{G}(t) = 1 - G(t). \quad (17)$$

Suppose that there are x_i outstanding calls generated from the i th microcell, where $1 \leq i \leq M$. For $1 \leq j \leq x_i$, let $u_{i,j}$ be the age (i.e., elapsed holding time) of call j from the i th microcell. Without loss of generality, let $u_{i,1} \leq u_{i,2} \leq \dots \leq u_{i,x_i}$. Let

$$\mathbf{u} = (u_{1,1}, u_{1,2}, \dots, u_{1,x_1}, u_{2,1}, \dots, u_{2,x_2}, \dots, u_{M,1}, \dots, u_{M,x_M}).$$

Then \mathbf{u} forms a Markov process. By convention, let state \emptyset represent that the WLL is empty (i.e., there is no outstanding call

in the WLL). For $1 \leq j \leq x_i$, let $\mathbf{e}_{i,j}(\mathbf{u})$ be the vector by removing $u_{i,j}$ from \mathbf{u} . That is

$$\mathbf{e}_{i,j}(\mathbf{u}) = \begin{cases} (u_{1,2}, \dots, u_{M,x_M}), & \text{if } i=j=1 \\ (u_{1,1}, \dots, u_{i-1,x_{i-1}}, u_{i,2}, \dots, u_{M,x_M}), & \text{if } 2 \leq i \leq M \text{ and } j=1 \\ (u_{1,1}, \dots, u_{i,x_i-1}, u_{i+1,1}, \dots, u_{M,x_M}), & \text{if } 1 \leq i \leq M-1 \text{ and } j=x_i \\ (u_{1,1}, \dots, u_{M,x_M-1}), & \text{if } i=M \text{ and } j=x_M \\ (u_{1,1}, \dots, u_{i,j-1}, u_{i,j+1}, \dots, u_{M,x_M}), & \text{otherwise.} \end{cases}$$

Furthermore, we define $\mathbf{e}_{i,j}(\mathbf{u}) \rightarrow \mathbf{u}$ as the transition from state $\mathbf{e}_{i,j}(\mathbf{u})$ to \mathbf{u} .

The Markov process moves from \mathbf{u} to $\mathbf{e}_{i,j}(\mathbf{u})$ when the outstanding call with age $u_{i,j}$ completes instantaneously. From (17), the transition intensity for $\mathbf{u} \rightarrow \mathbf{e}_{i,j}(\mathbf{u})$ is $h(u_{i,j})$. The Markov process moves from $\mathbf{e}_{i,j}(\mathbf{u})$ to \mathbf{u} when the call with elapsed holding time $u_{i,j}$ arrives instantaneously. Thus, the transition intensity for $\mathbf{e}_{i,j}(\mathbf{u}) \rightarrow \mathbf{u}$ is $\lambda_i g(u_{i,j})$. Let $\pi(\mathbf{u})$ be the equilibrium probability for \mathbf{u} , then

$$\pi(\mathbf{u})h(u_{i,j}) = \pi(\mathbf{e}_{i,j}(\mathbf{u}))\lambda_i g(u_{i,j}). \quad (18)$$

From (17), (18) is rewritten as

$$\pi(\mathbf{u}) = \pi(\mathbf{e}_{i,j}(\mathbf{u}))\lambda_i \bar{G}(u_{i,j}). \quad (19)$$

Begin with $i = 1$ and $j = 1$, we iterate (19) to yield

$$\begin{aligned} \pi(\mathbf{u}) &= \pi(\mathbf{e}_{1,1}(\mathbf{u}))\lambda_1 \bar{G}(u_{1,1}) \\ &= \pi(\mathbf{e}_{1,2}(\mathbf{e}_{1,1}(\mathbf{u}))) [\lambda_1 \bar{G}(u_{1,1})] [\lambda_1 \bar{G}(u_{1,2})] \\ &\vdots \\ &= \pi(\emptyset) \prod_{i=1}^M \left[\lambda_i^{x_i} \prod_{j=1}^{x_i} \bar{G}(u_{i,j}) \right]. \end{aligned} \quad (20)$$

In the two-tier WLL, the system state is $\mathbf{x} = [x_1, x_2, \dots, x_M]$, and the set of all possible states in the system is A [see (8)]. Integrating (20) over all vectors \mathbf{u} , the equilibrium probability $P_A(\mathbf{X} = \mathbf{x})$ for the equilibrium state $\mathbf{X} \in A$ is

$$\begin{aligned} P_A(\mathbf{X} = \mathbf{x}) &= \int \pi(\mathbf{u}) d\mathbf{u} \\ &= \pi(\emptyset) \prod_{i=1}^M \left\{ \lambda_i^{x_i} \int \int \dots \int_{u_{i,1} \leq u_{i,2} \leq \dots \leq u_{i,x_i}} \left[\prod_{j=1}^{x_i} \bar{G}(u_{i,j}) \right] \right. \\ &\quad \left. \times du_{i,1} du_{i,2} \dots du_{i,x_i} \right\} \\ &= \pi(\emptyset) \prod_{i=1}^M \left\{ \left(\frac{\lambda_i^{x_i}}{x_i!} \right) \int \int \dots \int_{u_{i,1}, u_{i,2}, \dots, u_{i,x_i}} \left[\prod_{j=1}^{x_i} \bar{G}(u_{i,j}) \right] \right. \\ &\quad \left. \times du_{i,1} du_{i,2} \dots du_{i,x_i} \right\} \\ &= \pi(\emptyset) \prod_{i=1}^M \left[\left(\frac{\lambda_i^{x_i}}{x_i!} \right) \left(\frac{1}{\mu} \right)^{x_i} \right] \\ &= \pi(\emptyset) \prod_{i=1}^M \left(\frac{\rho_i^{x_i}}{x_i!} \right). \end{aligned} \quad (21)$$

From (21) and because $\sum_{\mathbf{x} \in A} P_A(\mathbf{X} = \mathbf{x}) = 1$, we obtain

$$\pi(\emptyset) = \left\{ \sum_{\mathbf{x} \in A} \left[\prod_{i=1}^M \left(\frac{\rho_i^{x_i}}{x_i!} \right) \right] \right\}^{-1}$$

and

$$P_A(\mathbf{X} = \mathbf{x}) = \frac{\prod_{i=1}^M \left(\frac{\rho_i^{x_i}}{x_i!} \right)}{\sum_{\mathbf{x}' \in A} \left[\prod_{i=1}^M \left(\frac{\rho_i^{x'_i}}{x'_i!} \right) \right]}. \quad (22)$$

From (22) and (3), for $\mathbf{x} \in A$, we have

$$P_A(\mathbf{X} = \mathbf{x}) = \frac{P(\mathbf{X} = \mathbf{x})}{\sum_{\mathbf{x}' \in A} P(\mathbf{X} = \mathbf{x}')}. \quad (23)$$

From (9), we have $P(A) = \sum_{\mathbf{x}' \in A} P(\mathbf{X} = \mathbf{x}')$, and (23) is rewritten as

$$P_A(\mathbf{X} = \mathbf{x}) = \frac{P(\mathbf{X} = \mathbf{x})}{P(A)}.$$

The above equation is independent of the distribution of the call holding times, and is only affected by the mean of the call holding times.

APPENDIX II PROOF FOR THEOREM 2

Consider a two-tier WLL system. The WLL is called WLL_{RoD} if the RoD scheme is exercised, and is called WLL_{AR} if the AR scheme is exercised. Consider a stochastic process for WLL_{RoD} with state $[(s_1, w_1), (s_2, w_2), \dots, (s_i, w_i), \dots, (s_M, w_M)]$. In a state, s_i represents the number of outstanding calls from the i th microcell, which are served by the i th microcell, and w_i represents the number of outstanding calls from the i th microcell, which are served by the macrocell (where, $1 \leq i \leq M$). In WLL_{RoD}, a legal state must satisfy the following inequalities:

$$\text{for } 1 \leq i \leq M, 0 \leq s_i \leq c_i, 0 \leq w_i \text{ and } \sum_{i=1}^M w_i \leq C. \quad (24)$$

From the definition of RoD in Section II, it is clear that a call attempt from the j th microcell is blocked if and only if the following three conditions are satisfied:

- Condition 1:** the j th microcell is blocked (i.e., $s_j = c_j$);
- Condition 2:** the macrocell is blocked (i.e., $\sum_{i=1}^M w_i = C$);
- Condition 3:** NR candidate is found (i.e., for $1 \leq i \leq M$, $w_i = 0$ or $s_i = c_i$).

As mentioned in Section III, a stochastic process for WLL_{AR} has states of form $[x_1, \dots, x_i, \dots, x_M]$, where x_i represents the number of outstanding calls from the i th microcell (for $1 \leq i \leq M$). In WLL_{AR}, a legal state must satisfy the following inequalities:

$$\text{for } 1 \leq i \leq M, 0 \leq x_i \text{ and } \sum_{i=1}^M \max(x_i - c_i, 0) \leq C. \quad (25)$$

In WLL_{AR}, a call attempt from the j th microcell is blocked if and only if the following two conditions are satisfied:

- Condition 4:** the j th microcell is blocked (i.e., $x_j \geq c_j$);
- Condition 5:** the macrocell is blocked (i.e., $\sum_{i=1}^M \max(x_i - c_i, 0) = C$).

Lemma 1: For the j th microcell, the possible values for (s_j, w_j) pair in WLL_{RoD} can be classified into five cases.

- Case 1) $0 \leq s_j < c_j, 0 \leq w_j < c_j - s_j$, and $\sum_{i=1}^M w_i \leq C$;
- Case 2) $0 \leq s_j < c_j, c_j - s_j \leq w_j$, and $\sum_{i=1}^M w_i \leq C$;
- Case 3) $s_j = c_j, 0 \leq w_j$, and $\sum_{i=1}^M w_i < C$;
- Case 4) $s_j = c_j, 0 \leq w_j, \sum_{i=1}^M w_i = C$, and $\exists r, 1 \leq r \neq j \leq M, w_r > 0$ and $s_r < c_r$;
- Case 5) $s_j = c_j, 0 \leq w_j, \sum_{i=1}^M w_i = C$, and for $1 \leq i \leq M, w_i = 0$ or $s_i = c_i$.

Proof: Directly from (24). Note that **Case 1** represents that the j th microcell is not blocked, and the number of current calls generated from the j th microcell is less than the number of radio channels in the j th microcell. **Case 2** represents that the j th microcell is not blocked, and the number of current calls generated from the j th microcell is no less than that of radio channels in the j th microcell. **Case 3** represents that the j th microcell is blocked but the macrocell is not. **Case 4** represents that both the j th microcell and the macrocell are blocked, but there are repacking candidates. **Case 5** represents that both the j th microcell and the macrocell are blocked, and the NR candidate is found.

Lemma 2: For the j th microcell, the possible values for x_j in WLL_{AR} can be classified into three cases.

- Case A. $0 \leq x_j < c_j$ and $\sum_{i=1}^M \max(x_i - c_i, 0) \leq C$;
- Case B. $x_j \geq c_j$ and $\sum_{i=1}^M \max(x_i - c_i, 0) < C$;
- Case C. $x_j \geq c_j$ and $\sum_{i=1}^M \max(x_i - c_i, 0) = C$.

Proof: Directly from (25). Note that **Case A** represents that the j th microcell is not blocked. **Case B** represents that the j th microcell is blocked but the macrocell is not. **Case C** represents that both the j th microcell and the macrocell are blocked.

Lemma 3: Consider the exercise of WLL_{RoD} and WLL_{AR} at a time point. If

$$s_i + w_i = x_i, \quad \text{for } 1 \leq i \leq M \quad (26)$$

then

- (I) If **Case 1** in WLL_{RoD} holds, then **Case A** in WLL_{AR} holds.
- (II) If **Case 2** in WLL_{RoD} holds, then **Case B** in WLL_{AR} holds.
- (III) If **Case 3** in WLL_{RoD} holds, then **Case B** in WLL_{AR} holds.
- (IV) If **Case 4** in WLL_{RoD} holds, then **Case B** in WLL_{AR} holds.
- (V) If **Case 5** in WLL_{RoD} holds, then **Case C** in WLL_{AR} holds.

Proof: From (26), $w_i = x_i - s_i \geq 0$ (for $1 \leq i \leq M$), we have $x_i - s_i = \max(x_i - s_i, 0)$ and

$$\sum_{i=1}^M w_i = \sum_{i=1}^M \max(x_i - s_i, 0). \quad (27)$$

Now we consider (I)–(V) as follows.

- (I) From Lemma 1, when **Case 1** in WLL_{RoD} holds, $0 \leq w_j < c_j - s_j$. From (26), we have $x_j < c_j$. From (27) and since $\sum_{i=1}^M w_i \leq C$ in **Case 1**, we have

$$\sum_{i=1}^M \max(x_i - s_i, 0) \leq C. \quad (28)$$

For $1 \leq i \leq M$, $s_i \leq c_i$ (i.e., $x_i - s_i \geq x_i - c_i$), (28) can be rewritten as $\sum_{i=1}^M \max(x_i - c_i, 0) \leq C$. From this inequality and because $x_j < c_j$, **Case A** in WLL_{AR} holds.

- (II) From Lemma 1, when **Case 2** in WLL_{RoD} holds, $c_j - s_j \leq w_j$. From (26), we have $x_j \geq c_j$. From (27) and since $\sum_{i=1}^M w_i \leq C$ in **Case 2**, we have

$$\sum_{i=1}^M \max(x_i - s_i, 0) \leq C. \quad (29)$$

Since $w_j = x_j - s_j > 0$ and $s_j < c_j$ (i.e., $x_j - s_j > x_j - c_j$), (29) can be rewritten as

$$\sum_{i=1}^M \max(x_i - c_i, 0) < C. \quad (30)$$

From (30) and because $x_j \geq c_j$, **Case B** in WLL_{AR} holds.

- (III) From Lemma 1, when **Case 3** in WLL_{RoD} holds, $s_j = c_j$. From (26), we have $x_j \geq c_j$. From (27) and since $\sum_{i=1}^M w_i < C$ in **Case 3**, we have

$$\sum_{i=1}^M \max(x_i - s_i, 0) < C. \quad (31)$$

For $1 \leq i \leq M$, $s_i \leq c_i$ (i.e., $x_i - s_i \geq x_i - c_i$), (31) can be rewritten as

$$\sum_{i=1}^M \max(x_i - c_i, 0) < C. \quad (32)$$

From (32) and because $x_j \geq c_j$, **Case B** in WLL_{AR} holds.

- (IV) From Lemma 1, when **Case 4** in WLL_{RoD} holds, $s_j = c_j$. From (26), we have $x_j \geq c_j$. From (27) and since $\sum_{i=1}^M w_i = C$ in **Case 4**, we have

$$\sum_{i=1}^M \max(x_i - s_i, 0) = C. \quad (33)$$

Since $\exists r$, $w_r = x_r - s_r > 0$ and $s_r < c_r$ (i.e., $x_r - s_r > x_r - c_r$), (33) can be rewritten as

$$\sum_{i=1}^M \max(x_i - c_i, 0) < C. \quad (34)$$

From (34) and $x_j \geq c_j$, **Case B** in WLL_{AR} holds.

- (V) From Lemma 1, when **Case 5** in WLL_{RoD} holds, $s_j = c_j$. From (26), we have $x_j \geq c_j$. From (27) and since $\sum_{i=1}^M w_i = C$ in **Case 5** in WLL_{RoD} , we have

$$\sum_{i=1}^M \max(x_i - s_i, 0) = C. \quad (35)$$

For $1 \leq i \leq M$, $w_i = x_i - s_i = 0$ or $s_i = c_i$ (i.e., $x_i - s_i = x_i - c_i$), (35) can be rewritten as

$$\sum_{i=1}^M \max(x_i - c_i, 0) = C. \quad (36)$$

From (36) and $x_j \geq c_j$, **Case C** in WLL_{AR} holds.

QED

Lemma 4: Consider the exercise of WLL_{RoD} and WLL_{AR} at a time point. If (26) holds, then

- (I) **Case 1** in WLL_{RoD} holds if and only if **Case A** in WLL_{AR} holds.
- (II) **Cases 2, 3, or 4** in WLL_{RoD} hold if and only if **Case B** in WLL_{AR} holds.
- (III) **Case 5** in WLL_{RoD} holds if and only if **Case C** in WLL_{AR} holds.

Proof:

- (I) From (I) of Lemma (3), we only need to prove that **Case 1** in WLL_{RoD} holds if **Case A** in WLL_{AR} holds. Note that

$$\sum_{i=1}^M w_i \leq C \quad (37)$$

always holds in WLL_{RoD} . From Lemma 2, when **Case A** in WLL_{AR} holds, we have $0 \leq x_j < c_j$. From (26), we have $0 \leq s_j + w_j < c_j$. That is, $0 \leq s_j < c_j$ and $0 \leq w_j < c_j - s_j$. From the above inequality and (37), **Case 1** in WLL_{AR} holds.

- (II) From (II)–(IV) of Lemma (3), we only need to prove that **Cases 2, 3, or 4** in WLL_{RoD} hold if **Case B** in WLL_{AR} holds. From Lemma 2, when **Case B** in WLL_{AR} holds, $x_j \geq c_j$. From (26), we have $w_j \geq c_j - s_j$. If $0 \leq s_j < c_j$, it is obvious that **Case 2** in WLL_{RoD} holds but **Case 1** in WLL_{RoD} does not hold. If $s_j = c_j$, we show that **Cases 3 or 4** may hold and **Case 5** does not hold. Assuming that **Case 5** holds, then (36) in Lemma (3) holds. However, (36) contradicts with the proposition $\sum_{i=1}^M \max(x_i - c_i, 0) < C$ in **Case B** in WLL_{AR} . Thus, **Case 5** does not hold. It suffices to show that if **Case B** in WLL_{AR} holds with $s_j = c_j$, then there are examples that **Cases 3 or 4** in WLL_{RoD} hold. Consider a scenario where $x_i = c_i$ (for $1 \leq i \leq M$) in **Case B** of WLL_{AR} . If $s_i = c_i$ and $w_i = 0$ (for $1 \leq i \leq M$), then (26) holds and **Case 3** in WLL_{RoD} holds. If for $1 \leq i \neq j \leq M$, $w_i > 0$, $s_i = c_i - w_i$, and $\sum_{i=1}^M w_i = C$, then (26) holds and **Case 4** in WLL_{RoD} holds. Therefore, **Cases 2, 3, or 4** in WLL_{RoD} holds.

- (III) From (V) of Lemma (3), we only need to prove that **Case 5** in WLL_{RoD} holds if **Case C** in WLL_{AR} holds. From Lemma 2, when **Case C** in WLL_{AR} holds, $x_j \geq c_j$. From (26), we have $w_j \geq c_j - s_j \geq 0$. We show that

$$s_j = c_j \quad (38)$$

by the contradiction method. Assume that $0 \leq s_j < c_j$. Since $w_j \geq c_j - s_j$ and from Lemma 1, **Case 2**

in WLL_{RoD} must hold, and from Lemma 3 (II), (30) must be satisfied. However, (30) contradicts with the requirement

$$\sum_{i=1}^M \max(x_i - c_i, 0) = C \quad (39)$$

in **Case C** of WLL_{AR} . Therefore, the assumption (i.e., $0 \leq s_j < c_j$) is false and (38) must hold. We next show that

$$\sum_{i=1}^M w_i = C \quad (40)$$

by the contradiction method. Assuming that $\sum_{i=1}^M w_i < C$, then from (38) and Lemma 1, **Case 3** in WLL_{RoD} must hold, and from Lemma 3 (III), (32) must be satisfied. However, (32) contradicts with the requirement (39) in **Case C** of WLL_{AR} . Thus, this assumption (i.e., $\sum_{i=1}^M w_i < C$) is false and (40) must hold. Finally, we show that

$$w_i = 0 \quad \text{or} \quad s_i = c_i, \quad \text{for } 1 \leq i \leq M \quad (41)$$

by contradiction. Assuming that

$$\exists r, 1 \leq r \neq j \leq M, w_r > 0 \quad \text{and} \quad s_r < c_r \quad (42)$$

then from (38), (40), and Lemma 1, **Case 4** in WLL_{RoD} must hold. From Lemma 3 (IV), (34) must be satisfied. However, (34) contradicts with the requirement (39). Thus, this assumption [i.e., (42)] is false and (41) must hold. From (38), (40), and (41), **Case 5** in WLL_{RoD} holds. **Q.E.D.**

Consider a sequence of call arrivals to WLL_{RoD} and WLL_{AR} . Let $a(n)$ be the n th call arrival. Let

$$\Delta_{\text{RoD}}(a(n)) = \langle d_{\text{RoD}}(n, 1), d_{\text{RoD}}(n, 2), \dots, d_{\text{RoD}}(n, m) \rangle$$

be the sequence of call completions for WLL_{RoD} occurring in the period $[t(a(n)), t(a(n+1))]$, where $m \geq 0$, $t(a(n)) \leq t(d_{\text{RoD}}(n, 1))$, $t(d_{\text{RoD}}(n, m)) < t(a(n+1))$, $t(a(n))$ is the time when call arrival $a(n)$ occurs, and $t(d_{\text{RoD}}(n, k))$ is the time when call completion $d_{\text{RoD}}(n, k)$ occurs. Similarly, let $\Delta_{\text{AR}}(a(n))$ be the sequence of call completions defined for WLL_{AR} . We say that $\Delta_{\text{RoD}}(a(n)) = \Delta_{\text{AR}}(a(n))$ if and only if

- the number m of call completions in $\Delta_{\text{RoD}}(a(n))$ is the same as that in $\Delta_{\text{AR}}(a(n))$;
- $t(d_{\text{RoD}}(n, k)) = t(d_{\text{AR}}(n, k))$ for $1 \leq k \leq M$;
- $(d_{\text{RoD}}(n, k))$ and $(d_{\text{AR}}(n, k))$ are generated from the same microcell.

Let $s_i^-(\mathbf{e})$, $w_i^-(\mathbf{e})$, and $x_i^-(\mathbf{e})$ be the s_i , w_i , and x_i values immediately before an event \mathbf{e} (e.g., \mathbf{e} is either a call arrival or a call completion) occurs. Similarly, let $s_i^+(\mathbf{e})$, $w_i^+(\mathbf{e})$, and $x_i^+(\mathbf{e})$ be the s_i , w_i , and x_i values immediately after an event \mathbf{e} occurs.

Lemma 5: If

$$s_i^+(a(n)) + w_i^+(a(n)) = x_i^+(a(n)), \quad \text{for } 1 \leq i \leq M \quad (43)$$

and

$$\Delta_{\text{RoD}}(a(n)) = \Delta_{\text{AR}}(a(n)) \quad (44)$$

then for $1 \leq k \leq m$ and $1 \leq i \leq M$, we have

$$s_i^+(d_{\text{RoD}}(n, k)) + w_i^+(d_{\text{RoD}}(n, k)) = x_i^+(d_{\text{AR}}(n, k)). \quad (45)$$

Proof: We prove by induction on the k th call completion $d_{\text{RoD}}(n, k)$ in WLL_{RoD} and $d_{\text{AR}}(n, k)$ in WLL_{AR} .

Basis: For $k = 1$, from (43), we have that

$$\begin{aligned} s_i^-(d_{\text{RoD}}(n, 1)) + w_i^-(d_{\text{RoD}}(n, 1)) \\ = x_i^-(d_{\text{AR}}(n, 1)), \quad \text{for } 1 \leq i \leq M. \end{aligned} \quad (46)$$

Suppose that $d_{\text{RoD}}(n, 1)$ is for a call generated from the j th microcell (where $1 \leq j \leq M$). It is clear that for $1 \leq i \neq j \leq M$, $s_i^+(d_{\text{RoD}}(n, 1)) = s_i^-(d_{\text{RoD}}(n, 1))$, $w_i^+(d_{\text{RoD}}(n, 1)) = w_i^-(d_{\text{RoD}}(n, 1))$, and $x_i^+(d_{\text{RoD}}(n, 1)) = x_i^-(d_{\text{RoD}}(n, 1))$. Now consider the j th microcell. If $d_{\text{RoD}}(n, 1)$ is completed at the j th microcell, we have

$$s_j^+(d_{\text{RoD}}(n, 1)) = s_j^-(d_{\text{RoD}}(n, 1)) - 1$$

and

$$w_j^+(d_{\text{RoD}}(n, 1)) = w_j^-(d_{\text{RoD}}(n, 1)). \quad (47)$$

If $d_{\text{RoD}}(n, 1)$ is completed at the macrocell, we have

$$s_j^+(d_{\text{RoD}}(n, 1)) = s_j^-(d_{\text{RoD}}(n, 1))$$

and

$$w_j^+(d_{\text{RoD}}(n, 1)) = w_j^-(d_{\text{RoD}}(n, 1)) - 1. \quad (48)$$

From (44), $d_{\text{AR}}(n, 1)$ is for a call generated from the j th microcell. When $d_{\text{AR}}(n, 1)$ occurs, we have

$$s_j^+(d_{\text{AR}}(n, 1)) = x_j^-(d_{\text{AR}}(n, 1)) - 1. \quad (49)$$

From (46)–(49), we have $s_j^+(d_{\text{RoD}}(n, 1)) + w_j^+(d_{\text{RoD}}(n, 1)) = x_j^+(d_{\text{AR}}(n, 1))$. Thus, (45) holds for $d_{\text{RoD}}(n, 1)$ and $d_{\text{AR}}(n, 1)$.

Induction: Assume that (45) holds for $d_{\text{RoD}}(n, k-1)$ and $d_{\text{AR}}(n, k-1)$. That is

$$\begin{aligned} s_i^+(d_{\text{RoD}}(n, k-1)) + w_i^+(d_{\text{RoD}}(n, k-1)) \\ = x_i^+(d_{\text{AR}}(n, k-1)), \quad \text{for } 1 \leq i \leq M. \end{aligned} \quad (50)$$

Equations (50) and (44) imply that $s_i^-(d_{\text{RoD}}(n, k)) + w_i^-(d_{\text{RoD}}(n, k)) = x_i^-(d_{\text{AR}}(n, k))$, for $1 \leq i \leq M$. Following an argument similar to the proof for the Basis case, we have $s_i^+(d_{\text{RoD}}(n, k)) + w_i^+(d_{\text{RoD}}(n, k)) = x_i^+(d_{\text{AR}}(n, k))$, for $1 \leq i \leq M$. Thus, (45) holds for $d_{\text{RoD}}(n, k)$ and $d_{\text{AR}}(n, k)$. **Q.E.D.**

Lemma 6: Consider a sequence of call arrivals $a(n)$, where $n = 1, 2, \dots$. For both WLL_{RoD} and WLL_{AR} , we have

- 1) for all n and $1 \leq i \leq M$

$$s_i^+(a(n)) + w_i^+(a(n)) = x_i^+(a(n)) \quad (51)$$

- 2) for all n , $a(n)$ is blocked in WLL_{RoD} if and only if $a(n)$ is blocked in WLL_{AR} ;
- 3) for all n , if $a(n)$ is accepted, then $a(n)$ is completed in WLL_{RoD} at the same time as in WLL_{AR} ;
- 4) for all n

$$\Delta_{\text{RoD}}(a(n)) = \Delta_{\text{AR}}(a(n)). \quad (52)$$

Proof: We prove by induction on the n th call arrival $a(n)$ that hypotheses (1)–(4) hold.

Basis: Initially, there is no outstanding call in both WLL_{RoD} and WLL_{AR} . That is, for $i = 1$ to M , $s_i^-(a(1)) = w_i^-(a(1)) = x_i^-(a(1)) = 0$. For $n = 1$, after the first call arrival $a(1)$ occurs, it is obvious that hypotheses (1) and (2) hold for $a(1)$. If $a(1)$ is accepted in WLL_{RoD} and in WLL_{AR} at time $t(a(1))$ with call holding time $\tau(a(1))$, then $a(1)$ is completed in both WLL_{RoD} and WLL_{AR} at time $t(a(1)) + \tau(a(1))$, and hypothesis (3) holds for $a(1)$. If $t(a(1)) \leq t(a(1)) + \tau(a(1)) < t(a(2))$, we have $\Delta_{RoD}(a(1)) = \langle d_{RoD}(1, 1) \rangle$ and $\Delta_{AR}(a(1)) = \langle d_{AR}(1, 1) \rangle$. Form hypothesis (3), $t(d_{RoD}(1, 1)) = t(d_{AR}(1, 1))$. If $t(a(1)) + \tau(a(1)) \geq t(a(2))$, we have $\Delta_{RoD}(a(1)) = \Delta_{AR}(a(1)) = \langle \rangle$. Thus, $\Delta_{RoD}(a(1)) = \Delta_{AR}(a(1))$ and hypothesis (4) holds for $a(1)$.

Induction: Assume that the four hypotheses hold for $a(n)$. Now we prove that the four hypotheses also hold for $a(n+1)$. We first show that

$$s_i^-(a(n+1)) + w_i^-(a(n+1)) = x_i^-(a(n+1)), \quad \text{for } 1 \leq i \leq M. \quad (53)$$

Since (52) holds for $a(n)$, if there is $m = 0$ call completion in $\Delta_{RoD}(a(n))$ and $\Delta_{AR}(a(n))$, it is clear that for $1 \leq i \leq M$, $s_i^-(a(n+1)) = s_i^-(a(n))$, $w_i^-(a(n+1)) = w_i^-(a(n))$, and $x_i^-(a(n+1)) = x_i^-(a(n))$. From (51), (53) holds if $m = 0$. Based on (51), (52), and from Lemma 5, (53) also holds for $m \geq 1$.

Now we show that (51) holds for $a(n+1)$. Let the $(n+1)$ th call arrival $a(n+1)$ be generated from the j th microcell (where $1 \leq j \leq M$). From Lemma 4, it suffices to show that (51) holds for three cases.

- (I) When **Case 1** in WLL_{RoD} holds, $a(n+1)$ to WLL_{RoD} uses a channel in the j th microcell. Therefore

$$s_j^+(a(n+1)) = s_j^-(a(n+1)) + 1 \quad \text{and} \\ w_j^+(a(n+1)) = w_j^-(a(n+1)). \quad (54)$$

From Lemma 4 (I), **Case A** in WLL_{AR} holds. In this case, $a(n+1)$ to WLL_{AR} uses a channel in the j th microcell and

$$x_j^+(a(n+1)) = x_j^-(a(n+1)) + 1. \quad (55)$$

From (54), (55), and (53), we have $s_j^+(a(n+1)) + w_j^+(a(n+1)) = x_j^+(a(n+1))$, for $a(n+1)$. For $1 \leq i \neq j \leq M$, it is clear that $s_i^+(a(n+1)) + w_i^+(a(n+1)) = x_i^+(a(n+1))$.

- (II) When **Case 2** in WLL_{RoD} holds, $a(n+1)$ to WLL_{RoD} uses a channel in the j th microcell. Then

$$s_j^+(a(n+1)) = s_j^-(a(n+1)) + 1 \quad \text{and} \\ w_j^+(a(n+1)) = w_j^-(a(n+1)). \quad (56)$$

When **Cases 3** or **4** in WLL_{RoD} hold, $a(n+1)$ to WLL_{RoD} uses a macrocell channel. Then

$$s_j^+(a(n+1)) = s_j^-(a(n+1)) \quad \text{and} \\ w_j^+(a(n+1)) = w_j^-(a(n+1)) + 1. \quad (57)$$

From Lemma 4 (II), **Case B** in WLL_{AR} holds. In this case, $a(n+1)$ to WLL_{AR} uses a macrocell channel and

$$x_j^+(a(n+1)) = x_j^-(a(n+1)) + 1. \quad (58)$$

From (56), (57), (58), and (53), we have $s_j^+(a(n+1)) + w_j^+(a(n+1)) = x_j^+(a(n+1))$, for $a(n+1)$. Note that when **Case 4** in WLL_{RoD} holds, a repacking candidate in the r th microcell is handed off from the macrocell to the r th microcell and $s_r^+(a(n+1)) = s_r^-(a(n+1)) + 1$ and $w_r^+(a(n+1)) = w_r^-(a(n+1)) - 1$. Therefore, $s_r^+(a(n+1)) + w_r^+(a(n+1)) = [s_r^-(a(n+1)) + 1] + [w_r^-(a(n+1)) - 1] = x_r^-(a(n+1)) = x_r^+(a(n+1))$. For $1 \leq i \neq j, r \leq M$, it is clear that $s_i^+(a(n+1)) + w_i^+(a(n+1)) = x_i^+(a(n+1))$.

- (III) When **Case 5** in WLL_{RoD} holds, since WLL_{RoD} satisfies **Conditions 1, 2, and 3**, $a(n+1)$ to WLL_{RoD} is blocked. Therefore

$$s_j^+(a(n+1)) = s_j^-(a(n+1)) \quad \text{and} \\ w_j^+(a(n+1)) = w_j^-(a(n+1)). \quad (59)$$

From Lemma 4 (III), **Case C** in WLL_{AR} holds. Since WLL_{AR} satisfies **Conditions 4** and **5**, $a(n+1)$ to WLL_{AR} is blocked. Thus

$$x_j^+(a(n+1)) = x_j^-(a(n+1)). \quad (60)$$

From (59), (60), and (53), we have $s_j^+(a(n+1)) + w_j^+(a(n+1)) = x_j^+(a(n+1))$ for $a(n+1)$. For $1 \leq i \neq j \leq M$, it is clear that $s_i^+(a(n+1)) + w_i^+(a(n+1)) = x_i^+(a(n+1))$.

Based on the above discussion, we have $s_i^+(a(n+1)) + w_i^+(a(n+1)) = x_i^+(a(n+1))$, for $1 \leq i \leq M$. Thus, hypothesis (1) holds for $a(n+1)$.

When **Case 5** in WLL_{RoD} holds and **Case C** in WLL_{AR} holds, $a(n+1)$ is blocked in both WLL_{RoD} and WLL_{AR} . Thus, hypothesis (2) holds for $a(n+1)$.

When **Cases 1, 2, 3, or 4** in WLL_{RoD} hold and **Cases A** or **B** in WLL_{AR} hold, $a(n+1)$ is accepted in WLL_{RoD} and in WLL_{AR} at the same time. Since $a(n+1)$ has the same call holding time in both WLL_{RoD} and WLL_{AR} , $a(n+1)$ is completed in WLL_{AR} at the same time as that in WLL_{RoD} . Thus, hypothesis (3) holds for $a(n+1)$.

Since hypotheses (2) and (3) hold for $a(n+1)$, by following an argument similar to the proof for hypothesis (4) in the Basis of Lemma 6, it is clear that hypothesis (4) also holds for $a(n+1)$. **Q.E.D.**

Directly from hypothesis (2) of Lemma 6, we have Theorem 2.

APPENDIX III SIMULATION MODEL

This appendix describes a discrete event simulation model for RoD, NR, and AR. In the simulation model, all events are inserted into an event list and are deleted/processed from the event list in the nondecreasing time stamp order. A simulation clock is maintained to indicate the progress of the simulation, which is the time stamp of the event being processed. The following attributes are defined for an event e :

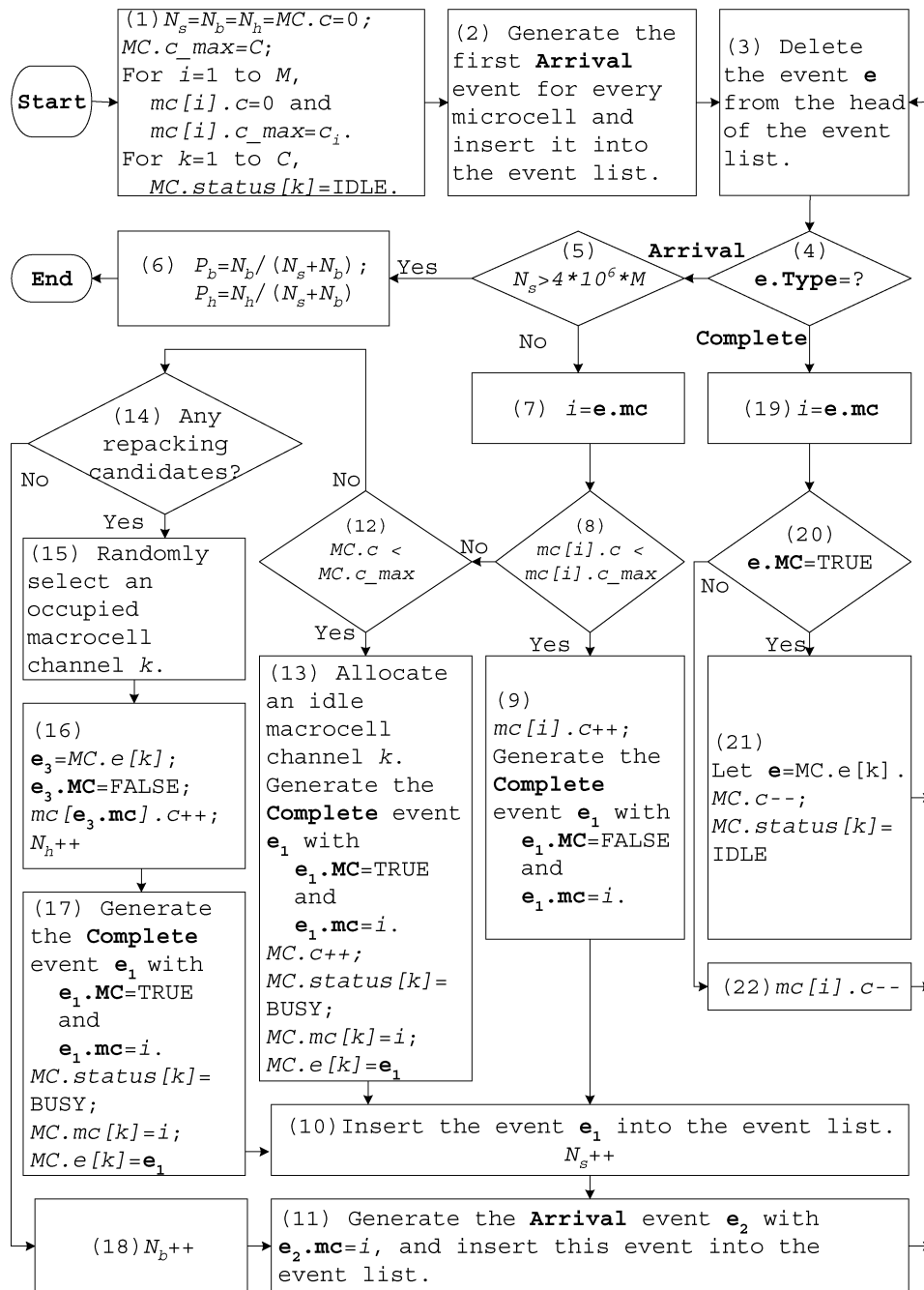


Fig. 6. Simulation flowchart for RoD-R.

- e.Type** attribute indicates the type of an event e , which can be **Arrival** or **Complete**;
- e.ts** attribute indicates the time when the event occurs;
- e.mc** attribute indicates the microcell where the event occurs;
- e.MC** attribute is a flag, which indicates whether the **Complete** event occurs at the macrocell.

A record MC is used to represent the macrocell. MC consists of five fields. MC.c is the number of busy channels in the macrocell. MC.c_max = C is the number of radio channels in the macrocell. MC.status[k] is the status of the kth radio channel (1 ≤ k ≤ C) in the macrocell, which can be

IDLE or BUSY. If a call occupies the kth macrocell channel (i.e., MC.status[k] = BUSY), MC.mc[k] is the microcell where the call is generated, and MC.e[k] is the pointer to the **Complete** event for the call. A record array mc[1...M] is used to represent the microcells. For the i th microcell, the record mc[i] consists of two fields: mc[i].c is the number of busy channels, and mc[i].c_max = c_i is the number of radio channels.

The outputs measured in the simulation are the number N_s of successful calls, the number N_b of blocking calls, and the number N_h of handoffs. From the above measures, the blocking probability P_b and the handoff probability P_h are computed as

$$P_b = \frac{N_b}{N_s + N_b} \quad \text{and} \quad P_h = \frac{N_h}{N_s + N_b}. \quad (61)$$

Fig. 6 shows the simulation flow chart for RoD-R. Step 1 initializes the variables. Step 2 generates the first **Arrival** event for each microcell and inserts the event into the event list. The next event e is then deleted from the head of the event list at Step 3. At Step 4, the event e is processed based on its type described as follows.

e.type=Arrival At Step 5, if $N_s > 4 \times 10^6 \times M$, then the simulation terminates. Step 6 computes the performance measures P_b and P_h using (61). Otherwise, let the **Arrival** event be generated from the i th microcell (i.e., $i = e.mc$) at Step 7. At Step 8, if the i th microcell has idle channels (i.e., $mc[i].c < mc[i].c_max$), one channel in the i th microcell is assigned to the incoming call, and Steps 9–11 are executed. Step 9 increments $mc[i].c$ by one and generates a new **Complete** event e_1 with $e_1.MC = FALSE$ and $e_1.mc = i$. The time stamp $e_1.ts$ is set to $e.ts$ plus the call holding time generated from a random number generator. Step 10 inserts the event e_1 into the event list and increments N_s by 1. Step 11 generates a new **Arrival** event e_2 with $e_2.mc = i$. The time stamp $e_2.ts$ is set to $e.ts$ plus the interarrival time generated from a random number generator. Event e_2 is inserted into the event list.

At Step 8, if the i th microcell is blocked (i.e., $mc[i].c = mc[i].c_max$), the call attempt overflows to the macrocell. At Step 12, if the macrocell has idle channels (i.e., $MC.c < MC.c_max$), then Step 13 assigns an idle macrocell channel k to the call arrival. A new **Complete** event e_1 is generated with $e_1.MC = TRUE$ and $e_1.mc = i$. Step 13 also increments $MC.c$ by one, sets $MC.status[k] = BUSY$, $MC.mc[k] = i$, and $MC.e[k] = e_1$. Then Steps 10 and 11 are executed as described before.

At Step 12, if the macrocell is blocked (i.e., $MC.c = MC.c_max$), Step 14 checks if there are any repacking candidates. If so, Step 15 randomly selects a repacking candidate. Assume that the candidate occupies the k th macrocell channel (i.e., $MC.status[k] = BUSY$ and $mc[MC.mc[k]].c < mc[MC.mc[k]].c_max$). At Step 16, the original **Complete** event e_3 of the selected repacking candidate is modified such that $e_3.MC = FALSE$. Step 16 also increments $mc[e_3.mc].c$ and N_h by one. Step 17 gener-

ates a new **Complete** event e_1 with $e_1.MC = TRUE$ and $e_1.mc = i$ for the call arrival event e . Step 17 also sets $MC.status[k] = BUSY$, $MC.mc[k] = i$, and $MC.e[k] = e_1$. Then Steps 10 and 11 are executed.

At Step 14, if NR candidate is found, the new call is blocked. In this case, N_b is incremented by one at Step 18, and the next **Arrival** event is generated at Step 11.

e.type=Complete At Step 19, let the **Complete** event be generated from the i th microcell (i.e., $i = e.mc$). Step 20 checks if the **Complete** event occurs at the macrocell. If so, Step 21 releases the macrocell channel k used by the call ($e = MC.e[k]$). That is, $MC.c$ is decremented by one and $MC.status[k]$ is set to IDLE. If the **Complete** event occurs at the i th microcell, Step 22 decrements $mc[i].c$ by one.

The above flow chart can be easily modified for NR, AR, RoD-L, and RoD-ST. In NR, Steps 14–17 are not executed. In AR, Steps 14–16 are performed after Step 22. In RoD-L, Step 15 selects the repacking candidate whose microcell has the least traffic loading (e.g., the call occupying the k th macrocell channel has the smallest $mc[MC.mc[k]].c$). In RoD-ST, Step 15 selects all repacking candidates and Step 16 is executed for all candidates.

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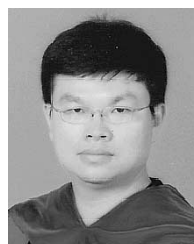
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