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ORIGINAL ARTICLE

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C_{pm} MPPAC for manufacturing quality control applied to precision voltage reference process

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Abstract A multiprocess performance analysis chart (MPPAC), based on the process capability index C_{pm}, called C_{pm} MPPAC, is developed to analyse the manufacturing quality of a group of processes in a multiple process environment. The C_{pm} MPPAC conveys critical information about multiple processes regarding the departure of the process and process variability on one single chart. Existing research on MPPAC has been restricted to obtaining quality information from one single sample of each process, ignoring sampling errors. The information provided from the existing MPPAC chart, therefore, is unreliable and misleading, resulting in incorrect decisions. In this paper, the natural estimator of C_{pm} is considered based on multiple samples. Based on the natural estimator of C_{pm}, sampling errors are considered by providing an explicit formula with Matlab to obtain the estimation accuracy of the C_{pm}. The sampling accuracy of C_{pm} is tablulated for sample size determination so that engineers/practitioners can use it for in-plant applications. An example of multiple PVR processes is presented to illustrate the applicability of C_{pm} MPPAC for manufacturing quality control.

Keywords Multiprocess performance analysis chart · Maximum likelihood estimator · Process capability index · Sample size determination

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1 Introduction

Process capability indices (PCIs) have been widely used in various manufacturing industries to provide a numerical measures of process potential and process performance. The two most commonly used process capability indices are C_p and C_{pk}, introduced by Kane [1]. These two indices are defined in the following:

$$C_p = \frac{USL - LSL}{6\sigma}$$

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\},$$

where USL and LSL are the upper and the lower specification limits, respectively, μ is the process mean, and σ is the process standard deviation. The index C_p measures the process variation relative to the manufacturing tolerance, which reflects only the process potential. The C_{pk} index measures process performance based on the process yield (percentage of conforming items) without considering the process loss (a new criteria for process quality championed by Hsiang and Taguchi [2]). Taking process departure into consideration (which reflects the process loss), Chan et al. [3] developed the index C_{pm}, which measures the ability of the process to cluster around the target. The C_{pm} index is defined as:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}},$$

where T is the target value, and $d = (USL - LSL)/2$ is half of the length of the specification interval (LSL, USL). Ruczinski [4] showed that $yield \geq 2\Phi\ (3C_{pm}) - 1$, or the fraction of nonconformities $\leq 2\Phi\ (-3C_{pm})$. Table 1 displays various values of $C_{pm} = 0.95(0.05)$ 2.00, and the corresponding nonconformities (in PPM). For example, if a process has capability with $C_{pm} = 1.25$, then the manufacturing proces yield would be at least 99.982%. Some commonly used values of

Table 1 Various values of $C_{pm}=0.95(0.01)2.00$ and the corresponding nonconformities (in PPM)

C_{pm}	PPM
0.95	4371.923
1.00	2699.796
1.05	1632.705
1.10	966.848
1.15	560.587
1.20	318.217
1.25	176.835
1.30	96.193
1.35	51.218
1.40	26.691
1.45	13.614
1.50	6.795
1.55	3.319
1.60	1.587
1.65	0.742
1.70	0.340
1.75	0.152
1.80	0.067
1.85	0.029
1.90	0.012
1.95	0.005
2.00	0.002

C_{pm} are 1/3 (process is incapable), 1/2 (process is incapable), 1.00 (process is normally called capable), 1.33 (process is normally called satisfactory), 1.67 (process is normally call good), and 2.00 (process is normally called super).

Statistical process control charts have been widely used to monitor individual factory manufacture processes on a routine basis. Those charts are essential tools for the control and improvement of these processes. In the multiprocess environment, where a group of processes need to be monitored and controlled, it could be difficult and time-consuming for factory engineers or supervisors to analyse the individual chart in order to evaluate overall performance of factory process control activities. Singhal [5, 6] introduced the multiprocess performance analysis chart (MPPAC) using the process capability indices C_P and C_{pk}, which can be implemented to illustrate and analyse the performance of a group of processes in a multiple process environment by including the departure of the process mean from the target value, process variability, capability zones, and expected fallout outside specification limits on a single chart. Pearn and Chen [7] proposed a modification to MPPAC that combined the more advanced process capability index, C_{pm}, to identify the problems causing the processes to fail to centre around the target. Pearn et al. [8] introduced MPPAC based on the incapability index. Chen et al. [9] presented a modification to the MPPAC.

With respect to these studies, there are some limitations and shortcomings included. First, the existing MPPACs based on the process capability indices are restricted to obtaining quality information by calculating one single sample data for each process. In practice, however, manufacturing information is often derived from multiple samples rather than one single sample, particularly, when a daily-based process control plan is implemented for monitoring process stability. Second, most existing MPPACs using process capability indices simply use the estimates of the indices on the chart and then make a conclusion as to whether processes meet the capability requirement and directions need to be taken for further quality improvement. Their approach is highly unreliable, since sampling errors are ignored. Therefore, in this paper, a new control chart is introduced for C_{pm} MPPAC. The natural estimator of C_{pm}, based on multiple samples, is investigated. The sampling errors are also considered and a Matlab program is developed to determine the overall number of observations and sub-samples required for a C_{pm} estimating accuracy. An example of PVRP is presented to illustrate the applicability of C_{pm} MPPAC for production quality control.

2 The C_{pm} MPPAC

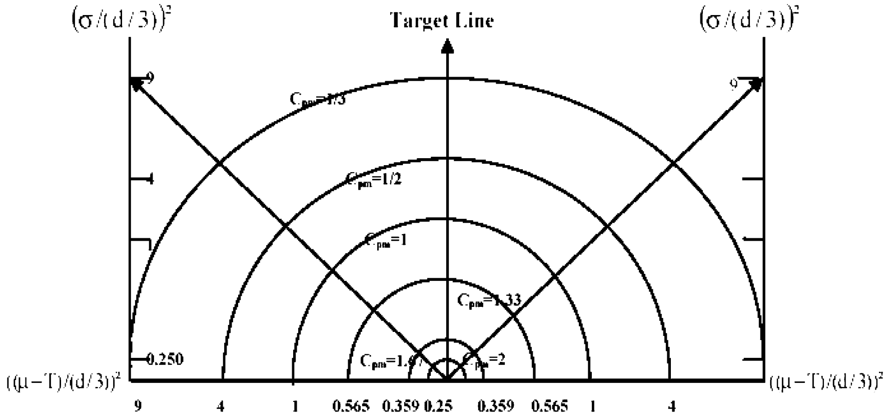
Singhal [5] indicated that the MPPAC can be used to evaluate the performance of a single process as well as multiple processes, to set the priorities among multiple processes for quality improvement, and to indicate whether reducing the variability or the departure of the process mean should be the focus, as well as to provide an easy way to qualify the process improvement by comparing the locations on the chart of the processes before and after the improvement effort. Since C_{pm} simultaneously measures process variability and centreing, a C_{pm} MPPAC would provide a convenient way to identify problems in process capability after statistical control is established. Based on the definition, first $C_{pm}=h$ is set for various h values, then a set of (μ,σ) values satisfying the equation:

$$\left(\frac{\sigma}{d/3}\right)^2+\left(\frac{\mu-T}{d/3}\right)^2=\left(\frac{1}{h}\right)^2$$

can be plotted on the contour (indifference curve) of $C_{pm}=h$. These contours are semicircles centered at $(T, 0)$ with radius $1/h$. The more capable the process, the smaller the semicircle. The six contours are plotted on the C_{pm} MPPAC for the six values, $C_{pm}=1/3$, 1/2, 1, 1.33, 1.67, and 2, as shown in Fig. 1. On the C_{pm} MPPAC, it is noted that:

(a) The parallel line and perpendicular line through the plotted point intersecting the vertical axis and horizontal axis at points represented $(\sigma/(d/3))^2$ and $((\mu-T)/(d/3))^2$, respectively.
(b) The distance between T and the point, which the perpendicular line through the plotted point intersecting the horizontal axis, denotes the departure of process mean from target.
(c) The distance between 0 and the point, which the parallel line through the plotted point intersecting the vertical axis, denotes the process variance.

Fig. 1 The contours of C_{pm} MPPAC



(d) For the points inside the semicircle of contour (indifference curve) $C_{pm}=h$, the corresponding C_{pm} values are larger than h. For the points outside the semicircle of contour $C_{pm}=h$, the corresponding C_{pm} values are smaller than h.

(e) As the point gets closer to the target, the value of the C_{pm} becomes larger, and the process performance is better.

(f) For processes with fixed values of C_{pm}, the points within the two 45° lines envelop, the process variability is contributed mainly by the process variance.

(g) For processes with fixed values of C_{pm}, the points outside the two 45° lines envelop, the process variability is contributed mainly by the process departure.

In general, the process parameters μ and σ^2 are unknown. But, in practice μ and σ^2 can be estimated by sample data obtained from stable processes. In the next section, estimating C_{pm} and estimation accuracy based on multiple samples is investigated.

3 Estimating C_{pm} based on multiple samples

Kirmani et al. [10] indicated that a common practice of process capability estimation in the manufacturing industry is to first implement a routine-basis data collection program for monitoring/controlling the process stability, then to analyse the past "in control" data. For multiple samples of m_s groups, each of size n, are chosen randomly from a stable process which follows a normal distribution $N(\mu,\sigma^2)$. Let $\bar{X}_i=\sum_{j=1}^{n}x_{ij}/n$ and $\left[S_i=(n)^{-1}\sum_{j=1}^{n}\left(x_{ij}-\bar{X}_i\right)^2\right]^{1/2}$ be the i-th sample mean and the sample standard deviation, respectively. The following natural estimator of C_{pm} is considered:

$$\tilde{C}_{\mathrm{pm}}^{M}=\frac{USL-LSL}{6\sqrt{\mathrm{S}_{\mathrm{p}}^{2}+\left(\overline{\overline{X}}-T\right)^{2}}},$$

where $\bar{\bar{X}}=\sum_{i=1}^{m_s}\bar{X}_i/m_s$ and $S_p^2=\sum_{i=1}^{m_s}S_i^2/m_s$.

If the process follows the normal distribution $N(\mu,\sigma^2)$, then $\tilde{C}_{pm}^{M}=\sqrt{N}\left[\frac{USL-LSL}{6\sigma}\right]\left[\frac{NS_P^2}{\sigma^2}+\frac{N\left(\overline{\overline{X}}-m\right)^2}{\sigma^2}+\frac{N(m-T)^2}{\sigma^2}\right]^{-1/2}$, where $\sum_{i=1}^{m_s}n=N$.

The NS_P^2/σ^2 and $N\left(\overline{\overline{X}}-\mu\right)/\sigma^2$ are distributed as ordinary central Chi-square distribution with $N{-}m_s$ and one degree of freedom, $\chi^2_{N-m_s}$ and χ^2_1, respectively. Therefore,

$$\tilde{C}_{pm}^{M}\sim\frac{USL-LSL}{6\sigma}\sqrt{\frac{N}{\chi^2_{N-m_s+1,\lambda}}}=C_P\sqrt{\frac{N}{\chi^2_{N-m_s+1,\lambda}}},$$

where $\chi^2_{N,\lambda}$ denotes the noncentral Chi-square distribution with N degrees of freedom and noncentral parameter $\lambda=N((\mu-T)/\sigma)^2$. The r-th moment (about zero) can be obtained as the following:

$$E\left[\tilde{C}_{Pm}^{M}\right]^r=\left(\sqrt{N}C_P\right)^rE\left(\chi^2_{N-m_s+1,\lambda}\right)^{-r/2}$$
$$=\left(\frac{\sqrt{N}C_P}{\sqrt{2}}\right)^r\exp\left(\frac{-\lambda}{2}\right)\sum_{j=0}^{\infty}\left\{\frac{(\lambda/2)^j}{j!}\times\frac{\Gamma((2j+N-m_s+1-r)/2)}{\Gamma((2j+N-m_s+1)/2)}\right\}$$

The probability density function (PDF) of natural estimator of C_{pm} can be easily attained as the following, where $C'=3\sqrt{N}C_p$, $N{-}m_s=N^*$, and $x>0$.

$$f(x)=\frac{2^{(1-N^*)/2}C'^{(N^*+1)}}{3^{(N^*+1)}x^{(N^*+2)}}\exp\left[-\frac{\lambda}{2}-\frac{C'^2}{18x^2}\right]\sum_{j=0}^{\infty}\left\{\left[\frac{\lambda C'^2}{36x^2}\right]^j\right.$$
$$\left.\times\left[j!\Gamma\left(\frac{N^*+1+2j}{2}\right)\right]^{-1}\right\}$$

Using the method similar to that presented in Vännman [11], an exact form of the cumulative distribution function of $\tilde{C}_{Pm}^{M}$ may be obtained. The cumulative distribution function of $\tilde{C}_{Pm}^{M}$ can be expressed in terms of a mixture of the ordinary central Chi-square distribution and the normal distribution:

$$F_{\tilde{C}_{pm}^{M}}(x)=1-\int_0^{b\sqrt{N}/(3x)}G\left(\frac{b^2N}{9x^2}-t^2\right)\left[\phi\left(t+\xi\sqrt{N}\right)\right.$$
$$\left.+\phi\left(t-\xi\sqrt{N}\right)\right]dt,$$

where $b = d/\sigma$, $\xi = (\mu - T)/\sigma$, $G(\cdot)$ is the cumulative distribution function of the ordinary central Chi-square distribution $\chi^2_{N-m_s}$, and $\phi(\cdot)$ is the probability density function of the standard normal distribution N (0, 1). Note that for cases with one single sample, $m_s = 1$, the special case of multiple samples, the statistical properties of the estimator of C_{pm} are proposed by Chan et al. [3], Boyles [12], Pearn et al. [13], Kotz and Johnson [14], Vännman and Kotz [15], and Vännman [11].

4 Estimation accuracy of C_{pm}

For processes with target value setting to the mid-point of the specification limits ($T = (USL + LSL)/2$), the index may be rewritten as the following. Note that when $C_{pm} = C$, $b = d/\sigma$ can be expressed as $b = 3C\sqrt{1+\xi^2}$. Thus, the index C_{pm} may be expressed as a function of the characteristic parameter ξ.

$$C_{pm} = \frac{d}{3\sigma\sqrt{1+\xi^2}} = \frac{d/\sigma}{3\sqrt{1+\xi^2}},$$

where $\xi = (\mu - T)/\sigma$. Hence, given the total number of observations N, the number of sub-samples m_s with the confidence level γ, the parameter ξ, and the estimating accuracy R_{pm} can be obtained using numerical integration technique with iterations, to solve the following Eq. 1. It should be noted, particularly, that Eq. 1 is an even function of ξ. Thus, for both $\xi = \xi_0$ and $\xi = -\xi_0$ the same total observations N may be obtained.

$$\int_0^{R_{pm}\sqrt{N(1+\xi^2)}} G\left(R_{pm}^2 N(1+\xi^2) - t^2\right) \times \left[\phi\left(t + \xi\sqrt{N}\right) + \phi\left(t - \xi\sqrt{N}\right)\right] dt = 1 - \gamma. \quad (1)$$

4.1 Estimation accuracy R_{pm} and parameter ξ

Since the process parameters μ and σ are unknown, then the distribution characteristic parameter, $\xi = (\mu - T)/\sigma$ is also unknown, which has to be estimated in real applications, naturally by substituting μ and σ with the sample mean $\overline{\overline{X}}$ and the sample standard deviation S_p. Such an approach introduces additional sampling errors from estimating ξ to determine the sample accuracy, and certainly would make this approach (and of course including all the existing methods) less reliable. Consequently, any decisions made would result in less production yield assurance to the factories, and provide less quality protection to the customers. To eliminate the need for further estimation of the distribution characteristic parameter $\xi = (\mu - T)/\sigma$, the behaviour of the sample accuracy R_{pm} must be examined against the parameter $\xi = (\mu - T)/\sigma$.

Extensive calculations are performed to obtain the R_{pm} for $\xi = 0(0.1)3.00$, the total number of observations

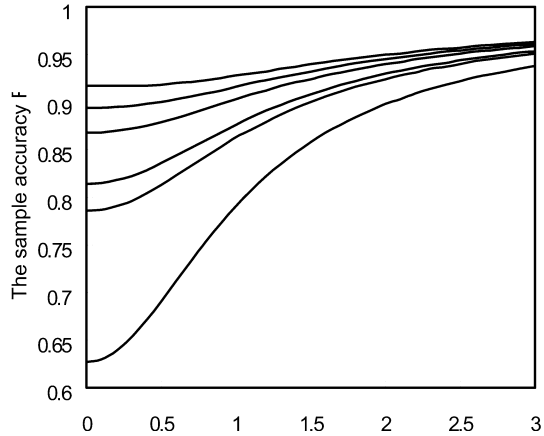


Fig. 2 Plots of R_{mp} vs $|\xi|$ for $N = 200$, $m_s = 1$, 10, 20, 40, 50, 100 (from *top* to **bottom**), $\gamma = 0.95$

$N = 200$, $m_s = 1$, 10, 20, 40, 50, and 100 with confidence level $\gamma = 0.90$, 0.95, 0.975, and 0.99. The results indicate that: 1. The sample precision is increasing in ξ and is decreasing in m_s, 2. The sample precision R_{pm} obtains its minimum at $\xi = 0$ in all cases. Hence, for practical purposes Eq. 1 may be solved with $\xi = 0$ to obtain the required sample accuracy for a given N, m_s and γ, without having to further estimate the parameter ξ. Thus, the level of confidence γ can be ensured, and the decisions made based on such approach are indeed more reliable. Fig. 2 plots the curves of the sample accuracy R_{pm} versus the parameter ξ for $N = 200$ and $m_s = 1$, 10, 20, 40, 50, and 100 with confidence level $\gamma = 0.95$. For bottom curve 1, $m_s = 100$; for bottom curve 2, $m_s = 50$; for bottom curve 3, $m_s = 40$; for top curve 3, $m_s = 20$; for top curve 2, $m_s = 10$; for top curve 1, $m_s = 1$.

5 Sample size determination for C_{pm} MPPAC

Using Eq. 1, the estimation accuracy of R_{pm} may be computed. Three auxiliary functions for evaluating R_{pm} are included here: (a) the cumulative distribution function of the chi-square $\chi^2_{N-m_s}$, $G(\cdot)$, (b) the probability density function of the standard normal distribution ϕ $(\cdot)$, and (c) the function of numerical evaluate integration using the recursive adaptive Simpson quardrature, or "quad". The program sets $(\mu - T)/\sigma = 0$, reads the number of sub-samples m_s, the total number of observations N, and the confidence level γ, then outputs with the estimating precision R_{pm}. The program, actual executed inputs, and outputs are listed below.

Matlab Program for computing the accuracy

```
%--------------
% This program calculates the sample
% accuracy of Cpm for given sample
% observations, the number of samples
% and confidence level.
%--------------
clear global
```

Table 2 Total number of sample observations, $nm_s = N$, number of samples, m_s, and precision of estimation with $\gamma = 0.90, 0.95, 0.975, 0.99$

n	4				5				6			
m_s	γ											
	0.90	0.95	0.975	0.99	0.9	0.95	0.975	0.99	0.90	0.95	0.975	0.99
5	0.682	0.630	0.587	0.538	0.727	0.680	0.641	0.596	0.759	0.715	0.679	0.637
6	0.692	0.649	0.608	0.563	0.740	0.697	0.661	0.619	0.771	0.731	0.697	0.659
7	0.708	0.663	0.626	0.583	0.751	0.711	0.676	0.637	0.781	0.744	0.712	0.676
8	0.717	0.675	0.640	0.599	0.760	0.722	0.689	0.652	0.789	0.754	0.724	0.690
9	0.725	0.685	0.651	0.613	0.767	0.731	0.700	0.665	0.795	0.762	0.734	0.702
10	0.731	0.694	0.661	0.625	0.773	0.739	0.709	0.676	0.801	0.770	0.743	0.712
11	0.737	0.701	0.670	0.635	0.778	0.745	0.718	0.685	0.806	0.776	0.750	0.721
12	0.742	0.708	0.678	0.644	0.783	0.751	0.725	0.694	0.810	0.781	0.757	0.728
13	0.747	0.713	0.685	0.652	0.787	0.757	0.731	0.701	0.814	0.786	0.763	0.735
14	0.751	0.719	0.691	0.659	0.791	0.761	0.736	0.708	0.818	0.791	0.768	0.741
15	0.755	0.723	0.696	0.666	0.794	0.766	0.741	0.714	0.821	0.795	0.772	0.747
16	0.758	0.728	0.702	0.672	0.797	0.770	0.746	0.719	0.823	0.798	0.777	0.752
17	0.761	0.731	0.706	0.677	0.800	0.773	0.750	0.724	0.826	0.802	0.781	0.756
18	0.764	0.735	0.710	0.682	0.802	0.776	0.754	0.728	0.828	0.805	0.784	0.760
19	0.766	0.738	0.714	0.687	0.805	0.779	0.758	0.733	0.830	0.807	0.787	0.764
20	0.769	0.741	0.718	0.691	0.807	0.782	0.761	0.737	0.832	0.810	0.790	0.768
21	0.771	0.744	0.721	0.695	0.809	0.785	0.764	0.740	0.834	0.812	0.793	0.771
22	0.773	0.747	0.724	0.699	0.811	0.787	0.767	0.744	0.836	0.814	0.796	0.774
23	0.775	0.749	0.727	0.702	0.812	0.789	0.770	0.747	0.838	0.817	0.798	0.777
24	0.777	0.752	0.730	0.705	0.814	0.791	0.772	0.750	0.839	0.818	0.801	0.778
25	0.778	0.754	0.733	0.708	0.816	0.793	0.774	0.752	0.841	0.820	0.803	0.783
26	0.780	0.756	0.735	0.711	0.817	0.795	0.777	0.755	0.842	0.822	0.805	0.785
27	0.782	0.758	0.738	0.714	0.818	0.797	0.779	0.758	0.843	0.824	0.807	0.787
28	0.783	0.760	0.740	0.717	0.820	0.799	0.781	0.760	0.844	0.825	0.809	0.789
29	0.784	0.762	0.742	0.719	0.821	0.800	0.783	0.762	0.846	0.827	0.810	0.792
30	0.786	0.763	0.744	0.721	0.822	0.802	0.784	0.764	0.847	0.828	0.812	0.793
31	0.787	0.765	0.746	0.724	0.823	0.803	0.786	0.766	0.848	0.829	0.814	0.795
32	0.788	0.766	0.748	0.726	0.824	0.805	0.788	0.768	0.849	0.831	0.815	0.797
33	0.789	0.768	0.749	0.728	0.825	0.806	0.789	0.770	0.850	0.832	0.817	0.799
34	0.790	0.769	0.751	0.730	0.826	0.807	0.791	0.772	0.851	0.833	0.818	0.800
35	0.791	0.771	0.752	0.732	0.827	0.809	0.792	0.774	0.851	0.834	0.819	0.802
36	0.792	0.772	0.754	0.733	0.828	0.810	0.794	0.775	0.852	0.835	0.821	0.804
37	0.793	0.773	0.755	0.735	0.829	0.811	0.795	0.777	0.853	0.836	0.822	0.805
38	0.794	0.774	0.757	0.737	0.830	0.812	0.796	0.778	0.854	0.837	0.823	0.806
39	0.795	0.775	0.758	0.738	0.831	0.813	0.797	0.780	0.855	0.838	0.824	0.808
40	0.796	0.776	0.760	0.740	0.832	0.814	0.799	0.781	0.855	0.839	0.825	0.809

```
[N1 ms1 r1]=read ('Enter the total observations, the number of subsamples, and confidence level:');
global b N epsilon ecpm ms
N=N1;
r=r1;
ms=ms1;
epsilon=0;
ecpm=1.0;
b=0; d=0;
c=0.2:0.025:3;
for i=1:1:113
b=0; d=0; y=0;
b=3*c(i)*sqrt(1+epsilon^2);
d=b*sqrt(N)/(3*ecpm);
y=quad('cpm',0,d);
if (y-(1-r))>0
break
end
end
c=0.2+0.025*(i-1):-0.001:0.2;
for k=1:(0.025*(i-1)*1000)+1
b=0; d=0; y=0;
b=3*c(k)*sqrt(1+epsilon^2);
d=b*sqrt(N)/(3*ecpm);
y=quad('cpm',0,d);
if ((1-r)-y)>0.0001
break
end
end
fprintf ('The Estimating Accuracy is %g\n',c(k)/ecpm)
%--------------
% read.m file.
%--------------
function [a1, a2, a3]=read(labl)
if nargin==0,
labl='?';
end
n=nargout;
```

Table 3 Total number of sample observations, $nm_s = N$, number of samples, m_s, and precision of estimation with $\gamma = 0.90, 0.95, 0.975, 0.99$

n	8				10				12			
m_s	γ											
	0.90	0.95	0.975	0.99	0.9	0.95	0.975	0.99	0.90	0.95	0.975	0.99
5	0.800	0.762	0.730	0.693	0.827	0.792	0.763	0.729	0.845	0.814	0.787	0.756
6	0.811	0.776	0.746	0.712	0.837	0.805	0.778	0.747	0.855	0.826	0.801	0.772
7	0.820	0.787	0.759	0.728	0.845	0.815	0.790	0.761	0.862	0.835	0.812	0.785
8	0.827	0.796	0.770	0.740	0.851	0.823	0.800	0.773	0.868	0.843	0.821	0.796
9	0.833	0.804	0.779	0.751	0.856	0.830	0.808	0.782	0.873	0.849	0.828	0.805
10	0.838	0.810	0.787	0.760	0.861	0.836	0.815	0.790	0.877	0.854	0.835	0.812
11	0.842	0.816	0.793	0.767	0.865	0.841	0.821	0.797	0.880	0.859	0.840	0.819
12	0.846	0.821	0.799	0.774	0.868	0.846	0.826	0.804	0.884	0.863	0.845	0.824
13	0.849	0.825	0.804	0.778	0.871	0.849	0.831	0.809	0.886	0.866	0.849	0.829
14	0.852	0.829	0.809	0.785	0.874	0.853	0.835	0.814	0.889	0.870	0.853	0.834
15	0.855	0.832	0.813	0.790	0.876	0.856	0.838	0.818	0.891	0.873	0.856	0.838
16	0.857	0.835	0.817	0.795	0.879	0.859	0.842	0.822	0.893	0.875	0.860	0.842
17	0.860	0.838	0.820	0.799	0.881	0.861	0.845	0.826	0.895	0.878	0.862	0.845
18	0.862	0.841	0.823	0.802	0.882	0.864	0.848	0.829	0.897	0.880	0.865	0.848
19	0.864	0.843	0.826	0.806	0.884	0.866	0.850	0.832	0.898	0.882	0.867	0.851
20	0.865	0.846	0.829	0.809	0.886	0.868	0.853	0.835	0.890	0.884	0.870	0.853
21	0.867	0.848	0.831	0.812	0.887	0.870	0.855	0.838	0.901	0.885	0.872	0.856
22	0.868	0.850	0.833	0.815	0.889	0.872	0.857	0.840	0.902	0.887	0.874	0.858
23	0.870	0.851	0.836	0.817	0.890	0.873	0.859	0.842	0.904	0.888	0.875	0.860
24	0.871	0.853	0.838	0.820	0.891	0.875	0.861	0.845	0.905	0.890	0.877	0.862
25	0.872	0.855	0.840	0.822	0.892	0.876	0.863	0.847	0.906	0.891	0.879	0.864
26	0.874	0.856	0.841	0.824	0.893	0.878	0.864	0.849	0.907	0.892	0.880	0.866
27	0.875	0.858	0.843	0.826	0.894	0.879	0.866	0.850	0.908	0.894	0.881	0.867
28	0.876	0.859	0.845	0.828	0.895	0.880	0.867	0.852	0.908	0.895	0.883	0.869
29	0.877	0.860	0.846	0.830	0.896	0.881	0.869	0.854	0.909	0.896	0.884	0.871
30	0.878	0.862	0.848	0.831	0.897	0.882	0.870	0.855	0.910	0.897	0.885	0.872
31	0.879	0.863	0.849	0.833	0.898	0.883	0.871	0.857	0.911	0.898	0.886	0.873
32	0.880	0.864	0.850	0.835	0.899	0.884	0.872	0.858	0.912	0.899	0.887	0.875
33	0.880	0.865	0.852	0.836	0.899	0.885	0.873	0.860	0.912	0.900	0.889	0.876
34	0.881	0.866	0.853	0.838	0.900	0.886	0.875	0.861	0.913	0.900	0.890	0.877
35	0.882	0.867	0.854	0.839	0.901	0.887	0.876	0.862	0.913	0.901	0.890	0.878
36	0.883	0.868	0.855	0.840	0.901	0.888	0.877	0.863	0.914	0.902	0.891	0.879
37	0.883	0.869	0.856	0.841	0.902	0.889	0.877	0.864	0.915	0.903	0.892	0.880
38	0.884	0.870	0.857	0.843	0.903	0.890	0.878	0.865	0.915	0.903	0.893	0.881
39	0.885	0.870	0.858	0.844	0.903	0.890	0.879	0.866	0.916	0.904	0.894	0.882
40	0.885	0.871	0.859	0.845	0.904	0.891	0.880	0.867	0.916	0.905	0.895	0.883

```
str=input(labl,'s'); str=['[',str,']'];
v=eval (str);
L=length(v);
if L>=n, v=v(1:n);
else, v=[v,zeros(1,n-L)];
end
for j=1:nargout
eval(['a', int2str(j),'=v(j);']);
end
%--------------
% cpm.m file.
%--------------
function Q1=cpm(t)
global N b epsilon ecpm ms
Q1=chi2cdf(((b^2*N/
(9*ecpm^2))-t.^2),N-ms).*...
(normpdf((t+epsilon*sqrt(N)))
+normpdf((t-epsilon*sqrt(N))));
%--------------
% The end.
%--------------
```

Input Enter the total observations, the number of subsamples, and confidence level: 100,20,0.95

Output The estimating accuracy is 0.782

The sample size determination is essential to most factory applications, particularly for those implementing a routine-basis data collection plan for monitoring and controlling process quality. It directly relates to the sampling cost of a data collection plan. Tables 2 and 3 display the sample size N and number of samples, m_s, required and the corresponding precision of the estimation R_{pm}. For example, $\gamma = 0.95$, $N = 150$, $m_s = 30$ gives $R_{pm} = 0.802$. Thus, the true value of C_{pm}, is no less than $\tilde{C}_{pm}^{M} \times 0.802$. On the other hand, if $R_{pm} = 0.802$ is chosen, then $N = 102$ may be determined with $m_s = 17$ (each sample with six observations). Similarly, if $R_{pm} = 0.85$ is chosen, then $N = 190$ may be determined with $m_s = 10$ and $\gamma = 0.975$, $N = 198$ with $m_s = 6$ and $\gamma = 0.90$, or $N = 256$ with $m_s = 32$ and $\gamma = 0.975$, depending on which sampling plan is more appropriate to the application.

6 Manufacturing quality control for multiple PVR processes

In the following, the production quality of a group of multiple processes is investigated for manufacturing the voltage reference devices. Voltage references are essential to the accuracy and performance of analog systems. They are used in many types of analog circuitry for signal processing, such as, analog-to-digital (AD) or digital-to-analog (DA) converters and smart sensors. Precision voltage references can be used in constructing an accuracy regulated supply that could have better characteristics than some regulator chips, which can sometimes dissipate too much power. Another application of the voltage references is creating a precision, constant, current supply. In addition, voltage references are needed in the test equipment, which must be accurate, such as voltmeters, ohmmeters and ammeters.

Consider the following case taken from a microelectronics manufacturing factory that produces precision voltage reference devices. Twelve specific types of precision voltage reference devices extensively used on the PC-based instrumentation and test equipment with different precision voltage references specifications are selected in this study. Their precision voltage reference specifications are displayed in Table 4. A sample data collection plan is implemented in the factory on a daily basis to monitor/control production quality. The factory resource and sampling schedule allow the data collection plan be implemented with $N=150$ with $m_s=15$ (each sample with 10 observations). Looking at Tables 2 and 3, the estimation accuracy $R_{mp}=0.856$ is obtained, with confidence $\gamma=0.95$. The calculated overall sample mean, pooled sample standard deviation, the estimated $\tilde{C}^{M}_{pm}$, the minimum true value, and the maximum nonconformities are displayed in Table 5 and Table 6. Fig. 3 provides a plot of the C_{pm} MPPAC for the twelve processes using the data summarised in Table 5. These process points were analysed in Fig. 3 and the following critical summary of the quality condition was obtained for all processes.

(a) The plotted point E is outside the contour of $C_{pm}=1/2$. It indicates that the process has a very low capability. Since point E is close to the target line, it signifies that the process mean is close to the target value, and the poor capability results mainly from the significant process variation. Thus, immediate quality improvement actions must be taken to reduce process variance.

(b) The plotted points H, D, and G lie outside of the contour of $C_{pm}=1$. This indicates that the capability C_{pm} is less than 1. Since the point lies inside the two 45° lines envelop range, it indicates that the process variation measure, $(\sigma/(d/3))^2$, is more significant than the departure measure, $((\mu-T)/(d/3))^2$. Thus, reducing the process variance should be set to higher priority than reducing the process departure.

(c) The plotted points C, F, and B lie outside the contour of $C_{pm}=1$. Since these points also lie outside the two 45° lines envelop range. It indicates that the departure measure, $((\mu-T)/(d/3))^2$ is higher than process variation measure, $(\sigma/(d/3))^2$. Thus, quality improvement effort for these processes should first focus on reducing their process departure from the target value T, then on the reduction of the process variance.

Table 5 The calculated statistics of the ten processes

Process	$\overline{\overline{X}}$	S_P	$\left[\left(\overline{\overline{X}}-T\right)/(d/3)\right]^2$	$[S_P/(d/3)]^2$
A	4.999529	0.001491	0.02	0.2
B	10.00111	0.000667	1.78	0.64
C	14.99325	0.004796	1.82	0.92
D	19.99795	0.002728	0.38	0.67
E	1.00003	0.00015	0.13	3.24
F	0.499996	1.49E−06	1.44	0.2
G	2.999946	7.87E−05	0.29	0.62
H	11.99864	0.002272	0.46	1.29
I	9.004948	0.005333	0.68	0.79
J	6.00337	0.0032	0.71	0.64
K	3.000087	0.000296	0.03	0.35
L	17.99944	0.002057	0.035	0.47

Table 4 The precision voltage reference specifications

Code	V	Precision	*LSL*	*USL*
A	5	±0.2%	4.99	5.01
B	10	±0.025%	9.9975	10.0025
C	15	±0.1%	14.985	15.015
D	20	±0.05%	19.99	20.01
E	1	±0.025%	0.99975	1.00025
F	0.5	±0.002%	0.49999	0.50001
G	3	±0.01%	2.9997	3.0003
H	12	±0.05%	8.994	9.006
I	9	±0.2%	8.982	9.018
J	6	±0.2%	5.988	6.012
K	3	±0.05%	2.9985	3.0015
L	18	±0.05%	17.991	18.009

Table 6 The $\tilde{C}^{M}_{pm}$, minimum true capability C_{pm}, and the maximum nonconformities (in PPM)

Process	$\tilde{C}^{M}_{pm}$	C_{pm}	PPM
A	2.132	1.825	0.0438
B	0.643	0.550	98943
C	0.604	0.517	120900
D	0.976	0.835	8439
E	0.545	0.467	161210
F	0.781	0.669	44750
G	1.048	0.897	4331
H	0.756	0.647	52258
I	0.825	0.706	34175
J	0.861	0.737	27036
K	1.622	1.389	30.86
L	1.407	1.205	300.35

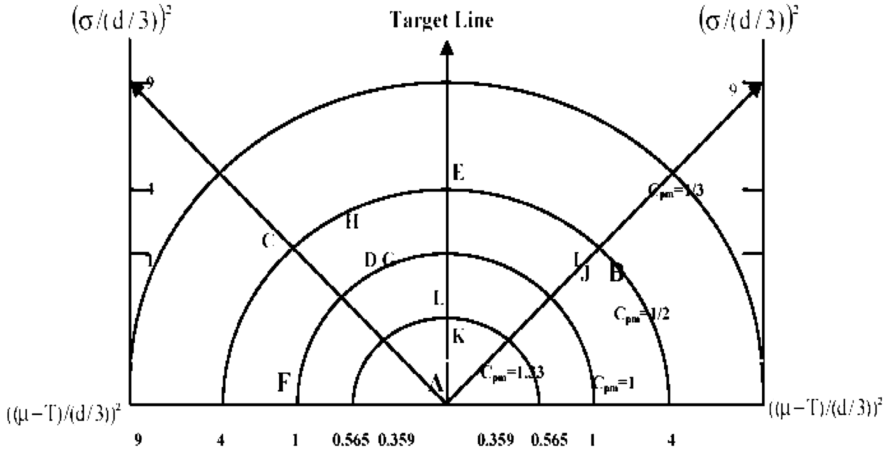


Fig. 3 The C_{pm} MPPAC for the application

(d) The plotted points I and J are very close to the two 45° lines, and are outside the contour of $C_{pm} = 1$. This indicates the contributions of process mean departure and process variance are equally significant factors for the poor capability of both processes.

(e) The plotted points K and L lie inside the contours of $C_{pm} = 1.33$ and $C_{pm} = 1$, respectively. It shows that both process capability values C_{pm} are greater than 1. Capabilities of both processes are consider satisfactory. They have lower priorities in allocating quality improvement efforts than other processes.

(f) Process A is close to T and the amount of variation is small. Therefore, process A is considered to be performing well. No immediate improvement actions need to be taken.

7 Conclusions

Conventional investigations on manufacturing quality control are restricted to obtaining quality information based on one single sample for each process ignoring sampling errors. The proposed C_{pm} MPPAC, using a process capability index C_{pm}, is useful for manufacturing quality control of a group of processes in a multiple process environment. In this paper, a new control chart, called C_{pm} MPPAC, was introduced that uses the natural estimator C_{pm} based on multiple samples. The accuracy of the estimation was investigated as a function of the process characteristic parameter $\xi = (\mu - m)/\sigma$, given a group of multiple control chart samples. Information regarding the true capability values and the maximum nonconformities (in PPM) is provided for production quality control. Appropriate sample sizes are then recommended to the proposed C_{pm} MPPAC for multiple processes production quality control. This approach ensures that the critical information conveyed from the C_{pm} MPPAC, based on multiple control chart samples, is more reliable than all other existing methods. A Matlab computer program was developed to calculate the estimating accuracy and provided convenient tables for practitioners to use in determining appropriate sample sizes needed for their factory applications. An example of PVR manufacturing process is given to illustrate the applicability and of the proposed C_{pm} MPPAC.

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