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Chaos synchronization and parameters identification of single time scale brushless DC motors

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Abstract

Chaos synchronization and parameters identification of single time scale brushless dc motors are studied in this paper. In order to analyze a variety of periodic and chaotic phenomena, we employ several numerical techniques such as phase portrait, bifurcation diagram, and Lyapunov exponents. By the adaptive control, the improved backstepping design method, the Gerschgorin theorem, and by addition of a monitor, chaos synchronization of two identical BLDCM systems are presented. Then, by the adaptive control, and the random optimization method, parameters identification is approached.

1. Introduction

Chaos synchronization has been applied in many fields such as secure communication [1,2], chemical and biological systems [3,4], etc.

The theme of this paper is brushless dc motor (BLDCM). The major advantage of BLDCM is the elimination of the physical contact between the brushes and the commutators. BLDCM has been widely applied in direct-drive applications such as robotics [5], aerospace [6], etc. In this paper, we investigate chaos synchronization and parameters identification of BLDCM. In order to analyze a variety of periodic and chaotic phenomena, we employ several numerical techniques such as time history, phase portrait, bifurcation diagram, and Lyapunov exponents.

This paper is organized as follows. Section 2 contains the dynamic characteristics of BLDCM [7–10]. First, the system model is described. Second, the system equations are transformed to a compact form. Finally, the numerical results of periodic and chaotic phenomena are presented. In Section 3, four methods are investigated to achieve chaos synchronization of identical systems: the adaptive control [11], the improved backstepping design method [12], the Gerschgorin theorem [13], and the addition of a monitor [14]. Two methods are investigated to achieve parameters identification in Section 4: the adaptive control [15], and the random optimization method [16]. Finally, the conclusions of the whole paper are briefly stated.

2. Regular and chaotic dynamics of brushless DC motor

In this section, the dynamic characteristics of BLDCM are investigated. First, the dynamic system model is given. Second, the state equations are transformed to a compact form. Finally, we present the numerical analysis of periodic and chaotic behavior of BLDCM.

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2.1. Description of the system model and differential equation of motion

BLDCM is an electromechanical system. The physical model of BLDCM is shown in Fig. 1 [7], where, $Q_{1,3}$, light transistor; $Q_{4,9}$, transistor; D, light diode; $L_{1,3}$, stator winding; $H_{1,3}$, light sensor.

The equation of electrical dynamics can be described by [8,9]

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{I}(t) = \frac{1}{\mathbf{L}(\theta)}\left[\mathbf{V}(t) - \mathbf{R}\mathbf{I}(t) - \left(\frac{\mathrm{d}\mathbf{L}(\theta)}{\mathrm{d}\theta}\mathbf{I}(t) + \frac{\mathrm{d}\Lambda_{\mathrm{M}}(\theta)}{\mathrm{d}\theta}\right)\frac{\mathrm{d}\theta}{\mathrm{d}t}\right] \tag{2.1.1}$$

where, $\mathbf{I}(t)$, the phase current vector; $\mathbf{L}(\theta)$, the inductance matrix; $\mathbf{V}(t)$, the vector corresponding to the voltages across the phase windings; $\mathbf{R}$, the winding resistance matrix; $\Lambda_{\mathrm{M}}(\theta)$, the flux linkage vector due to the presence of permanent magnets; θ, the displacement variable, and the equation of mechanical dynamics can be described by

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega = \frac{1}{J}[T(\mathbf{I},\theta) - T_{\mathrm{l}}(t)] \tag{2.1.2}$$

where, ω, the rotator angular velocity; J, the inertia of rotator; $T(\mathbf{I},\theta)$, the electromagnetic torque; $T_{\mathrm{l}}(t)$, the external torques imposed on the rotator shaft.

Accounting for viscous damping friction, the external torques can be described by

$$T_{\mathrm{l}}(t) = b\omega + T_{\mathrm{L}} \tag{2.1.3}$$

where, b, the viscous damping coefficient; T_{L}, the torque due to external load, cogging effect, coulomb friction, etc.

Up to now, Eqs. (2.1.1) and (2.1.2) explicitly depend on θ. This is not expected, since the solutions are hard to obtain. Therefore, we transform the above equations to the rotating frame via Park's transformation, and the explicit dependence on θ can be eliminated. We can obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}i_{\mathrm{q}} = \frac{1}{L_{\mathrm{q}}}[-Ri_{\mathrm{q}} - n\omega(L_{\mathrm{d}}i_{\mathrm{d}} + k_t) + v_{\mathrm{q}}] \tag{2.1.4}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}i_{\mathrm{d}} = \frac{1}{L_{\mathrm{d}}}[-Ri_{\mathrm{d}} + nL_{\mathrm{q}}\omega i_{\mathrm{q}} + v_{\mathrm{d}}] \tag{2.1.5}$$

and the electromagnetic torque is described by

$$T(i_{\mathrm{q}}, i_{\mathrm{d}}) = n[k_t i_{\mathrm{q}} + (L_{\mathrm{d}} - L_{\mathrm{q}})i_{\mathrm{q}}i_{\mathrm{d}}] \tag{2.1.6}$$

where, i_{q}, i_{d}, the quadrature-axis and direct-axis current; v_{q}, v_{d}, the quadrature-axis and direct-axis voltage; $L_{\mathrm{q}}q$, L_{d}, the fictitious inductance on the quadrature-axis and direct-axis; R, winding resistance; n, number of permanent pole pairs; $k_t = \sqrt{\frac{3}{2}}k_e$, k_e is the permanent-magnet flux constant.

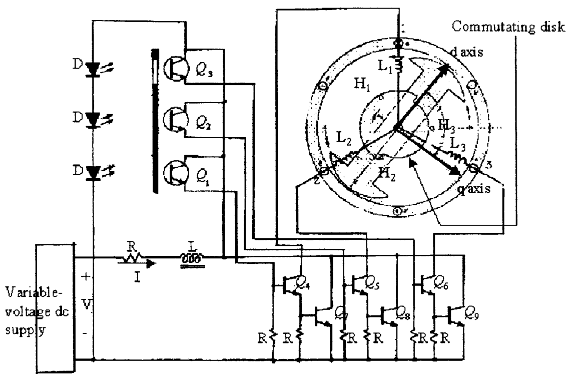

Fig. 1. A schematic diagram of typical brushless dc motor.

2.2. Single time scale representation of the equations of motion

In this section, we transform the system equations to a compact form, through an affine transformation and a single time scaling transformation [10].

$$\mathbf{x} = \mathbf{\Phi}\hat{\mathbf{x}} + \varsigma \tag{2.2.1}$$

$$t = \tau\hat{t} \tag{2.2.2}$$

where, $\mathbf{x}$, the m-dimensional state vector; $\mathbf{\Phi}$, $m \times m$ constant non-singular matrix; ς, $m \times 1$, constant vector.

Transformation matrix has not to be a specified form, for our purposes and simplicity, we choose

$$\mathbf{\Phi} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}, \quad \varsigma = \begin{bmatrix} \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \end{bmatrix} \tag{2.2.3}$$

where

$$\sigma_1 = \frac{-\delta k_t \pm \sqrt{\delta^2 k_t^2 - 4\rho\delta\Delta\tau b\sigma_3^2}}{2\rho\delta\Delta}, \quad \sigma_2 = \delta\sigma_1, \quad \sigma_3 = \frac{1}{n\tau}$$

$$\varsigma_1 = 0, \quad \varsigma_2 = -\rho\sigma_2 - \frac{k_t}{L_d}, \quad \varsigma_3 = 0$$

$$\tau = \frac{L_q}{R}, \quad \Delta = L_d - L_q, \quad \delta = \frac{L_q}{L_d}, \quad \rho \text{ is a free parameter}$$

Combining Eqs. (2.2.1)–(2.2.3) and (2.1.2)–(2.1.6), we obtain the equations in compact forms. The numbers of parameters are greatly reduced.

$$\begin{aligned} \frac{d}{d\hat{t}}\hat{x}_1 &= \hat{v}_q - \hat{x}_1 - \hat{x}_2\hat{x}_3 + \rho\hat{x}_3 \\ \frac{d}{d\hat{t}}\hat{x}_2 &= \hat{v}_d - \delta\hat{x}_2 + \hat{x}_1\hat{x}_3 \\ \frac{d}{d\hat{t}}\hat{x}_3 &= \sigma(\hat{x}_1 - \hat{x}_3) + \eta\hat{x}_1\hat{x}_2 - \widehat{T}_L \end{aligned} \tag{2.2.4}$$

where

$$\hat{v}_q = \frac{\tau}{\sigma_1 L_q} v_q, \quad \hat{v}_d = \frac{\tau}{\sigma_2 L_d}(v_d - R\varsigma_2), \quad \widehat{T}_L = \frac{\tau}{J\sigma_3} T_L$$

$$\sigma = \frac{\tau b}{J}, \quad \eta = \frac{\Delta\sigma_1\sigma_2}{J\sigma_3^2}$$

Here we have to assert that Eq. (2.2.4) is nondimensionalized. In the sections below, a variety of different control inputs added on Eq. (2.2.4) are also nondimensionalized. However, if we transform them to the original forms, each control input is dimensional and has its practically physical meaning.

In addition, BLDCM is an autonomous system. It means that the period of the system is not explicitly known, so different choice of Poincaré section would lead to different bifurcation diagram. In the sections below, adding control inputs changes the dynamics of the system, thus we have to modify the choice of Poincaré section. Modifying Poincaré section, we obtain almost the same bifurcation diagram. The only difference is the shift in $\hat{x}_3$ axis. Therefore, we just present the original bifurcation diagram.

At last, we present the numerical results. The parameters in numerical simulation are $\hat{v}_q = 0.168$, $\rho = 60$, $\hat{v}_d = 20.66$, $\delta = 0.875$, $\eta = 0.26$, $\widehat{T}_L = 0.53$, and the initial condition is $\hat{x}_1(0) = \hat{x}_2(0) = \hat{x}_3(0) = 0.01$. The phase portrait, bifurcation diagram, and Lyapunov exponents are shown in Figs. 2–4, respectively. It can be observed that the motion is period 1 for $\sigma = 4.05$, period 2 for $\sigma = 4.15$, and period 4 for $\sigma = 4.21$. For $\sigma = 4.55$, the motion is chaotic.

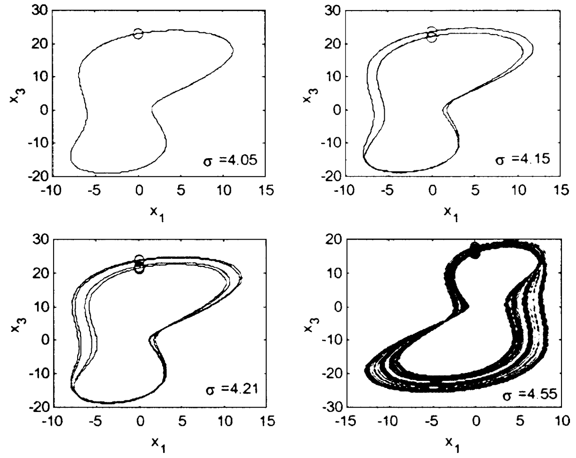

Fig. 2. Phase portrait for BLDCM.

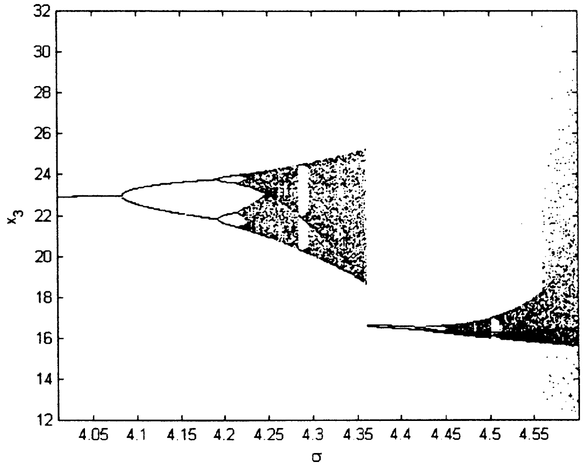

Fig. 3. Bifurcation diagram for BLDCM.

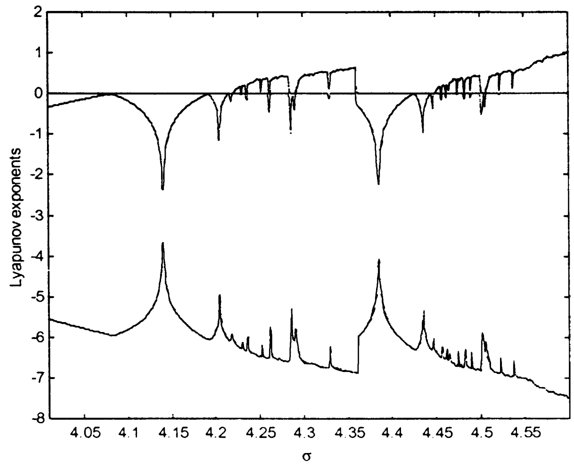

Fig. 4. Lyapunov exponents for BLDCM.

3. Chaos synchronization of identical systems

Chaos synchronization of identical systems is discussed in this section. Four methods are presented: the adaptive control [11], the backstepping design method [12], the Gerschgorin theorem [13], and the addition of a monitor [14].

3.1. Chaos synchronization of identical systems by adaptive control

We investigate two identical BLDCM systems in this section. Both systems have the same unknown parameters [11]. The master system is described by

$$\begin{aligned} \dot{x}_1 &= V_q - x_1 - x_2x_3 + px_3 \\ \dot{x}_2 &= V_d - Bx_2 + x_1x_3 \\ \dot{x}_3 &= a(x_1 - x_3) + hx_1x_2 - T_3 \end{aligned} \tag{3.1.1}$$

The slave system is described by

$$\begin{aligned} \dot{y}_1 &= V_q - y_1 - y_2y_3 + py_3 \\ \dot{y}_2 &= V_d - By_2 + y_1y_3 \\ \dot{y}_3 &= a(y_1 - y_3) + hy_1y_2 - T_3 \end{aligned} \tag{3.1.2}$$

The true value of "unknown" parameters are $V_q = 0.168$, $p = 60$, $V_d = 20.66$, $B = 0.875$, $a = 4.55$, $h = 0.26$, $T_3 = 0.53$ in numerical simulation. The initial conditions of the master and the slave systems are $x_1(0) = x_2(0) = x_3(0) = 0.01$, $y_1(0) = y_2(0) = y_3(0) = 0.1$, respectively. The initial values of estimate for "unknown" parameters are $\hat{p}(0) = \widehat{B}(0) = \hat{a}(0) = \hat{h}(0) = 0$.

To synchronize two identical BLDCM systems, we add three controllers, u_1, u_2, and u_3, on the first, second, and third equation of (3.1.2), respectively.

$$\begin{aligned} \dot{y}_1 &= V_q - y_1 - y_2y_3 + py_3 + u_1 \\ \dot{y}_2 &= V_d - By_2 + y_1y_3 + u_2 \\ \dot{y}_3 &= a(y_1 - y_3) + hy_1y_2 - T_3 + u_3 \end{aligned} \tag{3.1.3}$$

Subtracting Eq. (3.1.1) from Eq. (3.1.3), we can obtain the error dynamics

$$\begin{aligned} \dot{e}_1 &= -e_1 - e_2e_3 - x_3e_2 - x_2e_3 + pe_3 + u_1 \\ \dot{e}_2 &= -Be_2 + e_1e_3 + x_1e_3 + x_3e_1 + u_2 \\ \dot{e}_3 &= ae_1 - ae_3 + h(e_1e_2 + x_2e_1 + x_1e_2) + u_3 \end{aligned} \tag{3.1.4}$$

where $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$.

Choose a Lyapunov function of the form

$$V(e_1, e_2, e_3, \tilde{p}, \widetilde{B}, \tilde{a}, \tilde{h}) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + \tilde{p}^2 + \widetilde{B}^2 + \tilde{a}^2 + \tilde{h}^2) \tag{3.1.5}$$

where $\tilde{p} = p - \hat{p}$, $\widetilde{B} = B - \widehat{B}$, $\tilde{a} = a - \hat{a}$, $\tilde{h} = h - \hat{h}$, and $\hat{p}$, $\widehat{B}$, $\hat{a}$, $\hat{h}$ are estimate values of the unknown parameters p, B, a, h, respectively.

Its derivative along the solution of Eq. (3.1.4) is

$$\begin{aligned} \dot{V} = {} & e_1(-e_1 - e_2e_3 - x_3e_2 - x_2e_3 + pe_3 + u_1) + e_2(-Be_2 + e_1e_3 + x_1e_3 + x_3e_1 + u_2) + e_3(ae_1 - ae_3 + he_1e_2 \\ & + hx_2e_1 + hx_1e_2 + u_3) + \tilde{p}(-\dot{\hat{p}}) + \widetilde{B}(-\dot{\widehat{B}}) + \tilde{a}(-\dot{\hat{a}}) + \tilde{h}(-\dot{\hat{h}}) \end{aligned} \tag{3.1.6}$$

Choose

$$\begin{aligned} u_1 &= -\hat{p}e_3 \\ u_2 &= (\widehat{B} - 1)e_2 \\ u_3 &= (1 - \hat{h})x_2e_1 - (1 + \hat{h})x_1e_2 - \hat{h}e_1e_2 - \hat{a}e_1 - (1 - \hat{a})e_3 \end{aligned}$$

$$\dot{\hat{p}} = e_1 e_3$$
$$\dot{\hat{B}} = -e_2^2$$
$$\dot{\hat{a}} = e_3^2 + e_1 e_3$$
$$\dot{\hat{h}} = e_1 e_2 e_3 + x_1 e_2 e_3 + x_2 e_1 e_3$$

Eq. (3.1.6) can be rewritten as

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 < 0 \tag{3.1.7}$$

this means that chaos synchronization between two identical BLDCM systems can be achieved. The numerical results are shown in Fig. 5.

3.2. Chaos synchronization of identical systems by backstepping design

We investigate two identical BLDCM systems in this section. The parameters of both systems are known.

The master system is described by

$$\begin{aligned} \dot{x}_1 &= V_{\mathrm{q}} - x_1 - x_2 x_3 + p x_3 \\ \dot{x}_2 &= V_{\mathrm{d}} - B x_2 + x_1 x_3 \\ \dot{x}_3 &= a(x_1 - x_3) + h x_1 x_2 - T_3 \end{aligned} \tag{3.2.1}$$

The slave system is described by

$$\begin{aligned} \dot{y}_1 &= V_{\mathrm{q}} - y_1 - y_2 y_3 + p y_3 \\ \dot{y}_2 &= V_{\mathrm{d}} - B y_2 + y_1 y_3 \\ \dot{y}_3 &= a(y_1 - y_3) + h y_1 y_2 - T_3 \end{aligned} \tag{3.2.2}$$

By request of the backstepping design method [12], the system has to obey the special form. Otherwise, the method cannot be used. The pronounced achievement of using only one controller is doubtful. So we use the improved backstepping design method to achieve the chaos synchronization of two identical BLDCM systems. The system considered can be in general form.

The general form of our autonomous third-order BLDCM system can be described by

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, x_3) \\ \dot{x}_2 &= f_2(x_1, x_2, x_3) \\ \dot{x}_3 &= f_3(x_1, x_2, x_3) \end{aligned} \tag{3.2.3}$$

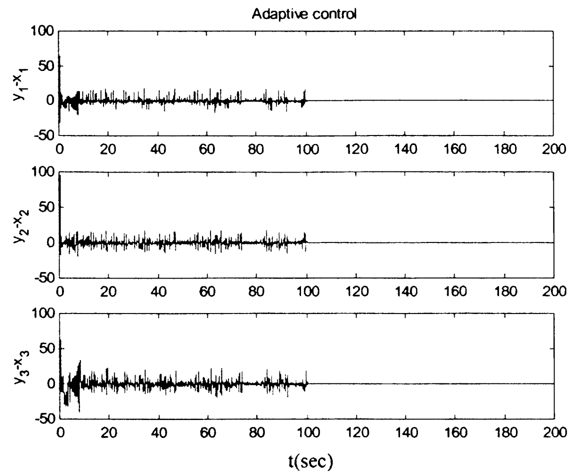

Fig. 5. Time history of errors.

f_1, f_2, and f_3 can be arbitrary functions, where the system solution exists and is unique.

To synchronize two identical BLDCM systems, we add two controllers, u_2 and u_3, on the second and third equation of (3.2.2), respectively.

$$\begin{aligned} \dot{y}_1 &= V_q - y_1 - y_2y_3 + py_3 \\ \dot{y}_2 &= V_d - By_2 + y_1y_3 + u_2 \\ \dot{y}_3 &= a(y_1 - y_3) + hy_1y_2 - T_3 + u_3 \end{aligned} \tag{3.2.4}$$

Subtracting Eq. (3.2.1) from Eq. (3.2.4), we can obtain the error dynamics

$$\begin{aligned} \dot{e}_1 &= -e_1 - e_2e_3 - x_3e_2 - x_2e_3 + pe_3 \\ \dot{e}_2 &= -Be_2 + e_1e_3 + x_1e_3 + x_3e_1 + u_2 \\ \dot{e}_3 &= a(e_1 - e_3) + h(e_1e_2 + x_2e_1 + x_1e_2) + u_3 \end{aligned} \tag{3.2.5}$$

where $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$, namely, $y_1 = e_1 + x_1$, $y_2 = e_2 + x_2$, $y_3 = e_3 + x_3$.

Variables x_1, x_2, x_3 in the error dynamics (3.2.5) can be considered as input signal from the master system (3.2.1). Without u_2 and u_3, the error dynamics (3.2.5) has an equilibrium point $(0, 0, 0)$. If we properly choose u_2 and u_3, the equilibrium point would not change. The problem of chaos synchronization between two identical BLDCM systems can be transformed to the problem of stabilization of the error dynamics (3.2.5).

First, we consider the stability of the first equation of Eq. (3.2.5)

$$\dot{e}_1 = -e_1 - x_3e_2 + (p - e_2 - x_2)e_3 \tag{3.2.6}$$

where e_2 and e_3 are regarded as controllers.

Choose a Lyapunov function of the form

$$V_1(e_1) = \frac{1}{2}e_1^2 \tag{3.2.7}$$

its derivative along the solution of Eq. (3.2.6) is

$$\dot{V}_1 = -e_1^2 + e_1[-x_3e_2 + (p - e_2 - x_2)e_3] \tag{3.2.8}$$

Assume controllers $e_2 = \alpha_1(e_1)$, $e_3 = \alpha_2(e_1)$, Eq. (3.2.8) can be rewritten as

$$\dot{V}_1 = -e_1^2 + e_1[-x_3\alpha_1 + (p - \alpha_1 - x_2)\alpha_2] \tag{3.2.9}$$

Choose

$$\begin{aligned} \alpha_1(e_1) &= 0 \\ \alpha_2(e_2) &= 0 \end{aligned}$$

Eq. (3.2.9) can be rewritten as

$$\dot{V}_1 = -e_1^2 < 0 \tag{3.2.10}$$

this means that the zero solution of Eq. (3.2.6) is asymptotically stable.

When e_2 and e_3 are considered as controllers, $\alpha_1(e_1)$ and $\alpha_2(e_1)$ are estimate functions.

Define

$$\begin{aligned} w_2 &= e_2 - \alpha_1(e_1) \\ w_3 &= e_3 - \alpha_2(e_1) \end{aligned} \tag{3.2.11}$$

Study the (e_1, w_2, w_3) system

$$\begin{aligned} \dot{e}_1 &= -e_1 - x_3w_2 + (p - w_2 - x_2)w_3 \\ \dot{w}_2 &= -Bw_2 + e_1w_3 + x_1w_3 + x_3e_1 + u_2 \\ \dot{w}_3 &= a(e_1 - w_3) + h(e_1w_2 + x_1w_2 + x_2e_1) + u_3 \end{aligned} \tag{3.2.12}$$

Choose a Lyapunov function of the form

$$V_2(e_1, w_2, w_3) = V_1(e_1) + \frac{1}{2}w_2^2 + \frac{1}{2}w_3^2 \tag{3.2.13}$$

its derivative along the solution of Eq. (3.2.12) is

$$\dot{V}_2 = -e_1^2 - Bw_2^2 - aw_3^2 + w_2(e_1x_3 + u_2) + w_3(ae_1 + he_1x_2 + (1+h)e_1w_2 + (1+h)x_1w_2 + u_3) \tag{3.2.14}$$

Choose

$$u_2 = -x_3e_1$$
$$u_3 = -ae_1 - he_1x_2 - (1+h)e_1w_2 - (1+h)x_1w_2$$

Eq. (3.2.14) can be rewritten as

$$\dot{V}_2 = -e_1^2 - Bw_2^2 - aw_3^2 < 0 \tag{3.2.15}$$

this means that the zero solution of Eq. (3.2.12) is asymptotically stable.

By addition of u_2 and u_3, the equilibrium point $(0, 0, 0)$ of the error dynamics (3.2.5) is unchanged. Chaos synchronization between two identical BLDCM systems can be achieved. The numerical results are shown in Fig. 6.

3.3. Chaos synchronization of identical systems by Gerschgorin theorem

We investigate two identical BLDCM systems in this section. The parameters of both systems are known.

The master system is described by

$$\begin{aligned}\dot{x}_1 &= V_q - x_1 - x_2x_3 + px_3\\ \dot{x}_2 &= V_d - Bx_2 + x_1x_3\\ \dot{x}_3 &= a(x_1 - x_3) + hx_1x_2 - T_3\end{aligned} \tag{3.3.1}$$

The slave system is described by

$$\begin{aligned}\dot{y}_1 &= V_q - y_1 - y_2y_3 + py_3\\ \dot{y}_2 &= V_d - By_2 + y_1y_3\\ \dot{y}_3 &= a(y_1 - y_3) + hy_1y_2 - T_3\end{aligned} \tag{3.3.2}$$

To synchronize two identical BLDCM systems, we add three coupling terms, $k_1(x_1 - y_1)$, $k_2(x_2 - y_2)$, and $k_3(x_3 - y_3)$, on the first, second, and third equation of (3.3.2), respectively.

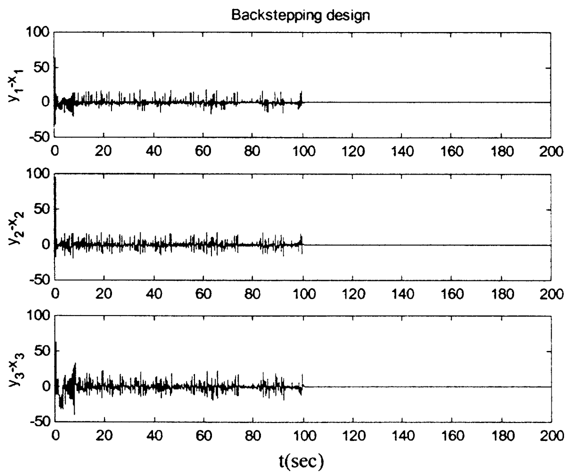

Fig. 6. Time history of errors.

$$\begin{aligned} \dot{y}_1 &= V_q - y_1 - y_2 y_3 + p y_3 + k_1(x_1 - y_1) \\ \dot{y}_2 &= V_d - B y_2 + y_1 y_3 + k_2(x_2 - y_2) \\ \dot{y}_3 &= a(y_1 - y_3) + h y_1 y_2 - T_3 + k_3(x_3 - y_3) \end{aligned} \tag{3.3.3}$$

Eqs. (3.3.1) and (3.3.3) can be written as [13]

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + g(\mathbf{x}) + \mathbf{u} \\ \dot{\mathbf{y}} &= \mathbf{A}\mathbf{y} + g(\mathbf{y}) + \mathbf{u} + \mathbf{K}(\mathbf{x} - \mathbf{y}) \end{aligned} \tag{3.3.4}$$

where $\mathbf{A} \in R^{n\times n}$ is a constant matrix, $g(\mathbf{x})$ is a nonlinear function, and $\mathbf{u} \in R^n$ is the external input vector.

Assume

$$g(\mathbf{x}) - g(\mathbf{y}) = \mathbf{M}_{\mathbf{x},\mathbf{y}}(\mathbf{x} - \mathbf{y}) \tag{3.3.5}$$

where the elements in $\mathbf{M}_{\mathbf{x},\mathbf{y}}$ are dependent on $\mathbf{x}$ and $\mathbf{y}$.

From Eq. (3.3.4), we can obtain the error dynamics

$$\dot{\mathbf{e}} = (\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{x},\mathbf{y}})\mathbf{e} \tag{3.3.6}$$

where $\mathbf{e} = \mathbf{x} - \mathbf{y}$.

Choose a Lyapunov function of the form

$$V = \mathbf{e}^{\mathrm{T}}\mathbf{P}\mathbf{e} \tag{3.3.7}$$

where $\mathbf{P}$ is a positive definite diagonal constant matrix.

Its derivative along the solution of Eq. (3.3.6) is

$$\begin{aligned} \dot{V} &= \dot{\mathbf{e}}^{\mathrm{T}}\mathbf{P}\mathbf{e} + \mathbf{e}^{\mathrm{T}}\mathbf{P}\dot{\mathbf{e}} = \mathbf{e}^{\mathrm{T}}(\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{x},\mathbf{y}})^{\mathrm{T}}\mathbf{P}\mathbf{e} + \mathbf{e}^{\mathrm{T}}\mathbf{P}(\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{x},\mathbf{y}})\mathbf{e} \\ &= \mathbf{e}^{\mathrm{T}}[(\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{x},\mathbf{y}})^{\mathrm{T}}\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{x},\mathbf{y}})]\mathbf{e} = \mathbf{e}^{\mathrm{T}}\mathbf{Q}\mathbf{e} \end{aligned} \tag{3.3.8}$$

where $\mathbf{Q} = (\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{x},\mathbf{y}})^{\mathrm{T}}\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{x},\mathbf{y}})$.

Rewrite $\mathbf{Q}$ as

$$\begin{aligned} \mathbf{Q} &= (\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{x},\mathbf{y}})^{\mathrm{T}}\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{K} + \mathbf{M}_{\mathbf{x},\mathbf{y}}) = [\mathbf{P}(\mathbf{A} + \mathbf{M}_{\mathbf{x},\mathbf{y}}) + (\mathbf{A} + \mathbf{M}_{\mathbf{x},\mathbf{y}})^{\mathrm{T}}\mathbf{P}] - [\mathbf{P}\mathbf{K} + \mathbf{K}^{\mathrm{T}}\mathbf{P}] \\ &= [\bar{a}_{ij}] - [b_{ij}] \end{aligned} \tag{3.3.9}$$

where $[b_{ij}] = \mathrm{diag}(2k_1p_1, 2k_2p_2, \ldots, 2k_np_n)$.

Gerschgorin theorem guarantees that each eigenvalue of $\mathbf{Q}$, when plotted in the complex plane, must lie on or within Gerschgorin's circle. The center of circle is $\bar{a}_{ii} - 2k_ip_i$, the radii are r_i, where $r_i = \sum_{j=1,j\neq i}^{n} |\bar{a}_{ij}|$.

Since $\mathbf{Q} = \mathbf{Q}^{\mathrm{T}}$, if all eigenvalues of $\mathbf{Q}$ are negative, $\dot{V}$ would be negative definite. This means that the error dynamics (3.3.6) would be asymptotically stable about $(0, 0, 0)$. In the other word, two identical BLDCM systems would be synchronized.

To achieve synchronization, we assume all eigenvalues of $\mathbf{Q}$ are negative

$$\lambda_i \leqslant \mu < 0, \quad i = 1, 2, \ldots, n \tag{3.3.10}$$

where μ is a negative constant.

From Gerschgorin theorem and Eq. (3.3.10), we can get that $\bar{a}_{ii} - 2k_ip_i + r_i \leqslant \mu$, and the range of k_i can be obtained.

$$k_i \geqslant \frac{1}{2p_i}(\bar{a}_{ii} + r_i - \mu), \quad i = 1, 2, \ldots, n \tag{3.3.11}$$

Choose $\mathbf{P} = \mathbf{I}$, and Eq. (3.3.11) can be rewritten as

$$k_i \geqslant \frac{1}{2}(\bar{a}_{ii} + r_i - \mu), \quad i = 1, 2, \ldots, n \tag{3.3.12}$$

Consider two identical BLDCM systems investigated in this section, we can obtain

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & p \\ 0 & -B & 0 \\ a & 0 & -a \end{bmatrix}, \quad \mathbf{M}_{\mathbf{x},\mathbf{y}} = \begin{bmatrix} 0 & -y_3 & -x_2 \\ y_3 & 0 & x_1 \\ hy_2 & hx_1 & 0 \end{bmatrix} \tag{3.3.13}$$

$$[\bar{a}_{ij}] = \mathbf{P}(\mathbf{A}+\mathbf{M}) + (\mathbf{A}+\mathbf{M})^{\mathrm{T}}\mathbf{P} = \begin{bmatrix} -2 & 0 & a+p-x_2+hy_2 \\ 0 & -2B & (1+h)x_1 \\ a+p-x_2+hy_2 & (1+h)x_1 & -2a \end{bmatrix} \tag{3.3.14}$$

Choose $\mu = -0.5$, we can obtain the coupling strength as $k_1 = 14$, $k_2 = 8$, $k_3 = 18$. Chaos synchronization between two identical BLDCM systems can be achieved. The numerical results are shown in Fig. 7.

3.4. Chaos synchronization of identical systems by addition of a monitor

We investigate two identical BLDCM systems in this section. The parameters of both systems are known.

The master system is described by

$$\begin{aligned} \dot{x}_1 &= V_{\mathrm{q}} - x_1 - y_1 z_1 + p z_1 \\ \dot{y}_1 &= V_{\mathrm{d}} - B y_1 + x_1 z_1 \\ \dot{z}_1 &= a(x_1 - z_1) + \eta x_1 y_1 - T_3 \end{aligned} \tag{3.4.1}$$

The slave system is described by

$$\begin{aligned} \dot{x}_2 &= V_{\mathrm{q}} - x_2 - y_2 z_2 + p z_2 \\ \dot{y}_2 &= V_{\mathrm{d}} - B y_2 + x_2 z_2 \\ \dot{z}_2 &= a(x_2 - z_2) + \eta x_2 y_2 - T_3 \end{aligned} \tag{3.4.2}$$

Regard the master system (3.4.1) as a sender, and the slave system (3.4.2) as a receiver [14]. Then we send z_1 to the receiver, i.e., $z_2 = z_1$. All the knowledge the receiver knows is only z_1, x_2, and y_2, i.e., x_1 and y_1 are unknown.

The receiver (3.4.2) can be rewritten as

$$\begin{aligned} \dot{x}_2 &= V_{\mathrm{q}} - x_2 - y_2 z_1 + p z_1 \\ \dot{y}_2 &= V_{\mathrm{d}} - B y_2 + x_2 z_1 \end{aligned} \tag{3.4.3}$$

To judge whether the synchronization occurs, we have to estimate the errors between x_1 and x_2, y_1 and y_2.

If x_1 and y_1 are available, subtract Eq. (3.4.3) from the first two equations of (3.4.1), we can obtain the error dynamics

$$\begin{aligned} \dot{e}_x &= -e_x - z_1 e_y \\ \dot{e}_y &= -B e_y + z_1 e_x \end{aligned} \tag{3.4.4}$$

where $e_x = x_1 - x_2$, $e_y = y_1 - y_2$.

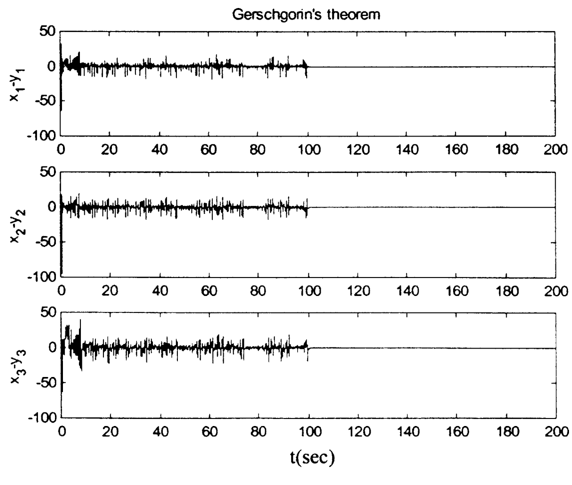

Fig. 7. Time history of errors for $k_1 = 14$, $k_2 = 8$, $k_3 = 18$.

Choose a Lyapunov function of the form

$$V(e_x, e_y) = \frac{1}{2}(e_x^2 + e_y^2) \tag{3.4.5}$$

its derivative along the solution of Eq. (3.4.4) is

$$\dot{V} = -e_x^2 - Be_y^2 < 0 \tag{3.4.6}$$

this means that chaos synchronization of two identical BLDCM systems can be achieved.

But, in fact, x_1 and y_1 are unknown, thus e_x and e_y are unavailable. To estimate e_x and e_y, we add a monitor to the receiver. Regard x_2 and y_2 as inputs, the monitor system can be written by

$$\dot{z}_3 = a(x_2 - z_3) + \eta x_2 y_2 - T_3 \tag{3.4.7}$$

Define

$$\gamma = z_1 - z_3 \tag{3.4.8}$$

since γ is available, we can obtain the first and second derivatives of γ, with the order of error $= h^8$

$$\begin{aligned} \gamma_i' &= [672(\gamma_{i+1} - \gamma_{i-1}) - 168(\gamma_{i+2} - \gamma_{i-2}) + 32(\gamma_{i+3} - \gamma_{i-3}) - 3(\gamma_{i+4} - \gamma_{i-4})]/840h \\ \gamma_i'' &= [8064(\gamma_{i+1} + \gamma_{i-1}) - 1008(\gamma_{i+2} + \gamma_{i-2}) + 128(\gamma_{i+3} + \gamma_{i-3}) - 9(\gamma_{i+4} - \gamma_{i-4}) - 14,350\gamma_i]/5040h^2 \end{aligned} \tag{3.4.9}$$

where h is the step size, and $i = 5, 6, \ldots, n-4$.

Let α and β be the estimate of e_x and e_y. Taking the first and second derivatives of Eq. (3.4.8), we can obtain

$$\begin{aligned} \dot{\gamma} &= (a + \eta y_2)\alpha + \eta x_2 \beta + \eta \alpha \beta - a\gamma \\ \ddot{\gamma} &= (-a - a^2 - a\eta y_2 - \eta y_2 - B\eta y_2 + 2\eta z_1 x_2 + \eta V_{\mathrm{d}})\alpha \\ &\quad + (-a\eta x_2 - \eta x_2 - B\eta x_2 + \eta p z_1 - a z_1 - 2\eta z_1 y_2 + \eta V_{\mathrm{q}})\beta \\ &\quad + (-\eta B - a\eta - \eta)\alpha\beta + \eta z_1 \alpha^2 - \eta z_1 \beta^2 + a^2\gamma \end{aligned} \tag{3.4.10}$$

Solving Eq. (3.4.10), we can obtain a four powers equation of β and four sets of solution for (α, β). But only one solution is what we want. For simplifying the problem, we consider the special case for $\eta = 0$. Thus α and β can be obtained

$$\alpha = \frac{\ddot{\gamma} + (1 + a)\dot{\gamma} + a\gamma}{-a z_1}, \quad \beta = \frac{\dot{\gamma} + a\gamma}{a} \tag{3.4.11}$$

For $\eta = 0$, the true values of errors are shown in Fig. 8, γ, γ', and γ'' are shown in Fig. 9. Since γ, γ', and γ'' all tend to zero, from Eq. (3.4.11), we can find that α and β tend to zero, too. This means that the process of chaos synchronization can be monitored. The numerical results are shown in Fig. 10.

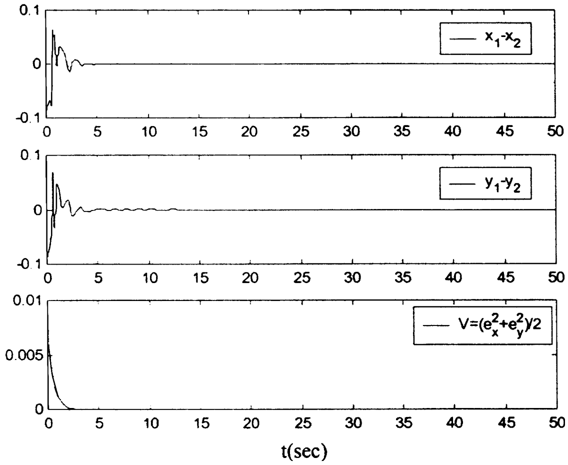

Fig. 8. Time history of errors and Lyapunov function.

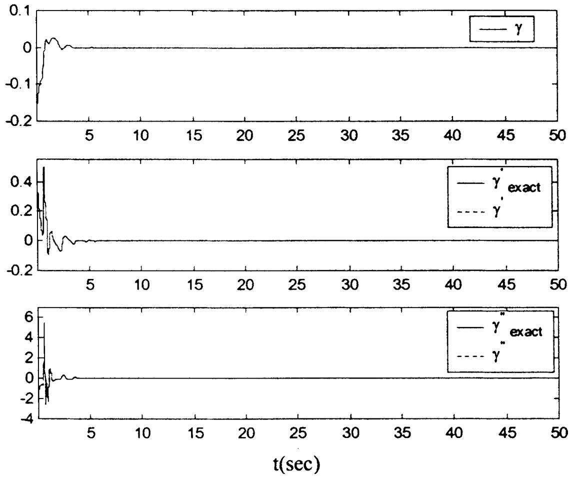

Fig. 9. Time history of exact and approximate derivatives.

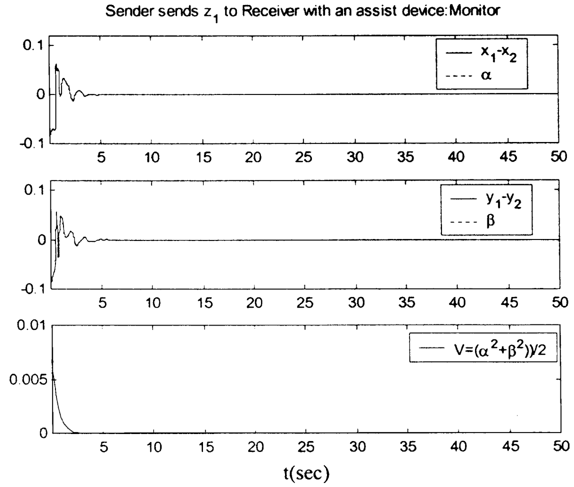

Fig. 10. Time history of true and estimate errors.

4. Parameters identification

We investigate parameters identification in this section. Two methods are presented: the adaptive control [15], and the random optimization method [16].

4.1. Parameters identification by adaptive control

We investigate two identical BLDCM systems in this section. Both systems have the same unknown parameters, and some parameters of the slave system are uncertain. Our work is to identify the uncertain parameters.

The master system is described by

$$\begin{aligned}
\dot{x}_1 &= V_q - x_1 - x_2x_3 + px_3 \\
\dot{x}_2 &= V_d - Bx_2 + x_1x_3 \\
\dot{x}_3 &= a(x_1 - x_3) + hx_1x_2 - T_3
\end{aligned} \tag{4.1.1}$$

The slave system is described by

$$\begin{aligned}\dot{y}_1 &= V_q - y_1 - y_2y_3 + p_h(t)y_3\\ \dot{y}_2 &= V_d - B_h(t)y_2 + y_1y_3\\ \dot{y}_3 &= a_h(t)(y_1 - y_3) + h_h(t)y_1y_2 - T_3\end{aligned} \tag{4.1.2}$$

The true value of "unknown" parameters are $V_q = 0.168$, $p = 60$, $V_d = 20.66$, $B = 0.875$, $a = 4.55$, $h = 0.26$, $T_3 = 0.53$ in numerical simulation. The initial conditions of the master and the slave systems are $x_1(0) = x_2(0) = x_3(0) = 0.01$, $y_1(0) = y_2(0) = y_3(0) = 0.1$, respectively.

If the parameters of both systems are known, the master and slave systems can be rewritten as [15]

$$\begin{aligned}\dot{\mathbf{x}} &= f(\mathbf{x}) + (F_1(\mathbf{x})p + F_2(\mathbf{x})B + F_3(\mathbf{x})a + F_4(\mathbf{x})h)\\ \dot{\mathbf{y}} &= f(\mathbf{y}) + (F_1(\mathbf{y})p + F_2(\mathbf{y})B + F_3(\mathbf{y})a + F_4(\mathbf{y})h) + \mathbf{U}\end{aligned} \tag{4.1.3}$$

where

$$F_1(\mathbf{x}) = \begin{bmatrix} x_3\\ 0\\ 0\end{bmatrix},\quad F_2(\mathbf{x}) = \begin{bmatrix} 0\\ -x_2\\ 0\end{bmatrix},\quad F_3(\mathbf{x}) = \begin{bmatrix} 0\\ 0\\ x_1 - x_3\end{bmatrix},\quad F_4(\mathbf{x}) = \begin{bmatrix} 0\\ 0\\ x_1x_2\end{bmatrix},$$
$$f(\mathbf{x}) = \begin{bmatrix} V_q - x_1 - x_2x_3\\ V_d + x_1x_3\\ -T_3\end{bmatrix},\quad \text{and}\quad \mathbf{U} = \begin{bmatrix} u_1\\ u_2\\ u_3\end{bmatrix}$$

From Eq. (4.1.3), we can obtain the error dynamics

$$\dot{\mathbf{e}} = f(\mathbf{y}) - f(\mathbf{x}) + (F_1(\mathbf{y}) - F_1(\mathbf{x}))p + (F_2(\mathbf{y}) - F_2(\mathbf{x}))B + (F_3(\mathbf{y}) - F_3(\mathbf{x}))a + (F_4(\mathbf{y}) - F_4(\mathbf{x}))h + \mathbf{U} \tag{4.1.4}$$

Choose a Lyapunov function of the form

$$V(\mathbf{e}) = \frac{1}{2}\mathbf{e}^{\mathrm{T}}\mathbf{e} \tag{4.1.5}$$

Its derivative along the solution of Eq. (4.1.4) is

$$\dot{V} = \mathbf{e}^{\mathrm{T}}\{f(\mathbf{y}) - f(\mathbf{x}) + [F_1(\mathbf{y}) - F_1(\mathbf{x})]p + [F_2(\mathbf{y}) - F_2(\mathbf{x})]B + [F_3(\mathbf{y}) - F_3(\mathbf{x})]a + [F_4(\mathbf{y}) - F_4(\mathbf{x})]h + \mathbf{U}\} \tag{4.1.6}$$

Choose

$$\mathbf{U} = \begin{bmatrix} -he_2e_3\\ -(1+h)x_1e_3 + (B-1)e_2\\ (1-h)x_2e_1 - (p+a)e_1 + (a-1)e_3\end{bmatrix}$$

Eq. (4.1.6) can be rewritten as

$$\dot{V} = -\mathbf{e}^{\mathrm{T}}\mathbf{e} < 0 \tag{4.1.7}$$

this means that chaos synchronization can be achieved.

If both systems have the same unknown parameters, and some parameters of the slave system are uncertain, the master and slave systems can be rewritten as

$$\begin{aligned}\dot{\mathbf{x}} &= f(\mathbf{x}) + (F_1(\mathbf{x})p + F_2(\mathbf{x})B + F_3(\mathbf{x})a + F_4(\mathbf{x})h)\\ \dot{\mathbf{y}} &= f(\mathbf{y}) + (F_1(\mathbf{y})p_h + F_2(\mathbf{y})B_h + F_3(\mathbf{y})a_h + F_4(\mathbf{y})h_h) + \mathbf{U}_h\end{aligned} \tag{4.1.8}$$

where

$$\mathbf{U}_h = \begin{bmatrix} u_{h1}\\ u_{h2}\\ u_{h3}\end{bmatrix}$$

From Eq. (4.1.8), we can obtain the error dynamics

$$\dot{\mathbf{e}} = f(\mathbf{y}) - f(\mathbf{x}) + (F_1(\mathbf{y})p_h - F_1(\mathbf{x})p) + (F_2(\mathbf{y})B_h - F_2(\mathbf{x})B) + (F_3(\mathbf{y})a_h - F_3(\mathbf{x})a) + (F_4(\mathbf{y})h_h - F_4(\mathbf{x})h) + \mathbf{U}_h \tag{4.1.9}$$

Choose a Lyapunov function of the form

$$V(\mathbf{e},\tilde{p},\widetilde{B},\tilde{a},\tilde{h}) = \frac{1}{2}\mathbf{e}^{\mathrm{T}}\mathbf{e} + \frac{1}{2}\tilde{p}^{\mathrm{T}}\tilde{p} + \frac{1}{2}\widetilde{B}^{\mathrm{T}}\widetilde{B} + \frac{1}{2}\tilde{a}^{\mathrm{T}}\tilde{a} + \frac{1}{2}\tilde{h}^{\mathrm{T}}\tilde{h} \tag{4.1.10}$$

where $\tilde{p} = p_h - p$, $\widetilde{B} = B_h - B$, $\tilde{a} = a_h - a$, $\tilde{h} = h_h - h$.

Its derivative along the solution of Eq. (4.1.9) is

$$\begin{aligned}
\dot{V} &= \mathbf{e}^{\mathrm{T}}\{f(\mathbf{y}) - f(\mathbf{x}) + [F_1(\mathbf{y})p_h - F_1(\mathbf{x})p] + [F_2(\mathbf{y})B_h - F_2(\mathbf{x})B] + [F_3(\mathbf{y})a_h - F_3(\mathbf{x})a] \\
&\quad + [F_4(\mathbf{y})h_h - F_4(\mathbf{x})h] + \mathbf{U}_h\} + \dot{p}_h\tilde{p} + \dot{B}_h\widetilde{B} + \dot{a}_h\tilde{a} + \dot{h}_h\tilde{h} \\
&= \mathbf{e}^{\mathrm{T}}\{f(\mathbf{y}) - f(\mathbf{x}) + [F_1(\mathbf{y}) - F_1(\mathbf{x})]p_h + [F_2(\mathbf{y}) - F_2(\mathbf{x})]B_h + [F_3(\mathbf{y}) - F_3(\mathbf{x})]a_h + [F_4(\mathbf{y}) - F_4(\mathbf{x})]h_h + \mathbf{U}_h\} \\
&\quad + \mathbf{e}^{\mathrm{T}}[F_1(\mathbf{x})(p_h - p) + F_2(\mathbf{x})(B_h - B) + F_3(\mathbf{x})(a_h - a) + F_4(\mathbf{x})(h_h - h)] \\
&\quad + \dot{p}_h(p_h - p) + \dot{B}_h(B_h - B) + \dot{a}_h(a_h - a) + \dot{h}_h(h_h - h)
\end{aligned} \tag{4.1.11}$$

Choose

$$\mathbf{U}_h = \begin{bmatrix} -h_h e_2 e_3 \\ -(1+h_h)x_1 e_3 + (B_h - 1)e_2 \\ (1-h_h)x_2 e_1 - (p_h + a_h)e_1 + (a_h - 1)e_3 \end{bmatrix}$$

$$\dot{p}_h = -F_1^{\mathrm{T}}(\mathbf{x})\mathbf{e} = -x_3 e_1$$

$$\dot{B}_h = -F_2^{\mathrm{T}}(\mathbf{x})\mathbf{e} = x_2 e_2$$

$$\dot{a}_h = -F_3^{\mathrm{T}}(\mathbf{x})\mathbf{e} = -(x_1 - x_3)e_3$$

$$\dot{h}_h = -F_4^{\mathrm{T}}(\mathbf{x})\mathbf{e} = -x_1 x_2 e_3$$

Eq. (4.1.11) can be rewritten as

$$\dot{V} = -\mathbf{e}^{\mathrm{T}}\mathbf{e} < 0 \tag{4.1.12}$$

The assumption of the adaptive control method [15] is satisfied. With the specific controller and parameters estimate update law, the parameters identification and chaos synchronization can be achieved. The numerical results are shown in Figs. 11–16.

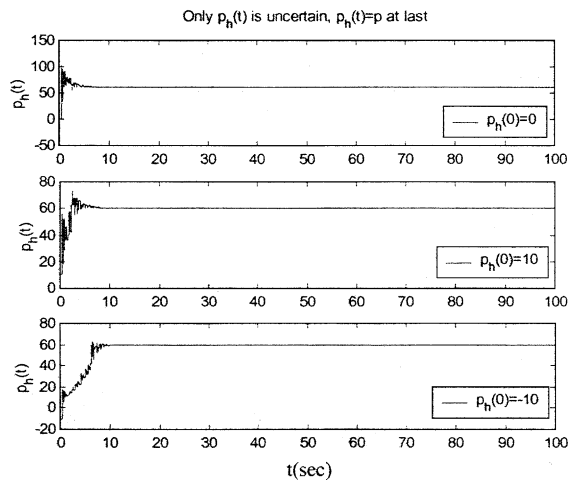

Fig. 11. Time history of $p_h(t)$.

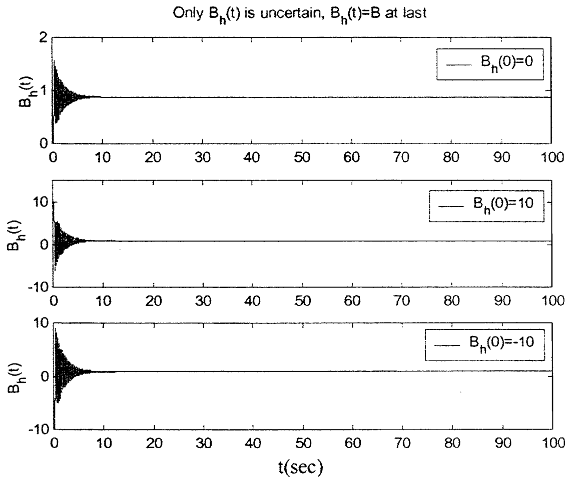

Fig. 12. Time history of $B_h(t)$.

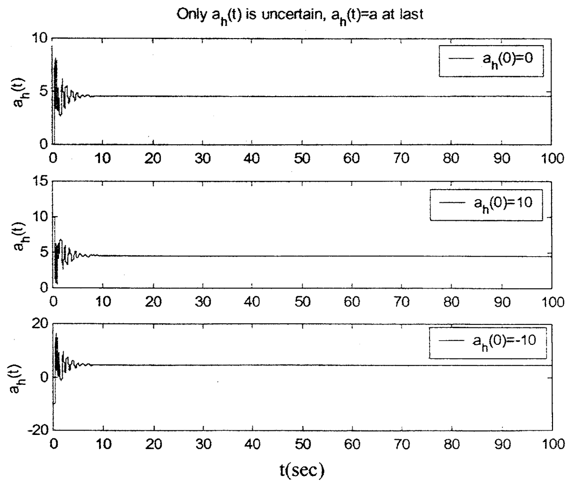

Fig. 13. Time history of $a_h(t)$.

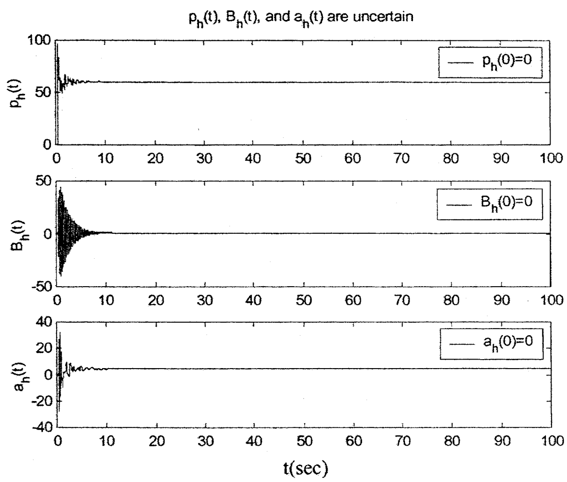

Fig. 14. Time history of $p_h(t)$, $B_h(t)$, and $a_h(t)$.

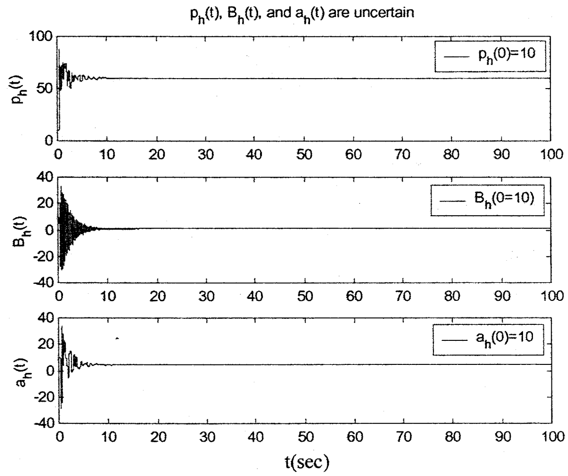

Fig. 15. Time history of $p_h(t)$, $B_h(t)$, and $a_h(t)$.

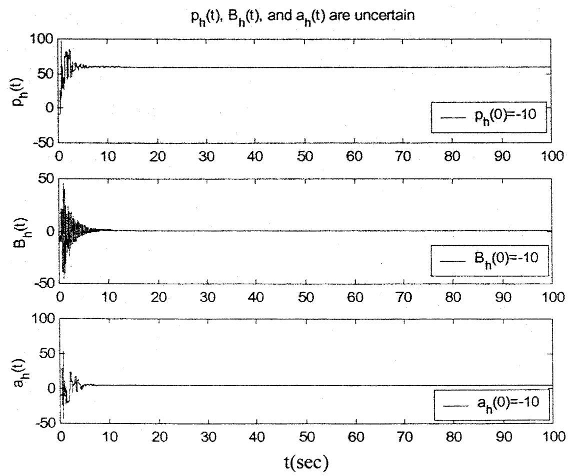

Fig. 16. Time history of $p_h(t)$, $B_h(t)$, and $a_h(t)$.

4.2. Parameters identification by random optimization

We investigate two identical BLDCM systems in this section. Both systems have the same parameters, but some parameters of the slave system are unknown. Our work is to identify the unknown parameters.

The master system is described by

$$\begin{aligned}\dot{x}_1 &= V_q - x_1 - x_2x_3 + px_3\\ \dot{x}_2 &= V_d - Bx_2 + x_1x_3\\ \dot{x}_3 &= a(x_1 - x_3) + hx_1x_2 - T_3\end{aligned} \tag{4.2.1}$$

The slave system is described by

$$\begin{aligned}\dot{y}_1 &= V_q - y_1 - y_2y_3 + p'y_3\\ \dot{y}_2 &= V_d - B'y_2 + y_1y_3\\ \dot{y}_3 &= a'(y_1 - y_3) + h'y_1y_2 - T_3\end{aligned} \tag{4.2.2}$$

To synchronize two identical BLDCM systems, we add one coupling term, $k(x_1 - y_1)$, on the first equation of (4.2.2).

$$\begin{aligned}
\dot{y}_1 &= V_q - y_1 - y_2 y_3 + p' y_3 + k(x_1 - y_1) \\
\dot{y}_2 &= V_d - B' y_2 + y_1 y_3 \\
\dot{y}_3 &= a'(y_1 - y_3) + h' y_1 y_2 - T_3
\end{aligned} \tag{4.2.3}$$

Eqs. (4.2.1) and (4.2.3) can be rewritten as [16]

$$\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \{c\}) \\
\dot{\mathbf{y}} &= \mathbf{f}(\mathbf{y}, \{c'\}) + \mathbf{K}(\mathbf{x} - \mathbf{y})
\end{aligned} \tag{4.2.4}$$

where $\{c\}$, $\{c'\}$ are parameter sets, and $\mathbf{K} = [k \quad 0 \quad 0]$.

Define the difference by

$$U = \int_{0.9T}^{T} |x_1 - y_1|^2 \, \mathrm{d}t \tag{4.2.5}$$

where T is the simulation time.

With the same parameter sets, $\{c'\} = \{c\}$, the synchronization can be achieved for $k > k_c$. In numerical simulation, we obtain that $k_c = 2.93$. The numerical result is shown in Fig. 17.

The difference U can be considered as a function of $\{c'\}$and k. If k is sufficiently large and $\{c'\}$ is close to $\{c\}$, the difference U would tend to zero. In the other word, with sufficiently large value of k, if U is small, $\{c'\}$ would be close to $\{c\}$. In numerical simulation, we assume that only one parameter of $\{c'\}$ is unknown. The result is shown in Figs. 18–21.

To identify the unknown parameters of the slave system, we use the random optimization method. The algorithm is as follows.

First, choose a sufficiently large value of k. In our case, we choose $k = 5$. By estimating each initial value of $\{c'\}$, we can calculate the difference U.

Each parameter c' in the parameter set $\{c'\}$ is randomly modified as

$$c'_m = c' + r \tag{4.2.6}$$

where r is a random number which obeys the Gaussian distribution with variance $\sigma = 0.01$.

Substituting the modified parameter set $\{c'_m\}$ into Eq. (4.2.3), we can obtain y'_1. The difference between two systems is

$$U' = \int_{0.9T}^{T} |x_1 - y'_1|^2 \, \mathrm{d}t \tag{4.2.7}$$

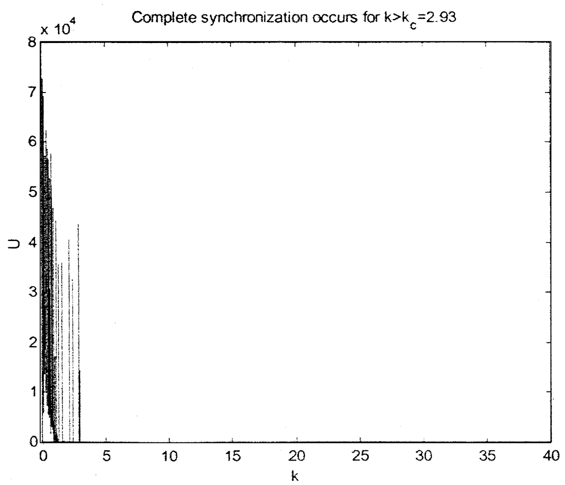

Fig. 17. Difference with respect to the coupling strength k.

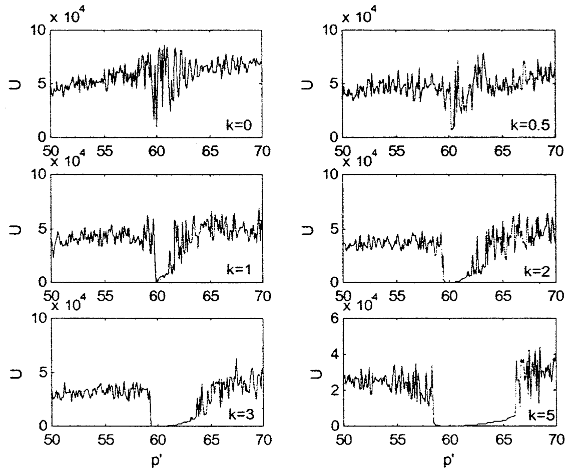

Fig. 18. Difference with respect to the parameter p' for different k.

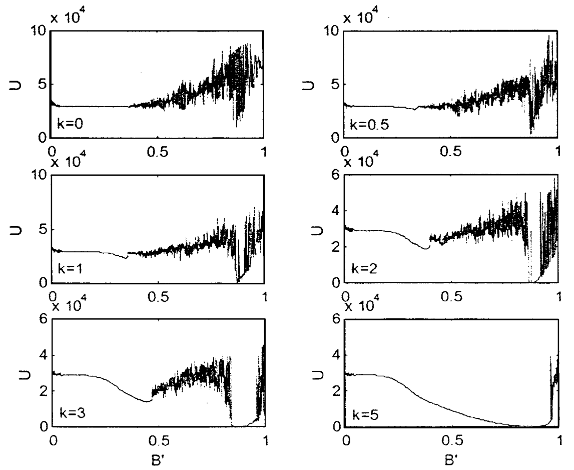

Fig. 19. Difference with respect to the parameter B' for different k.

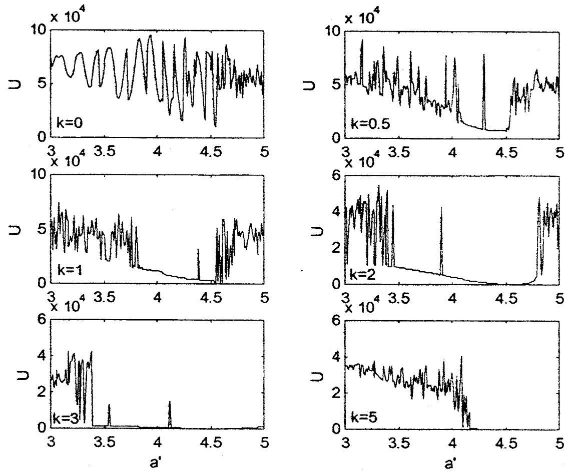

Fig. 20. Difference with respect to the parameter a' for different k.

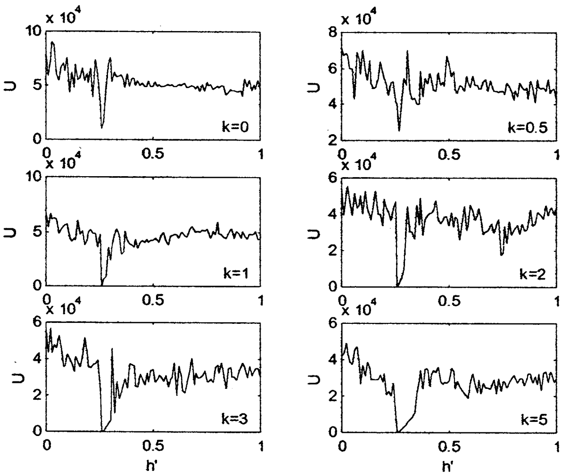

Fig. 21. Difference with respect to the parameter h' for different k.

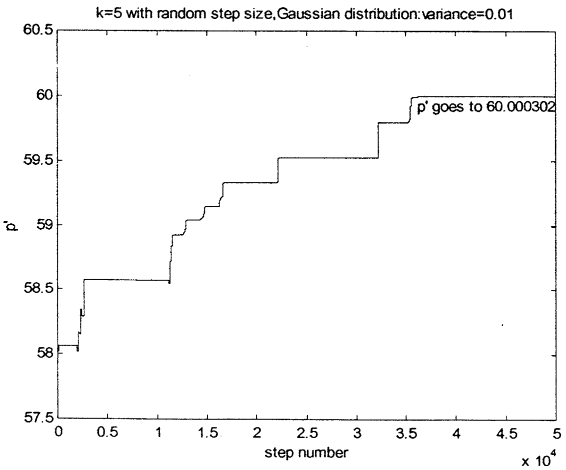

Fig. 22. Time evolution of p' by random optimization process.

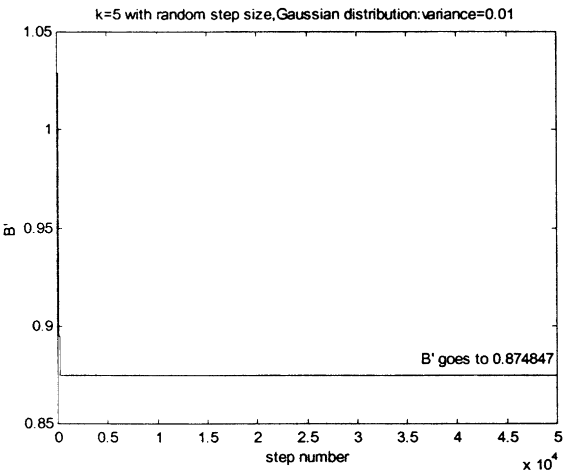

Fig. 23. Time evolution of B' by random optimization process.

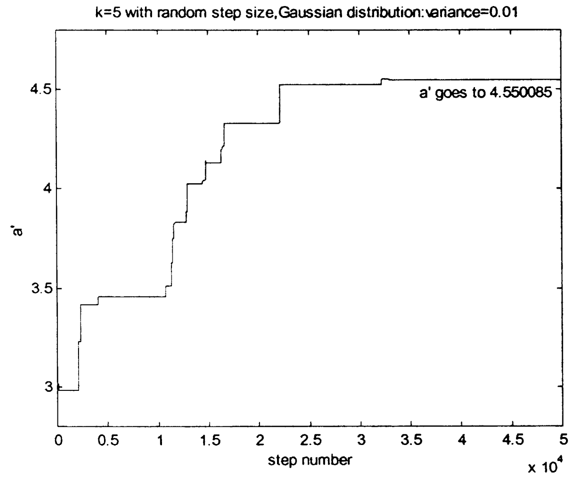

Fig. 24. Time evolution of a' by random optimization process.

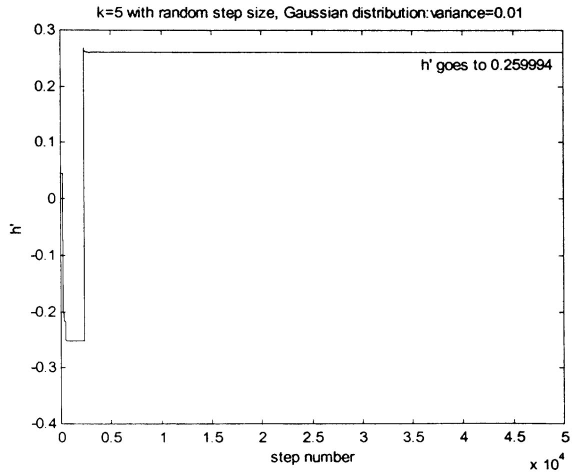

Fig. 25. Time evolution of h' by random optimization process.

If the difference U' is smaller than U, the parameter set is changed from $\{c'\}$ to $\{c'_{\mathrm{m}}\}$. On the other hand, if the difference U' is larger than U, the parameter set is unchanged and kept to be $\{c'\}$. The processes are repeated until the difference U tends to zero.

In numerical simulation, we assume that only one parameter of $\{c'\}$ is unknown. Parameters identification can be achieved. The result is shown in Figs. 22–25.

5. Conclusions

Brushless dc motor (BLDCM) is studied in this paper. It is an autonomous third-order electromechanical system. In order to analyze a variety of periodic and chaotic phenomenon, we employ several numerical techniques such as time history, phase portrait, bifurcation diagram, and Lyapunov exponents.

The dynamic characteristics of BLDCM are discussed in Section 2. The system model is described, and the numerical results of periodic and chaotic phenomenon are presented.

In Section 3, four methods are investigated to achieve chaos synchronization between two identical BLDCM systems. First, the adaptive control is used. Second, the improved backstepping design method is used. Third, Gerschgorin theorem is used. Finally, a monitor is added. Chaos synchronization of identical systems can be achieved by each method.

Two methods are investigated to achieve parameters identification in Section 4. First, the adaptive control is used. Second, the random optimization method is used. Parameters identification can be achieved by each method.

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