# New Formulas for Synthesizing Microstrip Bandpass Filters With Relatively Wide Bandwidths

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***Abstract*—New formulas are proposed for designing wideband parallel-coupled microstrip bandpass filters with improved prediction of bandwidth. When a fractional bandwidth $\Delta$ is required, a correction $\theta = (\pi/2)(1 \pm \Delta/2)$ is incorporated into the formulation for determining the dimensions of each coupled stage. Two filters with $\Delta = 50\%$ are designed and fabricated to show the improvement. The measurement shows a very good agreement with the simulation.**

***Index Terms*—Parallel-coupled filter, wide bandwidth.**

## I. Introduction

The ULTRA-wideband (UWB) technologies for commercial communication applications have created a need of a transmitter with bandwidths of up to or more than several gigahertz [1]. Microwave passive devices with such a wide bandwidth have been investigated recently [2]–[4]. Lumped elements are incorporated into the circuit design for a directional coupler with an octave-band [2]. The three-line structures in [3] and ground plane aperture compensation techniques in [4] are suitable for implementing filters of a wide bandwidth.

Consisting of a cascade of coupled stages, parallel-coupled line configuration is attractive for realizing microstrip bandpass filters in microwave frequencies [3]–[7]. Approximate design and synthesis formulas have been well documented for determining the dimensions of each stage for an all-pole bandpass filter [6], [7]. In deriving these formulas, one of the key steps is to establish the equivalence of a coupled stage to a two-port network of two quarter-wave transmission line sections with an admittance inverter in between. The approximation has a good accuracy when the filter has a relatively small bandwidth. This is because the frequency response of a coupled stage has a zero derivative at center frequency $f_o$, and thus is relatively insensitive to variation of frequency. When the designed bandwidth becomes larger, however, the coupling of the coupled stage is no longer a constant, and it apparently rolls off as the frequency moves away from $f_o$. Thus, a modification is required for the formulas when the microstrip filters are designed to have a wide bandwidth.

The $Q$ distribution method in [8] can provide correct solutions for filters with narrow- and wide-bandwidths. The entire procedure for finding the $Q$-distribution includes choosing the number of sections, creating the composite $ABCD$ matrix for a cascade of transmission line sections and coupled stages, and then solving for individual admittance values of the resonators. For direct-coupled microwave filters of $n = 2$ to $n = 12$ elements having $\Delta = 10\%$ to 43% and VSWR ripple levels from 1.01 to 1.50, the theory in [9] can give good agreement with computed response characteristics.

In this paper, simple formulas are proposed for improving prediction of the bandwidth of parallel-coupled microstrip filters. Two experimental Chebyshev filters are measured to demonstrate the significant improvement.

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## II. Design Formulas With Improved Accuracy

From the perspective of circuit synthesis, accurate dimensions of the coupled stage are the most important in implementing the filter. The coupled stage in Fig. 1(a) has an electrical length $\theta$, and even and odd mode characteristic impedances $Z_{oe}$ and $Z_{oo}$. The $ABCD$ matrix for the coupled-line stage can be derived as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}}\cos\theta & -\frac{j}{2}\left\{-(Z_{oe}-Z_{oo})\sin\theta + \frac{4Z_{oe}Z_{oo}}{Z_{oe}-Z_{oo}}\frac{\cos^2\theta}{\sin\theta}\right\} \\ j\frac{2}{Z_{oe}-Z_{oo}}\sin\theta & \frac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}}\cos\theta \end{bmatrix}. \tag{1}$$

Here, the even and odd modes are assumed to have identical phase velocities. The $ABCD$ matrix for the $J$ inverter circuit in Fig. 1(b) can be derived as [7]

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(JZ_o+\frac{1}{JZ_o}\right)\cos\theta\sin\theta & j\left(JZ_o^2\sin^2\theta - \frac{\cos^2\theta}{J}\right) \\ j\left(\frac{1}{JZ_o^2}\sin^2\theta - J\cos^2\theta\right) & \left(JZ_o+\frac{1}{JZ_o}\right)\cos\theta\sin\theta \end{bmatrix}. \tag{2}$$

Equating the right hand sides of (1) and (2), one can express $Z_{oe}$ and $Z_{oo}$ in terms of the circuit parameters of the admittance inverter as follows:

$$Z_{oe} = \frac{JZ_o^2\sin\theta}{\sin^2\theta - (JZ_o\cos\theta)^2} \times \left[\left(JZ_o + \frac{1}{JZ_o}\right)\sin\theta + 1\right] \tag{3a}$$

$$Z_{oo} = \frac{JZ_o^2\sin\theta}{\sin^2\theta - (JZ_o\cos\theta)^2} \times \left[\left(JZ_o + \frac{1}{JZ_o}\right)\sin\theta - 1\right]. \tag{3b}$$

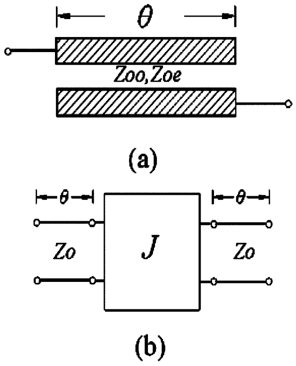


Fig. 1. (a) Coupled-line stage. (b) Equivalent circuit of (a).

TABLE I
EVEN AND ODD MODE CHARACTERISTIC IMPEDANCES FOR THE $n^{\mathrm{th}}$ COUPLED STAGES OF AN $N^{\mathrm{th}}$-ORDER CHEBYSHEV FILTER OBTAINED BY THE IMPROVED AND CLASSICAL FORMULAS. Ripple level $= 0.1$ dB AND $\Delta = 50\%$

| N | | Improved | | Classical | |
|---|---|---|---|---|---|
| | n | $Z_{oe}(\Omega)$ | $Z_{oo}(\Omega)$ | $Z_{oe}(\Omega)$ | $Z_{oo}(\Omega)$ |
| 3 | 1 | 155.6 | 47.0 | 131.7 | 44.4 |
| | 2 | 126.4 | 40.6 | 112.2 | 40.0 |
| 5 | 1 | 146.2 | 44.7 | 125.6 | 42.9 |
| | 2 | 111.0 | 38.3 | 100.9 | 38.3 |
| | 3 | 90.8 | 37.0 | 85.3 | 37.5 |
| 7 | 1 | 143.8 | 44.2 | 124.0 | 42.5 |
| | 2 | 107.9 | 38.0 | 98.6 | 38.1 |
| | 3 | 88.1 | 37.0 | 83.1 | 37.6 |
| | 4 | 85.5 | 37.1 | 81.0 | 37.7 |

It is difficult to implement a coupled microstrip stage having a frequency-dependent behavior as described in (3). In fact, constant values for $Z_{oe}$ and $Z_{oo}$ have to be used to determine the dimensions of each stage from the characteristic impedance design graphs. Note that if $\theta = \pi/2$ is used, (3a) and (3b) reduce to those given in [7]. Since the approximation $\theta = \pi/2$ is accurate only in the vicinity of the center frequency, this may lead to an error in estimating the filter bandwidth. Thus, when the required fractional bandwidth is $\Delta$

$$\theta = \frac{\pi}{2}\left(1 \pm \frac{\Delta}{2}\right) \tag{4}$$

can be used to calculate the $Z_{oe}$ and $Z_{oo}$ for each coupled stage. Obviously, an exact equivalence between the circuits in Fig. 1(a) and Fig. 1(b) is assured at the passband edges. This will make the prediction of filter bandwidths more accurate, which will be demonstrated later.

## III. RESULTS

To show the significant improvement in predicting the filter bandwidth provided by (3) and (4), we first examine the changes of $Z_{oe}$ and $Z_{oo}$ of coupled microstrip stages due to the deviation of $\theta$ from $\pi/2$. Table I lists their values for the $n^{\mathrm{th}}$ coupled stage in a third- and a fifth-order Chebyshev filters with 0.1-dB ripple level and 50% fractional bandwidth. It is noted that for an $N^{\mathrm{th}}$-order Chebyshev filter, the $n^{\mathrm{th}}$ coupled stage is identical to the $(N+2-n)^{\mathrm{th}}$ one. The numbers in Table I indicate that the end stages have the largest change in $Z_{oo}$, which is increased by no more than 6% for all cases shown here. On the other hand,

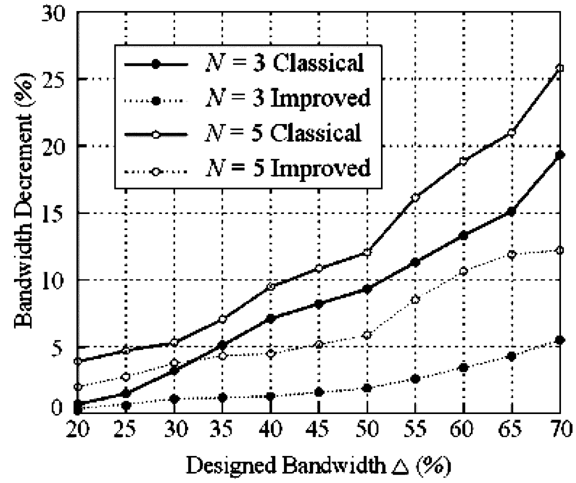


Fig. 2. Bandwidth decrement versus designed bandwidth from simulation responses of a third- and fifth-order Chebyshev filters with 0.1-dB ripple level.

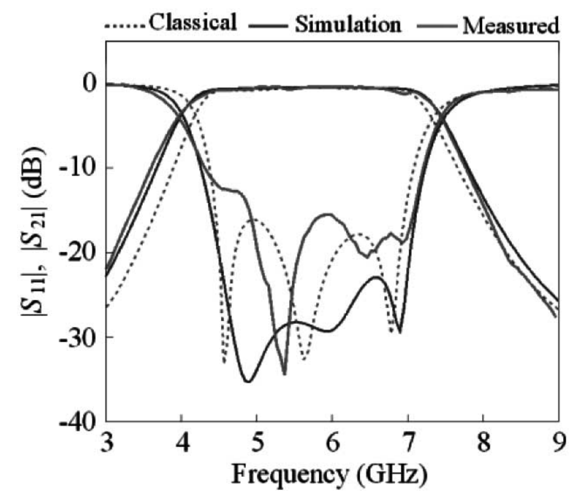


Fig. 3. Comparison of responses for third-order filters designed by the improved and classical formulas. The designed bandwidth is 50% and ripple level is 0.1 dB. The substrate has $\varepsilon_r = 10.2$ and thickness $h = 1.27$ mm.

the value of $Z_{oe}$ exhibits a significant change; for example, $Z_{oe}$ is increased by more than 20 Ω for the end stages.

Next, we proceed to synthesize the parallel-coupled microstrip wideband filters. All the filters are designed on an RT/duroid 6010 substrate with $\varepsilon_r = 10.2$ and thickness $h = 1.27$ mm. Fig. 2 plots the bandwidth decrement against the designed specification. The test vehicle includes a third-, a fifth- and a seventh-order Chebyshev filters of ripple level 0.1 dB. In simulation by the full-wave simulator IE3D [10], the responses are obtained by discretizing the circuits with 20 and 40 cells per wavelength, and they are found indistinguishable. In Fig. 2, the curves denoted by "classical" are of filters obtained by (3) with $\theta = \pi/2$, and those by "improved" are of filters synthesized by (3) and (4). When the filter order $N = 3$ and the designed bandwidth is less than 25%, the bandwidth decrement is insignificant. If $\Delta$ is increased to 35%, however, the classical formulas produce a fractional bandwidth with 5% less than the specification. The bandwidth decrement deteriorates as the filter order or the designed $\Delta$ is increased. Upon the requirement of $\Delta = 70\%$, in the classical design, the

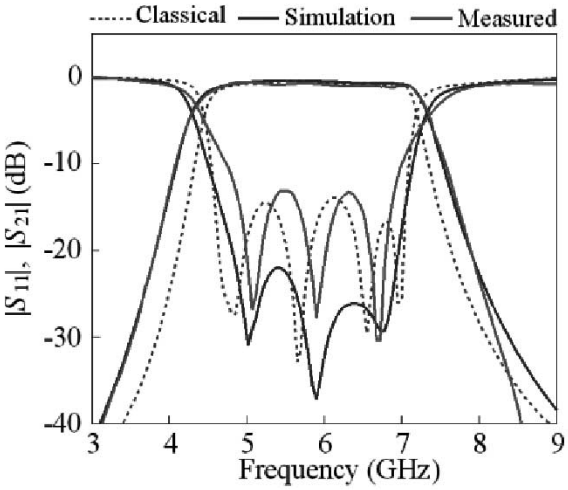


Fig. 4. Comparison of responses for fifth-order filters designed by the improved and classical formulas. The designed bandwidth is 50% and ripple level is 0.1 dB. The substrate has $\varepsilon_r = 10.2$ and thickness $h = 1.27$ mm.

bandwidth decrements are close to 19% and 25% for $N = 3$ and $N = 5$, respectively, while in our proposed equations, the decrements are only about 5% and 12.5%.

Finally, we examine the quality of the passband responses for filters designed with (3) and (4). Fig. 3 plots the simulation and measured responses for a third-order Chebyshev filter, and they show a very good agreement. Detailed data show that the simulated and measured results have fractional bandwidths of 48.4% and 48.2%, respectively, which are close to the designed bandwidth 50%. The measured results of a filter designed by classical formulas are also plotted for comparison. Its fractional bandwidth is only 41%.

Fig. 4 plots the results for a fifth-order filter. Again, the simulation and measured responses have a good agreement, and fractional bandwidths of 44.4% and 44%, respectively. For the filter based on the classical design, the measured response shows $\Delta = 38\%$.

## IV. Conclusion

Formulas for determining $Z_{oe}$ and $Z_{oo}$ of coupled stages are derived for synthesizing relatively wideband parallel-coupled microstrip filters with improved accuracy. A third- and a fifth-order Chebyshev filters with 50% designed bandwidth are fabricated and measured. The measurements show that the proposed formulas not only provide a significant improvement in predicting the filter bandwidth, but also preserve the quality of passband responses.

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