Density of states of the interacting two-dimensional electron gas

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We study the influence of electron-electron interactions on the density of states (DOS) of a clean two-dimensional electron gas. We find that the linear cusp in the DOS around the Fermi level, which was obtained previously, has an additional logarithmic factor. The cusp crosses over to a pure logarithmic dependence further away from the Fermi surface.

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It was established more than 20 years ago by Altshuler and Aronov[1] and Altshuler, Aronov, and Lee[2] that in low-dimensional diffusive systems the electron-electron interaction leads to the suppression of the single-particle density of states (DOS) at the Fermi level. In two dimensions their theory predicted a logarithmic cusp at the Fermi level due to the diffusion pole divergences in the vertex renormalizaion. The theory was later extended by Rudin, Aleiner and Glazman[3] to larger values of the energy measured from the Fermi surface.

Recently it was shown by Khveshchenko and Reizer[4] and by Mishchenko and Andreev[5] using the diagrammatic approach with the random phase approximation (RPA) dynamical susceptibility, that large electron-electron interaction induced correction to the DOS exists even in the absence of disorder. Both groups obtained a linear cusp at the Fermi level $\delta\nu(\epsilon)/\nu_0 \sim |\epsilon|/E_F$ (with the slope differing by a factor of 2), independent of the strength of the electron-electron interaction r_s. More recently Rollbuhler and Grabert[6] using a path integral technique (applied for relatively small couplings $r_s<1$) found numerically that the slope does depend on the coupling and flattens away from the Fermi surface.

Due to the apparent discrepancy between the two approaches we reconsider the problem in the framework of the diagrammatic method. Our aim is to spell explicitly and to clarify all the approximations which are made to obtain the transparent analytical formulas for the DOS correction. We also obtain the results which are partially different from those of Refs. 4 and 5.

The Hamiltonian of the two-dimensional electron gas is

$$H=\sum_{p\sigma}\varepsilon_p a^\dagger_{p\sigma}a_{p\sigma}+\frac{1}{2}\sum_{pp'\sigma\sigma'q}a^\dagger_{p+q\sigma}a^\dagger_{p'-q\sigma'}\times V(q)a_{p'\sigma'}a_p\sigma, \tag{1}$$

where

$$\varepsilon_p=\frac{p^2}{2m},\quad V(q)=\frac{2\pi e^2}{q}. \tag{2}$$

The Green's function $G(p,E)$ is given by the equation

$$G^{-1}(p,E)=G_0^{-1}(p,E)-\Sigma(p,E), \tag{3}$$

where $G_0(p,E)$ is the Green's function in the absence of electron-electron interaction:

$$G_0^{-1}(p,E)=E-\frac{p^2}{2m}; \tag{4}$$

the self-energy $\Sigma(p,E)$ in this paper will be calculated in the RPA at $T=0$:

$$\Sigma(p,E)=\int\frac{d^2q}{(2\pi)^2}\int\frac{d\omega}{2\pi}G_0(p+q,\,E+\omega)\frac{V(q)}{\varepsilon(q,\omega)}. \tag{5}$$

The DOS is

$$\nu(E)=-\frac{2}{\pi}\mathrm{Im}\int\frac{d^2p}{(2\pi)^2}G(p,\,E+i0). \tag{6}$$

Expanding the Green's function with respect to the self-energy and reversing the order of integrations, one obtains the electron-electron interaction induced correction to the DOS (Ref. 1):

$$\delta\nu(E)=-\frac{2}{\pi}\mathrm{Im}\int\frac{d^2p}{(2\pi)^2}\delta G(p,E+i0)\equiv\mathrm{Im}\,X(E+i0). \tag{7}$$

We will work in the Matrubara formalism, calculate X for the imaginary frequency,

$$X(i\Omega)=-\frac{2}{\pi}\int\frac{d^2q}{(2\pi)^2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{V(q)}{\varepsilon(q,i\omega)}\times\int\frac{d^2p}{(2\pi)^2}G_0^2(p,i\Omega)G_0[p+q,i(\Omega+\omega)], \tag{8}$$

and, at the end, will make an analytical continuation.

Using the fact that $G_0^2(p,i\Omega)=i\partial G_0(p,i\Omega)/\partial\Omega$, we can write the integral over p of the three Green's functions as

$$\int\frac{d^2p}{(2\pi)^2}G_0^2(p,i\Omega)G_0[p+q,i(\Omega+\omega)]=-i\left(\frac{\partial}{\partial\Omega}-\frac{\partial}{\partial\omega}\right)\Pi(q,i\omega,i\Omega), \tag{9}$$

where the polarization function $\Pi(\Omega,q,\omega)$ is given by the equation

$$\Pi(q,\ i\omega,\ i\Omega)=\int \frac{d^2p}{(2\pi)^2}G_0(p,\ i\Omega)G_0[p+q,\ i(\Omega+\omega)]. \tag{10}$$

The last equation in polar coordinates takes the form

$$\Pi(q,i\omega,i\Omega) = \frac{1}{(2\pi)^2}\int_0^{2\pi}d\theta\int_0^{\infty}pdp\frac{1}{i\Omega+\mu-\frac{p^2}{2m}} \times\frac{1}{i(\Omega+\omega)+\mu-\frac{p^2}{2m}-\frac{pq\cos(\theta)}{m}-\frac{q^2}{2m}}. \tag{11}$$

Since the main contributions come from fermionic momenta close to Fermi momentum, one can make the first approximation by replacing p in the term $pq\cos(\theta)/m$ by $\sqrt{2m\mu}=mv_F$ and ignoring the term $q^2/2m$. After that the integration over p can be easily performed:

$$\Pi(q,\ i\omega,\ i\Omega)=\frac{m}{(2\pi)^2}\int_0^{2\pi}\frac{d\theta}{i\omega-v_Fq\cos(\theta)}L(\omega,\Omega,q), \tag{12}$$

where

$$L(\omega,\Omega,q)=\log\frac{(\Omega+\omega)-i\mu+iv_Fq\cos(\theta)}{\Omega-i\mu}. \tag{13}$$

Explicitly, presenting the logarithm as

$$L(\omega,\Omega,q)=\frac{1}{2}\log\frac{(\Omega+\omega)^2+[\mu-v_Fq\cos(\theta)]^2}{\Omega^2+\mu^2} +i\left[\tan^{-1}\frac{\mu}{\Omega}-\tan^{-1}\frac{\mu-v_Fq\cos(\theta)}{\Omega+\omega}\right], \tag{14}$$

and taking into account that $\Omega,\omega,v_Fq\ll\mu$, we make the second approximation:

$$L=\pi i[\Theta(-\Omega)\Theta(\Omega+\omega)-\Theta(\Omega)\Theta(-\Omega-\omega)]. \tag{15}$$

After that, the integral over θ in Eq. (12) can be easily calculated, and the polarization function takes a form

$$\Pi(q,\ i\omega,\ i\Omega)=\frac{m}{2}\frac{\mathrm{sgn}(\omega)}{\sqrt{\omega^2+v_F^2q^2}}[\Theta(-\Omega)\Theta(\Omega+\omega) -\Theta(\Omega)\Theta(-\Omega-\omega)]. \tag{16}$$

Taking appropriate derivatives one obtains

$$\int \frac{d^2p}{(2\pi)^2}G_0^2(p,i\Omega)G_0[p+q,i(\Omega+\omega)] =\frac{-m|\omega|i}{2[\omega^2+v_F^2q^2]^{3/2}}[\Theta(-\Omega)\Theta(\Omega+\omega) -\Theta(\Omega)\Theta(-\Omega-\omega)]. \tag{17}$$

The dielectric constant $\varepsilon(q,\omega)$ is related to the polarization operator

$$P(q,i\omega)=-2\int\frac{d\Omega}{2\pi}\Pi(q,i\omega,i\Omega) \tag{18}$$

by the equation

$$\varepsilon(q,\ i\omega)=1-V(q)P(q,\ i\omega). \tag{19}$$

It is easier, however, not to use the approximate equation. (16) for the polarization function, but to insert in Eq. (18) exact Eq. (10) and (as it is traditionally done) integrate over Ω first, to obtain

$$P(q,i\omega)=2\sum_p\frac{n_p-n_{p+q}}{\varepsilon_{p+q}-\varepsilon_p-i\omega}, \tag{20}$$

where n_p is the Fermi distribution function. Obvious algebra gives

$$P(q,i\omega)=\frac{1}{\pi^2}\mathrm{Re}\int_{p<p_F}pdp\int d\theta\frac{1}{\frac{pq\cos\theta}{m}+\frac{q^2}{2m}-i\omega} =\frac{2}{\pi}\mathrm{Re}\int_{p<p_F}pdp\frac{1}{\sqrt{\left(\frac{q^2}{2m}-i\omega\right)^2-\frac{p^2q^2}{m^2}}} \tag{21}$$

Integrating over p,

$$P(q,\ i\omega)=\frac{2m^2}{\pi q^2}\mathrm{Re}\left[\frac{q^2}{2m}-i\omega-\sqrt{\left(\frac{q^2}{2m}-i\omega\right)^2-v_F^2q^2}\right], \tag{22}$$

and expanding the radical in small q, one obtains

$$\varepsilon(q,i\omega)=1+\frac{2e^2m}{q}\left[1-\frac{|\omega|}{\sqrt{\omega^2+v_F^2q^2}}\right]. \tag{23}$$

Now we are ready to return to the calculation of the DOS. Substituting Eqs. (17) and (23) into Eq. (8), considering $\Omega>0$ for definiteness, and subtracting an ''inessential'' constant, one has

$$X(i\Omega)=\frac{e^2mi}{2\pi^2}\int_{-\Omega}^{0}\omega d\omega\int_0^\infty qdq \times\frac{1}{q+2e^2m\left[1-\frac{\omega}{\sqrt{\omega^2+v_F^2q^2}}\right]}\times\frac{1}{[\omega^2+v_F^2q^2]^{3/2}}. \quad (24)$$

Equation. (24) coincides with those obtained in Refs. 4 and 5 (apart from the fact that we are considering imaginary energy). But we have noticed that the double integral can be calculated in a more rigorous way than it was done there. After we introduce the dimensionless variable $\bar{q}=v_Fq/|\omega|$, and change the order of integrations, the integral takes a form

$$X(i\Omega)=-\frac{i}{4\pi^2v_F^2}\int_0^\infty\frac{\bar{q}d\bar{q}}{[1+\bar{q}^2]^{3/2}}\int_0^\Omega \times\frac{d\omega}{\frac{\bar{q}}{2e^2mv_F}\omega+1-\frac{1}{\sqrt{1+\bar{q}^2}}}. \quad (25)$$

The integral over ω can be easily calculated and we obtain the following "scaling" form:

$$X(i\Omega)=-\frac{i\nu_0r_S}{2^{3/2}\pi}f\left(\frac{\Omega}{2\sqrt{2}r_SE_F}\right), \quad (26)$$

where $r_s=\sqrt{2}e^2/v_F$, $\nu_0=m/\pi$ is the DOS of the noninteracting two dimensional electron gas and the function f is

$$f(x)=\int_0^\infty\frac{d\bar{q}}{[1+\bar{q}^2]^{3/2}}\log\left[1+\frac{x\bar{q}}{1-\frac{1}{\sqrt{1+\bar{q}^2}}}\right]. \quad (27)$$

For $x\gg1$,

$$f(x)=\log x. \quad (28)$$

To obtain the density of states we should substitute $\Omega\to i\epsilon$, where $\epsilon=E-\mu$ is the energy measured from the Fermi surface, and take the imaginary part. Thus we obtain, for $|\epsilon|\gg r_sE_F$,

$$\frac{\delta\nu(\epsilon)}{\nu_0}=\frac{r_S}{2^{3/2}\pi}\log\left(\frac{|\epsilon|}{r_SE_F}\right). \quad (29)$$

To obtain asymptotic of $f(x)$ for $x\ll1$ we can chose arbitrary η satisfying $x\ll\eta\ll1$ and present the integral in Eq. (27) as some of two integrals: from 0 to η and from η to ∞. In the first integral we can expand the integrand with respect to q and in the second we can expand the logarithm in with respect to x. Thus we obtain

$$f(x)=\int_0^\eta d\bar{q}\log\left[1+\frac{2x}{\bar{q}}\right]+x\int_\eta^\infty\frac{d\bar{q}}{[1+\bar{q}^2]^{3/2}}\frac{\bar{q}}{1-\frac{q}{\sqrt{1+\bar{q}^2}}}. \quad (30)$$

After a simple algebra in a leading approximation with respect to x we obtain

$$f(x)=-2x\log x. \quad (31)$$

Thus, for $|\epsilon|\ll r_sE_F$,

$$\frac{\delta\nu(\epsilon)}{\nu_0}=-\frac{|\epsilon|}{4\pi E_F}\log\left(\frac{|\epsilon|}{r_SE_F}\right). \quad (32)$$

Equations (29) and (32) are our main results. The correction is smaller than ν_0, consistent with the perturbative assumption. At small $|\epsilon|$ the DOS has a linear downward cusp modified by a logarithmic factor. This factor also gives a weak dependence of the cusp upon the strength of the coupling. The quasilinear segment crosses over to the logarithmic one at the energy scale $|\epsilon|=r_sE_F$. This last statement is in good agreement with numerical results of Ref. 6.

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