Improved technique for measuring small angles

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Based on the total-internal-reflection effect and heterodyne interferometry, an improved technique for measuring small angles is proposed. This technique not only expands the measurement range but it also improves measurement performances. Its validity is demonstrated. © 1997 Optical Society of America

Key words: Total-internal-reflection effect, heterodyne interferometry.

1. Introduction

There are several optical methods[1–7] for measuring small angles. They are sensitive and accurate, but their sizes are too large to be used in some space-limited areas.[8] To solve this problem, Huang *et al.*[8,9] proposed an optical method for measuring the reflectance difference between two right-angle prisms based on the total-internal-reflection effect. Although it has some merits, as described by Huang *et al.*, there are some disadvantages, such as a small measurable range, operation limited to a darkroom with a highly stable light source, and a careful estimation to avoid influence from unnecessary reflectances at the entrance and the exit surfaces of the prisms. In this paper, an improved technique for measuring small angles by modifying Huang's method is proposed. Although both of them are based on the total-internal-reflection effect, we measure the phase difference instead of the reflectance difference. And the phase difference can be extracted and measured accurately with heterodyne interferometry despite surrounding light and an unstable light source. Hence its performance is not affected by surrounding light noise. Moreover its validity is demonstrated.

2. Principle

The schematic diagram of this technique is shown in Fig. 1. Huang *et al.*'s optical system is modified by introducing an optical heterodyne source and two analyzers before the detectors. The optical path arrangement is the same as that of Huang's system. The light beam coming from a heterodyne optical source[10] with the frequency difference f between s-(y-axis) and p-(x-axis) polarizations is incident on the beam splitter, BS, and is divided into two parts: the transmitted beam and the reflected beam. These two beams propagate nearly perpendicularly and are almost normally incident on the side surfaces of two right-angle prisms, P1 and P2, with the refractive index n_p placed on a rotary stage, respectively. The beams are totally internally reflected at the hypotenuse surfaces of the prisms. Then, they pass through analyzers, AN1 and AN2, which have the transmission axes at 45° with respect to the x axis and finally are detected by two detectors, D1 and D2. As shown in Fig. 2, if the rotary stage is rotated with a small angle θ, which is defined positive as it is rotated clockwise, the two interference signals I_1 and I_2 detected by D1 and D2, respectively, are

$$I_1 = I_{10}[1 + V_1 \cos(2\pi ft + \phi_1 + \phi_{BS})], \quad (1)$$

$$I_2 = I_{20}[1 + V_2 \cos(2\pi ft + \phi_2)], \quad (2)$$

where V_1 and V_2 are the visibilities of signals I_1 and I_2, ϕ_1 and ϕ_2 are the phase differences between p and s polarizations due to the total internal reflections within P1 and P2, respectively; ϕ_{BS} is the phase difference between the p and s polarizations of the light reflected from BS with the coating layer of a refractive index n and an extinction coefficient k. If α_1 and α_2 are the incident angles at the hypotenuse surfaces of prisms P1 and P2, according to

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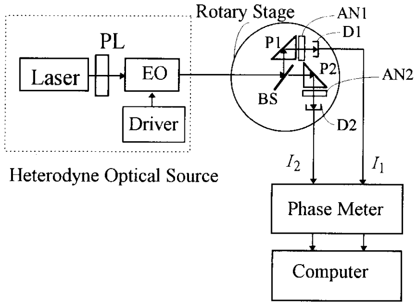

Fig. 1. Schematic diagram for measuring small angles: PL, polarizer; EOM, electro-optical modulator; AN, analyzer.

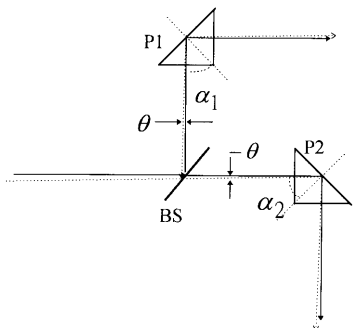

Fig. 2. Geometric relations between θ and α_1 or α_2.

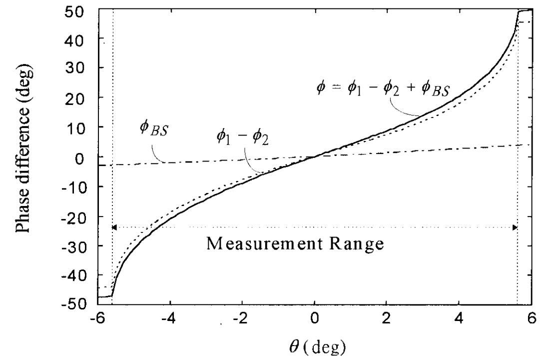

Fig. 3. Calculated curves of ϕ versus θ, $(\phi_1 - \phi_2)$, and ϕ_{BS}.

Fresnel's equation,[11] ϕ_1 and ϕ_2 can be written as

$$\phi_1 = 2\tan^{-1}\left\{\frac{[\sin^2\alpha_1 - (1/n_p{}^2)]^{1/2}}{\tan\alpha_1 \sin\alpha_1}\right\}, \tag{3}$$

$$\phi_2 = 2\tan^{-1}\left\{\frac{[\sin^2\alpha_2 - (1/n_p{}^2)]^{1/2}}{\tan\alpha_2 \sin\alpha_2}\right\}, \tag{4}$$

$$\phi_{BS} = \tan^{-1}\left(\frac{bc - ad}{bd + ac}\right), \tag{5}$$

where

$$a = 2v\cos\beta, \qquad b = \cos^2\beta - u^2 - v^2,$$

$$c = 2v\cos\beta(n^2 - k^2 - 2u^2), \qquad d = (n^2 + k^2)^2 \times \cos^2\beta - (u^2 + v^2),$$

$$2u^2 = n^2 - k^2 - \sin^2\beta + [(n^2 - k^2 - \sin^2\beta)^2 + 4n^2k^2]^{1/2},$$

$$2v^2 = -(m^2 - k^2 - \sin^2\beta) + [(n^2 - k^2 - \sin^2\beta)^2 + 4n^2k^2]^{1/2},$$

$$\beta = 45° + \theta.$$

From the geometric relations of Fig. 2, we have

$$\alpha_1 = 45° + \sin^{-1}\left(\frac{\sin\theta}{n_p}\right), \tag{6}$$

$$\alpha_2 = 45° - \sin^{-1}\left(\frac{\sin\theta}{n_p}\right). \tag{7}$$

From Eqs. (1) and (2), it is obvious that these two signals are sinusoidal with frequency f. When they are introduced into the phase meter, a phase difference ϕ between them can be obtained, and ϕ can be expressed as

$$\phi = \phi_1 - \phi_2 + \phi_{BS}. \tag{8}$$

Substituting the data ϕ into Eqs. (3)–(8), we can calculate the rotation angle θ. In our system the outputs of the phase meter are sent to a personal computer, so that the result can be obtained in real time.

As the rotary stage is rotated, one of the incident angles at the hypotenuse surfaces of the prisms is increased and the other is decreased. Because this technique is based on the effect of the total internal reflection, this technique can be performed only if both of them are larger than the critical angle. Hence the allowable measurement range is from $-\theta_{max}$ to θ_{max}, where θ_{max} should satisfy

$$\theta_{max} = \sin^{-1}\left\{n_p \sin\left[45° - \sin^{-1}\left(\frac{1}{n_p}\right)\right]\right\}. \tag{9}$$

Substituting the latter experimental conditions, $n_p = 1.51509$, $n = 3.57$, and $k = 4.36$, into Eqs. (3)–(5) and (8), we can obtain the relation curves of ϕ, $(\phi_1 - \phi_2)$, and ϕ_{BS} to θ as shown in Fig. 3. It shows that in this case the measurement range is between $-5.60°$ and $5.60°$.

3. Experiments and Results

To show the feasibility of this technique, two right-angle prisms of BK7 glass with $n_p = 1.51509$ and a beam splitter with a chromium layer with $n = 3.57$ and $k = 4.36$ are used, and they are mounted on a high-precision rotary stage (PS-θ-90) with an angular resolution of 0.005° manufactured by Japan Chuo

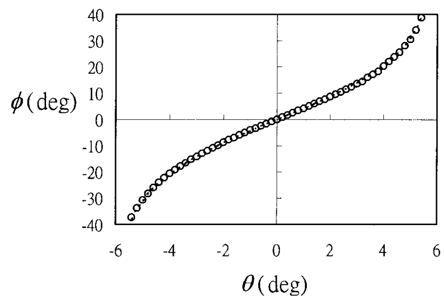

Fig. 4. Experimental curves of ϕ versus θ.

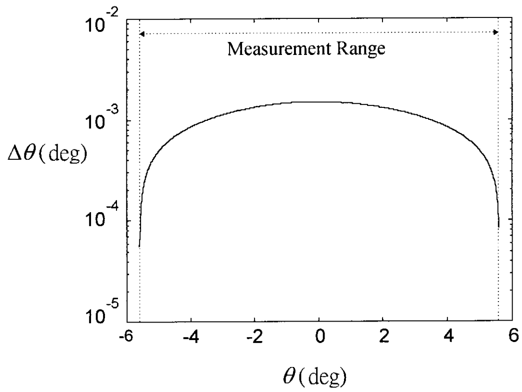

Fig. 5. Relation curves of $\Delta\theta$ versus θ.

Precision Industrial Company Ltd. The heterodyne optical source[10] with a wavelength of 632.8 nm and $f = 2$ kHz and a self-made phase meter with a resolution of 0.01° are used. The experimental curves of ϕ versus θ for the angle measurement are shown in Fig. 4. In this figure, the circles represent the evaluated values of the rotation angles, which are obtained by introducing the data of ϕ into Eqs. (3)–(8), and the dashed curve represents the direct angle readouts from the division mark of the rotary stage. It is clear that they have a good correspondence.

4. Discussion

From Eq. (8), we can obtain an expression for the measurement error:

$$\Delta\theta = \frac{1}{[d\phi_1 - d\phi_2)/d\theta] + (d\phi_{BS})/d\theta)}\,\Delta\phi, \qquad (10)$$

where $\Delta\phi$ is the angular resolution of the phase meter. In our experiments, the angular resolution of the phase meter is 0.01°. Consequently, the relation curve of $\Delta\theta$ versus θ can be obtained as shown in Fig. 5. Obviously, the error of the rotation angle becomes smaller as $|\theta|$ increases; the best resolution is $8 \times 10^{-5°}$ at $|\theta| = 5.6°$. And the resolution of this technique is better than $1.6 \times 10^{-3°}$ over the measurement range of $-5.60° \leq \theta \leq 5.60°$.

Unlike many angle-measurement techniques, this approach is absolute: The phase difference is precisely zero at zero angle. By using a higher-index glass, the angular range can be significantly extended (e.g., for $n_p = 1.8$, $\theta_{max} \approx 20°$).

5. Conclusion

An improved technique for measuring small angles is proposed. Using heterodyne interferometry, it makes uses of the effect of total internal reflection in right-angle prisms as well as the phase-difference measurement between the two light beams passing through the prisms. This technique not only expands the measurement range but also improves measurement performances. Its validity is demonstrated.

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