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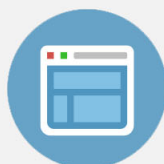
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# Spin-dependent Hall effect in semiconductor quantum wells

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We present a theoretical study of the spin-dependent scattering of electrons from screened attractive and repulsive impurities in III-V semiconductor quantum wells. The effective one-band Hamiltonian and the Rashba spin-orbit interaction are used. We demonstrated that the asymmetry of the spin-dependent skew-scattering and side-jump effect can lead to a quite large spin-dependent (anomalous) Hall effect at zero magnetic field in all-semiconductor quantum well structures. Our theory predicts a measurable spin-dependent Hall angle that reaches about  $2.5 \times 10^{-3}$  rad for a CdTe/InSb/CdTe quantum well with impurities doped in the center of the well. © 2004 American Institute of Physics. [DOI: 10.1063/1.1641147]

## I. INTRODUCTION

The extra degree of freedom provided by the electron spin may open up further enhancements for semiconductor devices. The spin-transistor proposed by Datta and Das<sup>1</sup> is an example of a spin-controlled device based on semiconductor two-dimensional (2-D) channels. For this reason, theoretical studies of spin-dependent electron processes in 2-D semiconductor structures have attracted a lot of interest since a particular branch of semiconductor electronics (so-called spintronics) has become a focus of study.<sup>2-4</sup>

Recently, detection of the electron polarization in paramagnetic metals<sup>5,6</sup> and semiconductors<sup>7</sup> through the spin-dependent Hall effect (SDHE) has been proposed. This is quite similar to the exploitation of the anomalous Hall effect (AHE), which can be observed in magnetic metals or semiconductors without external magnetic field (see, for instance Ref. 8, and references therein). The key point of the explanation of those effects is the presence of the spin-orbit interaction (SOI). Considerable work on the AHE has been done in the last 50 years since the pioneering work of Karplus and Luttinger.<sup>9</sup> It is generally recognized that two mechanisms contribute to the AHE. Those are the side-jump effect (SJ) proposed by Karplus and Luttinger<sup>9</sup> and Berger,<sup>10</sup> and the skew-scattering (SS) proposed by Smit.<sup>11</sup> It is commonly believed that the first mechanism can be more significant in metal alloys or semiconductors with relatively large resistivity, while the second one prevails in systems with low resistivity.

In the absence of magnetic impurities and at low temperatures, the main source of the spin-dependent scattering processes is the SO coupling to local defects. The effect of the SOI on the electron transport and relaxation in 2-D semiconductor systems has been studied for a long time.<sup>12-15</sup> We recently investigated the spin-dependent scattering processes in the bulk of nonmagnetic semiconductors in the presence of the SOI.<sup>16</sup> In semiconductor quantum wells (QWs) the effect of the SOI on the processes of scattering becomes even more stronger than in the bulk. This is a result of the local-

ization of electrons' wave functions in the conduction channel.<sup>17,18</sup>

In this article, we present a model of the spin-dependent electron scattering from impurities located in the center QWs of nonmagnetic III-V semiconductors. We calculate contributions from the SS and SJ mechanisms to the SDHE. Our calculation is based on the effective-one-band Hamiltonian<sup>19,20</sup> and Rashba-type model of the SOI.<sup>12,21,22</sup> For QWs of narrow-gap semiconductors (systems with large SO coupling parameters) and with impurities located in the center of the wells, we obtained relatively large spin-dependent Hall angles (SOHAs).

The article is organized as follows. Section II describes the method we use to calculate the spin-dependent (Mott) cross section for 2-D electrons scattered from impurities in semiconductor QWs. Section III presents the method of calculation of the off-diagonal element of the conductivity tensor in QWs with account of the Mott scattering. The calculation results are presented in Sec. IV and conclusions are given in Sec. V.

## II. BASIC EQUATIONS AND DESCRIPTION OF THE SPIN-DEPENDENT SCATTERING

We consider III-V semiconductor QWs with charged impurities and use the approximate one-electron-band effective Hamiltonian in the following form:<sup>19,20</sup>

$$\hat{H} = \hat{H}_0 + \hat{V}_{\text{im}}(\mathbf{r}), \quad (1)$$

where  $H_0$  is the Hamiltonian of the system without impurities:

$$\hat{H}_0 = -\frac{\hbar^2}{2} \nabla_{\mathbf{r}} \left[ \frac{1}{m(E, \mathbf{r})} \right] \nabla_{\mathbf{r}} + V(\mathbf{r}).$$

$\hat{V}_{\text{im}}(\mathbf{r})$  is the impurity potential,  $\nabla_{\mathbf{r}}$  stands for the spatial gradient,  $m(E, \mathbf{r})$  is the energy, and position-dependent electron effective mass is

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$$\frac{1}{m(E, \mathbf{r})} = \frac{2P^2}{3\hbar^2} \left[ \frac{2}{E + E_g(\mathbf{r}) - V(\mathbf{r})} + \frac{1}{E + E_g(\mathbf{r}) + \Delta(\mathbf{r}) - V(\mathbf{r})} \right],$$

where  $E$  is the electron energy,  $V(\mathbf{r})$  is the confinement potential of the well,  $E_g(\mathbf{r})$  and  $\Delta(\mathbf{r})$  stand for the position-dependent band gap and the SO splitting in the valence band, respectively, and  $P$  is the momentum matrix element.

The impurity potential consists of two parts,

$$\hat{V}_{\text{im}}(\mathbf{r}) = V_c(\mathbf{r}) + V_{\text{so}}(\mathbf{r}),$$

where  $V_c(\mathbf{r})$  is the Coulomb potential of the charged impurity and  $V_{\text{so}}(\mathbf{r})$  describes the SO coupling with the impurity

$$V_{\text{so}}(\mathbf{r}) = i\alpha(E, \mathbf{r}) \nabla V_c(\mathbf{r}) \cdot [\hat{\boldsymbol{\sigma}} \times \nabla \mathbf{r}],$$

where<sup>12,18,21,22</sup>

$$\alpha(E, \mathbf{r}) = \frac{P^2}{3} \left[ \frac{1}{[E + E_g(\mathbf{r}) - V(\mathbf{r})]^2} - \frac{1}{[E + E_g(\mathbf{r}) + \Delta(\mathbf{r}) - V(\mathbf{r})]^2} \right]. \quad (2)$$

We describe symmetrical QWs of thickness  $L$  and denote by  $z$  the direction perpendicular to the well interfaces. For systems with sharp discontinuity in the conduction band edge between the QW (material 1) and the barrier region (material 2), the confinement potential can be presented as

$$V(\mathbf{r}) = \begin{cases} 0, & -L/2 \leq z \leq L/2; (\mathbf{r} \in 1) \\ V_0, & |z| > L/2; (\mathbf{r} \in 2) \end{cases}. \quad (3)$$

We assume that an isolated impurity is located in the center of the wells ( $z=0$ ), and the unscreened Coulomb potential of the impurity is given as

$$V_c^0(\mathbf{r}) = -\frac{Ze^2}{4\pi\epsilon_s[\rho^2 + z^2]^{1/2}}, \quad (4)$$

where  $\boldsymbol{\rho}=(x,y)$  is the position vector parallel to the interfaces,  $\epsilon_s=(\epsilon_1+\epsilon_2)/2$  is the average permittivity of the system,  $Z$  is the charge of the impurity, and  $e$  is the electron charge. For most III-V semiconductor QWs, we can neglect the image potential, and we assume that for simplicity.

The Rashba term in  $\hat{H}_0$  does not occur due to the reflection symmetry of the quantum well.<sup>21,22</sup> Considering only the electrons' elastic scattering within the first subband of the well, we present the solution of the confinement problem with the Hamiltonian  $\hat{H}_0$  as<sup>19,23</sup>

$$\Psi(\boldsymbol{\rho}, z) = \psi^s(\boldsymbol{\rho}) \varphi_1(z), \quad (5)$$

where  $\varphi_1(z)$  is the normalized electron wave function in  $z$  direction, and  $s=\pm 1$  is the quantum number related to the spin states. The eigen-energy  $E_{nz}$  in  $z$  direction can be obtained easily from the well-known Ben-Daniel-Duke boundary conditions.<sup>20,21</sup>

By taking the average

$$\tilde{V}(\rho) = \int_{-\infty}^{+\infty} dz \varphi_1^*(z) V_{\text{im}}(r) \varphi_1(z),$$

and following the approach described in Refs. 19, 23 and 24, we obtain statistically screened and averaged quasi-2-D scattering potential in the following form:

$$\begin{aligned} \tilde{V}_c(\rho) &= -\frac{\hbar^2 Ze^2}{a_B^* m_1(0)} \int_0^\infty \frac{dq}{\epsilon(q)} J_0(q\rho) \\ &\quad \times \int_{-\infty}^{+\infty} dz |\varphi_1(z)|^2 e^{-q|z|}, \\ \tilde{V}_{\text{so}}(\rho) &= i \frac{\hbar^2 Ze^2}{a_B^* m_1(0)} \int_0^\infty \frac{q dq}{\epsilon(q)} J_1(q\rho) \\ &\quad \cdot \left[ \alpha_1(E) \int_{z \leq |L/2|} dz |\varphi_1(z)|^2 e^{-q|z|} \right. \\ &\quad \left. + \alpha_2(E) \int_{z \geq |L/2|} dz |\varphi_1(z)|^2 e^{-q|z|} \right] \times \frac{s}{\rho} \frac{\partial}{\partial \phi}, \end{aligned}$$

where  $a_B^* = \epsilon_s \hbar^2 / e^2 m_1(0)$  is the effective Bohr radius in the well,  $J_n(x)$  is the Bessel function,

$$\epsilon(q) = 1 + \frac{q_f}{q}$$

is the 2-D dielectric function,

$$q_f = \frac{1}{a_B^*} \frac{m_1(E_f)}{m_1(0)} \left\{ 1 + \frac{d}{dE} \ln[m_1(E)] \right\} \Big|_{E_f}$$

is the 2-D Thomas-Fermi screening constant,  $E_F$  is the Fermi energy of the system,<sup>24</sup> and

$$E = E_\rho + E_{1z}.$$

Due to the radial symmetry of the potentials  $\tilde{V}_c(\rho)$  and  $\tilde{V}_{\text{so}}(\rho)$ , we can present the wave function  $\psi^s(\boldsymbol{\rho})$  as the following:

$$\psi^s(\boldsymbol{\rho}) = \sum_{l=-\infty}^{l=+\infty} R_l^s(\rho) e^{il\phi} \chi^s.$$

where  $l$  is the orbital momentum number and  $\chi^s$  is the spin function upon which the Pauli matrix vector operates.

The quasi-2-D Schrödinger equation for the radial wave function is given by

$$\begin{aligned} &\left\{ \frac{\hbar^2}{2\tilde{m}(E)} \left[ \frac{1}{\rho} \frac{d}{d\rho} \left( \rho \frac{d}{d\rho} \right) - \frac{l^2}{\rho^2} + k^2 \right] - \tilde{V}_c(\rho) + l \tilde{V}_{\text{so}}(\rho) \right\} R_l^s(\rho) \\ &= 0, \end{aligned} \quad (6)$$

where

$$k = \frac{1}{\hbar} \sqrt{2\tilde{m}(E)E_\rho}$$

is the wave vector of the 2-D electrons, and

$$\frac{1}{\tilde{m}(E)} = \frac{1}{m_1(E)} \int_{z \leq |L/2|} dz |\varphi_1(z)|^2 + \frac{1}{m_2(E)} \int_{z \geq |L/2|} dz |\varphi_1(z)|^2.$$

At a large distance from the scattering center, the asymptotic of the radial function is given by

$$R_l^s(\rho) \rightarrow A_l^s [\cos \delta_l^s J_l(k\rho) - \sin \delta_l^s N_l(k\rho)]; \quad \rho \rightarrow \infty,$$

where  $\delta_l^s$  is the scattering phase shift<sup>25,26</sup> and  $N_l$  is the Neumann function. To solve the scattering problem we use the variable phase approach,<sup>26,27</sup> which assumes that the phase function  $\delta_l^s(\rho)$  at the point  $\rho$  determines the phase shift produced by the part of the potential contained within the cycle of a radius  $\rho$ . The scattering phase shift for the total potential is equal to the asymptotic value

$$\delta_l^s = \lim_{\rho \rightarrow \infty} \delta_l^s(\rho).$$

The phase function  $\delta_l^s(\rho)$  satisfies the following differential equation:

$$\frac{d\delta_l^s(\rho)}{d\rho} = -\frac{2\pi\tilde{m}(E)}{\hbar^2} \rho [\tilde{V}_c(\rho) - l\tilde{V}_{so}(\rho)] \times [\cos \delta_l^s(\rho) J_l(k\rho) - \sin \delta_l^s(\rho) N_l(k\rho)], \quad (7)$$

with the boundary condition

$$\delta_l^s(0) = 0. \quad (8)$$

The complex 2-D scattering amplitude we present as<sup>25,28</sup>

$$\mathbf{F}^s(E, \theta) = [f(E, \theta) + \sigma_z g(E, \theta)] \mathbf{X}^s, \quad (9)$$

where  $f^s(\theta)$  and  $g(\theta)$  describe scattering without and with electron spin reorientation, respectively, and they are determined by the following:<sup>28</sup>

$$f(E, \theta) = \sum_{l=0}^{\infty} f_l \cos(l\theta), \quad (10)$$

$$g(E, \theta) = \sum_{l \geq 1}^{\infty} g_l \sin(l\theta), \quad (11)$$

where

$$f_l = \sqrt{\frac{1}{2\pi k}} \begin{cases} \exp(2i\delta_0) - 1, & l=0 \\ \exp(2i\delta_l^+) + \exp(2i\delta_l^-) - 2, & l \geq 1 \end{cases}$$

$$g_l = i \sqrt{\frac{1}{2\pi k}} [\exp(2i\delta_l^+) - \exp(2i\delta_l^-)],$$

$\theta$  is the scattering angle between initial ( $\mathbf{k}_i$ ) and final ( $\mathbf{k}_f$ ) wave vectors. When electrons are spin polarized parallel to the  $z$  axis, the Mott scattering cross section<sup>29</sup> can be expressed in terms of the incident electron spin-polarization  $P$  as the following:

$$\sigma(E, \theta) = I(E, \theta) + G(E, \theta)P, \quad (12)$$

where  $I(\theta)$  is the differential cross section for the unpolarized incident electrons (the symmetric scattering part)

$$I(E, \theta) = |f(E, \theta)|^2 + |g(E, \theta)|^2,$$

and

$$G(E, \theta) = f^*(E, \theta)g(E, \theta) + f(E, \theta)g^*(E, \theta), \quad (13)$$

is the spin-flip part of the scattering cross section (the asymmetric scattering part).

### III. SPIN-DEPENDENT HALL EFFECT IN 2-D CHANNELS

In the Pauli approach to the explanation of the origin of the AHE,<sup>8</sup> the total electron velocity is presented as

$$\mathbf{v}_k^s = \frac{1}{\hbar} \nabla_{\mathbf{k}} E_{\rho}(\mathbf{k}) + \mathbf{w}_k^s \quad (14)$$

where  $E_{\rho}(\mathbf{k})$  is the dispersion relation of 2-D electrons in the well, and  $\mathbf{w}_k^s$  is the anomalous velocity, which can be written in the following form:<sup>8,21,22</sup>

$$\mathbf{w}_k^s = \alpha \frac{[\mathbf{s} \times \mathbf{k}]}{\tau_{\text{im}}}, \quad (15)$$

where  $\tau_{\text{im}}$  is the electron momentum relaxation time resulted from impurity scattering, and  $\mathbf{s}$  is the unit vector parallel to the spin polarization.

The total electron current can be obtained by averaging the total velocity over the electron distribution function  $f_s(\mathbf{k})$ . In the linear approximation with respect to the external electric field  $\mathbf{F}$ ,<sup>30-32</sup> this leads to

$$\mathbf{J} = -|e| \sum_{\mathbf{k}, s} \mathbf{v}_k^s f_s(\mathbf{k}) = \mathbf{J}^0 + \mathbf{J}^s = \sigma_c \mathbf{F} + \sigma_c^s \frac{[\mathbf{s} \times \mathbf{F}]}{s}, \quad (16)$$

where  $\sigma_c$  is the diagonal element of the conductivity tensor and  $\sigma_c^s$  is the spin-dependent Hall conductivity (off-diagonal element of the conductivity tensor). If the concentration of scatterers is low, one can assume these impurities scatter the electrons independently. In this approximation, the Boltzmann transport equation for the electron distribution function  $f_s(\mathbf{k})$  is given by

$$-\frac{|e|\hbar}{\hbar} \mathbf{F} \nabla_{\mathbf{k}} f_s(\mathbf{k}) = \sum_{\mathbf{k}, s} W_0(\mathbf{k}, \tilde{\mathbf{k}}) + W_s(\mathbf{k}, \tilde{\mathbf{k}}) \times [f_s(\mathbf{k}) - f_s(\tilde{\mathbf{k}})], \quad (17)$$

where

$$W_0(\mathbf{k}, \tilde{\mathbf{k}}) = \frac{8\pi\hbar^3}{A\tilde{m}^2(E)} k I(\mathbf{k}, \tilde{\mathbf{k}}) N_{\text{im}} \delta[E_{\rho}(\mathbf{k}) - E_{\rho}(\tilde{\mathbf{k}})], \quad (18)$$

$$W_s(\mathbf{k}, \tilde{\mathbf{k}}) = \frac{8\pi\hbar^3}{A\tilde{m}^2(E)} k P G(\mathbf{k}, \tilde{\mathbf{k}}) N_{\text{im}} \delta[E_{\rho}(\mathbf{k}) - E_{\rho}(\tilde{\mathbf{k}})], \quad (19)$$

$$I(\mathbf{k}, \tilde{\mathbf{k}}) = I[E_{\rho}(\mathbf{k}), \theta], \quad G(\mathbf{k}, \tilde{\mathbf{k}}) = G[E_{\rho}(\mathbf{k}), \theta], \quad (20)$$

$$P = \frac{1}{n} \sum_{s=\pm 1} s n_s, \quad n = n_{+1} + n_{-1}, \quad (21)$$

where  $W_0(\mathbf{k}, \tilde{\mathbf{k}})$  and  $W_s(\mathbf{k}, \tilde{\mathbf{k}})$  are the scattering transition probabilities per unit time due to symmetric and asymmetric

scattering, respectively,  $N_{\text{im}}$  is the impurity concentration,  $P$  is the polarization of the 2-D electronic gas,  $n_s$  is the concentration of the  $s$ -polarized electrons in the 2-D channel, and  $A$  is the system area. In the linear approximation, the electron distribution function can be written in the form

$$f_s(\mathbf{k}) \approx f_0(E) + P_0(E)(\mathbf{k} \cdot \mathbf{F}) + P_s(E)(\mathbf{k} \cdot [\mathbf{s} \times \mathbf{F}]). \quad (22)$$

Substituting  $f_s(\mathbf{k})$  into the Boltzmann equation, we obtain the coefficients  $P_0(E)$  and  $P_s(E)$ , and then the components of the conductivity tensor.

Finally, the off-diagonal element of the conductivity tensor obtained from Eq. (16) consists of two parts,

$$\sigma_c^s = \sigma_{\text{SS}}^s + \sigma_{\text{SJ}}^s \quad (23)$$

where  $\sigma_{\text{SS}}^s$  is the contribution from the skew-scattering (which comes from the spin-dependent part of the elastic scattering) and  $\sigma_{\text{SJ}}^s$  is the side-jump contribution (which comes with the anomalous velocity). For the case of the degenerated electronic system (low-temperature limit), those two contributions can be presented as the following:

$$\sigma_{\text{SS}}^s = \frac{\hbar e^2 N_{\text{im}}}{2\pi} \sum_{s=\pm 1} s \left( \frac{\tau_{\text{im}}^s}{\tilde{m}_s} \right)^2 \tilde{G}_s(k_F^s)^3, \quad (24)$$

$$\sigma_{\text{SJ}}^s = -\frac{e^2}{4\pi\hbar} \sum_{s=\pm 1} s \alpha_s (k_F^s)^2. \quad (25)$$

In Eqs. (24) and (25),

$$\tilde{G}_s = \int_0^{2\pi} G(E_F^s, \theta) [1 - \cos(\theta)] \sin(\theta) d\theta, \quad (26)$$

$$\frac{1}{\tau_{\text{im}}^s} = \frac{2\hbar N_{\text{im}}}{\tilde{m}_s} k_F^s \tilde{T}_s, \quad (27)$$

$$\tilde{T}_s = \int_0^{2\pi} I(E_F^s, \theta) [1 - \cos(\theta)] d\theta, \quad (28)$$

$\tilde{m}_s = \tilde{m}(E_F^s)$ ,  $\alpha_s = \alpha(E_F^s)$ , and the Fermi energy  $E_F^s$  for the  $s$ -group of the polarized electrons is the solution of the following equation:

$$E_F^s = \frac{\hbar^2}{2\tilde{m}_s} (k_F^s)^2 + E_{1z}, \quad (29)$$

with the electron Fermi wave vector defined as the following:

$$k_F^s = [2\pi n(1 + sP)]^{1/2}. \quad (30)$$

The tangent of the spin-dependent Hall angle (SDHA) is the sum of two tangents, and can be presented as

$$\tan(\theta_H) = \tan(\theta_H^{\text{SS}}) + \tan(\theta_H^{\text{SJ}}), \quad (31)$$

where

$$\tan(\theta_H^{\text{SS}}) = \frac{\sigma_{\text{SS}}^s}{\sigma}, \quad \tan(\theta_H^{\text{SJ}}) = \frac{\sigma_{\text{SJ}}^s}{\sigma}, \quad (32)$$

and

$$\sigma_c = \frac{e^2 k_F^s}{4\pi\tilde{m}} \tau_{\text{im}}^s \Big|_{P=0}. \quad (33)$$

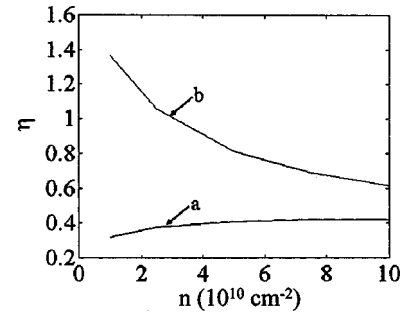


FIG. 1. The ratio  $\eta = |\theta_H^{\text{SJ}} / \theta_H^{\text{SS}}|$  ( $\theta_H^{\text{SJ}}$  and  $\theta_H^{\text{SS}}$  are contributions into the total SDHA from the side-jump and skew-scattering parts, correspondingly) as the function on the electron concentration for two types of impurities located in the center of the IGA QW [ $n = N_{\text{im}}$ ,  $L = 20$  nm;  $a$  is a repulsive impurity ( $Z = -1$ ),  $b$  is an attractive impurity ( $Z = +1$ )].

#### IV. CALCULATION RESULTS

To demonstrate the actual value of the SDHE in semiconductor QWs, we first present results of our simulations for  $\text{Al}_{0.48}\text{In}_{0.52}\text{As}/\text{In}_{0.53}\text{Ga}_{0.47}\text{As}/\text{Al}_{0.48}\text{In}_{0.52}\text{As}$  (IGA) symmetrical QWs, which possessed the well-developed growth technology. The parameters taken in this calculation are the following:  $E_{g1} = 0.813$  eV,  $E_{g2} = 1.508$  eV,  $\Delta_1 = 0.361$  eV,  $\Delta_2 = 0.332$  eV,  $m_1(0) = 0.041m_0$ ,  $m_2(0) = 0.075m_0$ ,  $\epsilon_1 = 14$ ,  $\epsilon_2 = 12.5$ , and  $V_0 = 0.504$  eV<sup>33</sup> ( $m_0$  is the free electron mass). Secondly, we present our calculation results for CdTe/InSb/CdTe (IS) QWs [where  $E_{g1} = 0.24$  eV,  $E_{g2} = 1.59$  eV,  $\Delta_1 = 0.81$  eV,  $\Delta_2 = 0.8$  eV,  $m_1(0) = 0.015m_0$ ,  $m_2(0) = 0.08m_0$ ,  $\epsilon_1 = 16.8$ ,  $\epsilon_2 = 10.2$ , and  $V_0 = 0.55$  eV<sup>34,35</sup>]; these demonstrate about the largest spin-coupling effects. In all calculations, we assure the validity of the one-subband scattering model, when the intersubband gap is larger than the energy of the  $\rho$ -direction motion:  $E_\rho < E_{2z} - E_{1z}$ . This allows us to consider scattering of electrons with the following wave vectors: for the IGA structures with  $L \leq 30$  nm- $k \leq k_F^1 = 2.5(a_B^*)^{-1}$  (the electron concentration  $n = 3.5 \times 10^{11}$  cm<sup>-2</sup>); for the IS structures with  $L \leq 30$  nm- $k \leq k_F^1 = 6.6(a_B^*)^{-1}$  ( $n = 3 \times 10^{11}$  cm<sup>-2</sup>). Notice, that  $a_B^*$  is taken different by definition for the different types of the systems. We also assume in all our calculations the polarization of the 2-D electron gas to be 50%.

Two contributions to the total SDHA ( $\theta_H^{\text{SS}}$  and  $\theta_H^{\text{SJ}}$ ) come with different signs and different dependencies on the QW width, the electron and impurity concentrations.  $\theta_H^{\text{SJ}}$  does not depend on the charge and concentration of the impurities, and obviously increases when the electron concentration increases. At the same time,  $\theta_H^{\text{SS}}$  depends on the charge of scatterers (see Ref. 18) and on their concentration (it decreases when  $N_{\text{im}}$  increases). This generates a complicated interplay between  $\theta_H^{\text{SS}}$  and  $\theta_H^{\text{SJ}}$  contributions to the total SDHA  $\theta_H$ . In Fig. 1, we present the ratio  $\eta = |\theta_H^{\text{SJ}} / \theta_H^{\text{SS}}|$  as a function on the electron concentration for two types of scatterers located in the center of the IGA well with  $L = 20$  nm. The concentration of scatterers and concentration of the electrons are taken to be equal:  $n = N_{\text{im}}$ . It follows from the figure that for the repulsive potential ( $Z = -1$ ) the skew-scattering mechanism is always predominant. For the case of the attractive impurities ( $Z = +1$ ), each of them can be pre-



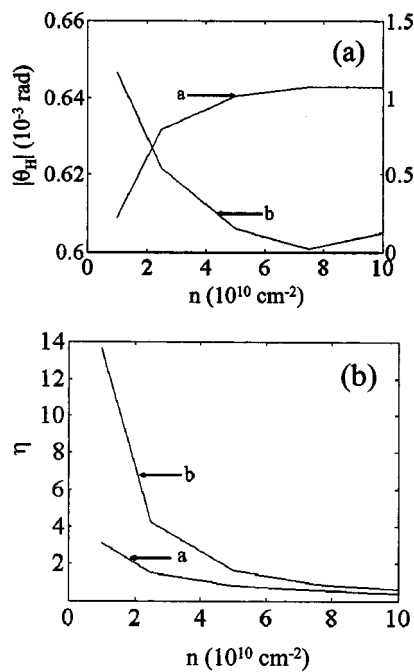


FIG. 2. (a) The absolute value of the SDHA in the IGA QW and (b) the ratio  $\eta$  as the function of the electron concentration ( $N_{\text{im}} = 10^{11} \text{ cm}^{-3}$ ,  $L = 20 \text{ nm}$ ; a:  $Z = -1$ , b:  $Z = +1$ ).

dominant for certain concentrations; for the concentrations near  $n = N_{\text{im}} \approx 3 \times 10^{10} \text{ cm}^{-2}$ , those two mechanisms can cancel each other.

The actual magnitude of the total SDHA as the function of the electronic concentration for the IGA well is presented in Fig. 2(a). In this figure we fixed the impurity concentration to be  $N_{\text{im}} = 10^{11} \text{ cm}^{-2}$ . Despite the different behavior of the angle for systems with attractive and repulsive impurities, both demonstrate quite measurable magnitudes. Figure 2(b) clearly shows that the skew-scattering mechanism dominates for systems both with attractive and repulsive impurities up to the very high concentrations of the 2-D electrons.

In addition, one can manipulate the effect in 2-D systems with a variation of the well width. The impact of the well width on the skew-scattering mechanism is discussed in detail in Ref. 18. The main result is the following: the effect always decreases when the well width increases. The dependencies of the total (SDHA) on the well width for the IGA wells are demonstrated in Fig. 3. The figure shows that the side-jump contribution can make the effect stable for the well width variations in the case of the attractive impurities.

The SOI is known to be larger in narrow-gap semiconductors. For this reason, we show in Fig. 4, as an example, the calculation results for the IS wells. We consider here only the QWs with repulsive impurities, in which we can expect (as it follows from the IGA wells) the most interesting result. In this case the total SDHA reaches about  $2.5 \times 10^{-3} \text{ rad}$  for the relatively narrow wells, and it increases when the electron concentration increases.

These results show how one can manipulate the formation of the effect mechanisms and magnitude as well by means of changes in the system parameters ( $n, N_{\text{im}}, L$ ). This possibility makes properties of the SDHE in 2-D semicon-

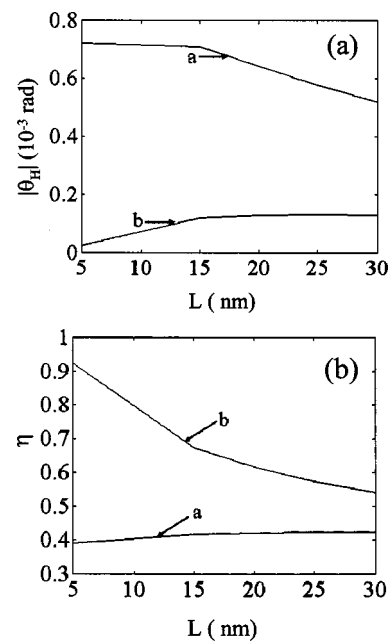


FIG. 3. (a) The absolute value of the SDHA in the IGA QW and (b) the ratio  $\eta$  as the function of the well width ( $n = N_{\text{im}} = 10^{11} \text{ cm}^{-3}$ ; a:  $Z = -1$ , b:  $Z = +1$ ).

ductor systems essentially different from those in the bulk. In three-dimensional systems, we only can manipulate  $N_{\text{im}}$ . In addition, we notice that the SDHA in the QWs is significantly larger (in few orders) than it was for the bulk (see Refs. 30, 31, and 36). In addition, the effect is easily tunable in QWs.

## V. CONCLUSIONS

We described theoretically the SDHE in semiconductor QWs when the 2-D electrons are scattered and form the screened Coulomb centers located in the center of the wells. The one-electronic-band effective Hamiltonian and SO coupling potential of the impurities allow us to solve the 2-D spin-dependent Boltzmann equation and to calculate the SDHA at zero magnetic field. We have found large SDHAs for AlInAs/InGaAsAs/AlInAs and CdTe/InSb/CdTe symmetrical QWs. For instance, in the CdTe/InSb/CdTe narrow QWs the SDHE can reach  $2.5 \times 10^{-3} \text{ rad}$ . This could be detected in the measurements of the Hall effect at low temperatures, and this is potentially useful in integrated electron

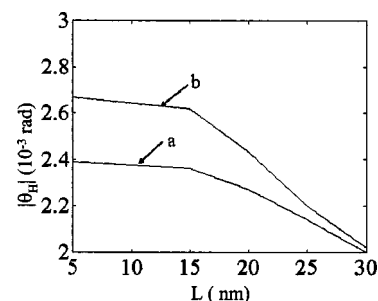


FIG. 4. The absolute value of the SDHA in the IS QW with repulsive impurities as the function of the well width ( $N_{\text{im}} = 10^{11} \text{ cm}^{-3}$ , a:  $n = 5 \times 10^{10} \text{ cm}^{-3}$ , b:  $n = 10^{11} \text{ cm}^{-3}$ ).

spin-polarization devices based on semiconductor heterostructures. It also can be used as a tool of determination of spin-coupling parameters in III-V narrow-gap semiconductor heterostructures. We suggest that experimental investigations should be conducted to verify our theory predictions.

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