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An automatic transmission for bicycles: a simulation

Huai-Ching Chien, Ching-Huan Tseng*

Department of Mechanical Engineering, National Chiao Tung University, 1001 Ta Hsuen Rd, Hsinchu 30050, Taiwan 300, Republic of China

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Abstract

A new design of an automatic transmission for bicycles was simulated. A sensitivity analysis found most environmental and bicycle variables had little effect but cycling velocity was very important while pedal rpm and gradient of the ground had some effect. The new design transmission therefore emphasizes velocity, pedal rpm and gradient of the ground when selecting the optimum gear ratios. Since the comfortable pedal rpm differs from cyclist to cyclist, this new automatic transmission continuously measures the pedal rpm and calculates an average to efficiently learn the cycling habit of different cyclists.

Relevance to industry

Recently, the multi-speed bicycles have become the main product of the bicycle market. However, some automatic gear-shifting systems are designed without a comprehensive ergonomic study.

Keywords: Automatic transmission; Bicycles; Human-generated power; Cycling cadence; Optimal gear ratio; Learning

1. Introduction

In the prior studies, the output power is generally considered an index of cyclists' comfort and cycling efficiency. A comfortable output power of an amateur cyclist is about 0.1 hp (Whitt and Wilson, 1982). A group of trained cyclists, trained runners and amateur cyclists have experimented with preferred cycling cadence in fixed output powers (Marsh et al., 2000). In this experiment, the trained cyclists and runners cycled at 100, 150, 200 and 250 W; the amateur cyclists cycled at 100 and 150 W. Besides, a simulation about the neuromuscular activation, force, stress and endurance based on a human endurance cycling model was carried out at a fixed output power of 265 W (Neptune and Hull, 1999). From these studies, it is reasonable to draw a conclusion that cycling in a fixed output power makes a cyclist comfortable. However, the comfortable output power differs with each cyclist and each cycling type. Hence, the function of a multi-speed transmission system of bicycles can be considered to provide cyclist choices between high cycling cadence with low cycling torque and low cycling cadence with high cycling torque under one's fixed comfortable output power (Kyle, 1988; Whitt and Wilson, 1982).

*Corresponding author. Tel.: +886-35-5726111; fax: +886-35-717243.

E-mail address: chtseng@cc.nctu.edu.tw (C.-H. Tseng).

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In the prior studies, the cycling cadence is also generally considered an index of cyclists' comfort and cycling efficiency. Some of these conclusions were determined by experiments. The experiments concluded that a cyclist cycling with a cadence of 50–65 rpm consumes the least oxygen (Seabury et al., 1977; Coast and Welch, 1985). Another experiment showed that most cyclists cycle at a cadence between 85 and 95 rpm (Hagberg et al., 1981). Furthermore, an experiment revealed that minimum individual muscle integrated electromyograms (iEMG) also occur at a cycling cadence of 90 rpm (Neptune et al., 1997). On the other hand, other studies are based on simulation. An optimization of cycling cadence, using a muscle stress-based cost function, revealed that the optimum cycling cadence is between 95 and 100 rpm (Hull et al., 1988). An optimized simulation based on a human endurance cycling model, carried out in a fixed output power of 265 W, showed that the minimum neuromuscular activation, force, stress and endurance occur at a cycling cadence of 90 rpm (Neptune and Hull, 1999). Hence, it is reasonable to draw a conclusion from these studies that cycling in a fixed cycling cadence makes the cyclist comfortable. However, the comfortable cycling cadence differs from each cyclist and each cycling type.

There is some prior research on the automatic gear-shifting logic with a learning function. Liu (1998) utilized multilayer neural networks and a space-decomposition optimization algorithm to design an automatic gear-shifting controller with a learning function. This controller calculates the optimal gear ratio based only on cycling velocity. By collecting the cycling velocities and corresponding gear ratios during a period of time, the controller learns the relationship between the cycling velocity and the optimal gear ratio. It takes the controller 10–30 s to learn the relationship for 50 s of training data, depending on the quality of the training data. Low learning efficiency is one disadvantage of this controller. Besides, the accuracy of the learned relationship strongly depends on the quality of the collected training data. An amateur cyclist often gives incorrect gear ratios at some cycling velocities so that the controller learns an inaccurate relationship. Furthermore, based on the prior studies, the cycling velocity is not the only parameter influencing the optimal gear ratio. Thus a gear ratio based only on the cycling velocity may not an optimal one. However, calculating an optimal gear ratio by using other parameters substantially decreases the learning efficiency and makes this controller impractical.

There are some commercial automatic gear-shifting systems on bicycles without a learning function. Most of them calculate the gear ratio based only on cycling velocity. Besides, the relationships between the cycling velocity and the gear ratio are fixed. Calculating the gear ratio based on cycling velocity has the cyclist cycle in a fixed reasonable region of cycling cadence. For example, Table 1 lists the relationship of one commercial product. Calculating the gear ratio without considering the output power is one reason that the system sometimes makes cyclists uncomfortable. Inability of the learning function to adapt itself to different cyclists is another reason.

Except for the commercial products, 20 US patents about automatic bicycle gear-shifting systems have been proposed so far (Matsumoto et al., 1984; Clem and Tretheway, 1986; Bellio and Eidelman, 1991; Colbert et al., 1993; Anderson, 1993; Browning, 1993; Ancarani Restelli, 1993; Bellio and Eidelman, 1994; Bellio and Eidelman, 1996a, b; Pikoulas, 1996; Gilbert, 1996; Ethington, 1997a, b; Bellio and Eidelman, 1998; Kimura, 1999; Matsuo, 2000; Spencer et al., 2000; Kimura, 2000a, b). All of these patents control based on cycling velocity or cycling cadence to keeps the cyclist cycling in a fixed reasonable region of cycling cadence. However, only Ethington (1997a, b) control based not only on cycling cadence but also on output power. Hence, most

Table 1
Relationships between the cycling velocity and the gear ratio of one commercial automatic gear-shifting system

Gear pairs	1	2	3	4
Gear ratios	1.619	2.007	2.429	2.979
Common mode (km/h)	<10	10–12	12–17	>17
Speedup mode (km/h)	<8	8–10	10–15	>15

systems proposed in the patents are not able to choose an optimal gear pair while cycling.

In this study, comfort and learning function are two chief design considerations of the automatic gear-shifting system on bicycles. This system calculates the optimal gear ratio based on the output power and comfortable cycling cadence to make cyclists cycle comfortably. By continuously measuring the cycling cadence and calculating an average cycling cadence in a past period of time, this system is able to efficiently, as well as accurately, learn the cycling habit of an individual in different cycling type.

2. Control logic

First, this system supposes that the cyclist wants to comfortably cycle in the next period of time. Second, based on the cycling velocity in the past period of time, comfortable output power and other environment parameters, this system calculates a proper cycling velocity in the next period of time. Third, the optimal gear ratio is calculated based on the comfortable cycling cadence and the proper cycling velocity in the next period of time. Finally, this system finds a practical gear ratio which is closest to the calculated optimal gear ratio and proceeds to gearshift. Therefore, the cyclist can cycle comfortably in the next period of time.

There are two methods to calculate the output power of a cyclist. The first one is to calculate, by means of a linear relationship, among the output power, cycling torque, and the cycling cadence (Takaishi et al., 1994). It is necessary to continuously measure the cycling torque and cycling cadence to calculate the output power by means of this linear relationship. However, this method is impractical since the torque transducer is too expensive to be applied to a commercial system. A second method is to calculate by means of an equation of conservation of energy (Whitt and Wilson, 1982):

$$P = \frac{C_V}{\eta_{mech}}\left\{\Sigma mg\left[C_R + \frac{s}{100} + \frac{a}{g}\left(1 + \frac{m_w}{\Sigma m}\right)\right] + 0.5C_D A\rho(C_V + C_W)^2\right\}, \quad (1)$$

where P is the power output from the cyclist (W), C_V is the cycling velocity (m/s), a is the cycling acceleration (m/s^2), s is the gradient of the ground (°), Σm is the system mass (kg), m_w is the mass of rotary parts (kg), A is the system area facing the air (m^2), ρ is the air density (kg/m^3), C_W the wind velocity (m/s), C_D is the air dynamic resistance coefficient, C_R is the rolling resistance coefficient, and η_{mech} is the mechanical efficiency coefficient.

From Eq. (1), $\Sigma mg(s/100)$ shows the approximate gravity force (F_g, N), $\Sigma mg(a/g)(1 + (m_w/\Sigma m))$ shows the system inertia (F_i, N), and finally $\Sigma mg \times C_R$ and $0.5C_D A\rho(C_V + C_W)^2$ represent the rolling resistance force and the air resistance force (F_r, N). Therefore, Eq. (1) can be simplified to:

$$P = \frac{C_V}{\eta_{mech}}\{F_g + F_i + F_r\}, \quad (2)$$

where F_g is the approximate gravity force (N), F_i the system inertia (N) and F_r the resistance force (N).

Hence

$$P = \frac{P_g + P_i + P_r}{\eta_{mech}}, \quad (3)$$

where P_g is the power consumed by the gravity force F_g (W), P_i is the power consumed by the system inertia F_i (W) and P_r is the power consumed by the resistance force F_r (W).

The output power P can be calculated based on the system behavior parameters (C_V and a) and environment parameters (s, Σm, m_w, A, ρ, C_W, C_D, C_R, and η_{mech}) by means of Eq. (1). Similarly, by means of Eq. (1), C_V can be calculated based on the environment parameters and the output power P. Among these parameters, only C_V and a are controllable parameters. However, acceleration a is the differential of velocity C_V. Hence, only C_V is an independently controlled parameter.

For the learning function, the automatic gear-shifting system continuously measures the cycling cadence, system behavior parameters, as well as the environment parameters and calculates the output power by means of Eq. (1) while a cyclist cycles. Besides, the system calculates the average cycling cadence and average output power of the several past periods of computing time. Then the system sets the comfortable cycling cadence and the comfortable output power of this cyclist in this cycling type.

At any point in time, the automatic gear-shifting system measures the environment parameters and the average cycling velocity in the past period of computing time. An assumption is made that the environment parameters in the next period of computing time are as same as those at this point in time. This assumption often stands because the measured environment parameters do not vary substantially in most situations. The logic hypothesizes that the cyclist wants to comfortably cycle in the next period of computing time. Then a proper cycling velocity in the next period of computing time can be calculated by substituting the comfortable output power and the environment parameters as follows:

$$P_{\mathrm{comfort}} = \frac{C_{\mathrm{V}}}{\eta_{\mathrm{mech}}}\left\{\Sigma mg\left[C_{\mathrm{R}} + \frac{s}{100} + \frac{a}{g}\left(1 + \frac{m_{\mathrm{w}}}{\Sigma m}\right)\right] + 0.5C_{\mathrm{D}}A\rho(C_{\mathrm{V}} + C_{\mathrm{W}})^2\right\}. \quad (4)$$

In Eq. (4), acceleration a (m/s^2) is the differential of velocity C_{V} (m/s) that can be substituted by

$$\frac{C_{\mathrm{V\_next}} - C_{\mathrm{V\_past}}}{\Delta t}, \quad (5)$$

where $C_{\mathrm{V\_next}}$ is the proper cycling velocity in the next period of computing time (m/s), $C_{\mathrm{V\_past}}$ is the average cycling velocity in the past period of computing time (m/s), and Δt is the computing period (s).

The relationship between comfortable output power and the proper cycling velocity in the next period of computing time can be obtained by rearranging Eqs. (4) and (5):

$$P_{\mathrm{comfort}} = k_1 \cdot C_{\mathrm{V\_next}} + k_2 \cdot C^2_{\mathrm{V\_next}} + k_3 \cdot C^3_{\mathrm{V\_next}}, \quad (6)$$

where

$$k_1 = \frac{\Sigma mgC_{\mathrm{R}} + (\Sigma mgs/100) - ((\Sigma m + m_{\mathrm{w}})/\Delta t)C_{\mathrm{V\_past}} + 0.5C_{\mathrm{D}}A\rho C^2_{\mathrm{W}}}{\eta_{\mathrm{mech}}}, \quad \mathrm{N},$$

$$k_2 = \frac{(\Sigma m + m_{\mathrm{w}}/\Delta t) + C_{\mathrm{D}}A\rho C_{\mathrm{W}}}{\eta_{\mathrm{mech}}}, \quad \mathrm{N/m},$$

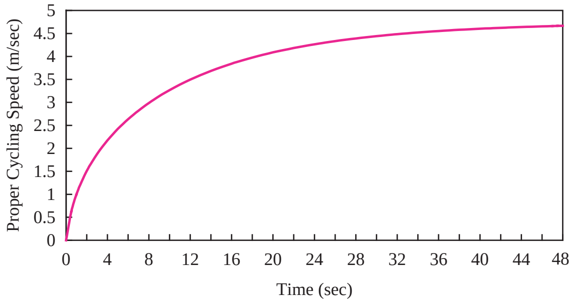


Fig. 1. Proper cycling velocity curve for 0.1 hp input for 0° gradient.

$$k_3 = \frac{0.5C_{\mathrm{D}}A\rho}{\eta_{\mathrm{mech}}}, \quad \mathrm{N^2/m^2}.$$

Based on Eq. (6), the system solves the only independent controllable variable $C_{\mathrm{V\_next}}$ according to the comfortable output power P_{comfort} and other measured environment variables. The different cycling situations, such as no gradient and a grade, generate different proper cycling velocities. In accordance with Eq. (6), a proper cycling velocity curve on the flat can be generated as shown in Fig. 1.

After the system calculates the proper cycling velocity in the next period of computing time, it calculates an optimum gear ratio in the next period of computing time as follows:

$$R = \frac{C_{\mathrm{V\_next}}/\pi \times D_{\mathrm{wheel}}}{\omega_{\mathrm{comfort}}}, \quad (7)$$

where R is the gear ratio, D_{wheel} is the diameter of the tire (m), $\omega_{\mathrm{comfort}}$ is the comfortable cycling cadence of the cyclist (rpm).

Since most of the transmission systems presently used on bicycles are discrete systems, the automatic gear-shifting system has to find a pair of gears that their gear ratio is closest to the optimum gear ratio derived in Eq. (7). Finally, the system proceeds to gearshift. There may be a gap between the gear ratio of the chosen pair of gears and the optimum gear ratio. The rider has to slightly modulate his output for this chosen pair of gears to let himself feel comfortable. The flowchart of control logic is shown in Fig. 2.

3. Parametric trend analysis

Since many parameters influence the optimal gear ratio, it is impossible, in a commercial system, to measure every environment parameters. A parameter trend analysis is proposed in this study to discover the key parameters for measuring and control. Then the automatic gear-shifting system can only measure these key parameters and set mean values for other parameters.

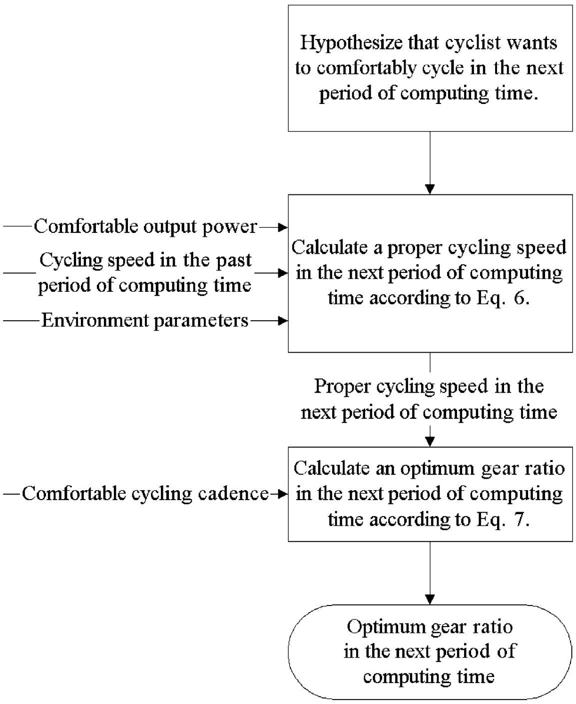


Fig. 2. Flowchart of control logic.

Parametric trend analysis first proceeds to one parameter and set other parameters' mean values. Three optimum gear ratios then are calculated through Eqs. (6) and (7), setting the lower bound value, mean value and the upper bound value. These values are determined based according to the general cyclist and cycling situation. The lower bound values, mean values and the upper bound values of the 13 parameters are listed in Table 2. This process is iteratively used to analysis all 13 parameters. Fig. 3 represents the optimum gear ratio corresponding to the lower bound values, mean values and the upper bound values of each parameter. Fig. 4 shows the differences between the optimum gear ratio corresponding to the lower bound value and the upper bound value of each parameter. From Fig. 4, the effects of every parameter upon the optimum gear ratio are revealed.

As a result, the average cycling velocity in the past period of computing time most influences the optimum gear ratio in a reasonable cycling situation. The comfortable cycling cadence and the gradient are the next. Hence, the automatic gear-shifting system need only measure the cycling velocity, cycling cadence and the gradient. Since the other 10 parameters have very small influence on the optimum gear ratio, setting the other parameters to mean values is acceptable. Furthermore, the learning of comfortable output power is not necessary since output power is not important either.

4. Simulation

According to the control logic described above, a simulation about the optimum gear ratio under different cycling cadence, gradient, cycling velocity

Table 2
The lower bound, mean, and upper bound values of the 13 parameters

	Σm (kg)	m_W (kg)	η_{mech}	C_R	C_W (m/s)	C_D	A (m^2)	ρ (kg/m^3)	s (°)	C_{V_past} (m/s)	$P_{comfort}$ (hp)	$\omega_{comfort}$ (rpm)	Δt (s)
Lower bound	40	0.4	0.9	0.000	−5	0.2	0.2	0.726	−10	0	0.05	1	0.2
Mean	80	0.8	0.95	0.008	0	1.2	0.5	1.226	0	4.166	0.1	1.25	0.5
Upper bound	120	1.2	1	0.016	5	2.2	0.8	1.726	10	8.333	0.15	1.5	0.8

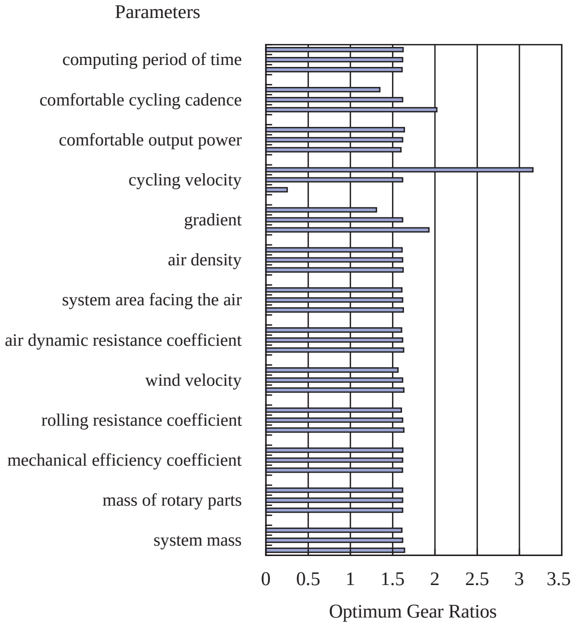


Fig. 3. Optimum gear ratio corresponding to the lower bound, mean, and upper bound values.

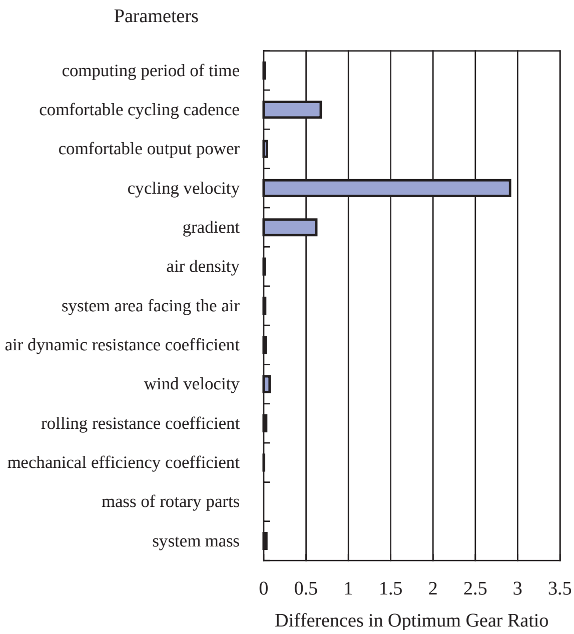


Fig. 4. Effects of parameters upon the optimum gear ratio.

in the past period of computing time and other mean parameters was carried out. The mean values of other parameters are listed in Table 2. Fig. 5 shows the distributions of optimum gear ratio under different comfortable cycling cadence at 60, 75 and 90 rpm.

This study proposes an automatic gear-shifting system by applying the control logic described above to a commercial multi-speed hub and its electrical gear-shifting mechanism. The gear ratios of this multi-speed hub are listed in Table 1. A simulation about the optimum gear pairs under different cycling cadence, gradient, cycling velocity in the past period of computing time and other mean parameters was carried out and is shown in Fig. 6. Fig. 6 represents the distributions of optimum gear pairs under different comfortable cycling cadence at 60, 75 and 90 rpm.

Supposing that cycling on a gradient of $-5°$, $0°$ and $5°$, Table 3 lists the corresponding cycling velocity region to each gear pairs under a cycling cadence at 60, 75 and 90 rpm according to the control logic and the commercial multi-speed hub.

As a result, the cycling velocity regions corresponding to each gear pairs modulate in accordance with the variation on the gradient at any cycling cadence. This result proves that calculating only the optimum gear ratio according to the cycling velocity in the past period of computing time is not enough. Calculating the optimum gear ratio based on output power is necessary. Furthermore, it explains why the commercial system sometimes makes cyclists uncomfortable.

Supposing that cycling on a flat, Table 4 lists the comparison on the corresponding cycling velocity region to each gear pairs between this study and the commercial system. As a result, the cycling velocity regions corresponding to each gear pairs modulate in accordance with the variation on the comfortable cycling cadence. This result proves that a control system without a learning function is not enough. Moreover, it explains yet again why the commercial system sometimes makes cyclists uncomfortable.

Furthermore, according to this study, the corresponding cycling velocity regions at a cycling cadence of 45 rpm are similar to those according to the common mode of the commercial system. And, according to this study, the corresponding cycling velocity regions at a cycling cadence of 40 rpm are

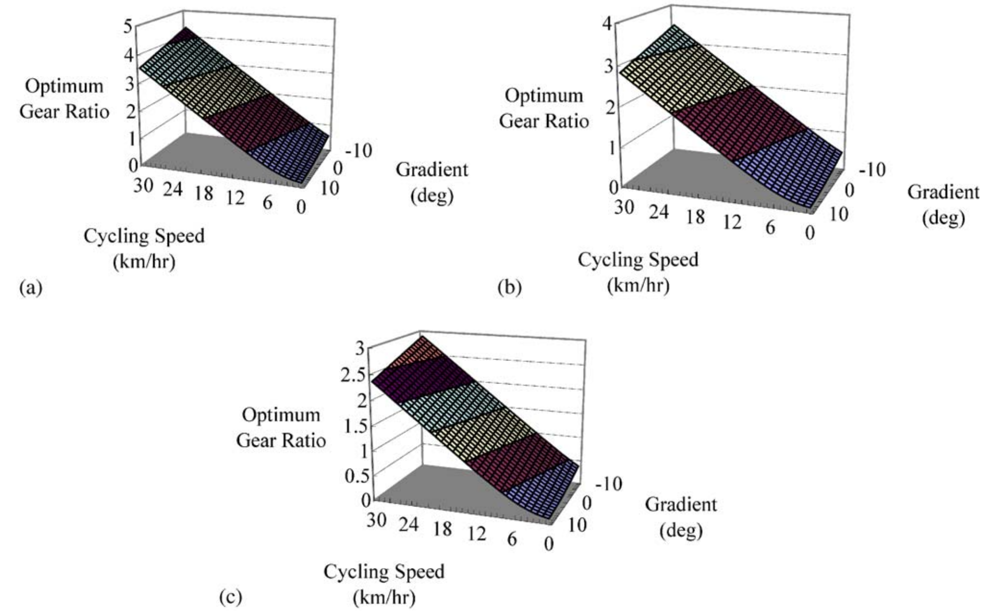


Fig. 5. Distributions of optimum gear ratio under different comfortable cycling cadence: (a) cadence at 60 rpm; (b) cadence at 75 rpm; and (c) cadence at 90 rpm.

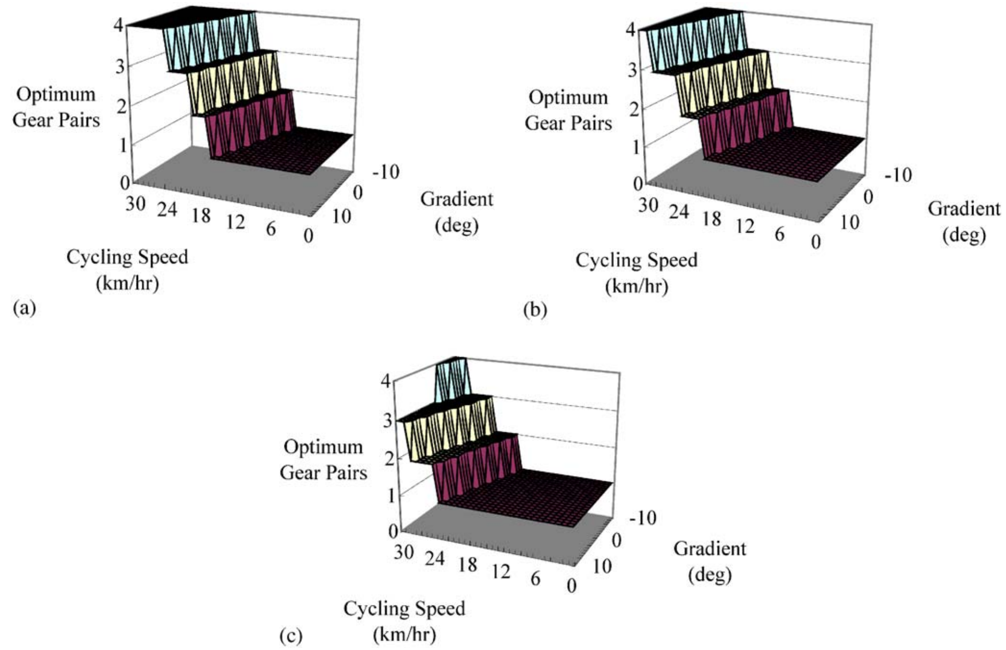


Fig. 6. Distributions of optimum gear pairs under different comfortable cycling cadence: (a) cadence at 60 rpm; (b) cadence at 75 rpm; and (c) cadence at 90 rpm.

similar to those according to the speedup mode of the commercial system. Based on the prior studies, a cyclist cycling with a cadence of 50–65 rpm consumes the least oxygen (Seabury et al., 1977; Coast and Welch, 1985) and generates minimum individual muscle iEMG, neuromuscular activation, force, stress and endurance at a cycling cadence of 90–100 rpm (Hull et al., 1988; Neptune et al., 1997; Neptune and Hull, 1999). The designed comfortable cycling cadence at 40–45 rpm is much lower than those proposed in the prior studies. Consequently a cyclist has to generate a higher cycling torque to maintain the comfortable output power (Takaishi et al., 1994). It makes the cyclist suffer improperly higher iEMG, neuromuscular activation, force and stress as well as endurance. This is the third reason that the commercial system makes cyclists uncomfortable.

Table 3
Corresponding cycling velocity regions to each gear pairs

Cadence (rpm)	Gradient (°)	Gear pairs			
		Cycling speed region (km/h)			
		1	2	3	4
90	−5	< 18	18–23	23–29	> 29
	0	< 20	20–25	> 25	infeasible
	5	< 21	21–26	> 26	infeasible
75	−5	< 15	15–19	19–24	> 24
	0	< 16	16–20	20–25	> 25
	5	< 18	18–22	22–27	> 27
60	−5	< 11	11–15	15–18	> 18
	0	< 13	13–16	16–20	> 20
	5	< 14	14–18	18–21	> 21

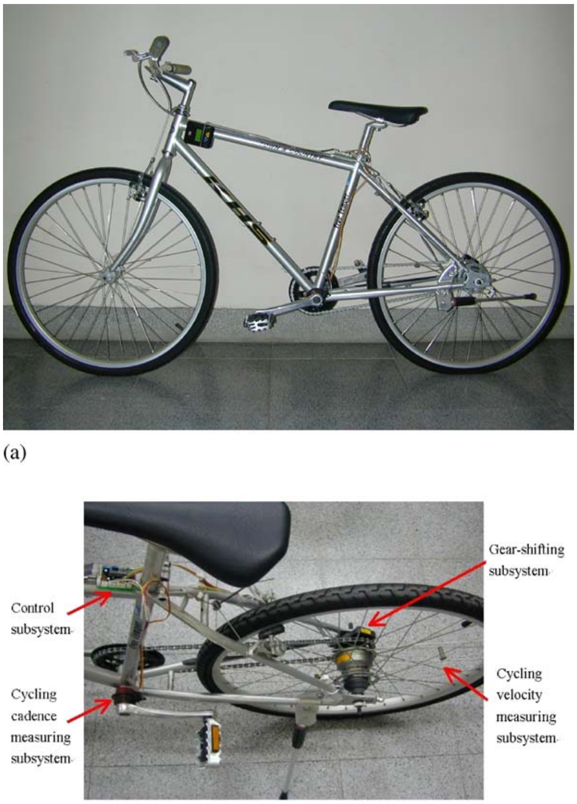
5. Conclusions

This study proposes an automatic gear-shifting logic on bicycles. This automatic gear-shifting logic not only takes the comfort of the individual into account, but also gives consideration to the difference in comfort among different cyclists. In the prior studies, the output power and cycling cadence are generally treated as indexes of cyclists' comfort and cycling efficiency. The cyclist who cycles in a stable cycling cadence and output power will feel comfortable. To keep a cyclist cycling in a comfortable output power, this controller measures the system behavior parameters, environment parameters, and the comfortable output power for calculating the proper cycling velocity. To keep a cyclist cycling in a comfortable cycling cadence, this controller measures the comfortable cycling cadence for calculating the optimal gear ratio. Since the comfortable output power and cycling cadence differ for each cyclist, this controller possesses a learning function to adapt itself to different cyclists.

Table 4
Comparison on the corresponding cycling velocity region to each gear pairs

	Cadence (rpm)	Gear pairs			
		Cycling speed region (km/h)			
		1	2	3	4
This study	90	< 20	20–25	> 25	
	75	< 16	17–20	20–25	> 25
	60	< 13	13–16	16–20	> 20
	45	< 9	9–12	12–15	> 15
	40	< 8	8–10	10–13	> 13
Commercial system	Common mode	< 10	10–12	12–17	> 17
	Speedup mode	< 8	8–10	10–15	> 15

According to the parameter trend analysis, the average cycling velocity in the past period of computing time, comfortable cycling cadence and the gradient most influence the optimum gear ratio. Hence this logic calculated the optimum gear ratio based only on the measured cycling velocity, comfortable cycling cadence as well as the gradient and it sets the other parameters to mean values. Besides, this logic only learns the comfortable cycling cadence. The comfortable output power, then, is set at mean value. This automatic gear-shifting system continuously measures the cycling cadence of one cyclist and calculates an average as one's comfortable cycling cadence. Hence, this automatic gear-shifting logic can efficiently learn the characteristic of comfort of different cyclists.



(b)

Fig. 7. Prototype of this automatic gear-shifting system: (a) bicycle equipped with the automatic gear-shifting system; and (b) subsystems of the automatic gear-shifting system.

First, this logic supposes that the cyclist wants to comfortably cycle in next period of time. Second, based on the average cycling velocity in the past period of time and the gradient, this system calculates a proper cycling velocity in the next period of time. Third, the optimal gear ratio is calculated based on the comfortable cycling cadence and the proper cycling velocity in the next period of time. Finally, this system finds a practical gear ratio which is closest to the calculated optimal gear ratio and proceeds to gearshift. Therefore, the cyclist can cycle comfortably in the next period of time. The prototype of this automatic gear-shifting system is shown in Fig. 7.

As a simulation result, the cycling velocity regions corresponding to each gear pairs modulate in accordance with the variation on the gradient and the comfortable cycling cadence. However, the commercial system is not able to measure the cycling cadence and the gradient, and it also set the comfortable cycling cadence excessively low. Therefore, it is the main reason that the commercial system sometimes makes cyclists uncomfortable.

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