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Improving contour accuracy by Fuzzy-logic enhanced cross-coupled precompensation method

Jih-Hua Chin*, Yuan-Ming Cheng, Jin-Huei Lin

Department of Mechanical Engineering, National Chiao Tung University, Hsinchu 300, Taiwan, ROC

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Abstract

Contour accuracy is obtained by efforts in algorithms. Most motion control algorithms reduce either position error or contour error to gain accuracy. In this paper, we investigate the idea of constructing a Fuzzy-logic controller to a proven algorithm, the cross-coupled precompensation method (CCPM), and using both position and contour error to generate compensation term. Simulation comparison with known methods and experimental comparisons among the uncoupled system, the cross-coupled system, the original CCPM and the proposed method were given. It was shown that the performance of the original CCPM was further enhanced. The proposed method also showed a better performance when subjected to external load.

Keywords: Fuzzy; Contour accuracy; Cross-coupling; Precompensation

1. Introduction

Trajectory accuracy is of fundamental importance for computerized numerical control (CNC) system. Continued research efforts to improve trajectory accuracy have brought out many efficient methodologies in the past years. Basically, they can be divided into two categories: (1) algorithms to improve the ability of each individual axis to pinpoint a desired position and (2) algorithms to make the contour converge to the desired trajectory faster.

The efforts of the first category reduce the position error, which also help to improve the contour fidelity because typical CNC trajectories are approximated by discrete points that are issued by interpolation. In this category, velocity feedforward controls were proposed to reduce positioning error [2,3]. Digital feedforward control [4–6] also brought improvement.

For continuous path machining, however, the algorithms directly addressing the contour precision are often of more interest. The concept of cross-coupled compensation is a successful approach in this respect.

The first cross-coupled system (CCS) was proposed by Sarachik and Ragazziai [7]. It was a “master-slave” and nonsymmetrical structure. Koren [8] proposed a cross-coupled biaxial system with a symmetric structure with the addition of two DDA integrators and digital comparator to a conventional biaxial control system. Srinivasan and Kulkarni [9] proposed a cross-coupled controller that involves an optimal control problem to improve high-speed contour accuracy independently of tracking accuracy. Koren and Lo [1] proposed a variable gain cross-coupled controller to deal with nonlinear contour such as circle and parabola. Ho et al. [10] decomposed the contour error to do a decoupled path-following control.

Parallel to the cross-coupled concept another path method was proposed by Chin and Tsai [11] to increase the contour accuracy in which the path parameters are preadapted before tracking. In that paper, the tracking algorithm for the multi-axis machine tool in Cartesian space was named as the path precompensation method (PPM). Chin and Lin [12] showed that the PPM can help flexible arm track a path with better precision. Chin and Tsai [13] proposed algorithms for PPM for tracking spatial curves in parameter form. Since a velocity modification is executed before trajectory tracking, and the most parts of motion mechanism are included

*Corresponding author. Tel.: +886-35-712121; fax: +886-35-720-634.

E-mail address: jhchin@mail.nctu.edu.tw (J.-H. Chin).

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Nomenclature

US	uncoupled system
VG	variable gain [1]
CCS	cross-coupled system
CCPM	cross-coupled precompensation method
FLC	Fuzzy-logic control
P_r, P_p	reference position and actual position
E_c	correct vector error
E_{cx}, E_{cy}	components of the correct error E_c in the x, y coordinate
E_r, E_p	contour error and position error
CE_r, CE_p	change of contour error and change of position error
GE, GCE, GU	parameters of FLC
V_b	command feeding velocity
V_{bx}, V_{by}	components of the federate V_b in the x, y coordinate
V_{kx}, V_{ky}	components of connections velocity V_k in the x, y coordinate
K_v	precompensation gain
K_{ex}, K_{ey}	gains of tracking error compensation in x, y-axial
K_{sx}, K_{sy}	gains of contour error compensation in x, y-axial
C_x, C_y	feedback contour error loop in x, y-axial [1]
K_c	digital to analog gain in volt/bit
U_x, U_y	x- and y-axial control input signals
K_x, K_y	x-and y-axial plant-loop gains
τ_x, τ_y	x- and y-axial plant-loop time constants
IAE	integral absolute error
ITAE	integral-of-time-multiplied absolute error
ISE	integral square error
ITSE	integral-of-time-multiplied error
RMS	root mean square
T	sampling time

within the feedback loop, the PPM has been proven accurate in tracking curvilinear trajectories.

Significant contour precision was obtained when this PPM was combined with the CCS to become a so-called cross-coupled precompensation method (CCPM) [14]. Chin and Lin [15] proposed the algorithms of involute-type scrolls for CCPM and tried to formulate a quasi-generalization approach for path generation.

The combined cross-coupled concept and the pre-compensation method has displayed a powerful ability in reducing the contour error for paths with curvature especially at high feedrates of up to 200 mm/s. The reason for its performance is that specific path algorithms were derived from exact mathematical equation of target curve so that accuracy is sustained from the phase of path generation, and the accuracy is then held tightly by both features of cross-coupling (contour accuracy) and precompensation (contour accuracy against curvature).

The precision of CCPM is a prize for difficult derivation of path generator and the analytical solution for contour error. Liu [16] developed path generator for cubic-spline-based CCPM to track higher order curves, but it was found that the accuracy of cubic-spline-based algorithms is less than that of the algorithms derived from exact equation of target profile. A kind of trade-off exists between the contour precision and the efforts devoted [5].

This study investigated a way to upgrade the precision of CCPM without devoting too much to the additional mathematical derivation.

Instead of pursuing exact mathematical or dynamical analysis, this paper enhanced the CCPM with Fuzzy-logic control. The proposed approach (CCPM with FLC) was compared with three kinds of control schemes: uncoupled system (US), CCS, and especially the original CCPM.

2. Fuzzy-logic enhanced CCPM

2.1. Basic considerations

A tracking status is shown in Fig. 1. P_r, P_p, E_r, E_p denotes reference position, actual position, contour error, and position error, respectively. CCS will create a vector normal to the path and push the actual tool point from P_p to P'_p (PPM and CCPM do similar things but using a velocity modification far beyond the position control loop). The contour error can be reduced in this way. But on the other hand, it may enlarge P_{px} and influence next tracking. This situation arises from (1) large curvature and (2) high ratio of E_p/E_r.

If we take the position error E_p into consideration and let P_p be taken to P''_p instead of P'_p, then position lag can be improved while keeping the contour error correction maneuvers. The correction vector $\mathbf{E}_c$ can be formed, for example, from the following equations:

$$E_{cx} = E_{rx} + E_{px}, \tag{1}$$

$$E_{cy} = E_{ry} + E_{py}. \tag{2}$$

2.2. Effects of using both contour and position errors

The idea of taking the position error into consideration when doing the compensation for contour error was examined in a preliminary study using the conditions of [8,1]. It was found that the average contour error was reduced from 4.86 to 3.06 μm for a circle of radius

50 mm, and from 7.86 to 4.54 μm for a circle with bigger curvature (radius 30 mm).

2.3. Strategies for Fuzzy logic

Instead of developing an accurate but complicated algorithm using position errors to enhance contour accuracy, a Fuzzy Logic may simply classify the contour and position errors into three ranges: "Small", "Medium" and "Large", and three basic strategies were considered.

(1) When contour error is "Small" and position error is "Large", then reducing position error is of primary concern.
(2) When contour and position errors are both "Small", "Medium" or "Large", then reducing both contour and position error at the same time.
(3) When contour error is "Large" and position error is "Small", then the compensation concentrates on reducing contour error.

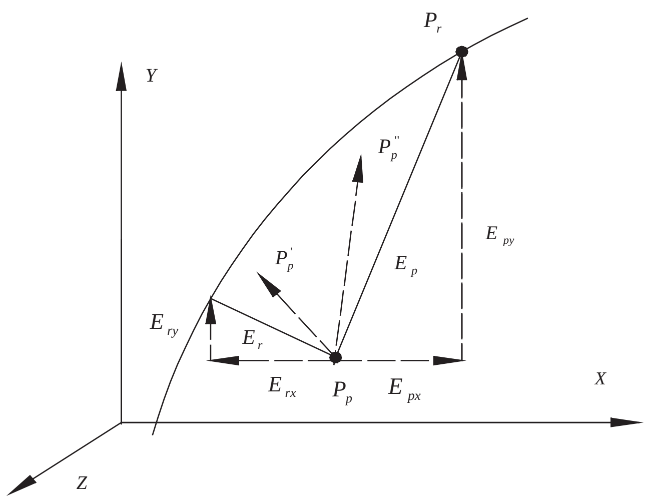

Fig. 1. Problem statement.

2.4. The construction of Fuzzy-logic enhanced CCPM

A Fuzzy-logic controller (FLC) is built to enhance the known CCPM as shown in Fig. 2. The contour error E_r, the change of contour error CE_r (defined as current E_r—previous E_r), the position error E_P, and the change of position error CE_P (defined as current E_p—previous E_p) are selected as input to FLC.

There are eight variables to be defined and used in FLC:

$$\varepsilon_{rx} = E_{rx}(n) \times GE, \tag{3}$$

$$d\varepsilon_{rx} = (E_{rx}(n) - E_{rx}(n-1)) \times GCE, \tag{4}$$

$$\varepsilon_{ry} = E_{ry}(n) \times GE, \tag{5}$$

$$d\varepsilon_{ry} = (E_{ry}(n) - E_{ry}(n-1)) \times GCE, \tag{6}$$

$$\varepsilon_{px} = E_{px}(n) \times GE, \tag{7}$$

$$d\varepsilon_{px} = (E_{px}(n) - E_{px}(n-1)) \times GCE, \tag{8}$$

$$\varepsilon_{py} = E_{py}(n) \times GE, \tag{9}$$

$$d\varepsilon_{py} = (E_{py}(n) - E_{py}(n-1)) \times GCE, \tag{10}$$

where GE and GCE are scale factors.

Five linguistic values were used, which are ZE = "-Zero", PS = "Positive Small", PL = "Positive Large", NS = "Negative Small" and NL = "Negative Large". The membership functions of these linguistic values on the domain of ε is shown in Fig. 3 in which ZE is chosen shrunk and NS's and PS's peaks are shifted toward center for higher sensitivity.

Rules like following are constructed and summarized in Table 1.

Principle 1: **IF** the contour error is large **THEN** provide a "*PL*" or "*NL*" component to the correction velocity. (Rule 1 & Rule 17)

Principle 2: **IF** the contour error is small **AND** the actual position tend to run to the opposite

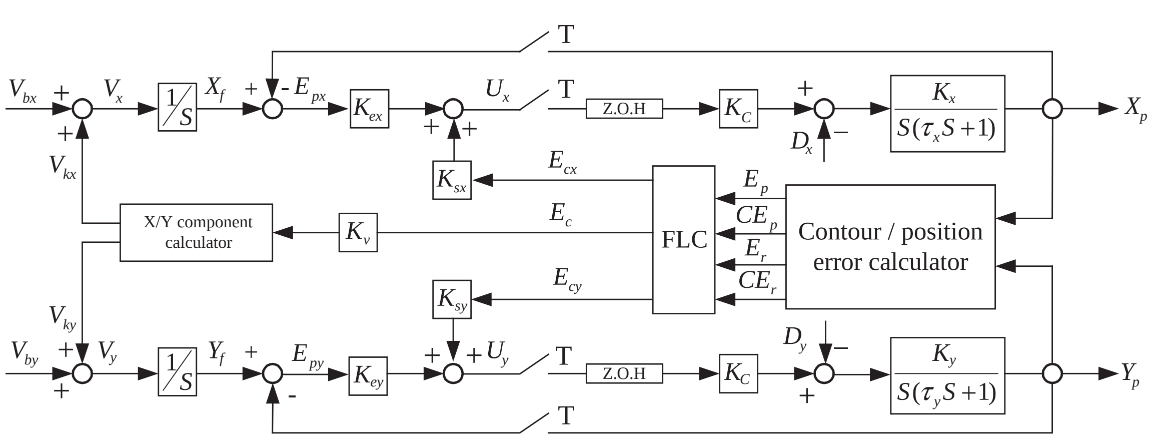

Fig. 2. Block diagram of FLC enhanced CCPM.

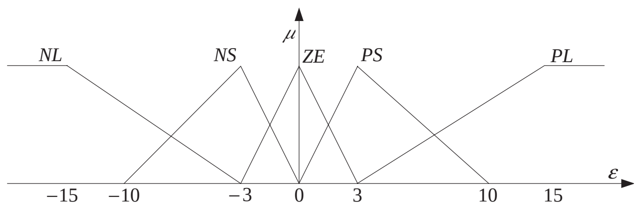

Fig. 3. Membership functions of five linguistic values on the domain of ε_r.

Table 1
Rule base of FLC

ε	dε				
	PL	*PS*	*ZE*	*NS*	*NL*
PL	PL^1	PL^1	PL^1	PL^1	PL^1
PS	PL^2	PL^3	PS^4	ZE^5	NS^6
ZE	PL^7	PS^8	ZE^9	NS^{10}	NL^{11}
NS	PS^{12}	ZE^{13}	NS^{14}	NL^{15}	NL^{16}
NL	NL^{17}	NL^{17}	NL^{17}	NL^{17}	NL^{17}

(i.e. superscripts indicate the number of rule.)

direction rapidly **THEN** provide a "*PL*" or "*NL*" component to the correction velocity. (Rule 2 & Rule 16)

Principle 3: **IF** the contour error is small **AND** the actual position tend to run to the opposite direction slowly **THEN** provide a "*PL*" or "*NL*" component to the correction velocity. (Rule 3 & Rule 15)

Fig. 4 shows the membership functions on dε. FLC can be regarded as a nonlinear PD controller in which dε plays a derivative role. The noise induces instability. For reducing the influence of $d\varepsilon_r$, we shift *NS*'s and *PS*'s peaks outward and magnify the range of "*ZE*". On the other hand, uniformly distributed membership functions of triangular forms are applied for linguistic members of output U_r as shown in Fig. 5.

The selection of parameters for Fuzzy logic in Figs. 3–5 is described in Fig. 6, the meaning of which is as follows:

Rule 1: IF ε is A_1 AND dε is B_1 THEN U is C_1.
Rule 2: IF ε is A_2 AND dε is B_2 THEN U is C_2.

Where A, B and C may be any of the linguistic terms.

Fuzzy controller inevitably depends on established practical knowledge. The actual numerical ranges used in Figs. 3–5 were based on former experiences [14,15].The truth values w_1 and w_2 can be calculated by

$$w_1 = \min(\mu_{A1}(\varepsilon), \quad \mu_{B1}(d\varepsilon)),$$

$$w_2 = \max(\mu_{A2}(\varepsilon), \quad \mu_{B2}(d\varepsilon)).$$

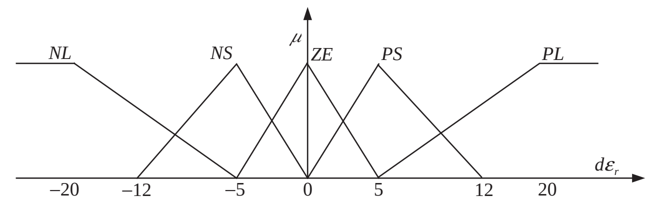

Fig. 4. Membership functions of five linguistic values on the domain of $d\varepsilon_r$.

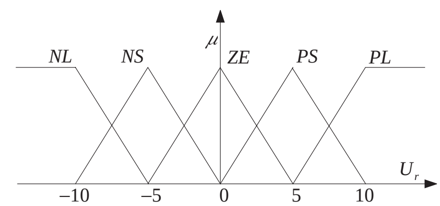

Fig. 5. Membership functions of five linguistic values on the domain of U_r.

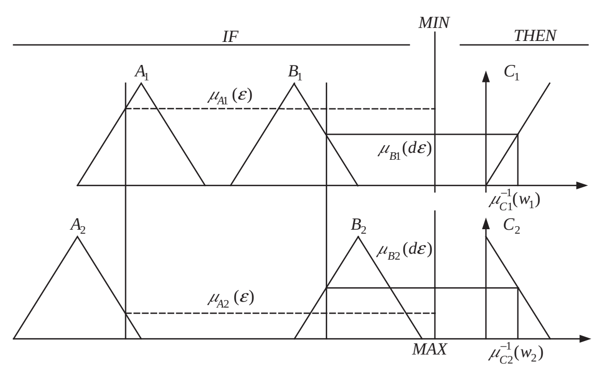

Fig. 6. Process of Fuzzy reasoning.

The Fuzzy controller calculates U_r, U_P according to its control rules:

$$U_{rx} = \frac{w_1 \times \mu_{C1}^{-1}(w_1) + w_2 \times \mu_{C2}^{-1}(w_2)}{w_1 + w_2} \times GU, \tag{11}$$

$$U_{ry} = \frac{w_1 \times \mu_{C1}^{-1}(w_1) + w_2 \times \mu_{C2}^{-1}(w_2)}{w_1 + w_2} \times GU, \tag{12}$$

$$U_{px} = \frac{w_1 \times \mu_{C1}^{-1}(w_1) + w_2 \times \mu_{C2}^{-1}(w_2)}{w_1 + w_2} \times GU \tag{13}$$

$$U_{py} = \frac{w_1 \times \mu_{C1}^{-1}(w_1) + w_2 \times \mu_{C2}^{-1}(w_2)}{w_1 + w_2} \times GU, \tag{14}$$

where GU is a scale factor.A new correction vector $\mathbf{E_c}$ is then generated as follows:

$$E_{cx} = U_{rx} + U_{px}, \tag{15}$$

$$E_{cy} = U_{ry} + U_{py}. \tag{16}$$

The control surface is shown in Fig. 7. The control surface for E_p can be constructed in a similar way.

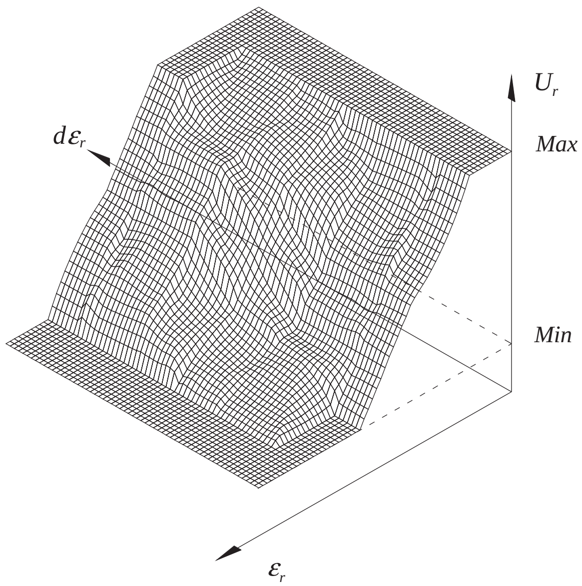

Fig. 7. Control surface of FLC.

2.5. Control signal for Fuzzy enhanced CCPM

The formation of correction velocity remains the same as in [14]

$$\mathbf{V} = \mathbf{V}_t + \mathbf{V}_c = V_b\mathbf{T} + K_v\mathbf{E_c}, \tag{17}$$

where V_b is the feedrate, K_v is the precompensation gain and $\mathbf{T}$ is the unit tangent vector.

From Eq. (17), the velocity component can be obtained

$$V_x(n) = V_{bx}(n-1) + K_v E_{cx}(n), \tag{18}$$

$$V_y(n) = V_{by}(n-1) + K_v E_{cy}(n), \tag{19}$$

where E_{cx}, E_{cy} denote the X and Y component of the correction vector.

A new reference position X_f and Y_f can be developed by integrating the V_x and V_y and the discrete forms of which are

$$X_f(n) = X_f(n-1) + TV_x, \tag{20}$$

$$Y_f(n) = Y_f(n-1) + TV_y, \tag{21}$$

where T is the sampling time, V_x and V_y denote the adjusted velocity component in x- and y-axis. The position errors are

$$E_{px}(n) = E_{px}(n-1) + X_f(n) - X_p(n), \tag{22}$$

$$E_{py}(n) = E_{py}(n-1) + Y_f(n) - Y_p(n), \tag{23}$$

where X_P, and Y_P are the actual position

The control signals are

$$U_x(n+1) = K_{ex}E_x(n) + K_{sx}E_{cx}(n), \tag{24}$$

$$U_y(n+1) = K_{ey}E_y(n) + K_{sy}E_{cy}(n). \tag{25}$$

Note that if E_r is used instead of E_c then the system becomes a cross-coupled precompensation system.

And if further the control gain K_v is set to zero, then the cross-coupled precompensation system reduces to a CCS. If K_v, K_{sx} and K_{sy} are all set to zero, the system reduces to an US.

2.6. Determination of gains

The determination of gains in the system shown in Fig. 2 depends on the workpiece geometry and the machine dynamics.

For precompensation loop [11]:

$$K_v = 1/\Delta T \quad \text{for linear trajectory}, \tag{26a}$$

$$K_v = \left| \frac{R(1-(\Delta S/R)^2)^{1/2} - R_i}{\Delta T(R - R_i)} \right| \quad \text{for circular trajectory}. \tag{26b}$$

For US system, the steady-state contour error can be shown to be the following:

$$\varepsilon_{ss}(t) = \frac{V\sin 2\theta}{2}\left[\frac{1}{k_{ey}k_c k_y} - \frac{1}{k_{ex}k_c k_x}\right]. \tag{27}$$

For CCS system, the transfer functions of tracking loop are as follows:

$$\frac{X_p(s)}{X_f(s)} = \frac{w_{nx}^2}{S^2 + 2\zeta_x w_{nx} S + (w_{nx}^2 + w_{dx}^2)}$$
$$\text{where } w_{dx}^2 = 0.5(k_v - 1)k_{ex}k_{sx}, \tag{28}$$

$$\frac{Y_p(s)}{Y_f(s)} = \frac{w_{ny}^2}{S^2 + 2\zeta_y w_{ny} S + (w_{ny}^2 + w_{dy}^2)}$$
$$\text{where } w_{dy}^2 = 0.5(1/k_v - 1)k_{ey}k_{sy}. \tag{29}$$

Eqs. (26)–(29) build a starting point which can be tuned to determine the required gains. Note that a meaningful determination of gains involved determination machine dynamics, which was not concretely addressed in this work.

3. Simulation and comparison

The performance of the Fuzzy-logic enhanced CCPM was put under comparison with its predecessors and a variable-gain CCS [1] in simulation. The variable-gain CCS [1] has ability to vary its gains according to the slope of linear trajectory or the radius of circular trajectory. For the sake of comparison, gains from the literature were used to evaluate different algorithms (Table 2 for linear, Table 3 for circular trajectory).

The simulation was performed with MATLAB. Fig. 8 shows the contour errors for tracking a 45° linear trajectory. The variable-gain method as proposed in [1] has bigger contour errors than the algorithms CCS

Table 2
Gains for linear trajectory

	K_v	C_x	C_y	K_{sx}	K_{sy}	K_{ex}	K_{ey}
US	×	×	×	×	×	0.04	0.04
VG	×	V_y/V	V_x/V	×	×	0.04	0.04
CCS	×	×	×	0.08	0.08	0.04	0.04
CCPM	0.1	×	×	0.08	0.08	0.04	0.04
CCPM + FLC	0.1	×	×	0.08	0.08	0.04	0.04

Table 3
Gains for circular trajectory

	K_v	C_x	C_y	K_{sx}	K_{sy}	K_{ex}	K_{ey}
US	×	×	×	×	×	0.8	0.8
VG	×	$\sin\theta - E_{px}/2R$	$\cos\theta + E_{py}/2R$	×	×	0.8	0.8
CCS	×	×	×	0.2	0.2	0.8	0.8
CCPM	0.1	×	×	0.2	0.2	0.8	0.8
CCPM + FLC	0.1	×	×	0.2	0.2	0.8	0.8

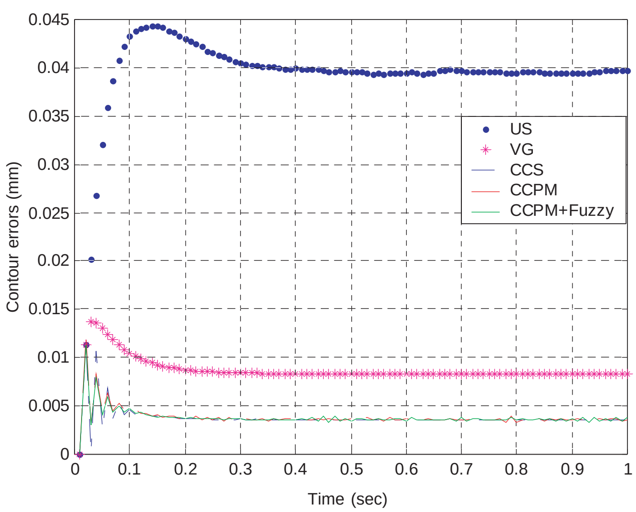

Fig. 8. Simulation results of five control methods for 45° linear trajectory.

and CCPM, and the Fuzzy-logic enhanced CCPM (CCPM + FLC) proves to be the best. This is because the Fuzzy-logic enhanced CCPM and its predecessors CCPM are more delicate in structure. They are equipped with a precompensation loop (K_v), in addition to the cross-coupled compensation (K_{sx},K_{sy}).

Fig. 9 is the results for tracking an R30 mm circle and Fig. 10 is a circular plot of the contour errors. Obviously, the variable gain is still the less efficient one while the Fuzzy-logic enhanced algorithm still prevails. Tables 4 and 5 list the maximum and the IAE contour errors. It is seen that the variable gain is less efficient in linear trajectory, roughly comparable with CCPM, but inferior to the Fuzzy-logic enhanced CCPM.

Parameter of variable gain (VG): $W_p = 1$; $W_i = 10$; $W_d = 0.005$.

Parameter of FLC: $GE = 7$; $GCE = 3$; $GU = 1.8$.

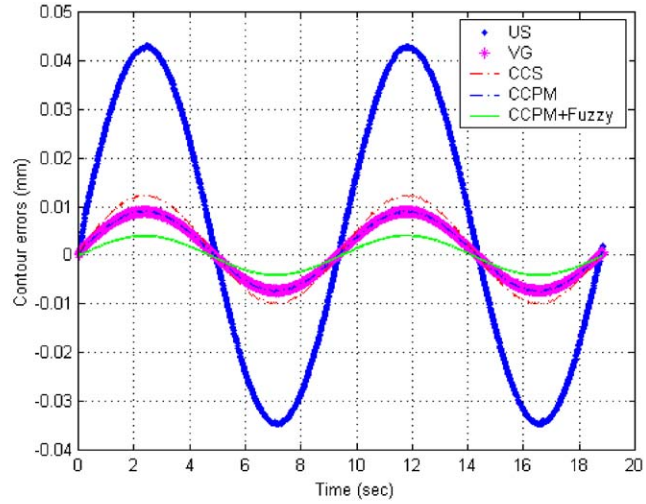

Fig. 9. Simulation results of five control methods for R30 mm circular trajectory.

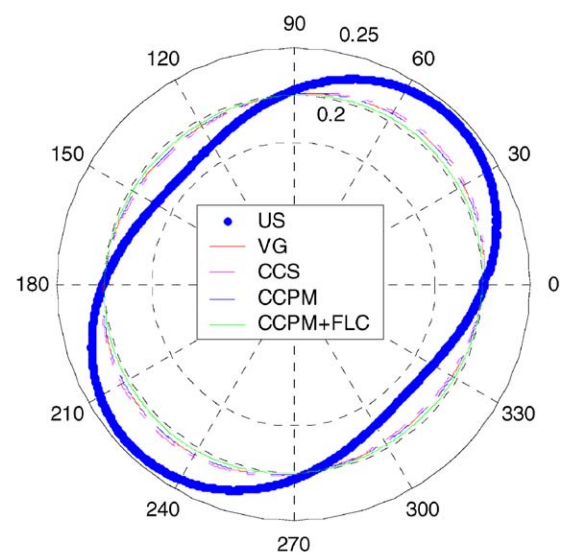

Fig. 10. Polar representation of contour errors for circular trajectory.

Table 4
Maximum contour errors of five control methods

MAX	US	VG	CCS	CCPM	CCPM + Fuzzy
Linear	0.044	0.13	0.012	0.011	0.011
Circular	0.043	0.009	0.012	0.009	0.004

Table 5
IAE contour errors of five control methods

IAE	US	VG	CCS	CCPM	CCPM + Fuzzy
Linear	0.039	0.009	0.004	0.004	0.004
Circular	0.025	0.005	0.007	0.005	0.002

4. Experimental study

Three kinds of experiments were performed as follows:

(1) Tracking linear path at different speeds.
(2) Tracking circular path of different curvature at different speeds.
(3) Tracking circular path under external load.

For the sake of comparison, we performed four kinds of control schemes, they are US, CCS, CCPM and CCPM with FLC.

The results were quantified by "integral absolute-error (IAE) criterion", "integral-of-time-multiplied absolute-error (ITAE) criterion", "integral square-error (ISE) criterion", "integral-of-time-multiplied (ITSE) criterion", and "root mean square "(RMS) [17]. They were defined as

$$\text{IAE}: \quad \frac{1}{N}\sum_{i=1}^{N}|e(i)|.$$

IAE is one of the most easily applied performance indexes.

$$\text{ITAE}: \quad \frac{1}{N}\sum_{i=1}^{N}t(i)|e(i)|.$$

A large initial error is weighed lightly by IATE, and errors occurring later in the transient response are penalized heavily.

$$\text{ISE}: \quad \frac{1}{N}\sum_{i=1}^{N}e^2(i).$$

A characteristic of ISE performance index is that it weighs large errors heavily and small errors lightly.

$$\text{ITSE}: \quad \frac{1}{N}\sum_{i=1}^{N}t(i)e^2(i).$$

A large initial error is weighed lightly by ITSE, while errors occurring late in the transient response are penalized heavily. This criterion has a better selectivity than the ISE.

$$\text{RMS}: \quad \sqrt{\frac{1}{N}\sum\nolimits_{i=1}^{N}e^2(i)},$$

where $e(i)$ is the contour error of ith sampling term.

4.1. Experimental instrument

The experiments were performed on a 600mm × 600mm × 300 mm X–Y–Z table with three linear guideways of 10 mm/rev lead (Fig. 11). The table was driven by SEM DC motors Type MT22G2-19, torque 0.7 N m, maximum rpm: 5000. The X–Y–Z table is of resolution 1 μm. APC-586 generated the path command and took charge of all control through an A/D–D/A card at a rate of 5 ms. Actual positions were detected by optical linear scales (resolution: 1 μm) and collected by the 16-bit counter HCTL-2016 (operating frequency: 14 MHz). The implementation of controllers was done in C language with a sampling time of 5 ms.

4.2. Linear tracking

A linear path of 164-mm length and 26° of inclination was tracked at speed of 10, 25, 50, 75 mm/s. The

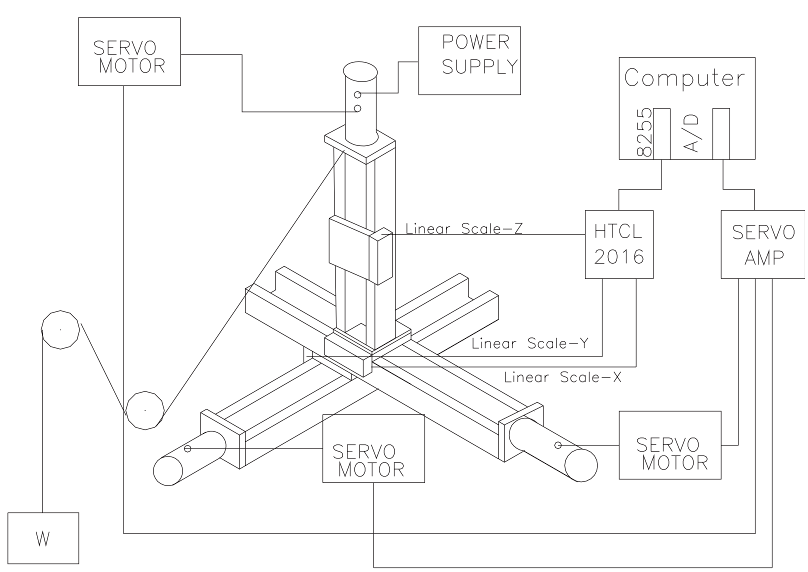

Fig. 11. The experimental setup.

values of K_{ex}, K_{ey}, K_{sx}, K_{sy} and K_v were empirically determined to be the ones that created the least contour errors.

4.3. Circular tracking

Circular tracking were experimented for three different radii: 50, 30, and 20 mm, and four different feedrates: 10, 25, 50, 75 mm/s.

4.4. Circular tracking under external loading

In practice, CNC servo system is operated under loading condition. Inertia of the workpiece or cutting force may pose influence on the feeding quality. An experiment was run in which an external loading of 40 kg was applied on the *y*-axis while tracking a circular path of 50 mm radius at a speed of 10 mm/s.

5. Discussion and comparison

5.1. For linear tracking

Error indices were plotted in Fig. 12. It is seen that the Fuzzy-logic enhanced CCPM brought out improvement against all other methods. For a simple comparison of performance, we take average of indices from feedrate 10–75 mm/s and compare control schemes in pairs. The results were listed in Table 6. The Fuzzy-logic enhanced CCPM and its direct predecessor CCPM bore similarity in the trend of error index, this speaks for the fact that the feature of CCPM was kept despite the Fuzzy nature. Besides, the Fuzzy CCPM outperforms CCPM in all error indices, which means the expected performance enhancement is reached. It is also seen that at lower feed rates, 10, 25 mm/s, the performance difference between CCPM and Fuzzy CCPM is closer, while at higher feed rates, 50, 75 mm/s, it becomes bigger.

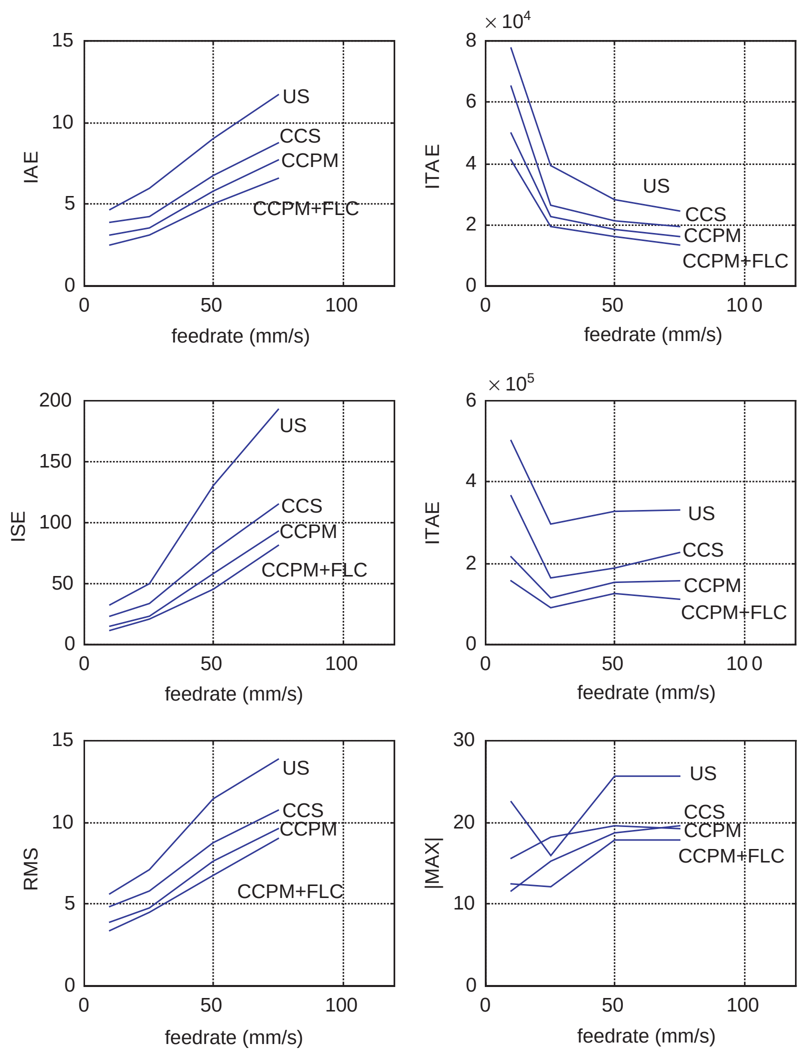

Fig. 12. The error indices of US, CCS, CCPM and CCPM + FLC for linear path. Feed-rate: 10, 25, 50, 75 mm/s.

Table 6 tells that the Fuzzy CCPM decreases the error index from 85.78(IAE)% to 90.77%(RMS) with respect to CCPM, which is a big margin if one considered the mathematic involvement in CCPM and its error improvement 84.64%(IAE), 85.08%(RMS) over CCS.

5.2. For circular tracking

Error indices were plotted in Figs. 13–15. Performance comparisons in pairs were listed in Tables 7–9.

Table 6
Comparisons for linear path by error index ratio (in %)

	IAE	ITAE	ISE	ITSE	RMS
CCPM + FLC/CCPM	85.78	85.00	82.50	75.95	90.77
CCPM/CCS	84.64	83.54	72.51	70.42	85.08
CCS/US	75.57	76.29	64.86	63.45	80.45

For tracking circular path of radius 50 mm, the IAE of the proposed method is about 79% of CCPM, 66% of CCS, and 55% of US. For tracking circular path of radius 30 mm, the IAE of the proposed method is about 77% of CCPM, 65% of CCS, and 52% of US. For tracking circular path of radius 20 mm, the IAE of the proposed method is about 75% of CCPM, 61% of CCS, and 50% of US.

One remarkable feature of CCPM is its efficiency in dealing with path of higher curvature. Looking at the first column of Tables 7–9, it is found that this feature was carried over to Fuzzy-logic enhanced CCPM. The error index IAE was 79.74%, 77% and 75.13% for radius of 50, 30 20 mm, respectively (error index decreasing for higher curvature). Other error indices showed similar descending trend. The higher the curvature, the better the results.

For linear path, the advantage of FLC enhanced CCPM (as well as the original CCPM) is comparatively

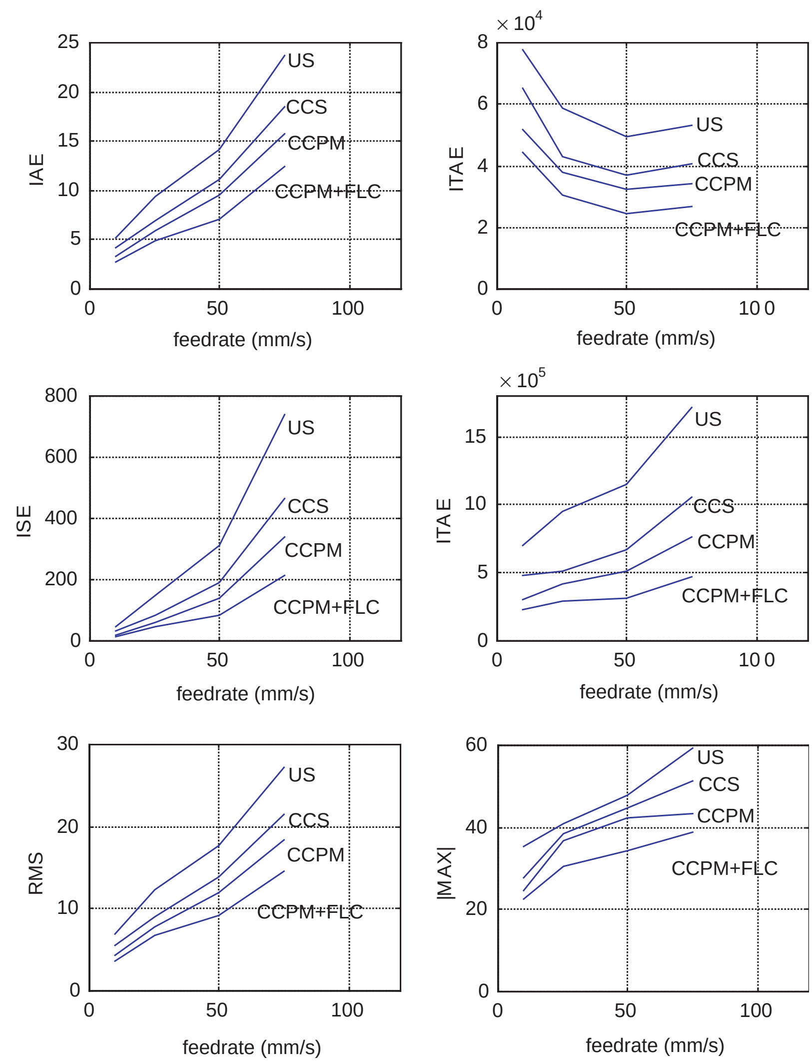

Fig. 13. The error indices of US, CCS, CCPM and CCPM + FLC for circular path of radius 50 mm.

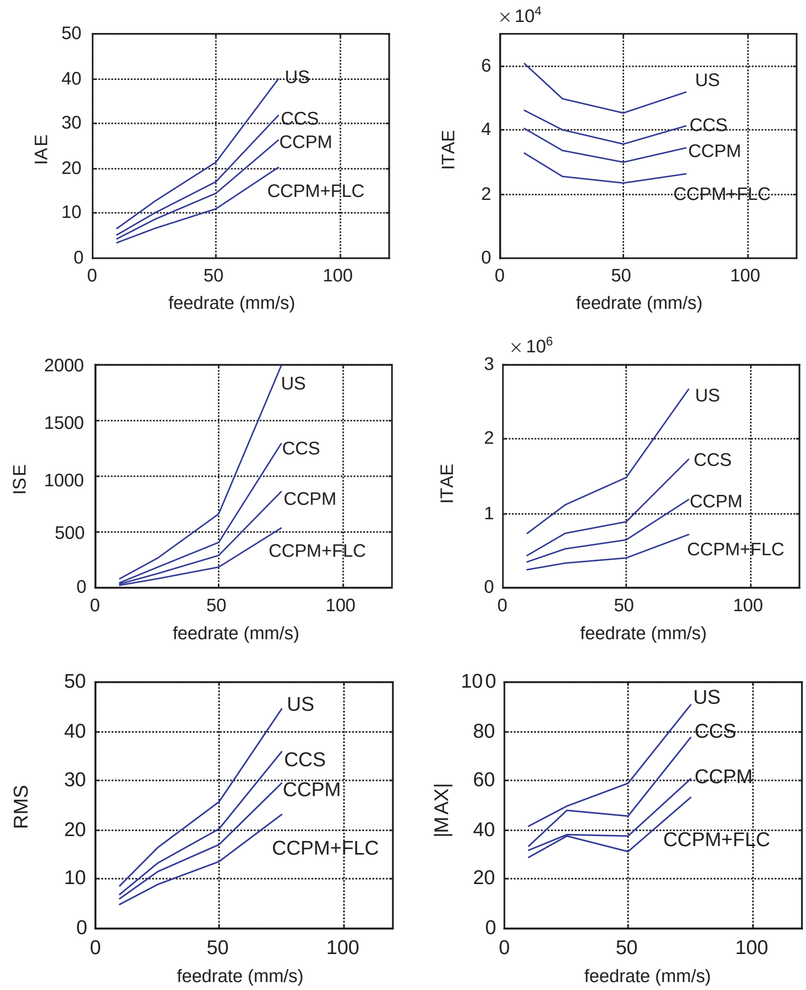

Fig. 14. The error indices of US, CCS, CCPM and CCPM + FLC for circular path of radius 30 mm.

moderate (compare the first row of Table 9 with that of Table 6).

While in linear trajectory the Fuzzy CCPM is moderately better than CCPM, in trajectory with curvature, the Fuzzy CCPM lead apparently in performance. The CCPM feature of suppressing curvature is not only preserved but further developed in Fuzzy CCPM.

5.3. *For circular tracking under external loading*

Table 10 lists the percentage error increase caused by 40 kg load in y-axis. The load worsened contour errors in all cases, however, the Fuzzy-logic enhanced CCPM suffered the least. Since load can be seen as a kind of disturbance, the Fuzzy-logic enhanced CCPM showed a better ability of rejecting disturbance.

6. Conclusion

Machine tools have made tremendous progress in trajectory tracking in the past decades. Old-fashioned position control has become more accurate because of the knowledge in machine dynamics and control schemes. The advent of control schemes aiming at contour error compensation brings more precision to continuous contours. However, precision is won by complicated algorithms which require mathematical or computational efforts. In this study we investigate a methodology of contour tracking without involving too much mathematics.

Position error and contour error are two separate targets to be reduced. The conventional tracking methods focused on reducing either one or the other. It is shown in this study that both can be considered to bring more contour accuracy.

The kind of so-called cross-coupled precompensation method (CCPM) has been proven to be powerful in tracking contour of curvature with precision. One of the reasons of its accuracy is the mathematical derivation of a specific path generator which is often difficult. We propose to use a FLC to manage both contour error and position error for creating compensation. Computer simulation was performed to compare the Fuzzy-logic

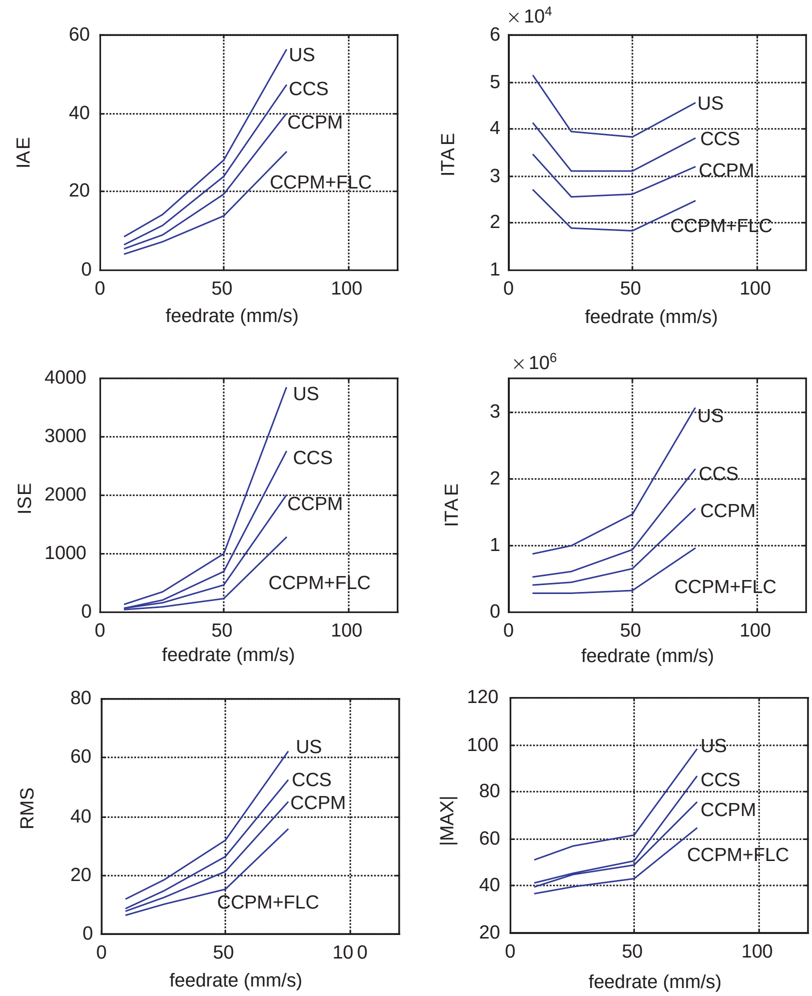

Fig. 15. The error indices of US, CCS, CCPM and CCPM + FLC for circular path of radius 20 mm.

Table 7
Comparisons for circular path of radius 50 mm by error index ratio

	IAE	ITAE	ISE	ITSE	RMS
CCPM + FLC/CCPM	79.74	80.01	67.25	67.47	81.93
CCPM/CCS	83.34	84.81	71.10	73.08	84.25
CCS/US	77.70	77.15	60.76	60.647	77.90

Table 8
Comparisons for circular path of radius 30 mm by error index ratio

	IAE	ITAE	ISE	ITSE	RMS
CCPM + FLC/CCPM	77.00	77.67	62.94	63.94	79.32
CCPM/CCS	84.43	84.79	72.90	72.75	85.34
CCS/US	80.08	78.74	63.75	62.50	79.83

Table 9
Comparisons for circular path of radius 20 mm by error index ratio

	IAE	ITAE	ISE	ITSE	RMS
CCPM + FLC/CCPM	75.13	75.18	63.03	62.26	79.25
CCPM/CCS	82.34	83.63	71.60	73.32	84.60
CCS/US	81.37	81.01	64.79	63.71	80.41

Table 10
Percentage error increase caused by 40 kg load in *y*-axis

	IAE	ITAE	ISE	ITSE	RMS
US	11.63	11.30	18.46	20.38	8.85
CCS	8.70	8.97	25.0	35.65	11.82
CCPM	8.17	8.77	23.04	25.30	10.93
CCPM + FLC	6.65	1.15	12.93	1.99	6.26

enhanced CCPM with known tracking algorithms. The algorithm was implemented and evaluated in experiments. The results showed that by using position and contour error, the contour accuracy of the original tracking method, here CCPM, can be further enhanced without involving too much mathematics. All original features were maintained, including the performance at higher path curvature and at higher feeding rate. The advantage of CCPM is mainly in tracking contour with curvature rather than tracking linear path of no curvature, this is also true after enhancing with FLC. Experiments with external y-axis load showed that the new idea had the best immunity to load disturbance. This feature could be studied in more detail in the future.

The idea of taking both position and contour error into consideration and using Fuzzy-logic control to enhance tracking algorithms is portable. It is conceivable that the idea investigated in this study can be used to enhance other tracking algorithms.

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