

Stability of the Anisotropic Brane Cosmology[‡]

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The stability of the Bianchi type I anisotropic brane cosmology is analyzed in this paper. We also study the effects of the brane solution by comparing the models on the 3-brane and the models in the conventional Einstein space. The analysis is presented for two different models: one with a perfect fluid and the other with a dilaton field. It is shown that the anisotropic expansion is smeared out dynamically for both theories in the large time limit, independent of the models of different types of matter. The initial states are, however, dramatically different. A primordial anisotropic expansion will grow for the conventional Einstein's theory. On the other hand, it is shown that the initial state is highly isotropic for the brane universe, except for a very particular case. Moreover, it is also shown that the Bianchi type I anisotropic cosmology is stable against any anisotropic perturbation for both theories in the large time limit.

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I. INTRODUCTION

The observation of the cosmic microwave background (CMB) radiation [1, 2] indicates that our universe is globally isotropic to a very high degree of precision. Therefore our universe is usually assumed to be described by the Friedmann-Robertson-Walker (FRW) metric in most of the literature. The origin of the isotropic universe has become an interesting research topic.

On the other hand, it is known that there are small large-angle CMB anisotropies, $\Delta T/T \simeq 10^{-5}$, in the CMB background [1, 2]. The isotropy of our Universe may have to do with the choice of initial conditions and the stability of the evolutionary solutions. Therefore, the present isotropic phase could be a dynamic result of the evolution of our universe, no matter what the initial starting state. In this paper, we will address this issue in both the standard Einstein theory and the newly developed brane world scenario [3, 4]. In particular, we will consider the evolution of an anisotropic cosmology described by the Bianchi type I models [5–12] for two different types of matter sources: one is the perfect fluid and the other is a dilaton field. Our result shows that the considered anisotropic models all evolve dynamically from the anisotropic Bianchi type I universe into the isotropic FRW space in the large time limit. This property reveals that the isotropy of the cosmological

[‡] Dedicated to Prof. James M. Nester on the occasion of his sixtieth birthday.

principle can be justified and made consistent with our current observational data.

These two different theories give, however, completely different initial anisotropic expansion at the very early stage of evolution. Indeed, for the conventional Einstein theory (CET), the anisotropic expansion tends to be large in the very early stage. In other words, the universe has to begin from a highly anisotropic initial expansion which then decays to zero as time increases. On the other hand, in the brane world scenario [13–16], the quadratic correction significantly changes the early time behavior of the Universe. As a result, any initially non-vanishing anisotropy parameter tends to vanish in the very early period. There is a characteristic time, t_c , that divides the evolution of the mean anisotropy parameter into two different stages. The mean anisotropy parameter is increasing when $t < t_c$ and reaches its maximal value at $t = t_c$. After that, the mean anisotropy parameter starts to decay. This result remains true for both the model with a perfect fluid and the model with a scalar field. It appears to be a general feature, independent of the types of matter considered.

In addition, stability analysis [17–24] indicates that the models in two different theories are all stable against any anisotropic perturbation. In particular, we also show that the system is stable against the dilaton perturbation when the dilaton potential is close to its local minimum. Note that the exact solution is known only when the dilaton potential is a constant. This also justifies the stability of the isotropic universe in the large time limit under the effect of a dilaton potential with local minima.

This paper is organized as follows. In Sec. II we briefly review the brane world formulation and the Bianchi type I model. Then we consider the evolution of the anisotropic cosmology in both the CET and the brane world theories. In section III analysis is presented for the model coupled to a perfect fluid. The model with a dilaton field and a constant potential is presented in section IV. Finally, the conclusion and possible implications are drawn in Sec. V.

II. BRANE WORLD AND ANISOTROPIC COSMOLOGY

The brane world scenario assumes that our universe is a four-dimensional space-time, a 3-brane, embedded in the 5D bulk space-time. All the matter fields and the gauge fields except the graviton are confined on the the 3-brane as a prior requirement in order to avoid any conflicts with the empirical results. Moreover, inspired by the string theory/M-theory, Z_2 -symmetry with respect to the brane is imposed [27]. A formal realization of the brane world scenario, which recovers Newton gravity in the linearized theory, is the Randall-Sundrum model [3, 4], in which the 4D flat brane(s) is (are) embedded in a 5D anti-de Sitter (AdS) space-time. Later on, a covariant formulation of the effective gravitational field equations on the 3-brane was obtained via a geometric approach by Shiromizu, Maeda and Sasaki [13, 15]. It was shown that the effective four-dimensional gravitational field equations on the brane take the following form

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + k_4^2 T_{\mu\nu} + k_5^4 S_{\mu\nu} - E_{\mu\nu}, \quad (1)$$

where $G_{\mu\nu}$ and $T_{\mu\nu}$ are the Einstein and energy-momentum tensors. $S_{\mu\nu}$ is a quadratic contribution from $T_{\mu\nu}$ defined as

$$S_{\mu\nu} = \frac{1}{12}TT_{\mu\nu} - \frac{1}{4}T_{\mu}^{\alpha}T_{\nu\alpha} + \frac{1}{24}g_{\mu\nu}\left(3T^{\alpha\beta}T_{\alpha\beta} - T^2\right). \quad (2)$$

The effective 4D parameters, e.g., the cosmological constant Λ and the gravitational coupling k_4 , are determined by the 5D bulk cosmological constant Λ_5 , the 5D gravitational coupling k_5 and the tension of the brane λ via the relations

$$\Lambda = k_5^2\left(\frac{\Lambda_5}{2} + \frac{k_5^2\lambda^2}{12}\right), \quad k_4^2 = \frac{k_5^4\lambda}{6}. \quad (3)$$

Here $g_{\mu\nu}$ is the metric tensor on the brane. In addition, the quantity $E_{\mu\nu}$ is a pure bulk effect defined by the bulk Weyl tensor [13].

From Eq. (1), it is easy to realize that the brane world scenario differs from the CET in two ways: (a) the matter fields contribute a local ‘‘quadratic’’ energy-momentum correction via the tensor $S_{\mu\nu}$, and (b) there are ‘‘nonlocal’’ effects from the free field in the bulk, transmitted via the projection of the bulk Weyl tensor $E_{\mu\nu}$. Therefore, the CET can be treated as a limit of the brane theory by taking $E_{\mu\nu} = 0$ and $k_5 \rightarrow 0$ along with properly adjusted values of the constants k_4 and Λ .

In addition to the generalized Einstein equations (1), the energy-momentum tensor also satisfies the conservation law $\nabla_{\mu}T^{\mu\nu} = 0$. Therefore, there is a constraint on the tensor $E_{\mu\nu}$, $\nabla_{\mu}E^{\mu\nu} = k_5^4\nabla_{\mu}S^{\mu\nu}$, due to the Bianchi identity on the brane. Here, the operator ∇ is the covariant derivative with respect to the metric $g_{\mu\nu}$ on the brane. One should point out here that the field equations on the brane, namely the generalized Einstein equations, the conservation of energy-momentum and the constraint on $E_{\mu\nu}$ are, in general, not a closed system in the 4D brane, since the quantity $E_{\mu\nu}$ is five-dimensional. They can only be evaluated by solving the field equations in the bulk. In this paper, we will, however, only consider the quadratic effect on the brane world in the anisotropic background. Therefore, we will set $E_{\mu\nu} = 0$ which is equivalent to embedding the 3-brane in a pure AdS bulk space-time.

Another important subject in this paper is the anisotropic cosmology described by the Bianchi type I metric. The line element of the Bianchi type I space, an anisotropic generalization of the flat FRW geometry, can be written as

$$ds^2 = -dt^2 + a_1^2(t)dx^2 + a_2^2(t)dy^2 + a_3^2(t)dz^2, \quad (4)$$

with $a_i(t)$, $i = 1, 2, 3$ being the expansion factors on each of the different spatial directions.

For later convenience, we introduce the following variables:

$$V \equiv \prod_{i=1}^3 a_i, \quad \text{volume scale factor,} \quad (5)$$

$$H_i \equiv \frac{\dot{a}_i}{a_i}, \quad i = 1, 2, 3, \quad \text{directional Hubble factors,} \quad (6)$$

$$H \equiv \frac{1}{3} \sum_{i=1}^3 H_i = \frac{\dot{V}}{3V}, \quad \text{mean Hubble factor.} \quad (7)$$

In addition, we also introduce two basic physical observational quantities in the cosmology:

$$A \equiv \sum_{i=1}^3 \frac{(H_i - H)^2}{3H^2}, \quad \text{mean anisotropy parameter,} \quad (8)$$

$$q \equiv \frac{d}{dt}(H^{-1}) - 1, \quad \text{deceleration parameter.} \quad (9)$$

Note that $A \equiv 0$ for an isotropic expansion. Moreover, the sign of the deceleration parameter indicates how the universe expands. A positive sign corresponds to “standard” decelerating models whereas a negative sign indicates an accelerating expansion.

III. ANISOTROPIC UNIVERSE WITH A PERFECT FLUID

In this section, we will consider the case where the matter energy-momentum tensor, $T_{\mu\nu}$, is a perfect fluid whose components are given by

$$T^0_0 = -\rho, \quad T^1_1 = T^2_2 = T^3_3 = p. \quad (10)$$

Here the energy density ρ and the pressure p of the cosmological fluid obey a linear barytropic equation of state of the form $p = (\gamma - 1)\rho$ with γ being a constant in the range $1 \leq \gamma \leq 2$.

III-1. Conventional Einstein's Theory

For the CET, the dynamics of the space-time is determined by the Einstein equations and the energy-momentum conservation law

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + k_4^2 T_{\mu\nu}, \quad \nabla_\mu T^{\mu\nu} = 0, \quad (11)$$

with $G_{\mu\nu}$ being the Einstein tensor, Λ the cosmological constant, and k_4 the gravitational coupling $k_4^2 = 8\pi G$. For the Bianchi type I cosmology with a perfect fluid the tt - and

ii -components of the Einstein equations ¹ and the conservation law become

$$3\dot{H} + \sum_{i=1}^3 H_i^2 = \Lambda - \frac{3\gamma-2}{2} k_4^2 \rho, \quad (12)$$

$$\frac{1}{V} \frac{d}{dt} (V H_i) = \Lambda - \frac{\gamma-2}{2} k_4^2 \rho, \quad i = 1, 2, 3, \quad (13)$$

$$\dot{\rho} + 3\gamma H \rho = 0. \quad (14)$$

First of all, Eq. (14) can be easily solved to obtain the time evolution law of the energy density of the fluid:

$$\rho = \rho_0 V^{-\gamma}, \quad \rho_0 = \text{constant} > 0. \quad (15)$$

Next, by summing the Eq. (13) with respect to the index i , we obtain the relation

$$\frac{1}{V} \frac{d}{dt} (V H) = \Lambda - \frac{\gamma-2}{2} k_4^2 \rho. \quad (16)$$

Subtracting this result from Eq. (13), we get

$$H_i = H + \frac{K_i}{V}, \quad i = 1, 2, 3, \quad (17)$$

with K_i , $i = 1, 2, 3$ being the integration constants satisfying the consistency condition $\sum_{i=1}^3 K_i = 0$. Moreover, by using the evolution law of the matter energy density (15), the basic equation (16) describing the dynamics of the anisotropic universe can be written as

$$\ddot{V} = 3\Lambda V - \frac{3(\gamma-2)}{2} k_4^2 \rho_0 V^{1-\gamma}, \quad (18)$$

with the general solution

$$t - t_0 = \int G(V)^{-1/2} dV, \quad (19)$$

where the function $G(V)$ is defined by

$$G(V) \equiv 3\Lambda V^2 + 3k_4^2 \rho_0 V^{2-\gamma} + C. \quad (20)$$

Here C is a constant of integration. Based on this result, the other variables can be calculated straightforwardly, giving answers are

$$H = \frac{G(V)^{1/2}}{3V}, \quad (21)$$

$$a_i = a_{0i} V^{1/3} \exp \left[K_i \int V^{-1} G(V)^{-1/2} dV \right], \quad i = 1, 2, 3, \quad (22)$$

$$A = 3K^2 G(V)^{-1}, \quad (23)$$

$$q = \frac{18\gamma k_4^2 \rho_0 V^{2-\gamma} + 12C}{4G(V)} - 1. \quad (24)$$

¹ Actually, equations (12) and (13) are the time and spatial components of the following version of the Einstein equation: $R_{\mu\nu} = \Lambda g_{\mu\nu} + k_4^2 (T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu})$, with $R_{\mu\nu}$ being the Ricci tensor and T the trace of $T_{\mu\nu}$.

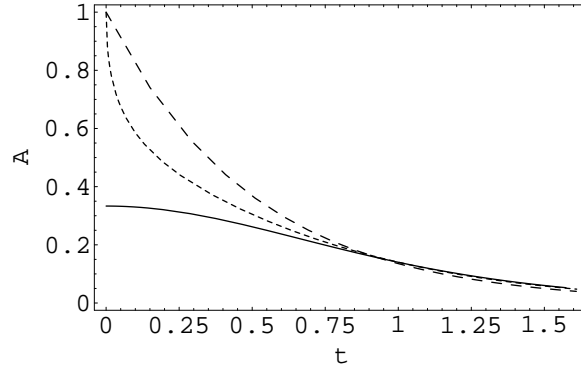


FIG. 1: Mean anisotropy parameter of the Bianchi type I model in the conventional Einstein theory with a perfect fluid: $\gamma = 2$ (solid curve), $\gamma = 1.5$ (dotted curve), and $\gamma = 1$ (dashed curve). The normalization of the parameters is chosen as $3\Lambda = 1$, $3k_4^2\rho_0 = 2$, and $C = 1$.

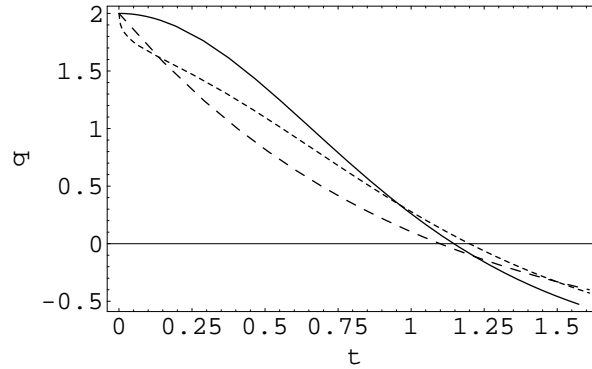


FIG. 2: Deceleration parameter of the Bianchi type I model in the conventional Einstein theory with a perfect fluid: $\gamma = 2$ (solid curve), $\gamma = 1.5$ (dotted curve), and $\gamma = 1$ (dashed curve). The normalization of the parameters is chosen as $3\Lambda = 1$, $3k_4^2\rho_0 = 2$, and $C = 1$.

Here a_{0i} , K_i , $i = 1, 2, 3$ are constants of integration and $K^2 = \sum_{i=1}^3 K_i^2$. In addition, the arbitrary integration constants K_i and C must satisfy the consistency condition

$$K^2 = \frac{2}{3}C. \quad (25)$$

Although the general solution can only be expressed in parametric form via the volume scale factor V , the behavior of the physical variables, namely the mean anisotropy parameter A and the deceleration parameter q , can be plotted with respect to the time as shown in Fig. 1 and Fig. 2.

Moreover, from the the above parametric expressions, we can still analyze the physics in two different interesting limits. First of all, let us consider the large t (or equivalently large V) limit. Since the value of parameter γ is in the range $1 \leq \gamma \leq 2$, the asymptotic value of $G(V)$ will approach $G(V) \rightarrow 3\Lambda V^2$ when the cosmological constant is non-vanishing

($\Lambda > 0$). This leads the volume scale parameter of our universe to expand exponentially in the large time limit, i.e. $V \propto \exp[\sqrt{3\Lambda}t]$. Therefore, in this limit, the mean anisotropy parameter decays to zero exponentially, $A \propto \exp[-2\sqrt{3\Lambda}t] \rightarrow 0$, and the deceleration parameter becomes negative $q \propto \exp[-\gamma\sqrt{3\Lambda}t] - 1 < 0$. Hence the universe can be dynamically isotropized and undergoes an accelerated expansion in the large time limit, due to the presence of a positive cosmological constant Λ . In fact, the value of the cosmological constant can only change the expansion rate of the universe, but will not, in general, affect its isotropization effect.

In order to illustrate this point, let us consider the model with a vanishing cosmological constant $\Lambda = 0$. Therefore, we have $G(V) \propto V^{2-\gamma}$ in the large time limit. This leads to the results $V \propto t^{2/\gamma}$, $A \propto t^{2-4/\gamma}$, and $q \rightarrow 3\gamma/2 - 1 > 0$. Hence, the universe can still be dynamically isotropized, except for the case when $\gamma = 2$. Note that the evolution will, however, be deaccelerated in the case where $\gamma = 2$.

Next, let us focus on the earlier stages of the above exact solutions. For simplicity, we will assume $C = 0$ in this case. One can show that $G(V) \sim V^{2-\gamma}$. Therefore, one can solve for $V \propto t^{2/\gamma}$ from Eq. (21). Hence one has $A \propto t^{2-4/\gamma}$, and $q \rightarrow 3\gamma/2 - 1 > 0$ from the rest of the field equations. Therefore, at the earlier stage of the universe, the evolution is decelerating even if a cosmological constant is present. This is also shown numerically in Fig. 1 and Fig. 2. Moreover, the mean anisotropy parameter A is always non-vanishing, independent of the values of γ . This means that the early universe is always anisotropic. Therefore, for the universe with perfect fluid matter in the CET, the initial state is always anisotropic and this primordial anisotropy is smeared away as a consequence of the evolution of the universe.

III-2. Brane Cosmology

We will focus on the brane effect for the model with a perfect fluid in this subsection. The same constraint for the perfect fluid conservation law (14) still holds on the brane. In addition, the gravitational field equations and the conservation law on the brane take the form, in terms of the variables (5)-(7),

$$3\dot{H} + \sum_{i=1}^3 H_i^2 = \Lambda - \frac{3\gamma-2}{2} k_4^2 \rho - \frac{3\gamma-1}{12} k_5^4 \rho^2, \quad (26)$$

$$\frac{1}{V} \frac{d}{dt} (V H_i) = \Lambda - \frac{\gamma-2}{2} k_4^2 \rho - \frac{\gamma-1}{12} k_5^4 \rho^2, \quad i = 1, 2, 3, \quad (27)$$

$$\dot{\rho} + 3\gamma H \rho = 0. \quad (28)$$

By comparing the above equations with the ones we considered in the previous section for the CET, it is easy to realize that the difference is the quadratic effect due to the energy density ρ . Moreover, the brane cosmology will reduce to the CET if we take the limit $k_5 \rightarrow 0$ and adjust the value of k_4 accordingly. In the next section we will perform a stability analysis section for the brane universe.

The general solution of the above system was obtained in the exact parametric form in [14] by the same approach used in the previous section. Instead of expressing results

as functions of time, we are able to present all variables, including time, in terms of the volume scale factor, V , with $V \geq 0$. For instance, the time variable can be expressed as

$$t - t_0 = \int F(V)^{-1/2} dV, \quad (29)$$

where $F(V)$ is defined as

$$F(V) \equiv 3\Lambda V^2 + 3k_4^2 \rho_0 V^{2-\gamma} + \frac{1}{4} k_5^4 \rho_0^2 V^{2-2\gamma} + C, \quad (30)$$

with ρ_0 and C being constants of integration. The other variables are

$$H = \frac{F(V)^{1/2}}{3V}, \quad (31)$$

$$a_i = a_{0i} V^{1/3} \exp \left[K_i \int V^{-1} F(V)^{-1/2} dV \right], \quad i = 1, 2, 3, \quad (32)$$

$$A = 3K^2 F(V)^{-1}, \quad (33)$$

$$q = \frac{3\gamma(6k_4^2 \rho_0 V^{2-\gamma} + k_5^4 \rho_0^2 V^{2-2\gamma}) + 12C}{4F(V)} - 1, \quad (34)$$

where a_{0i} , K_i , $i = 1, 2, 3$ are constants of integration and $K^2 = \sum_{i=1}^3 K_i^2$. In addition, the arbitrary integration constants K_i and C must satisfy the same consistency condition $K^2 = 2C/3$.

One can immediately show that the effect of the energy density quadratic term becomes significant at the high energy epoch, or in other words, at the early stages of the universe by looking at Eq. (30). Indeed, $F(V) \propto V^{2-2\gamma}$ in the limit $t \rightarrow 0$, when V is extremely small. As a result, F diverges as $t \rightarrow 0$. Therefore, the mean anisotropy parameter $A \rightarrow 0$ in the early universe. On the other hand, in the large time limit, by looking at Eq. (30) we can see that the properties of the universe should be more or less the same as the case discussed for the CET in the previous section. Hence the early evolution of the anisotropic Bianchi type I brane universe is dramatically changed, due to the presence of the brane correction terms proportional to the square of the energy density. The time variation of the mean anisotropy parameter of the Bianchi type I space-time is presented, for different values of γ , in Fig. 3. From Fig. 3, it is clear that at high energy density the evolution of the brane Universe always starts out from an isotropic state with $A \rightarrow 0$. The mean anisotropy parameter increases and reaches a maximum value after a finite time interval t_{\max} . One can show that, when $t > t_{\max}$, the mean anisotropy parameter is a monotonically decreasing function, approaching zero in the large time limit. This behavior is in sharp contrast to the usual evolution in the CET, as shown in Fig. 1, in which the universe has to kick off from state of maximum anisotropy, due to the constraint from the field equation.

In addition, the early time evolution of the brane universe is normally not in an inflationary phase. On the other hand, the brane universe always ends up in an inflationary phase in the large time limit in the presence of a nonvanishing cosmological constant, see

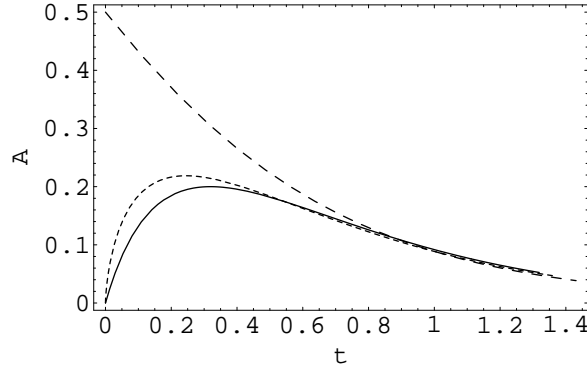


FIG. 3: Mean anisotropy parameter of the Bianchi type I brane universe with a perfect fluid: $\gamma = 2$ (solid curve), $\gamma = 1.5$ (dotted curve), and $\gamma = 1$ (dashed curve). The normalization of the parameters is chosen as $3\Lambda = 1$, $3k_4^2\rho_0 = 2$, $k_5^4\rho_0^2 = 4$, and $C = 1$.

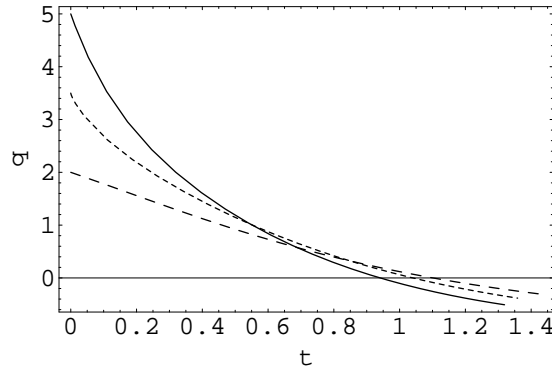


FIG. 4: Deceleration parameter of the Bianchi type I brane universe with a perfect fluid: $\gamma = 2$ (solid curve), $\gamma = 1.5$ (dotted curve), and $\gamma = 1$ (dashed curve). The normalization of the parameters is chosen as $3\Lambda = 1$, $3k_4^2\rho_0 = 2$, $k_5^4\rho_0^2 = 4$, and $C = 1$.

Fig. 4. These are generic features of the brane universe due to the constraint of the field equations on the brane cosmology. More detailed information can be extracted from the Eq. (29) in the limit $t \rightarrow 0$, or equivalently, the case with a small V . For simplicity, we will again take $C = 0$. We can show that $V \propto t^{1/\gamma}$ as $t \rightarrow 0$. Hence, the expansion of the early universe is of the form of a power law expansion. In addition, in the early stages of the evolution of the brane universe the mean anisotropy parameter varies as $A \propto t^{2-2/\gamma}$, approaching zero as $t \sim 0$. Moreover, the deceleration parameter is given by $q = 3\gamma - 1$, which is always positive for all possible values of γ for the case $C = 0$.

III-3. Stability Analysis

The general perturbations for the FRW background with a perfect fluid can be found in Ref. [16]. The same consideration is, however, more complicated for the anisotropic

background. For the primary effect, we will only consider the scalar mode and neglect the vector and tensor modes [18, 19]. The metric perturbation considered here is

$$a_i \rightarrow a_{Bi} + \delta a_i = a_{Bi}(1 + \delta b_i), \quad (35)$$

and the perturbation of the perfect fluid considered is

$$\rho \rightarrow \rho_B + \delta\rho, \quad p \rightarrow p_B + (\gamma - 1)\delta\rho. \quad (36)$$

Here the variables with subscript B are the exact solutions presented in the previous sections. For technical convenience, we use the variables δb_i instead of δa_i in our analysis. Therefore, the perturbations of the following quantities can be shown to be

$$\begin{aligned} H_i &\rightarrow H_{Bi} + \delta\dot{b}_i, & H &\rightarrow H_B + \frac{1}{3} \sum_i \delta\dot{b}_i, \\ \sum_i H_i^2 &\rightarrow \sum_i H_{Bi}^2 + 2 \sum_i H_{Bi} \delta\dot{b}_i, & V &\rightarrow V_B + V_B \sum_i \delta b_i. \end{aligned} \quad (37)$$

As discussed in the previous sections, the results in the CET are special cases for the brane universe, in the limit $k_5 = 0$. Therefore, we first perform our stability analysis for the brane universe. The conclusion for the CET can be easily extracted from the brane world case.

The perturbation equations for various quantities, can be obtained by substituting the perturbations (36, IIIII-3) into the field equations (26, 27, 28). The leading terms will reproduce the field equations. Therefore, one obtains the following perturbation equations from the first order terms $O(\delta b_i \delta\rho)$,

$$\sum_{i=1}^3 \delta\ddot{b}_i + 2 \sum_{i=1}^3 H_{Bi} \delta\dot{b}_i = -\frac{3\gamma-2}{2} k_4^2 \delta\rho - \frac{3\gamma-1}{6} k_5^4 \rho_B \delta\rho, \quad (38)$$

$$\delta\ddot{b}_i + \frac{\dot{V}_B}{V_B} \delta\dot{b}_i + H_{Bi} \sum_{j=1}^3 \delta\dot{b}_j = -\frac{\gamma-2}{2} k_4^2 \delta\rho - \frac{\gamma-1}{6} k_5^4 \rho_B \delta\rho, \quad i = 1, 2, 3, \quad (39)$$

$$\delta\dot{\rho} + 3\gamma H_B \delta\rho + \gamma \sum_{i=1}^3 \delta\dot{b}_i \rho_B = 0. \quad (40)$$

In order to solve the above system of differential equations, we need some inspiration from the process of constructing the exact solutions. First of all, from the general results (15, 17) in both the CET and the brane theory, their dynamics for the perturbation variables follow the constraints

$$\delta\rho = -\gamma\rho_B \sum_{j=1}^3 \delta b_j, \quad (41)$$

$$\delta\dot{b}_i = \frac{1}{3} \sum_{j=1}^3 \delta\dot{b}_j - \frac{K_i}{V_B} \sum_{i=1}^3 \dot{b}_j. \quad (42)$$

By summing Eq. (39) and then using the result (41), we end up with a second order differential equation for the variable combination $\sum_{i=1}^3 \delta b_i$:

$$\sum_{i=1}^3 \delta \ddot{b}_i + 6H_B \sum_{i=1}^3 \delta \dot{b}_i - \frac{1}{2} \gamma \rho_B [3(\gamma - 2)k_4^2 + (\gamma - 1)k_5^4 \rho_B] \sum_{i=1}^3 \delta b_i = 0. \quad (43)$$

The task is to solve for $\sum b_i$ from the above equation and then to construct δb_i and $\delta \rho$ from Eqs. (42) and (41). For the purpose of stability analysis in the final stage, it is sufficient to consider the large time limit behavior of the perturbation variables δb_i and $\delta \rho$. From the discussion of the asymptotic behavior of the exact solutions, the qualitative outcome in both the CET and the brane theory, we should divide our analysis into two different cases: in the presence or absence of the cosmological constant.

For the case with a positive cosmological constant, one can extract the asymptotic forms of the background variables from the exact solutions, which gives

$$H_B \rightarrow \sqrt{\Lambda/3}, \quad V_B \propto \exp(\sqrt{3\Lambda} t), \quad \rho_B \propto \exp(-\sqrt{3\Lambda} \gamma t). \quad (44)$$

The asymptotic expression for ρ_B indicates that the third term in Eq. (43) can be neglected in the large time limit. As a result, we have the following equation for the asymptotic $\sum_{i=1}^3 \delta b_i$, which remains valid in both the CET and the brane world:

$$\sum_{i=1}^3 \delta \ddot{b}_i + \sqrt{12\Lambda} \sum_{i=1}^3 \delta \dot{b}_i = 0. \quad (45)$$

This in turn leads to the final result:

$$\sum_{i=1}^3 \delta b_i \propto \exp(-\sqrt{12\Lambda} t). \quad (46)$$

Therefore, from Eqs. (41, 42) we can obtain the asymptotic expressions of the following perturbation variables:

$$\delta b_i \propto \exp(-\sqrt{12\Lambda} t), \quad \delta \rho \propto \exp[-\sqrt{3\Lambda} (\gamma + 2)t]. \quad (47)$$

This indicates that that the background solutions are stable.

Similarly, for the case with $\Lambda = 0$, we have

$$H_B \rightarrow \frac{2}{3\gamma} t^{-1}, \quad V_B \rightarrow \left(\frac{\sqrt{3\rho_0 k_4^2 \gamma}}{2} t \right)^{2/\gamma}, \quad \rho_B \rightarrow \frac{4}{3k_4^2 \gamma^2} t^{-2}, \quad (48)$$

where the preceding coefficients become important. Therefore, Eq. (43), when the brane correction due to the quadratic energy density term signified by the parameter k_5 can be neglected in the large time limit, reduces to

$$\sum_{i=1}^3 \delta \ddot{b}_i + \frac{4}{\gamma} t^{-1} \sum_{i=1}^3 \delta \dot{b}_i - \frac{2(\gamma - 2)}{\gamma} t^{-2} \sum_{i=1}^3 \delta b_i = 0. \quad (49)$$

The solution of the above equation has the form

$$\sum_{i=1}^3 \delta b_i \rightarrow t^\alpha, \quad (50)$$

where the exponent parameter is determined by

$$\alpha(\alpha - 1) + \frac{4}{\gamma}\alpha - \frac{2(\gamma - 2)}{\gamma} = 0. \quad (51)$$

The explicit expression for α is

$$\alpha = 2 - \frac{4}{\gamma}, \quad \text{or} \quad \alpha = -1, \quad (52)$$

which is always negative for all possible values of γ . The only exception is the case when $\gamma = 2$ such that $\alpha = 0$ is a possible solution. In summary, the asymptotic behavior of the perturbation variables δb_i and $\delta\rho$ is

$$\delta b_i \rightarrow t^\alpha, \quad \delta\rho \rightarrow t^{\alpha-2}, \quad (53)$$

which shows that the background solutions are always stable even when the cosmological constant is vanishing.

IV. ANISOTROPIC UNIVERSE WITH A SCALAR FIELD

In the previous section we considered the evolution of the universe with a perfect fluid, obeying a barytropic equation of state, in the anisotropic background. In this section we study the stability problem of the anisotropic universe with a scalar field. It is known that the scalar field ϕ with a potential can be thought of as a perfect fluid with the energy density and pressure given by $\rho_\phi = \dot{\phi}^2/2 + U(\phi)$ and $p_\phi = \dot{\phi}^2/2 - U(\phi)$, respectively. Here $U(\phi)$ is the scalar field potential. The scalar field is expected to play a fundamental role in the evolution of the early Universe.

In this section, we will only consider the case where the scalar field potential is a constant, $U(\phi) = \Lambda = \text{constant} > 0$, acting as a cosmological constant. As discussed earlier, the related anisotropic cosmology in the CET is just a particular limit of the brane universe. Therefore, we will consider the brane universe in detail for the moment. The CET can be recovered by taking a suitable limit.

When we discuss the stability problem of the system, we will focus on the system with a scalar potential that admits at least a local minimum ϕ_0 such that $U(\phi = \phi_0) = \Lambda$. We will assume that ϕ_0 is the asymptotic solution to the field equation in the large time limit. We will then be able to show that the system remains stable in the large time limit even though the evolutionary solution is only known when the scalar potential is a constant.

IV-1. Brane Cosmology

For a Bianchi type I brane universe with a scalar field the gravitational field equations take the following form [14]:

$$3\dot{H} + \sum_{i=1}^3 H_i^2 = k_4^2 [U(\phi) - \dot{\phi}^2] - k_5^4 \left[\frac{1}{6} \left(\frac{\dot{\phi}^2}{2} + U(\phi) \right)^2 + \frac{1}{4} \left(\frac{\dot{\phi}^4}{4} - U^2(\phi) \right) \right], \quad (54)$$

$$\frac{1}{V} \frac{d}{dt} (V H_i) = k_4^2 U(\phi) - \frac{1}{12} k_5^4 \left(\frac{\dot{\phi}^4}{4} - U^2(\phi) \right), \quad i = 1, 2, 3. \quad (55)$$

Note that the scalar field also obeys the following evolution equation:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dU(\phi)}{d\phi} = 0. \quad (56)$$

By imposing the constraint $U(\phi) = \Lambda$, Eq. (56) gives $\dot{\phi} = 2\phi_0/V$, with $\phi_0 > 0$ being a constant of integration. By summing Eq. (55) over the index i and comparing with the Eq. (54), one can extract the following equation:

$$\ddot{V} = \kappa^2 V - \kappa_0^2 V^{-3}. \quad (57)$$

Here we have denoted the positive constants κ and κ_0 as $\kappa^2 = 3k_4^2\Lambda + k_5^4\Lambda^2/4$ and $\kappa_0^2 = k_5^4\phi_0^4$. The general solution of Eq. (57) can be shown to be [14]

$$V(t) = \frac{1}{2\kappa} e^{-\kappa(t-t_0)} \sqrt{[e^{2\kappa(t-t_0)} - C]^2 - 4\kappa_0^2\kappa^2}. \quad (58)$$

Here C is an integration constant.

The time evolution of the expansion, scale factors, mean anisotropy, shear and deceleration parameter are given by

$$H(t) = \frac{\kappa}{3} \frac{e^{4\kappa(t-t_0)} - C^2 + 4\kappa_0^2\kappa^2}{(e^{2\kappa(t-t_0)} - C)^2 - 4\kappa_0^2\kappa^2}, \quad (59)$$

$$a_i(t) = a_{0i} e^{-\kappa(t-t_0)/3} \left[(e^{2\kappa(t-t_0)} - C)^2 - 4\kappa_0^2\kappa^2 \right]^{1/6} \exp \left[2K_i F \left(e^{\kappa(t-t_0)} \right) \right], \quad (60)$$

$i = 1, 2, 3,$

$$A(t) = \frac{12K^2 e^{2\kappa(t-t_0)} \left[(e^{2\kappa(t-t_0)} - C)^2 - 4\kappa_0^2\kappa^2 \right]}{\left[e^{4\kappa(t-t_0)} - C^2 + 4\kappa_0^2\kappa^2 \right]^2}, \quad (61)$$

$$q(t) = 12e^{2\kappa(t-t_0)} \frac{C e^{4\kappa(t-t_0)} + (C^2 - 4\kappa_0^2\kappa^2) [C - 2e^{2\kappa(t-t_0)}]}{\left[e^{4\kappa(t-t_0)} - C^2 + 4\kappa_0^2\kappa^2 \right]^2} - 1, \quad (62)$$

where a_{0i} , $i = 1, 2, 3$ are arbitrary constants of integration and $F(x) = \int [(x^2 - C)^2 - 4\kappa_0^2\kappa^2]^{-1/2} dx$. We do expect that the universe starts its evolution from a singular initial

condition. Therefore, according to the value $V(0)$, the parameter t_0 should be $\exp(-2\kappa t_0) = C + 2\kappa_0\kappa$.

Note that we can analyze the evolution of the universe in two different regions. First of all, at the very early time, the parameters can be shown to be

$$V \sim \sqrt{2\kappa_0 t} \sim 0, \quad A \sim 6K^2 t / \kappa_0 \sim 0, \quad q \sim 5 > 0. \quad (63)$$

Second, the asymptotic behavior of the parameters can be shown to be

$$V \rightarrow e^{\kappa t}, \quad A \rightarrow e^{-2\kappa t}, \quad q \rightarrow -1 < 0. \quad (64)$$

This result is similar to the model with a perfect fluid: so for the brane universe both models evolve from an isotropic singular state initially. Later on, the mean anisotropy parameter A increases dynamically and decays to zero after the mean anisotropy parameter reaches its maximal value.

IV-2. Conventional Einstein's Theory

The conventional Einstein's theory (CET) can be easily deduced from the above results by taking the limit $k_5 = 0$, or, in this case, $\kappa_0 = 0$. The result is

$$V(t) = \frac{\sqrt{C}}{2\kappa} (e^{\kappa t} - e^{-\kappa t}), \quad (65)$$

$$H(t) = \frac{\kappa e^{2\kappa t} + 1}{3 e^{2\kappa t} - 1}, \quad (66)$$

$$a_i(t) = a_{0i} C^{1/6} (e^{\kappa t} - e^{-\kappa t})^{1/3} \exp \left[2K_i C^{-1} \int (e^{2\kappa x} - 1)^{-1} dx \right], \quad i = 1, 2, 3, \quad (67)$$

$$A(t) = 12K^2 C^{-1} (e^{\kappa t} + e^{-\kappa t})^{-2}, \quad (68)$$

$$q(t) = 12 (e^{\kappa t} + e^{-\kappa t})^{-2} - 1. \quad (69)$$

Note that $\exp(-2\kappa t_0) = C$. The asymptotic behavior is the same as the case in the brane world. The initial state is, however, very different being such that

$$V \sim \sqrt{C} t \sim 0, \quad A \sim 3K^2 / C \sim \text{constant}, \quad q \sim 2 > 0. \quad (70)$$

Therefore, for the CET, the Universe has to start out from an anisotropic initial expansion and then decays all the way to the phase of isotropic expansion in the large time limit.

The evolution of the mean anisotropy parameter $A(t)$ and the deceleration parameter $q(t)$ for the CET and the brane world are presented in Fig. 5 and Fig. 6.

IV-3. Stability Analysis

The quadratic brane correction plays a significant role only at the early stage of the evolution of the universe. Whereas our stability analysis is in the large time limit, so we focus on the asymptotic behavior of the fields. Therefore, it is sufficient to consider the background solutions in the conventional Einstein's theory.

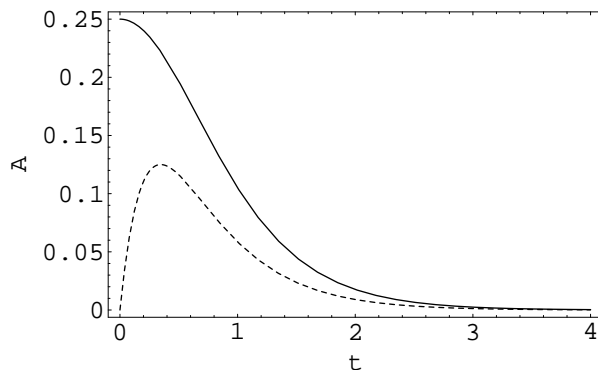


FIG. 5: Mean anisotropy parameter for the Bianchi type I universe with a scalar field: for the CET (solid curve) and for the brane world (dotted curve). The normalization of the parameters is set as $\kappa = 1, 2\kappa_0\kappa = C$.

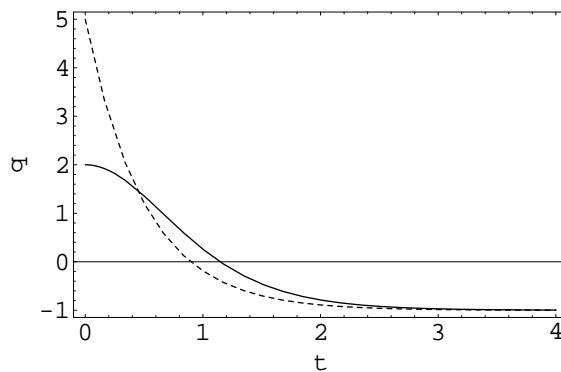


FIG. 6: Deceleration parameter for the Bianchi type I universe with a scalar field: for the CET (solid curve) and for the brane world (dotted curve). The normalization of the parameters is set as $\kappa = 1, 2\kappa_0\kappa = C$.

Perturbations of the fields of a gravitational system against the background evolutionary solution should be checked, to ensure the stability of the exact or approximated background solution. In principle, stability analysis should be performed against the perturbations of all possible fields in all possible manners, subject to the field equations and boundary conditions of the system. In the following section, we will divide the perturbations into two disjoint classes: (a) the perturbations of the scale factors, or equivalently, the metric field; and (b) the perturbations of the dilaton field.

We will argue that the most complete stability conditions we are looking for can be obtained from only class (a) and class (b) perturbations. Although the backreaction of the scalar field perturbation on the metric field perturbations is known to be important [25], we can show that this backreaction does not bring in any further restriction on the stability conditions.

The reason is rather straightforward. One can write the linearized perturbation equation as

$$D_{a_j}^i \delta a_j + D_\phi^i \delta \phi = 0, \quad (71)$$

for the system we are interested in. Moreover, perturbations are defined as $a_i = a_i^0 + \delta a_i$ and $\phi = \phi_0 + \delta \phi$ with the index 0 denoting the background field solution. Note also that the operators $D_{a_j}^i$ and D_ϕ^i denote the differential operator obtained from the linearized perturbation equation, with all fields evaluated at the background solutions. The exact form of these differential operators will be shown later.

One is looking for the stability conditions that the field parameters must obey in order to keep the evolutionary solution stable. One can show that the class (a) and class (b) solutions are good enough to cover all of the domain of the stability conditions. Let us denote the domain of solutions to the class (a), (b), and (a+b), stability conditions as $S(a)$, $S(b)$, and $S(a+b)$ respectively. Specifically, the definition of these domains are defined by $S(a) \equiv \{\delta a_i | D_{a_j}^i \delta a_j = 0\}$, $S(b) \equiv \{\delta \phi | D_\phi^i \delta \phi = 0\}$, and $S(a+b) \equiv \{(\delta a_i, \delta \phi) | D_{a_j}^i \delta a_j + D_\phi^i \delta \phi = 0\}$.

Therefore, one only needs to show that $S(a) \cap S(b) \subset S(a+b)$. This is because “ $D_{a_j}^i \delta a_j = 0$ and $D_\phi^i \delta \phi = 0$ ” imply that “ $D_{a_j}^i \delta a_j + D_\phi^i \delta \phi = 0$.” On the other hand, “ $D_{a_j}^i \delta a_j + D_\phi^i \delta \phi = 0$ ” does not imply that “ $D_{a_j}^i \delta a_j = 0$ or $D_\phi^i \delta \phi = 0$.” Hence the class (a) and class (b) solutions cover all the required stability conditions we are looking for. Hence we only need to consider these two separate cases for simplicity.

In addition, any small time-dependent perturbation against the metric field is known to be equivalent to a gauge choice [26]. This can be clarified as follows. Indeed, one can show that any small coordinate change of the form $x'^\mu = x^\mu - \epsilon^\mu$ will induce a gauge transformation on the metric field according to $g'_{\mu\nu} = g_{\mu\nu} + D_\mu \epsilon_\nu + D_\nu \epsilon_\mu$. Therefore, a small metric perturbation against a background metric amounts to a gauge transformation of the form $a'_i = a_i + \epsilon^t \dot{a}_i$ for the Bianchi type-I metric with $\epsilon_\mu = (\text{constant}, \epsilon_i(t))$. This is then equivalent to small metric perturbations. If a background solution is stable against small perturbations with respect to small field perturbations, one in fact has done nothing but a field redefinition.

If the background solution is, however, unstable against small perturbations, i.e. the small perturbation will grow exponentially, as we will show momentarily, the resulting large perturbations can no longer be classified as small gauge transformation. Therefore, the stability analysis performed in the literature [10, 11, 20–24, 28, 29] for various models against the unstable background solution served as a very simple method to check whether the system supports a stable metric field background. This is the reason why we still perform a perturbation on the metric field for stability analysis; even a small perturbation is equivalent to a gauge redefinition.

Note that one should also consider a more general perturbation with space perturbations included. That formulation is, however, much more complicated than the one we will show in this paper. We will focus on the time-dependent case for simplicity in this paper. The space-dependent perturbation analysis is still under investigation. The time-dependent

analysis alone will, however, bring us much useful information for the stability conditions for the models of interest. For example, we will show in the following subsection that the solution found in Ref. [14] remains stable as long as the scalar field falls close to any local minimum of the potential $U(\phi)$. Note again that the solution found in Ref. [14] is an exact solution only when $U = \text{constant}$.

Dilaton Perturbation

Let us consider a perturbation of the dilaton field of the following form

$$\phi \rightarrow \phi_B + \delta\phi, \quad U(\phi) \rightarrow \Lambda + \partial_\phi \delta\phi. \quad (72)$$

By setting $k_5 = 0$ in Eqs. (54, 55, 56), the field equations for $\delta\phi$ become

$$-2\dot{\phi}_B \delta\dot{\phi} + \partial_\phi U \delta\phi = 0, \quad (73)$$

$$\partial_\phi U \delta\phi = 0, \quad (74)$$

$$\delta\ddot{\phi} + 3H_B \delta\dot{\phi} + \partial_\phi^2 U \delta\phi = 0. \quad (75)$$

Due to the fact that $H_{Bi} \rightarrow \frac{\kappa}{3}$, the asymptotic solution for $\delta\phi$ can be shown to be

$$\delta\phi \propto \exp \left[(-\kappa \pm \sqrt{\kappa^2 - 4\beta})t/2 \right]. \quad (76)$$

Here $\beta \equiv \partial_\phi^2 U|_{t \rightarrow \infty}$. Therefore, the perturbation of the dilaton field decays to zero if $\beta \geq 0$. Hence we show that the system is stable with respect to the scalar perturbation if $\beta \geq 0$, then the scalar potential is capable of confining the scalar field to its local minimum.

Note that there are two additional equations $U' = 0$ and $\dot{\phi} = 0$, which are required for the consistency of the stability of the system. These constraints simply imply that the scalar field must be at rest at the local minimum of the scalar field potential. As pointed out earlier, the background solution we have at hand is an exact solution for the model with a constant cosmological constant. Therefore, the background solution we used for our stability analysis is only an approximate solution, which remains valid only when the scalar field is close to the local minimum of the the scalar field potential. As a result, one should not take these two further constraints too serious. In fact, these constraints are both negligible in the large time limit, when the scalar field falls close to the local minimum of the scalar field potential, namely, $U' \rightarrow 0$ and $\dot{\phi} \rightarrow 0$. Hence one is able to show that these additional constraints can be satisfied approximately in the large time limit.

Metric Perturbation

Using the metric perturbation (IIII-3), the equations for the metric perturbation δb_i can be obtained from perturbing the Eqs. (54, 55, 56). The result is

$$\sum_i \delta\ddot{b}_i + 2 \sum_i H_{Bi} \delta\dot{b}_i = 0, \quad (77)$$

$$\delta\ddot{b}_i + \frac{\dot{V}_B}{V_B} \delta\dot{b}_i + H_{Bi} \sum_j \delta\dot{b}_j = 0, \quad (78)$$

$$\dot{\phi}_B \sum_i \delta\dot{b}_i = 0. \quad (79)$$

Here we also choose the limit $k_5 = 0$. Note that the background variables V_B, a_{Bi} and H_{Bi} approach

$$V_B \propto e^{\kappa t}, \quad a_{Bi} \propto e^{\kappa t/3}, \quad H_{Bi} \rightarrow \frac{\kappa}{3}. \quad (80)$$

Therefore, the asymptotic behavior of the metric perturbation δb_i can be found from the second equation. In addition, the other two field equations will provide a constraint for the system. The results are

$$\delta b_i \rightarrow c_i e^{-\kappa t}, \quad \sum_i c_i = 0. \quad (81)$$

Hence, the metric perturbation

$$\delta a_i \equiv a_{Bi} \delta b_i \rightarrow e^{-2\kappa t/3} \rightarrow 0, \quad (82)$$

in the large time limit. This indicates that the background solution of the system is stable against the metric perturbation, as shown above.

V. CONCLUSION

We have discussed the anisotropic property of cosmological models in the CET and brane theory. A realistic model, being consistent with the current observations, should produce an anisotropic parameter with a small value at the later stage of the evolution of our universe near the last scattering surface. By assuming the Bianchi type I space-time for the evolution of our universe, we find that the final state of the evolving universe always approaches the phase of isotropic expansion in both theories.

These two different theories give completely different initial anisotropy at the very early stage of evolution. Indeed, for the CET, the anisotropy tends to be large in the very early stage. In another words, the universe tends to begin from a highly anisotropic initial state. The mean anisotropy parameter A will then decay to zero as the time increases. On the other hand, the early time behavior of the universe in the brane world scenario changes significantly due to the quadratic correction on the brane. As a result, any non-vanishing mean anisotropy parameter, $A(t)$, tends to vanish in the very early period. There is a characteristic time, t_c , that divides the evolution of $A(t)$ into two different stages. The mean anisotropy parameter is increasing when $t < t_c$ and reaches its maximal value at $t = t_c$. After that, $A(t)$ starts to decay. This kind of behavior is clearly shown in Fig. 2 and Fig. 5. This result remains true for both the model with a perfect fluid and the model with a scalar field. This appears to be a general feature independent of the types of matter considered.

It is worth noting that the only exception is the model with $p = 0$ (i.e. $\gamma = 1$) for the perfect fluid model. The mean anisotropy of this model behaves similarly to the models in the CET, where the mean anisotropy parameter is large in the very early times. We also analyzed the stability problem for those exactly solved anisotropic models shown in this

paper. The result indicates that all of the solutions known to us are stable in the large time limit. Therefore, the evolution of the universe in the CET starts with high initial anisotropic expansion. The dynamics of the system will take the universe to the phase of isotropic expansion in the large time limit. We also show that the final isotropic expansion will remain stable in the large time limit. In addition, the mean anisotropy parameter will keep decreasing as the time increases. The model provided here is a useful and explicit model that is capable of providing us with a universe that has a tiny anisotropy left over near the last scattering surface.

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