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ORIGINAL ARTICLE

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Measuring manufacturing capability based on lower confidence bounds of C_{pmk} applied to current transmitter process

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Abstract Several process capability indices, including C_p, C_{pk}, and C_{pm}, have been proposed to provide numerical measures on manufacturing potential and actual performance. Combining the advantages of those indices, a more advanced index C_{pmk} is proposed, taking the process variation, centre of the specification tolerance, and the proximity to the target value into account, which has been shown to be a useful capability index for manufacturing processes with two-sided specification limits. In this paper, we consider the estimation of C_{pmk}, and we develop an efficient algorithm to compute the lower confidence bounds on C_{pmk} based on the estimation, which presents a measure on the minimum manufacturing capability of the process based on the sample data. We also provide tables for practitioners to use in measuring their processes. A real-world example of current transmitters taken from a microelectronics device manufacturing process is investigated to illustrate the applicability of the proposed approach. Our implementation of the existing statistical theory for manufacturing capability assessment bridges the gap between the theoretical development and the in-plant applications.

Keywords Process capability index · Lower confidence bound

1 Introduction

Process capability indices, including C_p, C_{pk}, and C_{pm} [1, 4], have been proposed in the manufacturing industry to provide numerical measures on whether a process is capable of reproducing items meeting the manufacturing quality requirement preset in the factory. Combining the advantages of those indices, Pearn et al. [5] proposed a more advanced capability index called C_{pmk}, which has been shown to be a useful capability index for processes with two-sided specification limits. These indices are defined as:

$$C_p = \frac{USL - LSL}{6\sigma},$$

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\},$$

$$C_{pm} = \frac{USL}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

$$C_{pmk} = \min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\}$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation, and T is the target value predetermined by the product designer or the manufacturing engineer.

Criteria that have been considered for measuring manufacturing capability include: process variation (product quality consistency), process departure, process yield, and process loss. The index C_p considers the overall process variability relative to the manufacturing tolerance; therefore, it only reflects the consistency of the product quality characteristic. The index C_{pk} takes the process mean into consideration but it can fail to distinguish between on-target processes and off-target processes. It is a yield-based index providing lower bounds on process yield. The index C_{pm} takes the proximity of process mean from the target value into account and is more sensitive to process departure than C_{pk}. Since the design of C_{pm} is based on the average process loss relative to the manufacturing tolerance, the

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index C_{pm} provides an upper bound on the average process loss; C_{pm} has alternatively been called the Taguchi index. The index C_{pmk} is constructed from combining the modifications to C_p that produced C_{pk} and C_{pm}, inheriting the merits of both indices. The ranking of the four basic indices in terms of sensitivity to the departure of process mean from the target value, from the most sensitive one to the least sensitive, are (1) C_{pmk}, (2) C_{pm}, (3) C_{pk}, and (4) C_p. For semiconductor manufacturing, the index is appropriate for capability measures due to high standards and stringent requirements on product quality and reliability.

We note that a manufacturing process satisfying the capability requirement "$C_{pk} \geq c_0$" may not satisfy the capability condition "$C_{pm} \geq c_0$". On the other hand, a process satisfying the capability requirement "$C_{pm} \geq c_0$" may not satisfy the capability requirement "$C_{pk} \geq c_0$" either. But, a manufacturing process does satisfy both capability requirements "$C_{pk} \geq c_0$" and "$C_{pm} \geq c_0$" if the process satisfies the capability requirement "$C_{pmk} \geq c_0$" since $C_{pmk} \leq C_{pk}$ and $C_{pmk} \leq C_{pm}$. Thus, the index C_{pmk}does provide more capability assurance with respect to process yield and process loss to the customers than the other two indices C_{pk} and C_{pm}. This is a desired property according to today's modern quality improvement theory, as reduction of process loss (variation from the target) is just as important as increasing the process yield (meeting the specifications). While C_{pk} is still the more popular and widely used index, the index C_{pmk} is considered to be the most useful index to date for processes with two-sided manufacturing specifications. Chen and Hsu [2] investigated the asymptotic sampling distribution of the estimated C_{pmk}. Wright [14] derived an explicit but rather complicated expression for the probability density function of the estimated C_{pmk}. Pearn et al. [7] considered an extension of C_{pmk} for handling process with asymmetric tolerances. Jessenberger and Weihs [3] studied the behaviour of C_{pmk}, looking for processes with asymmetric tolerances. Pearn et al. [8] obtained an alternative, simpler form of the probability density function of the estimated C_{pmk} and considered capability testing based on C_{pmk}. Pearn et al. [9] investigated the statistical properties of the estimated C_{pmk}. Pearn and Lin [10] focused on a Bayesian-like estimator of C_{pmk} under a different manufacturing condition, in which the probability $p(\mu > m)$ is available. Pearn and Lin [11] developed efficient SAS/Maple computer programs to calculate the critical values and the p-value for testing manufacturing capability based on C_{pmk}.

2 Manufacturing capability of a current transmitter process

In practice, a manufacturing process is said to be inadequate if $C_{pmk} < 1.00$; this indicates that the process is not adequate with respect to the manufacturing tolerances, and the process variation σ^2 needs to be reduced (often by changing the design of the experiments). The fraction of nonconformities for such a process exceeds 2700 ppm (parts per million). A manufacturing process is said to be marginally capable if $1.00 \leq C_{pmk} < 1.33$; this indicates that caution needs to be taken regarding the process consistency and some process control is required (usually using R or S control charts). The fraction of nonconformities for such a process is within 66–2700 ppm. A manufacturing process is said to be satisfactory if $1.33 \leq C_{pmk} < 1.67$; this indicates that process consistency is satisfactory, material substitution may be allowed, and no stringent precision control is required. The fraction of nonconformities for such a process is within 0.54–66 ppm. A manufacturing process is said to be excellent if $1.67 \leq C_{pmk} < 2.00$; this indicates that process precision exceeds satisfactory. The fraction of nonconformities for such a process is within 0.002–0.54 ppm. Finally, a manufacturing process is said to be super if $C_{pmk} \geq 2.00$. The fraction of nonconformities for such a process is less than 0.002 ppm.

Table 1 summarizes the above five capability requirements and the corresponding C_{pmk} values. Some minimum capability requirements have been recommended in the manufacturing industry [15] for specific process types, which must run under more designated stringent quality conditions. For existing manufacturing processes, the capability must be no less than 1.33, and for new manufacturing processes, the capability must be no less than 1.50. For existing manufacturing processes on safety, strength, or critical parameters (such as manufacturing soft drinks or chemical solutions bottled with glass containers), the capability must be no less than 1.50, and for new manufacturing processes on safety, strength, or critical parameters, the capability must be no less than 1.67.

We consider the following case taken from a microelectronic manufacturing factory making various types of microelectronic devices. There is one production line, controlled and monitored in the factory, that makes current transmitters. The process investigated is the making of a monolithic 4–20 mA, two-wire current transmitter integrated circuit (2WCT IC) designed for bridge input signals. This device provides complete bridge excitation, instrumentation amplifier, linearization, and the current output circuitry necessary for high impedance strain gage sensors. The instrumentation amplifier can be used over a wide range of gain, accommodating a variety of input signals and sensors. Linearization circuitry consists of a

Table 1 Some commonly used capability requirement and the corresponding precision conditions

Precision Condition	C_{pmk} values
Inadequate	$C_{pmk} < 1.00$
Marginally capable	$1.00 \leq C_{pmk} < 1.33$
Satisfactory	$1.33 \leq C_{pmk} < 1.67$
Excellent	$1.67 \leq C_{pmk} < 2.00$
Super	$2.00 \leq C_{pmk}$

second, fully independent instrumentation amplifier that controls the bridge excitation voltage. It provides the second-order correction to the transfer function, typically achieving a 20:1 improvement in nonlinearity, even with low cost transducers. Total unadjusted error of the complete current transmitter, including the linearized bridge, is low enough to permit use without adjustment in many applications such as industrial process control, factory automation, SCADA remote data acquisition, weighting systems, and accelerometers. This 2WCT IC product is available in 16-pin plastic DIP and SOL-16 surface-mount packages, as depicted in Fig. 1.

The total unadjusted error of the 2WCT IC is an essential product characteristic, which has significant impact on product quality. Because the total unadjusted error is a two-sided specification, the upper specification limit, USL, is set to 5 μA, and the lower specification limit, LSL, is set to −5 μA,; therefore, the factory engineers recommend using C_{pmk} for determining whether products meet specifications and taking action to improve the process if necessary. In practice, we never know the true values of μ and σ^2 nor C_{pmk}. Hence, these parameters need to be estimated and sampling error needs to be considered.

For a normally distributed process that is demonstrably stable (under statistical control), Pearn et al. [5] considered the maximum likelihood estimator (MLE) of C_{pmk} as defined below:

$$\hat{C}_{\text{pmk}} = \min\left\{\frac{\text{USL}-\bar{X}}{3\sqrt{S_n^2+(\bar{X}-T)^2}}, \frac{\bar{X}-\text{LSL}}{3\sqrt{S_n^2+(\bar{X}-T)^2}}\right\}$$

where $\bar{X}=\sum_{i=1}^{n}X_i/n$ and $S_n^2=\sum_{i=1}^{n}(X_i-\bar{X})^2/n$ are the MLEs of μ and σ^2, respectively. We note that $S_n^2+(\bar{X}-T)^2=\sum_{i=1}^{n}(X_i-T)^2/n$, which is in the denominator of $\hat{C}_{\text{pmk}}$, is the uniformly minimum variance unbiased estimator (UMVUE) of $\sigma^2+(\mu-T)^2=E\left[(X-T)^2\right]$ in the denominator of C_{pmk}.

3 Sampling distribution of C_{pmk}

For symmetric manufacturing tolerance ($T=m$), Pearn et al. [5] expressed the natural estimator $\hat{C}_{\text{pmk}}$, alternatively, as the following:

$$\hat{C}_{\text{pmk}} = \frac{d-|\bar{X}-m|}{3\sqrt{S_n^2+(\bar{X}-T)^2}}$$

and showed that the distribution of the natural estimator $\hat{C}_{\text{pmk}}$ is a mixture of the chi-square distribution and the non-central chi-square distribution, as expressed in the following:

$$\hat{C}_{\text{pmk}} \sim \frac{\frac{d\sqrt{n}}{\sigma}-\chi_1'(\lambda)}{3\sqrt{\chi^2{}_{n-1}+\chi_{n-1}'^2(\lambda)}}$$

where χ_{n-1}^2 is the chi-square distribution with n–1 degrees of freedom, $\chi_1'(\lambda)$ is the non-central chi-square distribution with one degree of freedom and non-centrality parameter λ, and $\chi_{n-1}'^2(\lambda)$ is the non-central chi-square distribution with n–1 degrees of freedom and non-centrality parameter λ, where $\lambda=n(\mu-T)^2/\sigma^2$. The cumulative distribution of $\hat{C}_{\text{pmk}}$, therefore, can be found as the following [7, 8, 9]:

$$F_{C_{\text{pmk}}}(x)=\begin{cases}0, & x<\frac{-1}{3},\\ \sum_{j=0}^{\infty}\left\{p_j\int_{\beta}^{\infty}F_K\left(\left[(D-\sqrt{y})^2\big/(9x^2)\right]-y\right)f_{Y_j}(y)\mathrm{d}y\right\}, & \frac{-1}{3}\leqslant x<0,\\ 1-\sum_{j=0}^{\infty}\left\{p_j\int_0^{D^2}f_{Y_j}(y)\mathrm{d}y\right\}, & x=0,\\ 1-\sum_{j=0}^{\infty}\left\{p_j\int_0^{\beta}F_K\left(\left[(D-\sqrt{y})^2\big/(9x^2)\right]-y\right)f_{Y_j}(y)\mathrm{d}y\right\}, & x>0,\end{cases}$$

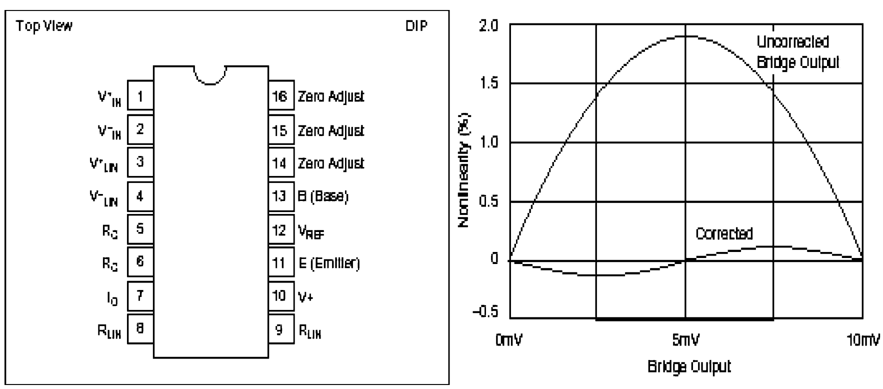

Fig. 1 A current transmitter with bridge excitation and linearization

where $\beta = [D/(1+3x)]^2$, $D = n^{1/2}(d/\sigma)$, $K = nS_n^2/\sigma^2$, F_K is the cumulative distribution function of K, and the probability density function (PDF) is:

$$f_{C_{\text{pmk}}}(x) = \begin{cases} B\sum_{j=0}^{\infty}\left\{p_j B_j \int_0^{\infty} I_j(x,z)\,\mathrm{d}z\right\}, & -\frac{1}{3}<x<0, \\ B\sum_{j=0}^{\infty}\left\{p_j B_j \int_0^{1/x} I_j(x,z)\,\mathrm{d}z\right\}, & x>0, \\ 0, & \text{otherwise}, \end{cases}$$

where $p_j = (\lambda/2)^j \mathrm{e}^{-\lambda/2}/j!$, $B = 12(n^{1/2}d)^n/\left\{(18\sigma^2)^{n/2}\Gamma \times [(n-1)/2]\right\}$, $B_j = (n^{1/2}d)^{2j}/\left\{(2\sigma^2)^j \Gamma\left[1/2+j\right]\right\}$, and

$$I_j(x,z) = \frac{(1-xz)^{2j}(z-3)^2 z^{(n-3)/2}[z+3(2-xz)]^{(n-3)/2}}{(1+3x)^{2j+(n+3)/2}} \times \exp\left\{-\frac{D^2(z+3)^2}{18(1+3x)^2}\right\}.$$

Using variable transformation and integration technique, for $x>0$, the cumulative distribution function (CDF) of the estimated C_{pmk} may be alternatively expressed as the following, which can be used for calculating the critical values c_0, the p-values, and the lower confidence bounds C on C_{pmk}.

$$F_{\hat{C}_{\text{pmk}}}(x) = 1 - \int_0^{b\sqrt{n}/(1+3x)} G\left(\frac{(b\sqrt{n}-t)^2}{9x^2} - t^2\right) \times \left[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right]\mathrm{d}t,$$

where $b = d/\sigma$, $\xi = (\mu - T)/\sigma$, $G(\cdot)$ is the cumulative distribution function of the chi-square distribution χ^2_{n-1}, and $\phi(\cdot)$ is the probability density function of the standard normal distribution $N(0,1)$. Note that for $\mu >$ USL or $\mu <$ LSL, the capability $C_{\text{pmk}} < 0.0$, and for $\mu =$ USL or $\mu =$ LSL, the capability $C_{\text{pmk}} = 0.0$. The requirement of LSL $< \mu <$ USL is a minimum capability requirement applying to most start-up engineering applications or new processes. Figure 2a, b displays the PDF plots of the MLE estimator $\hat{C}_{\text{pmk}}$ with $\xi = 0.5$ and 1, $b = 3$, $d = 2$, and $n = 10$, 20, 50. Figure 2c, d displays the CDF plots of the natural estimator $\hat{C}_{\text{pmk}}$ with $\xi = 0.5$ and 1, $b = 3$, $d = 2$, and $n = 10$, 20, 50.

4 Calculating manufacturing capability

Critical values are used for making decisions in manufacturing capability testing with designated type-I error α, which is the risk of misjudging an incapable process (H_0: $C_{\text{pmk}} \le c_0$) as a capable one (H_1: $C_{\text{pmk}} > c_0$). The p-values are used for making decisions in manufacturing capability testing, which presents the actual risk of misjudging an incapable process (H_0: $C_{\text{pmk}} \le c_0$) as a capable one (H_1: $C_{\text{pmk}} > c_0$). Thus, if $p < \alpha$, we reject the null hypothesis and conclude that the process is capable with actual type-I error p (rather than α). Both approaches, the critical values and the p-values, do not convey any information regarding the minimal value of the actual manufacturing capability (lower confidence bound). The development of the lower confidence bound on the actual manufacturing capability is essential. The lower confidence bound not

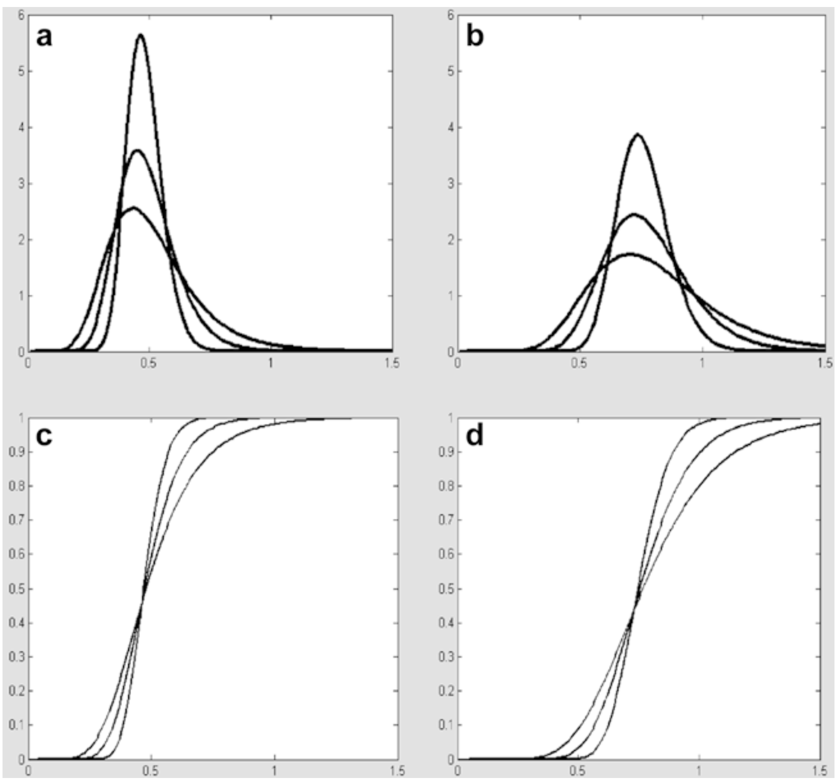

Fig. 2 a PDF plots of $\hat{C}_{\text{pmk}}$ with $\xi = 1$, $b = 3$, $d = 2$, and $n = 10$, 20, 50 (*bottom* to *top*). **b** PDF plots of $\hat{C}_{\text{pmk}}$ with $\xi = 0.5$, $b = 3$, $d = 2$, and $n = 10$, 20, 50 (*bottom* to *top*). **c** CDF plots of $\hat{C}_{\text{pmk}}$ with $\xi = 1$, $b = 3$, $d = 2$, and $n = 10$, 20, 50 (*bottom* to *top*). **d** CDF plots of $\hat{C}_{\text{pmk}}$ with $\xi = 0.5$, $b = 3$, $d = 2$, and $n = 10$, 20, 50 (*bottom* to *top*) only gives us a clue about the minimal level of the actual manufacturing performance, which is closely related to the fraction of nonconforming units (defectives), but is also useful in making decisions for manufacturing capability testing. For processes with a target value set to the mid-point of the manufacturing specifications ($T=m$), the index C_{pmk} may be rewritten as the following. When $C_{pmk}=C$, $b=d/\sigma$ can be expressed as $b = 3C\sqrt{1+\xi^2}+|\xi|$.

$$C_{pmk} = C_{pmk} = \frac{d-|\mu-T|}{3\sqrt{\sigma^2+(\mu-T)^2}} = \frac{d/\sigma-|\xi|}{3\sqrt{1+\xi^2}},$$

where $\xi = (\mu - T)/\sigma$.

Hence, given the sample of size n, the confidence level γ, the estimated value, $\hat{C}_{pmk}$, and the parameter ξ, the lower confidence bounds C can be obtained using the numerical integration technique with iterations to solve the following equation. In practice, the parameter $\xi=(\mu-T)/\sigma$ is unknown, but it can be calculated from the sample data as $\hat{\xi} = (\bar{X}-T)/S_n$. It should be noted that the equation is an even function of ξ. Thus, for both $\xi=\xi_0$ and $\xi=-\xi_0$ we have the same lower confidence bounds.

$$\int_0^{b\sqrt{n}/(1+3\hat{C}_{pmk})} G\left(\frac{(b\sqrt{n}-t)^2}{9\hat{C}_{pmk}^2}-t^2\right) \times \left[\phi(t+\xi\sqrt{n})+\phi(t-\xi\sqrt{n})\right]\mathrm{d}t = 1-\gamma \quad (1)$$

4.1 Algorithm for the LCB

Using Eq. 1, we may compute the lower confidence bounds, C, and a Matlab algorithm called the LCB is developed. Three auxiliary functions for evaluating C are included here: (a) the cumulative distribution function of the chi-square χ^2_{n-1}, $G(\cdot)$, (b) the probability density function of the standard normal distribution $N(0,1)$, $\phi(\cdot)$, and (c) the function of numerical evaluate integration using the recursive adaptive Simpson quardrature – "quad". The algorithm used commonly is known as the direct search.

1. Read the sample data (X_1, X_2, ..., X_n), LSL, USL, T, and γ.
2. Calculate $\bar{X}$, S_n, $\hat{\xi}$, and $\hat{C}_{pmk}$.
3. Compute an initial guess for C.
4. Find the lower confidence bound C on C_{pmk}.
5. Output the conclusive message, "The true value of the manufacturing capability C_{pmk} is no less than the C with 100γ% level of confidence."

We implement the algorithm and develop the following Matlab computer program to compute the minimal manufacturing capability.

4.2 Matlab program for LCB

```
%————————————————————————————————————
  % Input the sample data (X1, X2,..., Xn),
LSL, USL, T, and γ.
  %——————————————————————————————————
  [n1 usl lsl T r1]=read('Enter values of
sample size, lower specification limit,
  upper specification limit, target value,
confidence level:');
  global b n epsilon ecpmk
  n=n1;r=r1;
  [data(1:n,1)]=
  textread ('eeprom.dat','%f',n);
  %——————————————————————————————————
  % Compute X, Sn, ξ, and Cpmk.
  %——————————————————————————————————
  mdata=mean(data);
stddata=std(data)*sqrt((n-1)/n);
  epsilon=(mdata-T)/stddata;
  ecpmk=(min(usl-mdata,mdata-lsl))/
(3*sqrt(stddata^2+(mdata-T)^2));
  fprintf('The Sample Mean is %g.\n',
mdata);
  fprintf('The Sample Standard Deviation
is %g.\n',stddata)
  fprintf('The Epsilon is %g.\n',epsilon)
  fprintf('The Estimate of Cpmk is
%g.\n',ecpmk)
  %
————————————————————————————————————
  % Compute a good initial value of C.
  %
————————————————————————————————————
  c=0.2:0.025:3;
  for i=1:1:113
  b=0;d=0;y=0;b=3*c(i)*sqrt(1+epsilon^2)
  +abs(epsilon);
  d=b*sqrt(n)/(1+3*ecpmk);
  y=quad('cpmk',0,d);
  if (y-(1-r))>0 break
  end; end
  %——————————————————————————————————
  %Evaluate the lower confidence bound C on
Cpmk.
  %——————————————————————————————————
  c=0.2+0.025*(i-1):-0.001:0.2;
  for k=1:(0.025*(i-1)*1000)+1
  b=0;d=0;y=0;b=3*c(k)*sqrt(1+epsilon^2)
  +abs(epsilon);
  d=b*sqrt(n)/(1+3*ecpmk);
  y=quad('cpmk',0,d);
  if ((1-r)-y)>0.0001 break
  end; end
  %——————————————————————————————————
  % Output the conclusive message, ''The
true value of the process
  % capability Cpmk is no less than C with
100γ% level of confidence.''
  %——————————————————————————————————
```

```
fprintf('The true value of the
manufacturing capability Cpmk is no
less than %g', c(k))
fprintf('with %g',r)
fprintf('level of confidence.')
%———————————————————————————————————
%Two function files included—read.m and
cpmk.m
%———————————————————————————————————
function Q1=cpmk(t)
global n b epsilon ecpmk
Q1=chi2cdf(((((b*sqrt(n)-t).^2)./
(9*ecpmk^2))-t.^2),n-1).*…
(normpdf((t+epsilon*sqrt(n)))
+normpdf((t-epsilon*sqrt(n))));
function [a1, a2, a3, a4, a5]=read(labl)
if nargin==0, labl='?'; end
n=nargout;str=input(labl,'s');
str=['[',str,']'];v=eval(str);
L=length(v);
if L>=n, v=v(1:n);
else, v=[v,zeros(1,n-L)]; end
for j=1:nargout
eval(['a', int2str(j),'=v(j);']); end
%———————————————————————————————————
%The End
%———————————————————————————————————
```

5 Manufacturing capability and process parameter ξ

Since the process parameters μ and σ are unknown, the process characteristic parameter $\xi=(\mu-T)/\sigma$ is also unknown. Thus, it has to be estimated further in real applications, normally by substituting μ and σ with its sample mean and sample standard deviation. Such an approach certainly would make our approach less reliable as the level of the confidence γ cannot be ensured. To eliminate the need to further estimate the parameter ξ, we examine the behaviour of the lower confidence bounds C as the function of the process characteristic ξ.

We perform extensive computations to calculate the lower confidence bounds C for $\xi=0(0.05)$ 3.00, $\hat{C}_{\text{pmk}}=0.7(0.1)3.0$, and $n=5(5)200$. We note that $\xi=(\mu-T)/\sigma=0(0.05)3.00$ covers a wide range of applications with process capability $C_{\text{pmk}}\geq 0$. The result indicates that the lower confidence bound C obtains its minimum either at $\xi=0.50$ (for most cases), or at 0.45 (in a few cases), and the difference between the two lower confidence bounds is less than 5×10^{-4} (possibly due to computational precision errors). In fact, the lower confidence bound value C first decreases as ξ increases, obtains its minimum value at $\xi=0.45$ or 0.5, then increases again within the range of $\xi\in[0.5, 3.0]$. Hence, for practical purposes we may solve Eq. 1 for $\xi=0.50$ to obtain the required lower confidence bounds for a given $\hat{C}_{\text{pmk}}$, n and γ, without having to further estimate the parameter ξ. Thus, the level of confidence γ can be ensured, and the decisions made based on such approach must be reliable.

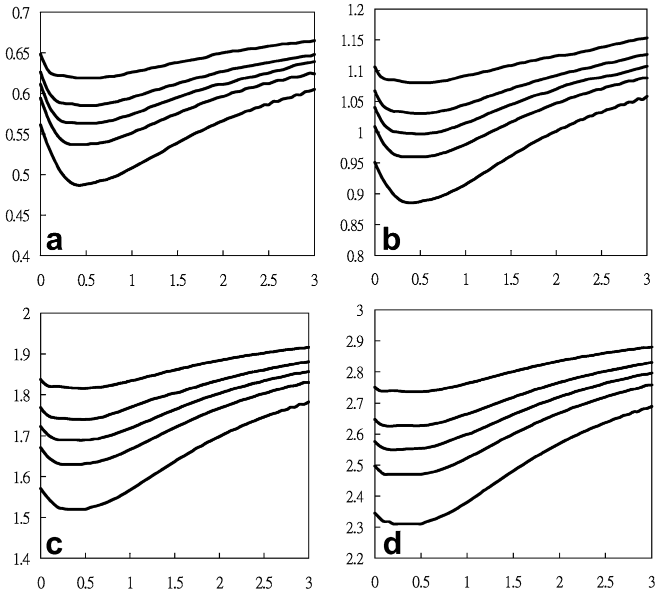

Fig. 3 a Plots of C versus $|\xi|$ for $\hat{C}_{\text{pmk}}=0.7$, $\gamma=0.95$, $n=30$, 50, 70, 100, 200 (*bottom* to *top*). **b** Plots of C versus $|\xi|$ for $\hat{C}_{\text{pmk}}=1.2$, $\gamma=0.95$, $n=$ 30, 50, 70, 100, 200 (*bottom* to *top*). **c** Plots of C versus $|\xi|$ for $\hat{C}_{\text{pmk}}=2.0$, $\gamma=0.95$, $n=$ 30, 50, 70, 100, 200 (*bottom* to *top*). **d** Plots of C versus $|\xi|$ for $\hat{C}_{\text{pmk}}=3.0$, $\gamma=0.95$, $n=$ 30, 50, 70, 100, 200 (*bottom* to *top*)

We note the above result is almost impossible to prove mathematically. It is important to recognize that the lower confidence bounds obtained from solving Eq. 1 with $\xi=0.50$ takes the minimal value among all possible C values, and therefore the capability estimation is conservative. But, given that the process parameter ξ is always unknown in real applications (as the process mean μ and the process variation σ are unknown), C must be the maximum value for which the confidence level γ can be assured. Any other value of C' that is greater than the minimal value C ($C'>C$) would certainly result in a confidence level γ' less than the preset γ ($\gamma'<\gamma$). Therefore, our approach provides the most accurate solution among other existing estimations for the manufacturing capability.

Figure 3a–d plots the curves of the lower confidence bound, C, versus the parameter ξ for $\hat{C}_{\text{pmk}}=0.7$, 1.2, 2.0, 3.0, respectively, with $\gamma=0.95$. For bottom curve 1, sample size $n=30$. For bottom curve 2, sample size $n=50$; for bottom curve 3, sample size $n=70$; for top curve 2, sample size $n=100$; for top curve 1, sample size $n=200$. We note that for all $\hat{C}_{\text{pmk}}$ values we investigated, as n increases the discrepancy between those C values with different ξ values decreases. Tables 2 and 3 tabulate the lower confidence bound, C, for $\hat{C}_{\text{pmk}}=0.7(0.1)3.0$, $n=5(5)200$, and $\gamma=0.95$ with the process parameter ξ set to $\xi=0.50$. For example, if $\hat{C}_{\text{pmk}}=1.4$, then with $n=100$ we find the lower confidence bound $C=1.208$, and so the minimal manufacturing capability is no less than 1.297,

Table 2 Lower confidence bounds C of C_{pmk} for $\hat{C}_{pmk}=0.7(0.1)1.8$, $n=5(5)200$, $\gamma=0.95$

n	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
5	0.247	0.297	0.346	0.395	0.444	0.493	0.542	0.591	0.637	0.682	0.729	0.791
10	0.358	0.423	0.488	0.551	0.616	0.679	0.741	0.803	0.867	0.930	0.993	1.056
15	0.413	0.484	0.556	0.627	0.698	0.770	0.841	0.912	0.983	1.054	1.125	1.195
20	0.447	0.523	0.598	0.673	0.747	0.822	0.897	0.972	1.047	1.121	1.196	1.270
25	0.466	0.545	0.623	0.700	0.778	0.855	0.933	1.010	1.087	1.164	1.241	1.318
30	0.488	0.568	0.648	0.728	0.808	0.887	0.966	1.046	1.125	1.204	1.284	1.363
35	0.505	0.586	0.668	0.749	0.830	0.911	0.992	1.073	1.154	1.235	1.316	1.397
40	0.518	0.601	0.683	0.766	0.848	0.931	1.013	1.095	1.177	1.259	1.342	1.424
45	0.529	0.612	0.696	0.780	0.863	0.946	1.030	1.113	1.196	1.279	1.363	1.446
50	0.537	0.622	0.707	0.791	0.875	0.960	1044	1128	1.212	1.296	1.380	1.464
55	0.545	0.631	0.716	0.801	0.886	0.971	1.056	1.141	1.226	1.310	1.395	1.480
60	0.552	0.638	0.724	0.809	0.895	0.981	1.066	1.152	1.237	1.323	1.408	1.494
65	0.558	0.644	0.731	0.817	0.903	0.989	1.076	1.162	1.248	1.334	1.420	1.506
70	0.563	0.650	0.737	0.824	0.910	0.997	1.084	1.170	1.257	1.344	1.430	1.517
75	0.567	0.655	0.742	0.830	0.917	1.004	1.091	1.178	1.265	1.352	1.439	1.526
80	0.572	0.660	0.747	0.835	0.923	1.010	1.098	1.185	1.273	1.360	1.448	1.535
85	0.575	0.664	0.752	0.840	0.928	1.016	1.104	1.192	1.279	1.367	1.455	1.543
90	0.579	0.668	0.756	0.844	0.933	1.021	1.109	1.198	1.286	1.374	1.462	1.550
95	0.582	0.671	0.760	0.849	0.937	1.026	1.114	1.203	1.291	1.380	1.468	1.557
100	0.585	0.675	0.763	0.852	0.941	1.030	1.119	1.208	1.297	1.385	1.474	1.563
105	0.588	0.677	0.767	0.856	0.945	1.034	1.123	1.213	1.302	1.391	1.480	1.569
110	0.590	0.680	0.770	0.859	0.949	1.038	1.128	1.217	1.306	1.395	1.485	1.574
115	0.593	0.683	0.773	0.862	0.952	1.042	1.131	1.221	1.310	1.400	1.489	1.579
120	0.595	0.685	0.775	0.865	0.955	1.045	1.135	1.225	1.314	1.404	1.494	1.584
125	0.597	0.697	0.778	0.868	0.958	1.048	1.138	1.228	1.318	1.408	1.498	1.588
130	0.599	0.690	0.780	0.870	0.961	1.051	1.141	1.232	1.322	1.412	1.502	1.592
135	0.601	0.692	0.782	0.873	0.963	1.054	1.144	1.235	1.325	1.415	1.506	1.596
140	0.603	0.694	0.784	0.875	0.966	1.056	1.147	1.238	1.328	1.419	1.509	1.600
145	0.604	0.695	0.786	0.877	0.968	1.059	1.150	1.240	1.331	1.422	1.513	1.603
150	0.606	0.697	0.788	0.879	0.970	1.061	1.152	1.243	1.334	1.425	1.516	1.606
155	0.608	0.699	0.790	0.881	0.972	1.064	1.155	1.246	1.337	1.428	1.519	1.610
160	0.609	0.701	0.792	0.883	0.974	1.066	1.157	1.248	1.339	1.430	1.522	1.613
165	0.610	0.702	0.794	0.885	0.976	1.068	1.159	1.250	1.342	1.433	1.524	1.616
170	0.612	0.704	0.795	0.887	0.978	1.070	1.161	1.253	1.344	1.435	1.527	1.618
175	0.613	0.705	0.797	0.888	0.980	1.072	1.163	1.255	1.346	1.438	1.529	1.621
180	0.614	0.706	0.798	0.890	0.982	1.073	1.165	1.257	1.348	1.440	1.532	1.623
185	0.615	0.707	0.799	0.891	0.983	1.075	1.167	1.259	1.351	1.442	1.534	1.626
190	0.616	0.709	0.801	0.893	0.985	1.077	1.169	1.261	1.352	1.444	1.536	1.628
195	0.617	0.710	0.802	0.894	0.986	1.078	1.170	1.262	1.354	1.446	1.538	1.630
200	0.619	0.711	0.803	0.895	0.988	1.080	1.172	1.264	1.356	1.448	1.540	1.632

i.e. $C_{pmk} > 1.208$. Consequently, the manufacturing yield is no less than 99.97% and the fraction of nonconformities is no greater than 290.08 ppm.

6 Data analysis and manufacturing capability computation

We collected sample data of the total unadjusted error from 150 current transmitters (displayed in Table 4). Figures 4 and 5 display the histogram and normal probability plot of the 150 observations. The sample data appears to be normal. The Shapiro-Wilk test is also used to check whether the sample data is normal. The statistic W is found to be 0.9934 with p-value 0.7283. Thus, we conclude that the sample data can be regarded as taken from a normal process.

In order to measure manufacturing capability of the current transmitter process, we execute the Matlab program to obtain the lower confidence bound on C_{pmk} (with process characteristic set to $\xi=(\mu-T)/\sigma=0.5$). The program reads the sample data file, and the input of sample size $n=150$, LSL $=-5$, USL $=5$, target value $T=0$, and confidence level $\gamma=0.95$, then outputs with the sample mean, $\bar{X}=0.187$, the sample standard deviation $S_n=1.081$, the estimator $\hat{C}_{pmk}=1.463$, and the lower confidence bound $C=1.299$. The actual program execution output is listed below. We therefore conclude that the true value of the process capability C_{pmk} is no less than 1.299 with a 95% level of confidence. The 2WCT manufacturing process is capable of reproducing products with a yield no less than 99.9902% and the fraction of nonconformities is no greater than 97.39ppm. We note that the conclusions made here have used the particular value of $\xi=0.5$ in finding the lower confidence bound; thus, the confidence level is ensured to be no less than 0.95 (or the Type I error is no greater than 0.05).

Table 3 Lower confidence bounds C of C_{pmk} for $\hat{C}_{\mathrm{pmk}} = 1.9(0.1)3.0$, $n = 5(5)200$, $\gamma = 0.95$

n	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
5	0.870	0.916	0.961	0.1007	1.057	1.107	1.157	1.218	1.274	1.329	1.402	1.492
10	1.133	1.195	1.257	1.319	1.381	1.453	1.525	1.587	1.649	1.700	1.780	1.850
15	1.268	1.338	1.417	1.487	1.557	1.626	1.696	1.765	1.840	1.909	1.984	2.050
20	1.342	1.421	1.495	1.569	1.643	1.720	1.796	1.870	1.944	2.020	2.094	2.168
25	1.396	1.473	1.550	1.626	1.703	1.780	1.857	1.934	2.011	2.088	2.165	2.242
30	1.442	1.521	1.600	1.679	1.758	1.837	1.916	1.995	2.074	2.153	2.232	2.311
35	1.477	1.558	1.639	1.720	1.800	1.881	1.962	2.042	2.123	2.203	2.284	2.365
40	1.506	1.588	1.670	1.752	1.834	1.916	1.998	2.079	2.161	2.243	2.325	2.407
45	1.529	1.612	1.695	1.778	1.861	1.944	2.027	2.110	2.193	2.276	2.359	2.442
50	1.548	1.632	1.716	1.800	1.884	1.968	2.052	2.136	2.219	2.303	2.387	2.471
55	1.565	1.649	1.734	1.819	1.903	1.988	2.073	2.157	2.242	2.327	2.411	2.496
60	1.579	1.664	1.750	1.835	1.921	2.006	2.091	2.176	2.262	2.347	2.432	2.518
65	1.592	1.678	1.764	1.850	1.936	2.021	2.107	2.193	2.279	2.365	2.451	2.537
70	1.603	1.690	1.776	1.862	1.949	2.035	2.122	2.208	2.295	2.381	2.467	2.554
75	1.613	1.700	1.787	1.874	1.961	2.048	2.135	2.221	2.308	2.395	2.482	2.569
80	1.622	1.710	1.797	1.884	1.972	2.059	2.146	2.234	2.321	2.408	2.495	2.583
85	1.631	1.718	1.806	1.894	1.981	2.069	2.157	2.245	2.332	2.420	2.508	2.595
90	1.638	1.726	1.814	1.902	1.990	2.079	2.167	2.255	2.343	2.431	2.519	2.607
95	1.645	1.734	1.822	1.910	1.999	2.087	2.175	2.264	2.352	2.441	2.529	2.617
100	1.652	1.740	1.829	1.918	2.006	2.095	2.184	2.272	2.361	2.450	2.538	2.627
105	1.658	1.747	1.836	1.925	2.014	2.102	2.191	2.280	2.369	2.458	2.547	2.636
110	1.663	1.752	1.842	1.931	2.020	2.109	2.198	2.288	2.377	2.466	2.555	2.644
115	1.668	1.758	1.847	1.937	2.026	2.116	2.205	2.295	2.384	2.473	2.563	2.652
120	1.673	1.763	1.853	1.942	2.032	2.122	2.211	2.300	2.391	2.480	2.570	2.660
125	1.678	1.768	1.858	1.948	2.037	2.127	2.217	2.307	2.397	2.487	2.577	2.666
130	1.682	1.772	1.862	1.953	2.043	2.133	2.223	2.313	2.403	2.493	2.583	2.673
135	1.686	1.777	1.867	1.957	2.047	2.138	2.228	2.318	2.408	2.499	2.589	2.679
140	1.690	1.781	1.871	1.962	2.052	2.142	2.233	2.323	2.414	2.504	2.594	2.685
145	1.694	1.784	1.875	1.966	2.056	2.147	2.237	2.328	2.419	2.509	2.600	2.690
150	1.697	1.788	1.879	1.970	2.060	2.151	2.242	2.333	2.423	2.514	2.605	2.696
155	1.700	1.792	1.882	1.973	2.064	2.155	2.246	2.337	2.428	2.519	2.610	2.700
160	1.704	1.795	1.886	1.977	2.068	2.159	2.250	2.341	2.432	2.523	2.614	2.705
165	1.707	1.798	1.889	1.980	2.072	2.163	2.254	2.345	2.436	2.527	2.619	2.710
170	1.710	1.800	1.892	1.984	2.075	2.166	2.258	2.349	2.440	2.532	2.623	2.714
175	1.712	1.804	1.895	1.987	2.078	2.170	2.261	2.353	2.444	2.535	2.627	2.718
180	1.715	1.807	1.898	1.990	2.081	2.173	2.264	2.356	2.448	2.539	2.631	2.722
185	1.717	1.809	1.901	1.993	2.084	2.176	2.268	2.359	2.451	2.543	2.634	2.726
190	1.720	1.812	1.904	1.995	2.087	2.179	2.271	2.363	2.454	2.546	2.638	2.730
195	1.722	1.814	1.906	1.998	2.090	2.182	2.274	2.366	2.457	2.549	2.641	2.733
200	1.724	1.816	1.909	2.000	2.093	2.185	2.277	2.369	2.461	2.553	2.644	2.736

Table 4 The collected 150 sample observations (μA)

0.10	0.84	−0.28	−0.14	−0.46	−0.54	0.76	0.08	−0.90	0.58
0.16	0.01	0.64	−1.02	−2.33	0.24	0.22	−1.17	0.50	0.78
0.76	−2.03	1.03	0.00	−1.12	−0.63	−0.07	−1.60	−1.15	1.64
−0.43	0.38	2.55	1.54	1.39	0.88	1.63	−0.54	−0.15	−0.37
0.07	1.98	−1.26	−1.00	0.11	−0.05	2.28	0.54	−0.81	0.52
−0.25	1.35	−0.89	0.93	0.65	0.76	−0.34	−0.37	−1.06	0.22
0.14	−1.51	1.37	−0.43	1.27	0.97	0.34	−1.24	−0.89	−0.41
1.92	0.14	−0.20	0.84	−2.10	0.14	−0.66	1.41	−0.21	2.58
−0.44	−0.52	−1.29	−0.98	−0.48	1.21	0.98	−0.55	0.42	−0.05
−1.25	−0.90	0.58	0.32	−0.54	2.77	−2.37	0.22	0.10	−1.32
0.75	−1.13	1.94	−1.98	−0.89	0.81	1.32	0.23	1.40	2.18
−0.76	0.55	1.01	−0.31	0.03	0.22	0.47	−0.04	−0.04	0.59
0.27	−0.24	2.38	0.74	1.90	1.23	0.52	0.67	−1.44	−1.00
−0.46	0.29	0.79	−0.12	0.19	0.29	1.56	−0.06	0.24	0.91
0.82	−0.17	2.28	1.59	1.58	−0.99	3.07	−1.60	0.31	1.63

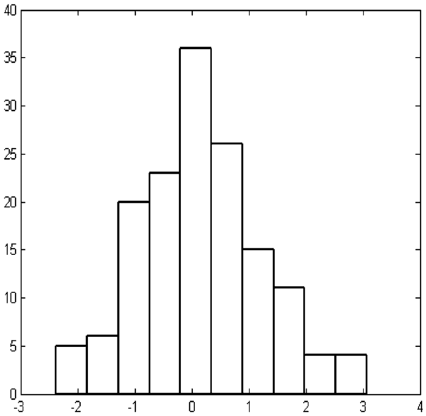

Fig. 4 Histogram of the sample data

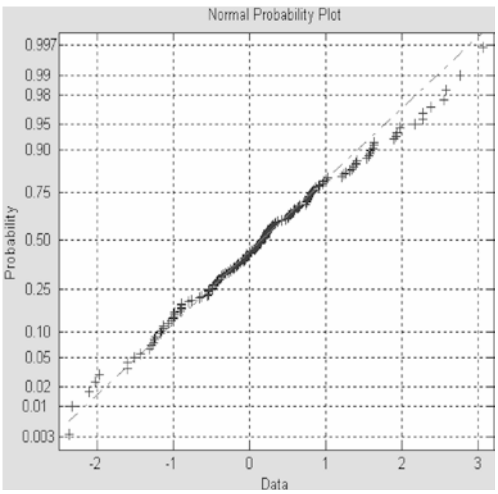

Fig. 5 The normal probability plot

6.1 Matlab execution input and output

```
  Input:
  Enter values of sample size, lower
specification limit, upper specification
limit, target value, confidence level:
150,-5,5,0,0.95
  Output:
  The Sample Mean is 0.186589.
  The   Sample   Standard   Deviation   is
1.08109.
  The Epsilon 0.5.
  The Estimate of Cpmk is 1.4625.
  The  true  value  of  the  manufacturing
capability Cpmk is no less than 1.299 with
0.95 level of confidence.
```

6.2 Multiple control chart samples application

Many of the existing 2WCT IC manufacturing factories have implemented a daily-based production control plan for monitoring/controlling process stability. A routine data collection procedure is executed to run $\bar{X}$ and S^2 control charts (for moderate sample sizes). The past "in control" data, consisting of multiple samples of m groups with variable sample size $n_i = (x_{i1}, x_{i2}, \ldots, x_{in_i}))$, is then analysed to compute the manufacturing capability. Thus, manufacturing information regarding product quality characteristics is derived from multiple samples rather than one single sample. Under the assumption that these samples are taken from the normal distribution $N(\mu, \sigma^2)$, we consider the following estimators of process mean and process standard deviation:

$$\bar{X}_i = \sum_{j=1}^{n_i} x_{ij}/n_i$$

$$S_i = \left[(n_i)^{-1}\sum_{j=1}^{n_i}\left(x_{ij} - \bar{X}_i\right)^2\right]^{1/2}$$

for the ith sample mean and the sample standard deviation, respectively. Then, $\bar{\bar{X}} = \sum_{i=1}^{m_s} \bar{X}_i/m_s$ and $S_p^2 = \sum_{i=1}^{m_s} n_i S_i^2 / \sum_{i=1}^{m_s} n_i$ are used for calculating the manufacturing capability C_{pmk}. For cases with multiple samples the natural estimator of C_{pmk} can be expressed as below. The derivations of the sampling distribution, lower confidence bounds, and the manufacturing capability calculations for cases with multiple samples can be performed using the same techniques for cases with one single sample, although the derivations and calculations may be more tedious and complicated.

$$\hat{C}_{\text{pmk}} = \min\left\{\frac{\text{USL} - \bar{\bar{X}}}{3\sqrt{S_p^2 + (\bar{\bar{X}} - T)^2}}, \frac{\bar{\bar{X}} - \text{LSL}}{3\sqrt{S_p^2 + (\bar{\bar{X}} - T)^2}}\right\}.$$

6.3 MPPAC control chart application

For factories having a group of processes that need to be monitored and controlled, it would be effective to use the MPPAC (multi-process performance analysis chart). The MPPAC can be used to illustrate and analyse the manufacturing capability for multiple processes, which conveys critical information regarding the departure of the process mean from the target value, process variability, and capability levels, and provides a guideline of directions for capability improvement. Singhal [13] introduced the C_{pk} MPPAC for monitoring multiple processes. Pearn and Chen [6] proposed a modification to the C_{pk} MPPAC, adding the more advanced capability index C_{pm} to identify the problems causing the processes to fail to centre around the target. Pearn et al. [9] developed the MPPAC based on the incapability index C_{pp}. Using the same technique, the C_{pmk} MPPAC can be developed to monitor the capability for multiple 2WCT IC manufacturing processes. Using the C_{pmk} MPPAC, practitioners or engineers can simultaneously analyse the performance of multiple processes based on one single chart. The C_{pmk} MPPAC also prioritizes the order of the processes that the quality improvement effort should focus on, either to move the process mean closer to the target value or reduce the process variation. The developed confidence lower bounds can then be applied to the C_{pmk} MPPAC to ensure the accuracy of the MPPAC for given sample sizes.

7 Conclusions

Combining the merits of the two earlier indices, C_{pk} and C_{pm}, the index C_{pmk} has been proposed to provide numerical measures of process performance. The index C_{pmk} takes into account the location of the process mean between the two specification limits, the proximity to the target value, and the process variation. It has been shown to be a useful capability index for processes with two-sided specification limits. Based on the complicated probability density function of the natural estimator of C_{pmk}, we developed an efficient algorithm to compute the lower confidence bounds on C_{pmk}. The lower confidence bound presents a measure of the minimum capability of the process based on the sample data. We investigated the behaviour of the lower confidence bound values versus the process characteristic parameter, $\xi = (\mu - T)/\sigma$, and concluded that the lower confidence bound obtains its minimal value at $\xi = 0.5$. The proposed decision making procedure ensures that the risk of making wrong decision will be no greater than the preset Type I error $1-\gamma$. We also provided a Matlab computer program for engineers or practitioners to use in measuring their processes. A real-world example on two-wire current transmitter integrated circuit (2WCT IC) manufacturing process, taken from a microelectronics device manufacturing factory, was investigated.

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