Spin-current generation and detection in the presence of an ac gate

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We predict that in a narrow gap III-V semiconductor quantum well or quantum wire, an observable electron spin current can be generated with a time-dependent gate to modify the Rashba spin-orbit coupling constant. Methods to rectify the so generated ac current are discussed. An all-electric method of spin-current detection is suggested, which measures the voltage on the gate in the vicinity of a two-dimensional electron gas carrying a time-dependent spin current. Both the generation and detection do not involve any optical or magnetic mediator.

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One key issue in spintronics based on semiconductor is the efficient control of the spin degrees of freedom. Datta and Das[1] suggested the use of gate voltage to control the strength of Rashba spin-orbit interaction (SOI)[2] which is strong in narrow gap semiconductor heterostructures. In InAs-based quantum wells a variation of 50% of the SOI coupling constant was observed experimentally.[3,4] Consequently, much interest has been attracted to the realization of spin-polarized transistors and other devices based on using electric gate to control the spin-dependent transport.[5]

In addition to using a static gate to control the SOI strength and so control the stationary spin transport, new physical phenomena can be observed in time-dependent spin transport under the influence of a fast varying gate voltage. Along this line, in this article we will consider a mechanism of ac spin current generation using time-dependent gate. This mechanism employs a simple fact that the time variation of Rashba SOI creates a force which acts on opposite spin electrons in opposite directions. Inversely, when a gate is coupled to a nearby electron gas, the spin current in this electron gas also induces a variation of the gate voltage, and hence affects the electric current in the gate circuit. We will use a simple model to clarify the principle of such a new detection mechanism without any optical or magnetic mediator. The systems to be studied will be 1D electron gas in a semiconductor quantum wire (QWR) and 2D electron gas in a semiconductor quantum well (QW).

We consider a model in which the Rashba SOI is described by the time-dependent Hamiltonian $H_{\mathrm{SO}}(t)=\hbar\alpha(t)\times(\vec{k}\times\hat{\nu})\cdot\vec{s}$, where $\vec{k}$ is the wave vector of an electron, $\hbar\vec{s}$ is the spin operator, and $\hat{\nu}$ is the unit vector. For a QWR $\hat{\nu}$ is perpendicular to the wire axis, and for a QW perpendicular to the interfaces. The time dependence of the coupling parameter $\alpha(t)$ is caused by a time-dependent gate.[6] To explain clearly the physical mechanisms leading to the spin-current generation, we will first consider the 1D electron gas in a QWR, and assume $\alpha(t)$ to be a constant α for $t<0$, and $\alpha(t)=0$ for $t>0$. For the 1D system we choose the x direction as the QWR axis and y axis parallel to $\hat{\nu}$, to write the SOI coupling in the form $H_{\mathrm{SO}}(t)=\hbar\alpha(t)k_x s_z$. For $t<0$ the spin degeneracy of conduction electrons is lifted by SOI, producing a splitting $\Delta=\hbar\alpha k_x$ between $s_z=1/2$ and $s_z=-1/2$ bands, as shown in Fig. 1 by solid curves together with the Fermi energy E_F. The spin current in this state is zero, as it should be under thermal equilibrium.

Indeed, the spin current is defined as $I_s(t)=I_\uparrow(t)-I_\downarrow(t)$, where $I_\uparrow(t)$ [or $I_\downarrow(t)$] is the partial current associated with the spin projections $s_z=1/2$ (or $s_z=-1/2$). Hence,

$$I_s(t)=\frac{\hbar}{2L}\sum_{E(k_x)<E_F}[v_\uparrow(k_x)-v_\downarrow(k_x)], \tag{1}$$

where L is the length of the QWR. Taking the momentum derivative of the Hamiltonian, we obtain the velocity as

$$v_{\uparrow,\downarrow}(k_x)=\hbar k_x/m^*\pm\alpha(t)/2. \tag{2}$$

The spin current is then readily obtained as

$$I_s(t)=(\hbar n/4m^*)(\hbar k_\uparrow-\hbar k_\downarrow)+\hbar\alpha(t)n/4, \tag{3}$$

where n is the 1D electron density, and $k_\uparrow$ (or $k_\downarrow$) is the average momentum in the $\uparrow$-spin (or $\downarrow$-spin) band.

For a parabolic band $\hbar k_\uparrow=-m^*\alpha/2$ and $\hbar k_\downarrow=m^*\alpha/2$. Although $\hbar k_\uparrow-\hbar k_\downarrow$ gives a finite contribution to $I_s(t)$ in Eq. (3), for $t<0$ where $\alpha(t)=\alpha$, this contribution is compensated by the contribution $\hbar\alpha n/4$ due to the SOI. Hence, the total spin current $I_s(t)=0$ for $t<0$. However, when the SOI is switched off at $t=0$, $\alpha(t)=0$ and so the spin current is finite, because the average electron momenta retain the same as they were at $t<0$. As time goes on, the electron momenta relax with a relaxation time τ. Therefore, $I_s(t)=-(\hbar\alpha n/4)\exp(-t/\tau)$ for $t>0$.

It is instructive to make a Fourier transform of $I_s(t)$ to obtain a Drude-like expression

$$I_s(\Omega)=\left[\frac{\tau\hbar n}{2m^*(i\Omega\tau-1)}\right]\left[\frac{m^*}{2}i\Omega\alpha(\Omega)\right]. \tag{4}$$

Since the units of our spin current is $\hbar/2$, the above expression is a complete analogy to the electric conductivity. Instead of an electric driving force eE, here we have an

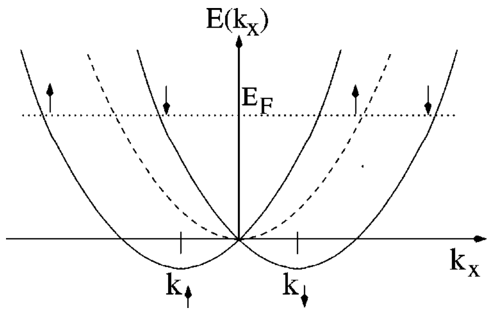

FIG. 1. The dashed curve is the electron energy band without SOI. The SOI splits the energy band into the ↑-spin and the ↓-spin bands, as shown by the solid curves, with corresponding average wave vectors $k_\uparrow$ and $k_\downarrow$.

equivalent driving force $(m^*/2)[d\alpha(t)/dt]$, the Fourier component of which is $(m^*/2)i\Omega\alpha(\Omega)$. Under this driving force we have the classic equation of motion

$$m^*\frac{dv_{\uparrow,\downarrow}}{dt}=\pm\frac{m^*}{2}\frac{d\alpha(t)}{dt}. \tag{5}$$

This force acts in opposite directions on electrons with opposite spin projections. When such a force creates a spin current, it does not induce an electric current.

The above conclusion of spin-current generation can be demonstrated with a rigorous linear response analysis, which will be performed on a 2D electron gas (2DEG). The simple Drude expression (4) will then appear as a general result. Let the 2DEG be in the xy plane with the unit vector $\hat{\nu}$ along the z axis, which is the spin-quantization axis. We will use the equation of motion for the spin-density operator to generalize the 1D expressions (1), (2) for the spin current. For a homogeneous system the spin-current density operators can be expressed in terms of the electron creation operator $c^\dagger_{\vec{k},\gamma}$ and destruction operator $c_{\vec{k},\gamma}$, where γ labels the spin projection onto the z axis. This current is then derived as

$$\mathcal{J}^i_j=J^i_j+J^i_{j,\mathrm{SOI}}, \tag{6}$$

where the superscript $i=x,y,z$ specifies the direction of spin polarization, and the subscript $j=x,y$ refers to the direction of the spin-current flow. The partial current

$$J^i_j=\sum_{\vec{k}}\sum_{\gamma\beta}\frac{\hbar^2 k_j}{m^*}c^\dagger_{\vec{k},\gamma}s^i_{\gamma\beta}c_{\vec{k},\beta} \tag{7}$$

is the ordinary kinematic term and

$$J^i_{j,\mathrm{SOI}}=\varepsilon^{ijz}\hbar\alpha n/4 \tag{8}$$

is the contribution of SOI.[7] Here ε^{ijz} denotes the Levy-Civita symbol. The SOI induced current resembles the diamagnetic current of electrons under the action of an external electromagnetic vector potential.

We note that the SOI Hamiltonian can be conveniently written in terms of the kinematic current as

$$H_{\mathrm{SO}}(t)=[m^*\alpha(t)/\hbar](J^x_y-J^y_x). \tag{9}$$

When an ac bias with frequency Ω is applied to the front or the back gate of a 2DEG,[3,4] the Rashba coupling constant contains two terms $\alpha(t)=\alpha_0+\delta\alpha(t)$, where α_0 is constant in time and $\delta\alpha(t)=\delta\alpha e^{i\Omega t}$. We assume that the only effect of the ac bias is to add a time-dependent component to the SOI coupling constant, although in practice it is not simple to avoid the bias effect on the electron density.[4] The SOI Hamiltonian is separated correspondingly into two parts $H_{\mathrm{SO}}(t)=H^0_{\mathrm{SO}}+H'_{\mathrm{SO}}(t)$. The time-independent part H^0_{SO} does not produce a net spin current in the thermodynamically equilibrium state. However, as pointed out in the above analysis on the 1DEG system, the time-dependent $H_{\mathrm{SO}}(t)$ can give rise to a spin current.

We will incorporate H^0_{SO} into our unperturbed Hamiltonian and treat $H'_{\mathrm{SO}}(t)$ within the linear response regime. The so-generated ac spin current $\langle\mathcal{J}^i_j(t)\rangle$ has the form

$$\langle\mathcal{J}^i_j(t)\rangle=\frac{i}{\hbar}\int_{-\infty}^{t}dt'\langle[H'_{\mathrm{SO}}(t'),J^i_j(t)]\rangle+\varepsilon^{ijz}\hbar\,\delta\alpha(t)n/4. \tag{10}$$

In the above equation the first term can be written in the form $\delta\alpha(t)\mathcal{R}^i_j(\Omega)$. For zero temperature and with $\Omega>0$, the response function $\mathcal{R}^i_j(\Omega)$ can be represented as the Fourier transform of the correlator

$$\mathcal{R}^i_j(t)=-i\frac{\hbar^2}{m^*}\sum_{\vec{k}'\alpha'\beta'}k'_j s^i_{\alpha'\beta'}\times\sum_{\vec{k}\alpha\beta}\vec{h}_{\vec{k}}\cdot\vec{s}_{\alpha\beta}\overline{\langle T\{c^\dagger_{\vec{k}',\alpha'}(t)c_{\vec{k}\beta}\}\rangle\langle T\{c_{\vec{k}'\beta'}(t)c^\dagger_{\vec{k}\alpha}\}\rangle}, \tag{11}$$

where $\vec{h}_{\vec{k}}=\vec{k}\times\hat{\nu}$. In the above equation, the bar over the product of two one-particle Green functions means an ensemble average over impurity positions.

We will use the standard perturbation theory[9] to calculate this ensemble average, which is valid when the elastic scattering time τ due to impurities is sufficiently long such that $E_F\tau\gg\hbar$. We will assume that the electron Fermi energy E_F is much larger than both $\hbar\Omega$ and $\hbar\alpha_0 h_{\vec{k}}$. To the first-order approximation, we neglect the weak localization corrections to the correlator (11), since these corrections simply renormalize the spin-diffusion constant.[8] Consequently, the configuration average of the pair product of Green functions is expressed in the so-called ladder series.[9] We found that since $\vec{h}_{\vec{k}}=-\vec{h}_{-\vec{k}}$ many of such ladder diagrams vanish after angular integration in Eq. (11), similar to suppression of ladders in the electric current driven by the vector potential.[9] At the same time, some of nondiagonal on spin indice diagrams do not turn to 0 after the angular integration. Employing the analysis of similar diagrams done in it can be shown that they cancel each other.[8] Hence, the configuration average in Eq. (11) decouples into a product of average Green functions and Eq. (11) becomes

$$\mathcal{R}_j^i(\Omega) = -i\frac{\hbar^2}{m^*}\sum_{l,n}\varepsilon^{lnz}\sum_{\vec{k}} k_j k_n \times \int \frac{d\omega}{2\pi}\mathrm{Tr}[s^l G(\vec{k},\omega)s^i G(\vec{k},\omega+\Omega)], \quad (12)$$

where $G(\vec{k},\omega)$ is the average Green's function which contains fully the effect of $H^0_{\rm SO}$. This function is represented by the 2×2 matrix

$$G(\vec{k},\omega) = [\omega - E_{\vec{k}}/\hbar - \alpha_0 \vec{h}_{\vec{k}}\cdot\vec{s} + i\Gamma\,\mathrm{sgn}(\omega)]^{-1}, \quad (13)$$

where $\Gamma = 1/2\tau$, and $E_{\vec{k}}$ is defined with respect to E_F. Substituting Eq. (13) into Eq. (12), and then into Eq. (10), we obtain the spin current

$$\langle \mathcal{J}_j^i(\Omega)\rangle = \varepsilon^{ijz}\frac{\hbar}{4}\delta\alpha n \frac{\Omega}{\Omega + 2i\Gamma}. \quad (14)$$

It is important to point out that the spin density under the gate area is zero. This is the reason why even in a 2DEG the D'yakonov-Perel spin relaxation[10] does not appear in Eq. (14) for the generated spin current, although this spin current is determined by the response function (12) which involves spin degrees of freedom. Hence, in the homogeneous system with zero spin density, only electron momentum relaxation occurs in the process of spin-current generation by a time-dependent gate.

Unlike the spin current (4) in a 1D system, in a 2DEG the current given by Eq. (14) has no specific direction. To clarify the spatial distribution of the spin flux induced by an ac gate, let us take the chiral component $\mathcal{J}_{\rm chir}(t)$ of the spin current

$$\mathcal{J}_{\rm chir}(t) = [\langle \mathcal{J}_y^x(t)\rangle - \langle \mathcal{J}_x^y(t)\rangle]/2. \quad (15)$$

It is easily seen that this chiral projection has the same form as the expression (4) for a 1D system, if n represents the electron density of the 2DEG. In Fig. 2 we illustrate the spin-current distribution for a circular gate which is marked as the gray area. The spin polarization at any point under the gate has two components parallel to the 2DEG. For any direction specified by the unit vector $\vec{N}$, the two spin-polarized fluxes with polarization directions parallel and antiparallel to $\vec{N}$ will oscillate out of phase by the amount of π along the direction perpendicular to $\vec{N}$. Such out of phase oscillation is schematically plotted in Fig. 2. The amplitude of the spin density flow in each of the opposite directions, as marked by the dashed-line arrows, is just $\mathcal{J}_{\rm chir}(t)$. In the 2DEG outside the gate area, the spin current can be supported only by spin diffusion. Therefore the chiral ac spin polarization is accumulated in the vicinity of the circumference of the gate, and from where diffuses away from the gate area. It can also diffuse under the gate. For small gates such back diffusion can diminish the efficiency of the spin generation. On the other hand, for large gates with the size larger than the spin-diffusion length the diffusion counterflow does not reduce much the total spin current.

The so-generated current amplitude can be easily estimated. With $\delta\alpha = 3\times 10^6$ cm/s,[4] for $\Omega = 2\pi\times 10^9\ \mathrm{s}^{-1}$, $n = 10^{12}\ \mathrm{cm}^{-2}$, and $\tau = 1$ ps, from (14) we derive $(2e/\hbar)\times\langle \mathcal{J}_j^i(\Omega)\rangle \simeq 10^{-3}$ Amp/cm. This ac spin current can be detected by various methods. For example, if holes can tunnel into the neighborhood of the gate edge, their recombination with spin-polarized electrons will produce the emission of circular-polarized light.[11]

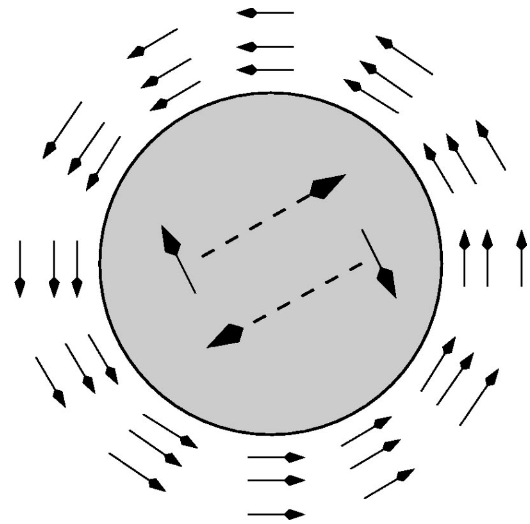

FIG. 2. Distribution of spin currents induced by a time-dependent circular gate which is marked as the gray region. Under the gate, electrons with opposite spins (solid arrows) move in opposite directions indicated by the dashed-line arrows. Arrows outside the gate area show the accumulated spin polarization during a half period of ac gate voltage oscillation.

However, here we will discuss a method of direct electric detection of the dc or the ac spin current. This method is based on a simple fact that the Rashba SOI couples the spin current to the gate voltage. We have shown in our above analysis that due to this coupling, spin current can be induced by a time-dependent gate voltage. In this case the voltage variation plays the role of a source which drives electrons out of thermodynamic equilibrium, and the spin current is the linear response to this perturbation. The reverse process is to create a spin current in a 2DEG by some source, and so inducing a voltage shift in a nearby gate. This is also possible to realize. We thus consider a model where the SOI constant $\alpha(U)$ is a function of the gate voltage $U(t) = U_0 + V(t)$. U_0 is the static equilibrium value in the absence of a spin current, while $V(t)$ is a dynamic variable. The mean value $\langle V\rangle$ of $V(t)$ has to be calculated as a linear response to the perturbation associated with the presence of the spin-polarization flow. The explicit form of this perturbation can be obtained by averaging the Hamiltonian of the system over an electronic state with the given time-dependent spin current.

Let $\langle\cdots\rangle_J$ be such type of average. To the lowest order with respect to SOI, the coupling of the gate voltage to the spin current is thus determined by the average of the Rashba interaction in Eq. (9) with $\alpha = \alpha(U)$. The coupling between the gate voltage $U(t)$ and the spin current $\mathcal{J}_j^i$ is via the kinetic current J_j^i. To derive the coupling Hamiltonian $H_{\rm int}$, we use Eq. (6) to express J_j^i in terms of $\mathcal{J}_j^i$, and expand

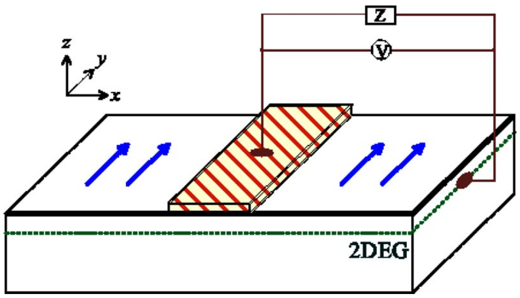

FIG. 3. Schematic illustration of spin current detection. ac spin current flows from the right to the left under the gate with spin polarized as shown by arrows. V denotes the voltmeter and Z is the outer circuit impedance.

$\alpha(U)=\alpha(U_0)+\alpha' V(t)$ for small $V(t)$. The coupling Hamiltonian is then derived from Eq. (9) as

$$H_{\mathrm{int}}=\frac{m^*\alpha'}{\hbar}V[\langle \mathcal{J}_y^x\rangle_J-\langle \mathcal{J}_j^y\rangle_J]. \quad (16)$$

The charging of the gate $Q=CV$ is related to the gate capacitance. Hence, Eq. (16) can be expressed in the convenient form $H_{\mathrm{int}}=Q\mathcal{E}$, where

$$\mathcal{E}=\frac{m^*\alpha'}{\hbar C}[\langle \mathcal{J}_y^x\rangle_J-\langle \mathcal{J}_j^y\rangle_J] \quad (17)$$

is the effective electromotive force.

To illustrate our proposed method of direct electric detection, let us consider a circuit connected to the gate. The principal scheme of the spin current detection is shown in Fig. 3. In it, an additional back gate can be utilized to tune the electron density (not shown). The circuit is characterized by a frequency-dependent impedance $Z(\Omega)$. The voltage induced on the gate by the electromotive force (17) is then easily obtained as

$$\langle V\rangle=\mathcal{E}\frac{i\Omega CZ(\Omega)}{1+i\Omega CZ(\Omega)}. \quad (18)$$

When the spin-current frequency is in resonance with the circuit eigenmode, the gate voltage becomes very large. In the limit of high impedance (open circuit), $\langle V\rangle=\mathcal{E}$. Using the spin current $(2e/\hbar)\langle \mathcal{J}_j^i(\Omega)\rangle \simeq 10^{-3}$ Amp/cm derived above, and the fact that $\langle \mathcal{J}_j^i\rangle_J=A\langle \mathcal{J}_j^i(\Omega)\rangle$, where A is the area under the gate, let us estimate the electromotive force induced in a probe gate by this spin current generated by a nearby source gate. For the reasonable parameter values $\alpha'=3\times10^7$ cm/Vs,[4] $m^*=0.03\, m_e$, and $C=\kappa\epsilon_0 A/l$ with $\kappa=10$ and $l=10^{-5}$ cm, from Eqs. (14) and (18) we obtain $\mathcal{E}\simeq 10^{-5}$ V.

The generated ac spin current can be rectified with various methods. For example, one can use a shutter gate which is $\pi/2$ phase shifted with respect to the generation gate. The shutter gate can be placed in the neighborhood of the generation gate or between two such gates. The evaluation of the rectifying efficiency of such a setup requires a thorough analysis of spin relaxation and diffusion processes caused by the spin accumulation during the shutter cycle.

We would like to add one relevant piece of information which we became aware of after we completed this paper. The preprint of Governale *et al.* on the quantum-spin pumping in a 1D wire is also based on the idea of creating spin current via a time-dependent gate.[12] However, our results involving dissipative transport in 2DEG and 1DEG cannot be compared directly to those in Ref. 12.

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