

## FREE-CARRIER ABSORPTION IN QUANTUM WELL STRUCTURES FOR ACOUSTIC PHONON SCATTERING

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(Received 21 August 1994)

Free-carrier absorption has been studied for quantum well structures fabricated from III-V semiconducting materials where the acoustic phonon scattering is important. The energy band of carriers is assumed to be nonparabolic. We discuss the effect of acoustic phonon scattering on the free-carrier absorption for both deformation-potential coupling and piezoelectric coupling. It is found that the free-carrier absorption coefficient depends upon the polarization of the electromagnetic radiation relative to the layer plane or quantum well, the photon frequency, and the temperature. When the deformation-potential coupling is dominant, the free-carrier absorption coefficient increases with increasing temperature for photons polarized in the layer plane or perpendicular to the layer plane. However, when the piezoelectric coupling is dominant, the free-carrier absorption coefficient increases with increasing temperature for photons polarized in the layer plane, but for photons polarized perpendicularly to the layer plane, the free-carrier absorption coefficient decreases with increasing temperature. Moreover, at high temperatures such as  $T = 300$  K, the free-carrier absorption coefficient oscillates with the film thickness in a small quantum well region and then decreases monotonically with increasing the film thickness. This is different from the result for three-dimensional semiconducting solids.

### 1. Introduction

Due to the confinement of carriers in quasi-two-dimensional structures, the size quantization begins to play an important role in determining the electronic and optical properties of carriers in semiconductors. In a magnetic field where the carrier motion is confined to a plane perpendicular to the magnetic field, the free-carrier absorption coefficient ( $\alpha$ ) depends upon the radiation field polarization relative to the magnetic field [1]. For carriers confined in a quasi-2D structure, it was found that  $\alpha$  depends upon the radiation field polarization relative to the direction normal to the quasi-2D structure [2]. In III-V semiconductors, the phonon-carrier interaction is dominated by the deformation-potential and piezoelectric couplings [3]. In this work, we investigate the quantum theory of the free-carrier absorption in III-V semiconductors with the nonparabolic energy band of electrons with a quasi-2D structure.

### 2. Theory

For a square well potential along the  $z$  axis with infinitely high barriers at  $z = 0$  and  $z = d$ , the electron field operator  $\Psi^\dagger(\mathbf{r})$  is given by [4]

$$\Psi^\dagger(\mathbf{r}) = \left(\frac{2}{V}\right)^{1/2} \sum_{n=1}^{\infty} \sum_{\mathbf{k}} b_{\mathbf{k}n}^\dagger \exp(-i\mathbf{k} \cdot \mathbf{x}) \sin\left(\frac{n\pi z}{d}\right) \quad (1)$$

where  $\mathbf{r} = (\mathbf{x}, z) = (x, y, z)$ ,  $V = dS$  is the film volume with the surface area  $S$  and the film thickness  $d$ ,  $\mathbf{k} = (k_x, k_y)$  is the electron wave vector in the  $x$ - $y$  plane, and  $b_{\mathbf{k}n}^\dagger$  is the electron creation operator. The electron energy  $E_{\mathbf{k}n}$  is given by the relation [3]

$$E_{\mathbf{k}n} \left(1 + E_{\mathbf{k}n} / E_g\right) = \frac{\hbar^2 \mathbf{k}^2}{2m^*} + \frac{\pi^2 \hbar^2 n^2}{2m^* d^2}, n = 1, 2, 3, \dots \quad (2)$$

where  $m^*$  is the effective mass of electron. Since

$(\hbar k_{\max})^2/2m^* + (\pi\hbar n)^2/2m^*d^2 \cong k_B T < E_g$  for  $T \leq 300$  K, then

$$E_{kn} \cong -\frac{1}{2}E_g + \frac{1}{2}E_g a_n + \hbar^2 \mathbf{k}^2 / 2m^* a_n \quad (3)$$

with

$$a_n \cong \left[ 1 + 2(\pi\hbar n)^2 / m^* d^2 E_g \right]^{\frac{1}{2}} \quad (4)$$

The absorption coefficient  $\alpha$  for the absorption of photons can be expressed as [2]

$$\alpha = \frac{\epsilon^{1/2}}{n_0 c} \sum_i W_i f_i \quad (5)$$

where  $\epsilon$  is the dielectric constant of material,  $n_0$  is the number of photons in the radiation field,  $f_i$  is the distribution function of carriers and  $W_i$  is the transition probability. Using the Born approximation, we have

$$W_i \cong \frac{2\pi}{\hbar} \sum_f \left[ |\langle f|M_x|i \rangle|^2 \delta(E_f - E_i - \hbar\Omega - \hbar\omega_q) + |\langle f|M_z|i \rangle|^2 \delta(E_f - E_i - \hbar\Omega + \hbar\omega_q) \right] \quad (6)$$

where  $E_i$  and  $E_f$  are the initial and final electron energies,  $\hbar\Omega$  is the photon energy, and  $\hbar\omega_q$  is the phonon energy. For interaction between electrons, photons, and phonons, the transition matrix elements are given by

$$\langle f|M_x|i \rangle = \sum_j \left[ \frac{\langle f|H_{rad}|j \rangle \langle j|V_s|i \rangle}{E_j - E_i \mp \hbar\omega_q} + \frac{\langle f|V_s|j \rangle \langle j|H_{rad}|i \rangle}{E_j - E_i - \hbar\Omega} \right] \quad (7)$$

where  $H_{rad}$  is the electron-photon interaction and  $V_s$  is the scattering potential due to the electron-phonon interaction.

(i) When the photon is polarized in the layer plane,

$$\langle \mathbf{k}'n'|H_{rad}|\mathbf{k}n \rangle = -\frac{e\hbar^{3/2}}{m^*} \left( \frac{2m_0}{\epsilon\Omega V} \right)^{1/2} \mathbf{e} \cdot \mathbf{k} \delta_{n',n} \delta_{\mathbf{k}',\mathbf{k}} \quad (8)$$

(ii) When the photon is polarized perpendicularly to the layer plane,

$$\langle \mathbf{k}'n'|H_{rad}|\mathbf{k}n \rangle = -\frac{ie\hbar^{3/2}}{m^*} \left( \frac{2m_0}{\epsilon\Omega V} \right)^{1/2} \left( \frac{n}{d} \right) \delta_{\mathbf{k}',\mathbf{k}} \times \left\{ \frac{1 - \cos[\pi(n'+n)]}{n'+n} + \frac{1 - \cos[\pi(n'-n)]}{n'-n} \right\} \quad (9)$$

where  $\mathbf{e}$  is the photon polarization vector.

The distribution function for a quasi-2D nondegenerate electron gas can be expressed as

$$f_{kn} = \left( \frac{n_0 d \hbar^2}{2\pi^2 m^* k_B T} \right) \left[ \sum_{t=1}^{\infty} a_t \exp\left( -\frac{E_g a_t}{2k_B T} \right) \right]^{-1} \times \exp\left[ -\frac{E_g a_n}{2k_B T} - \frac{\hbar^2 \mathbf{k}^2}{2m^* k_B T a_n} \right] \quad (10)$$

where  $n_0$  is the concentration of electron.

For III-V semiconductors, there are two dominant electron-phonon scattering mechanisms:

(A) Deformation-Potential Coupling:

When the deformation-potential coupling is dominant,

$$\langle \mathbf{k}'n'|V_s|\mathbf{k}n \rangle = \frac{1}{2} \left( \frac{k_B T}{2\rho v_s^2 V} \right)^{1/2} E_d \delta_{k_x, k_x+q_x} \delta_{k_y, k_y+q_y} \times [\delta_{q_z, (\pi/d)(n'-n)} + \delta_{q_z, -(\pi/d)(n'-n)} - \delta_{q_z, (\pi/d)(n'+n)} - \delta_{q_z, -(\pi/d)(n'+n)}] \quad (11)$$

where  $\rho$  is the density of material,  $v_s$  is the sound velocity,  $\mathbf{q}$  is the phonon wave-vector, and  $E_d$  is the deformation potential.

(B) Piezoelectric Coupling:

When the piezoelectric coupling is dominant,

$$\langle \mathbf{k}'n'|V_s|\mathbf{k}n \rangle = \frac{1}{2} \left( \frac{k_B T}{2\rho v_s^2 V} \right)^{1/2} \frac{|e|\beta_p}{\epsilon q} \delta_{k_x, k_x+q_x} \delta_{k_y, k_y+q_y} \times [\delta_{q_z, (\pi/d)(n'-n)} + \delta_{q_z, -(\pi/d)(n'-n)} - \delta_{q_z, (\pi/d)(n'+n)} - \delta_{q_z, -(\pi/d)(n'+n)}] \quad (12)$$

where  $\beta_p$  is the appropriate piezoelectric constant. In here we consider the phonon scattering in which electrons are confined in the x-y plane, but acoustic phonons have a motion along z direction. From Eqs. (1), (3), and (5) - (11), we may obtain the absorption coefficient  $\alpha_d$  for the deformation-potential coupling and  $\alpha_p$  for the piezoelectric coupling.

### 3. Numerical Results and Conclusion

The relevant values of physical parameters for n-type GaAs thin films are taken to be [3]  $n_c = 1.73 \times 10^{15} \text{ cm}^{-3}$ ,

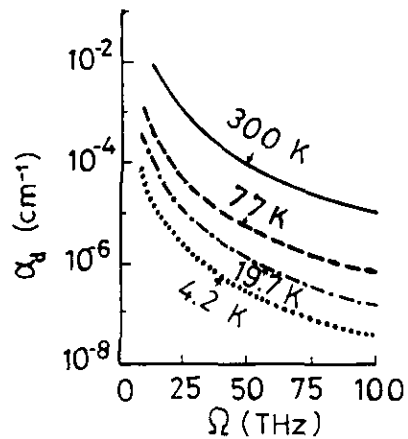


Fig. 1.  $\alpha_d$  as a function of the photon frequency with the quantum well  $d = 1 \mu\text{m}$  for photons polarized in the layer plane.

$m^* = 0.07m_0$  ( $m_0$  is the mass of free electron),  $\rho = 5.32$  gm/cm<sup>3</sup>,  $\epsilon = 12.9$ ,  $E_g = 1.51$  eV,  $E_d = 7$  eV,  $\beta_p = 4.71 \times 10^4$  esu/cm<sup>2</sup> and  $v_s = 3.6 \times 10^5$  cm/sec. Since we consider the effect of the acoustic phonon scattering with electrons and photons, thus the film thickness considered here will be several times of the usual quantum wells.

In Fig. 1, the free-carrier absorption coefficient for the deformation-potential coupling  $\alpha_d$  is plotted as a function of the photon frequency for photons polarized in the layer plane. It shows that  $\alpha_d$  decreases monotonically with increasing the photon frequency and increases with increasing temperature. We plot  $\alpha_d$  as a function of the photon frequency for photons polarized normally to the layer plane as shown in Fig. 2. It can be seen that the values of  $\alpha_d$  are

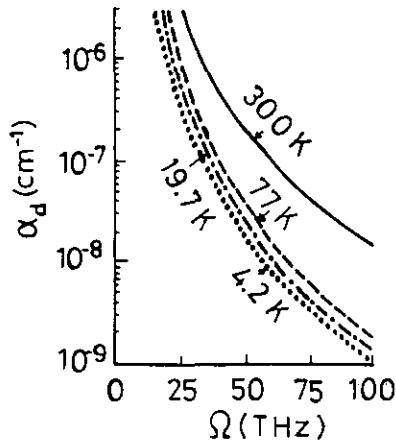


Fig. 2.  $\alpha_d$  as a function of the photon frequency with the quantum well  $d = 1 \mu\text{m}$  for photons polarized normally to the layer plane.

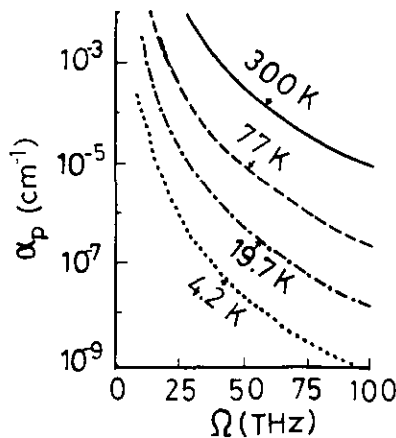


Fig. 3.  $\alpha_p$  as a function of the photon frequency with the quantum well  $d = 10 \mu\text{m}$  for photons polarized in the layer plane.

smaller than those for photons polarized in the layer plane. In Fig. 3, the free-carrier absorption coefficient for the piezoelectric coupling  $\alpha_p$  is plotted as a function of the photon frequency for photons polarized in the layer plane. It is shown that  $\alpha_p$  decreases monotonically with increasing the photon frequency and increases with increasing temperature. However, in Fig. 4 for photons polarized normally to the layer plane, it can be seen that  $\alpha_p$  decreases with increasing temperature. In Fig. 5, we plot  $\alpha_p$  as a function of the film thickness of quantum well for photons polarized normally to the layer plane with  $\Omega = 28$  THz. It shows that in low temperatures  $\alpha_p$  decreases monotonically with increasing the film thickness. However, at high temperatures such as  $T = 300$  K,  $\alpha_p$  oscillates with the film thickness at

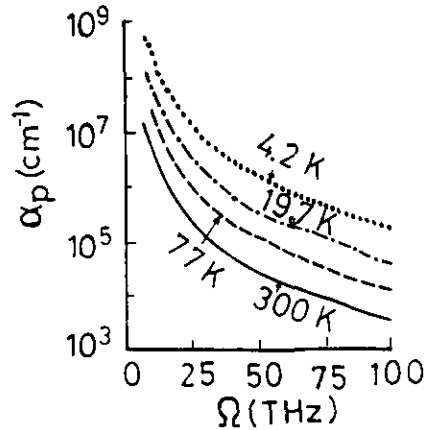


Fig. 4.  $\alpha_p$  as a function of the photon frequency with the quantum well  $d = 10 \mu\text{m}$  for photons polarized normally to the layer plane.

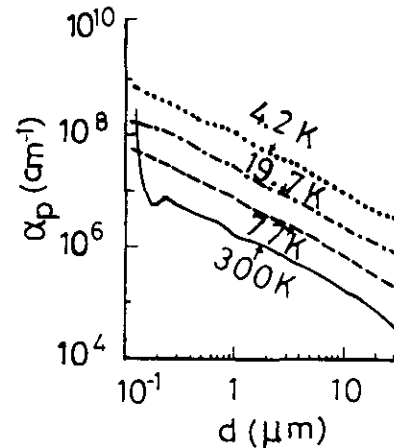


Fig. 5.  $\alpha_p$  as a function of the film thickness with  $\Omega = 28$  THz (or  $10.6 \mu\text{m}$  wavelength of  $\text{CO}_2$  laser) for photons polarized normally to the layer plane.

the region of small film thickness, and then decreases monotonically with increasing the film thickness. Since  $n_e = 1.73 \times 10^{15} \text{ cm}^{-3}$  can be considered as the nondegenerate electrons for the bulk case, thus the electrons are confined in the usual quantum wells may also be considered as the nondegenerate electron gas. However, the Maxwell-Boltzmann distribution will not be a good approximation at  $T = 4.2 \text{ K}$ .

*Acknowledgement* — This study was supported by National Science Council, Republic of China under contract number : NSC-81-0404-E009-539.

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