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# Output-feedback control of nonlinear systems using direct adaptive fuzzy-neural controller

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**Abstract**

In this paper, a direct adaptive fuzzy-neural output-feedback controller (DAFOC) for a class of uncertain nonlinear systems is developed under the constraint that only the system output is available for measurement. An output feedback control law and an update law are derived for on-line tuning the weighting factors of the DAFOC. By using strictly positive-real Lyapunov theory, the stability of the closed-loop system compensated by the DAFOC can be verified. Moreover, the proposed overall control scheme guarantees that all signals involved are bounded and the output of the closed-loop system asymptotically tracks the desired output trajectory. To demonstrate the effectiveness of the proposed method, simulation results are illustrated in this paper.


*Keywords:* Fuzzy-neural control; Direct adaptive control; Output feedback control; Nonlinear systems

## 1. Introduction

Adaptive control theory has been an active area of research for at least a quarter of a century [23,24,28,11,9,10,12,31,29,22,13]. For linear systems, there have been some researches on stability analysis of adaptive control systems, design of adaptive observers and adaptive control of plants, etc., all with satisfactory results [24,28]. There are also researches focusing on robust adaptive control that guarantee signal boundedness in the presence of modeling errors and bounded disturbances [11,9,10]. As for nonlinear systems, adaptive control schemes via feedback linearization have been reported

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on continuous-time or discrete-time systems [12,31,29,22,13]. The fundamental ideal of feedback linearization is to transform a nonlinear system into a linear one, so that linear control techniques can be employed to acquire desired performance.

Recently, neural networks and fuzzy systems are applied to several control problems with satisfactory results. Because both the neural networks [8] and fuzzy systems [38] are universal approximators, many adaptive control schemes for nonlinear systems based on fuzzy systems [35,14,32], or neural networks [25,16,26,20,6,5,21] have been proposed to obtain better control performance. For a class of nonlinear discrete-time systems, adaptive control using neural networks has been proposed in [27] by feedback linearization. Also, a dynamic recurrent neural-network-based adaptive observer for a class of nonlinear systems has been presented in [15]. In [39], an output feedback controller using multi-layer neural networks has been developed based on a high-gain observer used to estimate the time derivatives of the system output. Moreover, applications of fuzzy systems incorporated into neural networks in function approximation, decision systems and nonlinear control systems have been proposed in [7,37,36,30,18,19,34,2]. In [37,36,30,18,19], the *indirect* adaptive fuzzy-neural controllers for nonlinear systems have been proposed and in [30,18] the output feedback control laws, which also are tuned by the indirect adaptive methods, provide robust stability for the closed-loop systems.

Theoretical justification on the use of the direct adaptive fuzzy controllers [34,2,1,3,17,4] using a state feedback approach is valid when all of the system states are available for measurement. In practice, however, the state feedback control does not always hold because system states are not always available. Estimations of states from the system output for output feedback control design of the direct adaptive fuzzy-neural controller is required. Therefore, problem as to how a direct adaptive fuzzy-neural output-feedback controller (DAFOC) is designed remains to be solved. It is therefore the objective of this paper to develop a design algorithm of the DAFOC for uncertain nonlinear systems under the constraint that only the system output is available for measurement. Particularly, the output feedback control law and the update law can be on-line tuned. Moreover, the overall adaptive scheme guarantees that all signals involved are bounded and the output of the closed-loop system will asymptotically tracks the desired output trajectory.

The paper is organized as follows. In Section 2, the problem is formulated and a brief description of a fuzzy-neural network is presented. Design methodology of the DAFOC is included in Section 3. In Section 4, simulation results are demonstrated to show the effectiveness and applicability of the proposed method. Conclusions are included in Section 5.

## 2. Problem formulation and fuzzy-neural network

Consider the $n$th order nonlinear dynamical system of the form

$$x^{(n)} = f(x,\dot{x},\ldots,x^{(n-1)}) + g(x,\dot{x},\ldots,x^{(n-1)})u + d,$$
$$y = x, \tag{1}$$

where $d$ is the external bounded disturbance, and $u \in R$ and $y \in R$ are the control input and system output, respectively. We assume that $f$ and $g$ are uncertain functions, and $g$ is, without loss of generality, a strictly positive function. It is also assumed that a solution for (1) exists. In addition, only the system output $y$ is assumed to be measurable. The control objective is to design a DAFOC such that the system output $y$ follows a given bounded reference signal $y_m$, and all signals involved are bounded.

First, we convert the tracking problem to a regulation problem. Eq. (1) can be rewritten as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}(f(\mathbf{x}) + g(\mathbf{x})u + d),$$
$$y = \mathbf{C}^{\mathrm{T}}\mathbf{x}, \tag{2}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix},$$

and $\mathbf{x} = [x, \dot{x}, \ldots, x^{(n-1)}]^{\mathrm{T}} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in R^n$ is a vector of states. Define the output tracking error $e = y_m - y$, the reference vector $\mathbf{y}_m = [y_m, \dot{y}_m, \ldots, y_m^{(n-1)}]$ and the tracking error vector $\mathbf{e} = [e, \dot{e}, \ldots, \mathrm{e}^{(n-1)}]^{\mathrm{T}} = [e_1, e_2, \ldots, e_n]^{\mathrm{T}}$.

Based on the certainty equivalence approach, an optimal control law is

$$u^* = \frac{1}{g(\mathbf{x})}[-f(\mathbf{x}) + y_m^{(n)} + \mathbf{K}_c^{\mathrm{T}}\hat{\mathbf{e}}], \tag{3}$$

where $\hat{\mathbf{e}} = \mathbf{y}_m - \hat{\mathbf{x}}$, $\hat{\mathbf{e}}$ and $\hat{\mathbf{x}}$ denote the estimates of $\mathbf{e}$ and $\mathbf{x}$, respectively. $\mathbf{K}_c = [k_n^c k_{n-1}^c \ldots k_1^c]^{\mathrm{T}}$ is the feedback gain vector, chosen such that the characteristic polynomial of $\mathbf{A} - \mathbf{B}\mathbf{K}_c^{\mathrm{T}}$ is Hurwitz because $(\mathbf{A}, \mathbf{B})$ is controllable. Since only the system output $y$ is assumed to be measurable, and $f(\mathbf{x})$ and $g(\mathbf{x})$ are assumed to be uncertain, the optimal control law (3) cannot be implemented. Thus, suppose a control input $u$ is

$$u = u_f + u_s, \tag{4}$$

where $u_f$ is designed to approximate the optimal control law (3), and the control term $u_s$ is employed to compensate the external disturbance and the modeling error. From (2), (3) and (4), we have

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} - \mathbf{B}\mathbf{K}_c^{\mathrm{T}}\hat{\mathbf{e}} + \mathbf{B}[g(\mathbf{x})u^* - g(\mathbf{x})u_f - g(\mathbf{x})u_s - d],$$
$$e_1 = \mathbf{C}^{\mathrm{T}}\mathbf{e}. \tag{5}$$

Thus, the tracking problem has been converted into the regulation problem of designing a state observer for estimating the state vector $\mathbf{e}$ in (5) in order to regulate $e_1$ to zero.

In addition, the configuration of the fuzzy-neural network shown in Fig. 1 consists of a fuzzy system and neural network. The fuzzy system can be divided into two parts: some fuzzy IF-THEN rules and a fuzzy inference engine. The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input linguistic vector $\mathbf{e} = [e_1, e_2, \ldots, e_n] \in R^n$ to an output linguistic variable $u_f \in R$. The $i$th fuzzy IF-THEN rule is written as

$$\boldsymbol{R}^i\text{: If } e_1 \text{ is } A_1^i \text{ and} \ldots \text{ and } e_n \text{ is } A_n^i \text{ than } u_f \text{ is } B^i, \tag{6}$$

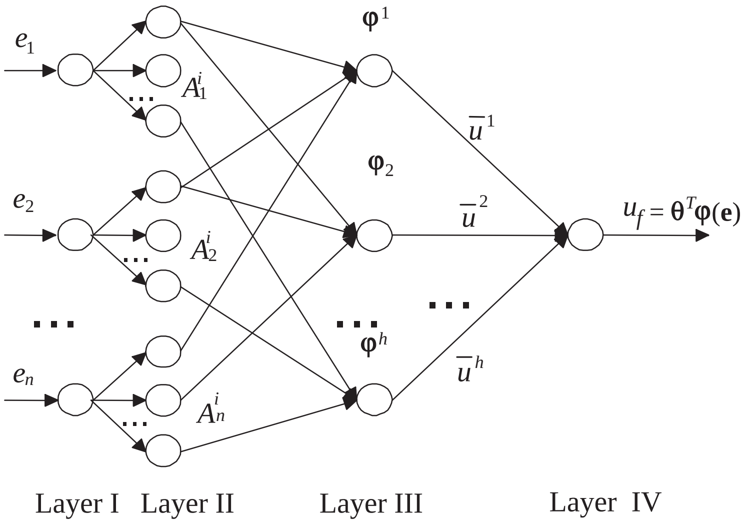


Fig. 1. Configuration of a fuzzy-neural approximator.

where $A_1^i, A_2^i, \ldots, A_n^i$ and $B^i$ are fuzzy sets [35,14]. By using produce inference, center average and singleton fuzzifier, the output of the fuzzy-neural network can be expressed as

$$u_f = \frac{\sum_{i=1}^{h} \bar{u}^i[\prod_{j=1}^{n} \mu_{A_j^i}(e_j)]}{\sum_{i=1}^{h}[\prod_{j=1}^{n} \mu_{A_j^i}(e_j)]} = \boldsymbol{\theta}^{\mathrm{T}}\varphi(\mathbf{e}), \tag{7}$$

where $\mu_{A_j^i}(e_j)$ is the membership function value of the fuzzy variable, $h$ is the total number of the IF-THEN rules, $\bar{u}^i$ is the point at which $\mu_{B^i}(\bar{u}^i) = 1$, $\boldsymbol{\theta} = [\bar{u}^1, \bar{u}^2, \ldots, \bar{u}^h]^{\mathrm{T}}$ is an adjustable parameter vector, and $\varphi = [\varphi^1, \varphi^2, \ldots, \varphi^h]^{\mathrm{T}}$ is a fuzzy basis vector, where $\varphi^i$ is defined as

$$\varphi^i(\mathbf{e}) = \frac{\prod_{j=1}^{n} \mu_{A_j^i}(e_j)}{\sum_{i=1}^{h}[\prod_{j=1}^{n} \mu_{A_j^i}(e_j)]}. \tag{8}$$

When the inputs are given into the fuzzy-neural network shown in Fig. 1, the truth value $\varphi^i$(layer III) of the antecedent part of the $i$th implication is calculated by (8). Among the commonly used deffuzzification strategies, the output (layer IV) of the fuzzy-neural network is expressed as (7). Therefore, the fuzzy logic approximator based on the nerual network can be established [37,19]. Fig. 1 shows the configuration of the fuzzy-neural function approximator. The approximator has four layers. At layer I, input nodes stand for the input linguistic variables $e_1, e_2, \ldots, e_n$. At layer II, nodes represent the values of the membership functions. At layer III, nodes are the values of the fuzzy basis vector $\varphi$. Each node of layer III performs a fuzzy rule. The links between layer III and layer IV are full connected by the weighting factors $\boldsymbol{\theta} = [\bar{u}^1, \bar{u}^2, \ldots, \bar{u}^h]^{\mathrm{T}}$, i.e., the adjusted parameters. At layer IV, the output stands for the value of $u_f$.

## 3. Output feedback control design of direct adaptive fuzzy-neural controller

In this section, our primary tasks are to design an observer that estimates the state vector $\mathbf{e}$ in (5), to use the fuzzy-neural network to approximate to the optimal control law $u^*$ in (3) and to develop the direct adaptive update law to adjust the parameters of the fuzzy neural network in order to achieve the control objective.

First, we replace $u_f$ in (4) by the output of the fuzzy-neural network, $\boldsymbol{\theta}^{\mathrm{T}}\varphi(\hat{\mathbf{e}})$ in (7), i.e.,

$$u_f(\hat{\mathbf{e}}|\boldsymbol{\theta}) = \boldsymbol{\theta}^{\mathrm{T}}\varphi(\hat{\mathbf{e}}), \tag{9}$$

where $\hat{\mathbf{e}}$ denotes the estimate of $\mathbf{e}$.

Next, consider the following observer that estimates the state vector $\mathbf{e}$ in (5)

$$\begin{aligned}\dot{\hat{\mathbf{e}}} &= \mathbf{A}\hat{\mathbf{e}} - \mathbf{B}\mathbf{K}_c^{\mathrm{T}}\hat{\mathbf{e}} + \mathbf{B}(gv - gu_s) + \mathbf{K}_{\mathrm{o}}(e_1 - \hat{e}_1),\\ \hat{e}_1 &= \mathbf{C}^{\mathrm{T}}\hat{\mathbf{e}},\end{aligned} \tag{10}$$

where $\mathbf{K}_{\mathrm{o}} = [k_1^{\mathrm{o}}, k_2^{\mathrm{o}}, \ldots, k_n^{\mathrm{o}}]^{\mathrm{T}}$ is the observer gain vector, chosen such that the characteristic polynomial of $\mathbf{A} - \mathbf{K}_{\mathrm{o}}\mathbf{C}^{\mathrm{T}}$ is strictly Hurwitz because $(\mathbf{C}, \mathbf{A})$ is observable. The control term $v$ is employed to compensate the external disturbance $d$ and the modeling error. We define the observation errors as $\tilde{\mathbf{e}} = \mathbf{e} - \hat{\mathbf{e}}$ and $\tilde{e}_1 = e_1 - \hat{e}_1$. Subtracting (10) from (5), we have

$$\begin{aligned}\dot{\tilde{\mathbf{e}}} &= (\mathbf{A} - \mathbf{K}_{\mathrm{o}}\mathbf{C}^{\mathrm{T}})\tilde{\mathbf{e}} + \mathbf{B}[gu^* - gu_f(\hat{\mathbf{e}}|\boldsymbol{\theta}) - gv - d],\\ \tilde{e}_1 &= \mathbf{C}^{\mathrm{T}}\tilde{\mathbf{e}}.\end{aligned} \tag{11}$$

Besides, the output error dynamics of (11) can be given as

$$\tilde{e}_1 = H(s)[gu^* - gu_f(\hat{\mathbf{e}}|\theta) - gv - d], \tag{12}$$

where $s$ is the Laplace variable, and $H(s) = \mathbf{C}^{\mathrm{T}}(s\mathbf{I} - (\mathbf{A} - \mathbf{K}_{\mathrm{o}}\mathbf{C}^{\mathrm{T}}))^{-1}\mathbf{B}$ is the transfer function of (11).

In order to derive the direct adaptive update law, the following assumption and lemma must be required.

**Assumption 1** (Tsakalis and Ioannou [33]). Let $\mathbf{e}$ and $\hat{\mathbf{e}}$ belong to compact sets $U_{\mathbf{e}} = \{\mathbf{e} \in \Re^n : \|\mathbf{e}\| \leqslant m_{\mathbf{e}} < \infty\}$ and $U_{\hat{\mathbf{e}}} = \{\hat{\mathbf{e}} \in \Re^n : \|\hat{\mathbf{e}}\| \leqslant m_{\hat{\mathbf{e}}} < \infty\}$, respectively, where $\hat{\mathbf{e}}$ denotes the estimate of $\mathbf{e}$ and $m_{\mathbf{e}}$ and $m_{\hat{\mathbf{e}}}$ are designed parameters. It is known a prior that the optimal parameter vector $\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta} \in M_{\boldsymbol{\theta}}}[\sup_{\mathbf{e} \in U_{\mathbf{e}}, \hat{\mathbf{e}} \in U_{\hat{\mathbf{e}}}} |u^* - u(\hat{\mathbf{e}}|\boldsymbol{\theta})|]$ lies in some convex region $M_{\boldsymbol{\theta}} = \{\boldsymbol{\theta} \in \Re^n : \|\boldsymbol{\theta}\| \leqslant m_{\boldsymbol{\theta}}\}$, where the radius $m_{\boldsymbol{\theta}}$ is constant.

**Lemma 1** (Ioannou and Sun [10]). *Consider the linear time-invariant system*

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \quad \mathbf{x}(0) = \mathbf{x}_0,$$

*where* $\mathbf{x} \in \Re^n, \mathbf{u}(t) \in \Re^m, \mathbf{A} \in \Re^{n\times n}, \mathbf{B} \in \Re^{n\times m}$. *Suppose that* $\mathbf{A}$ *is Hurwitz matrix and* $\mathbf{u}(t) \in L_{2e}$. *Let* $\alpha_0$ *and* $\lambda_0$ *be the positive constants that satisfy* $\|\mathrm{e}^{\mathbf{A}(t-\tau)}\| \leqslant \lambda_0 \mathrm{e}^{-\alpha_0(t-\tau)}$. *Then for any constant*

$\delta\in[0,\delta_1]$, *where* $0<\delta_1<2\alpha_0$,

$$\|\mathbf{x}(t)\| \leqslant \lambda_0 e^{-\alpha_0 t}\|\mathbf{x}_0\| + \frac{\|\mathbf{B}\|\lambda_0}{\sqrt{2\alpha_0-\delta}}\|\mathbf{u}_t\|_{2\delta},$$

*where* $\|\mathbf{u}_t\|_{2\delta}=(\int_0^t e^{-\delta(t-\tau)}\mathbf{u}^{\mathrm{T}}(\tau)\mathbf{u}(\tau)\,d\tau)^{1/2}$.

According to Assumption 1, (11) can be rewritten as

$$\begin{aligned}\dot{\tilde{\mathbf{e}}} &= (\mathbf{A}-\mathbf{K}_o\mathbf{C}^{\mathrm{T}})\tilde{\mathbf{e}} + \mathbf{B}[gu_f(\hat{\mathbf{e}}|\boldsymbol{\theta}^*) - gu_f(\hat{\mathbf{e}}|\boldsymbol{\theta}) - gv + w - d],\\ \tilde{e}_1 &= \mathbf{C}^{\mathrm{T}}\tilde{\mathbf{e}},\end{aligned} \tag{13}$$

where $w=gu^* - gu_f(\hat{\mathbf{e}}|\boldsymbol{\theta}^*)$ is an approximation error. According to (9), (13) can be rewritten as

$$\begin{aligned}\dot{\tilde{\mathbf{e}}} &= (\mathbf{A}-\mathbf{K}_o\mathbf{C}^{\mathrm{T}})\tilde{\mathbf{e}} + \mathbf{B}[g\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\varphi(\hat{\mathbf{e}}) - gv + w - d],\\ \tilde{e}_1 &= \mathbf{C}^{\mathrm{T}}\tilde{\mathbf{e}},\end{aligned} \tag{14}$$

where $\tilde{\boldsymbol{\theta}}=\boldsymbol{\theta}^*-\boldsymbol{\theta}$. Since only the output $\tilde{e}_1$, in (14) is assumed to be measurable, we use the SPR Lyapunov design approach to analyze the stability of (14) and generate the direct adaptive update law for $\boldsymbol{\theta}$. Eq. (14) can be rewritten as

$$\tilde{e}_1 = H(s)[g\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\varphi(\hat{\mathbf{e}}) - gv + w - d], \tag{15}$$

where $H(s)=\mathbf{C}^{\mathrm{T}}(s\mathbf{I}-(\mathbf{A}-\mathbf{K}_o\mathbf{C}^{\mathrm{T}}))^{-1}\mathbf{B}$ is a known stable transfer function. In order to employ the SPR-Lyapunov design approach, (15) can be written as

$$\tilde{e}_1 = H(s)L(s)[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\phi(\hat{\mathbf{e}}) - v_f + w_f], \tag{16}$$

where $v_f = L^{-1}(s)[gv]$, $w_f = L^{-1}(s)[w-d+g\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\varphi(\hat{\mathbf{e}})] - \tilde{\boldsymbol{\theta}}^{\mathrm{T}}\phi(\hat{\mathbf{e}})$, $\phi(\hat{\mathbf{e}}) = L^{-1}(s)[\varphi(\hat{\mathbf{e}})]$ and $L(s)$ is chosen so that $L^{-1}(s)$ is a proper stable transfer function and $H(s)L(s)$ is a proper SPR transfer function. Supposed that $L(s)=s^m+b_1s^{m-1}+b_2s^{m-2}+\cdots+b_m$, where $m<n$, such that $H(s)L(s)$ is a proper SPR transfer function. Then the state–space realization of (16) can be written as

$$\begin{aligned}\dot{\tilde{\mathbf{e}}} &= \mathbf{A}_c\tilde{\mathbf{e}} + \mathbf{B}_c[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\phi(\hat{\mathbf{e}}) - v_f + w_f],\\ \tilde{e}_1 &= \mathbf{C}_c^{\mathrm{T}}\tilde{\mathbf{e}},\end{aligned} \tag{17}$$

where $\mathbf{A}_c=(\mathbf{A}-\mathbf{K}_o\mathbf{C}^{\mathrm{T}})\in\Re^{n\times n}, \mathbf{B}_c^{\mathrm{T}}=[0\quad 0\cdots b_1\quad b_2\cdots b_m]\in\Re^n$ and $\mathbf{C}_c^{\mathrm{T}}=[10\cdots 0]\in\Re^n$. For the purpose of stability analysis of the DAFOC, the following assumptions and lemma must be required.

**Lemma 2** (Wang [35] and Leu et al. [18]). *Supposed that the update laws are chosen as*

$$\dot{\boldsymbol{\theta}} = \begin{cases} \gamma\tilde{e}_1\phi(\hat{\mathbf{e}}) & \textit{if } \|\boldsymbol{\theta}\| < m_{\boldsymbol{\theta}} \textit{ or } (\|\boldsymbol{\theta}\| = m_{\boldsymbol{\theta}} \textit{ and } \tilde{e}_1\boldsymbol{\theta}^{\mathrm{T}}\phi(\hat{\mathbf{e}}) \geqslant 0),\\ \Pr(\gamma\tilde{e}_1\phi(\hat{\mathbf{e}})) & \textit{if } \|\boldsymbol{\theta}\| = m_{\boldsymbol{\theta}} \textit{ and } \tilde{e}_1\boldsymbol{\theta}^{\mathrm{T}}\phi(\hat{\mathbf{e}}) < 0,\end{cases} \tag{18}$$

*where the projection operator* [35] *is given as*

$$\Pr(\gamma \tilde{e}_1 \phi(\hat{\mathbf{e}})) = \gamma \tilde{e}_1 \phi(\hat{\mathbf{e}}) - \gamma \frac{\tilde{e}_1 \boldsymbol{\theta}^{\mathrm{T}} \phi(\hat{\mathbf{e}})}{\|\boldsymbol{\theta}\|^2} \boldsymbol{\theta}.$$

*Then* $\|\boldsymbol{\theta}\| \leqslant m_{\boldsymbol{\theta}}$ *and* $\|\tilde{\boldsymbol{\theta}}\| \leqslant 2m_{\boldsymbol{\theta}}$.

**Assumption 2.** The uncertain function $g(\mathbf{x})$ is bounded by

$$\beta_1 \leqslant \|g(\mathbf{x})\| \leqslant \beta_2, \tag{19}$$

where $\beta_1$ and $\beta_2$ are positive constants.

**Assumption 3.** $w_f$ is assumed to satisfy

$$|w_f| \leqslant \varepsilon, \tag{20}$$

where $\varepsilon$ is a positive constant.

**Remark 1.** The assumption of $|w_f| \leqslant \varepsilon$ is reasonable because of Assumption 2, the universal approximate theorem and the external bounded disturbance.

On the basis of the above discussions, the following theorems can be obtained.

**Theorem 1.** *Consider system* (17) *that satisfies Assumptions* 1–3. *Let* $\boldsymbol{\theta}$ *be adjusted by the update law* (18), *and let* $v$ *be given as*

$$v = \begin{cases} \rho & \text{if } \tilde{e}_1 \geqslant 0, \\ -\rho & \text{if } \tilde{e}_1 < 0, \end{cases} \tag{21}$$

*where* $\rho \geqslant \varepsilon/\beta_1$. *Then* $\tilde{e}_1(t)$ *converges to zero as* $t \to \infty$.

**Proof.** Given in Appendix A. □

**Theorem 2.** *Consider the nonlinear system* (1) *that satisfies Assumptions* 1–3. *Let the control term* $u_s$ *in* (4) *and* (10) *be* $u_s = v$ *in* (21), *such that the state observer* (10) *becomes*

$$\dot{\hat{\mathbf{e}}} = (\mathbf{A} - \mathbf{B}\mathbf{K}_c^{\mathrm{T}})\hat{\mathbf{e}} + \mathbf{K}_o \tilde{e}_1. \tag{22}$$

*Suppose that the control law is*

$$u = u_f(\hat{\mathbf{e}}|\boldsymbol{\theta}) + u_s \tag{23}$$

*with the update law* (18). *Then all signals in the closed-loop system are bounded, and* $e_1(t)$ *converges to zero as* $t \to \infty$.

**Proof.** Given in Appendix B. □

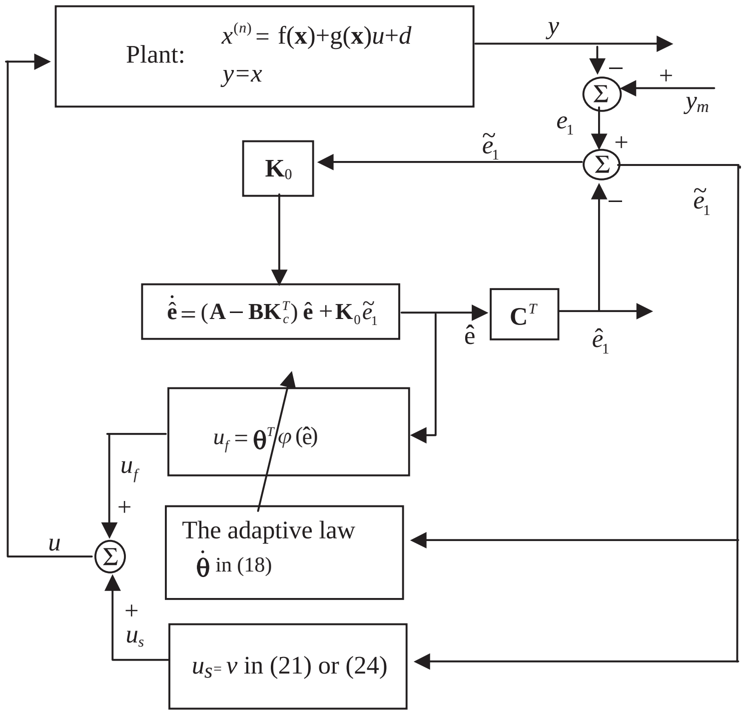


Fig. 2. Overall scheme of the proposed direct adaptive fuzzy-neural control system.

According to the above theorems, the design algorithm of the DAFOC is described as following.

**Design algorithm.** *Step* 1: Select the feedback and observer gain vectors $\mathbf{K}_c, \mathbf{K}_o$ such that the matrices $\mathbf{A} - \mathbf{B}\mathbf{K}_c^{\mathrm{T}}$ and $\mathbf{A} - \mathbf{K}_o\mathbf{C}^{\mathrm{T}}$ are Hurwitz matrices, respectively.

*Step* 2: Choose an appropriate value $\rho$ in (21) and $\gamma$ in (18). In order to remedy the control chattering, (21) can be modified as

$$v = \begin{cases} \rho & \text{if } \tilde{e}_1 \geqslant 0 \text{ and} |\tilde{e}_1| > \alpha, \\ -\rho & \text{if } \tilde{e}_1 < 0 \text{ and } |\tilde{e}_1| > \alpha, \text{ where } \alpha \text{ is a positive constant,} \\ \rho\tilde{e}_1/\alpha & \text{if } |\tilde{e}_1| < \alpha. \end{cases} \tag{24}$$

*Step* 3: Solve the state observer in (22), where $v$ in (21) or (24).

*Step* 4: Construct fuzzy sets for $\hat{\mathbf{e}}(t)$. Then, from (8) compute the fuzzy basis vector $\varphi$.

*Step* 5: Obtain the control law (23), and the update law (18).

To summarize, Fig. 2 shows the overall scheme of the direct adaptive fuzzy-neural output feedback control system proposed in this paper.

## 4. The illustrative example

This section presents simulation results of the proposed design algorithm to illustrate that stability of the closed-loop system is guaranteed, and all signals involved are bounded.

**Example.** Consider the Duffing forced oscillation system [34]

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -0.1x_2 - x_1^3 + 12\cos t + u + d,$$
$$y = x_1. \tag{25}$$

It is assumed that the external disturbance $d(t)$ is a square wave having an amplitude $\pm 1$ with a period of $2\pi$. The control objective is to control the state $x_1$ of the system to track the reference trajectory $y_m$, under the condition that only the system output $y$ is measurable. The design parameters are selected as $\gamma = 0.5\times 10^3$ and $\rho = 20$. The feedback and observer gain vectors are given as $\mathbf{K}_c = [144\ \ 24]^T$ and $\mathbf{K}_o = [60\ \ 900]^T$, respectively. The filter $L^{-1}(s)$ is given as $L^{-1}(s) = 1/(s+2)$. The following membership functions for $\hat{e}_j,\ j = 1,2$ are given as

$$\mu_{A_j^1}(\hat{e}_j) = 1/(1+\exp(5\times(\hat{e}_j+3))), \quad \mu_{A_j^2}(\hat{e}_j) = \exp(-(\hat{e}_j+2)^2),$$

$$\mu_{A_j^3}(\hat{e}_j) = \exp(-(\hat{e}_j+1)^2), \quad \mu_{A_j^4}(\hat{e}_j) = \exp(-(\hat{e}_j)^2),$$

$$\mu_{A_j^5}(\hat{e}_j) = \exp(-(\hat{e}_j-1)^2), \quad \mu_{A_j^6}(\hat{e}_j) = \exp(-(\hat{e}_j-2)^2),$$

$$\mu_{A_j^7}(\hat{e}_j) = 1/(1+\exp(-5\times(\hat{e}_j-3)).$$

The initial states are chosen to be $x_1(0) = x_2(0) = 3, \hat{x}_1(0) = \hat{x}_2(0) = -1$ and $\hat{\mathbf{e}}(0) = \mathbf{y}_m(0) - \hat{\mathbf{x}}(0)$. Simulation results are provided for three cases with different reference trajectories, i.e., $y_m = 0$ (Case 1), $y_m = \sin t$ (Case 2), and $y_m = 1-\exp(-t/2)$ (Case 3), respectively. Figs. 3–13 show the computer simulation results for these three cases. With reference to Figs. 3, 6, and 9, it is observed that the state observer correctly and responsively generates the estimated state $\hat{x}_1$. Referring to Figs. 4, 7, and 10, it is also observed that the tracking convergence is fast with only a relatively small tracking error by using the control term $v$ in (24). To avoid the chattering effect of control input, the control term $v$ in (24), instead of $v$ in (21), is used for these three cases as shown in Figs. 3–11. Comparing Figs. 4 with 12 (Case 1), we find that tracking performance using $v$ in (21) is slightly better than that using $v$ in (24). However, the chattering effect of control input using $v$ in (21) is much serious than that using $v$ in (24), as clearly demonstrated in Figs. 5 and 13. As shown in Figs. 5, 8, and 11, chattering effect of the control input for these three cases almost disappears by using the control term $v$ in (24). That is the reason that $v$ in (24), instead of $v$ in (21), is suggested to derive the control law for practical applications.

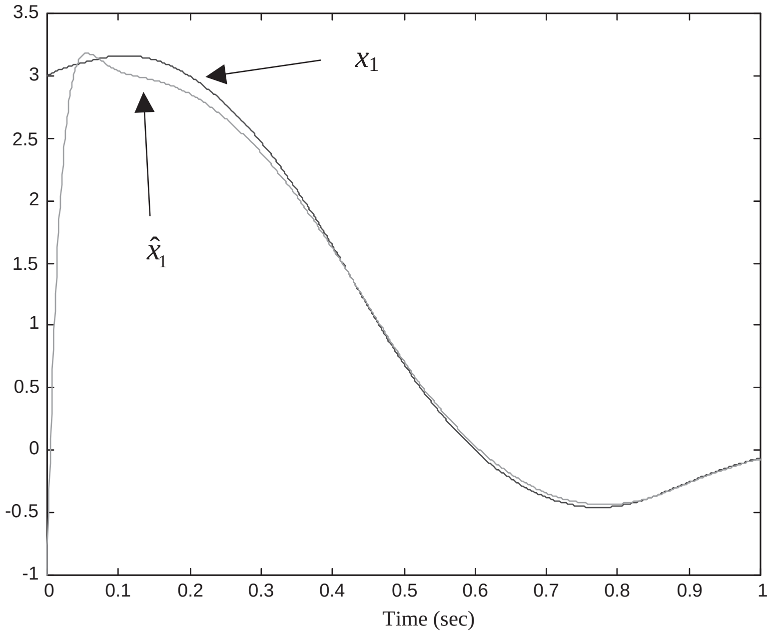


Fig. 3. Trajectories of the states $x_1$ and $\hat{x}_1$ of Case 1 using $v$ in (24).

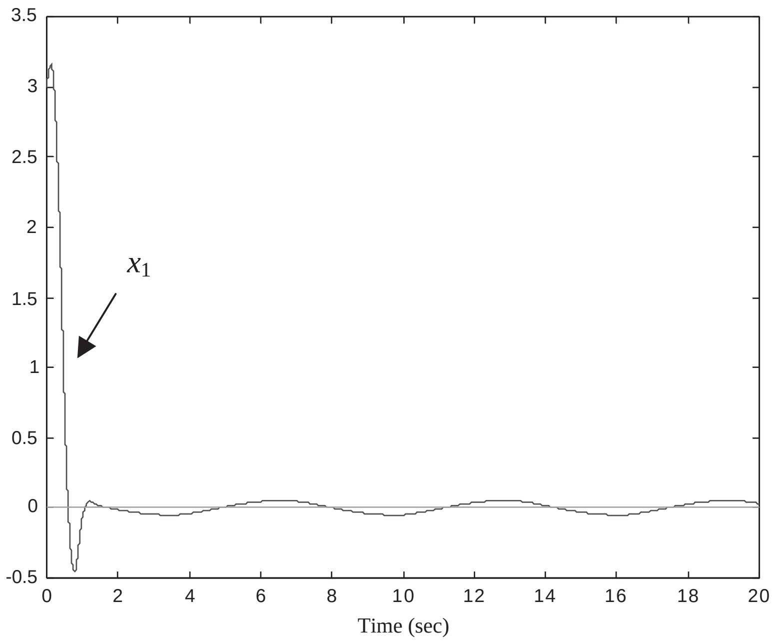


Fig. 4. Trajectories of the states $x_1$ and $y_m = 0$ of Case 1 using $v$ in (24).

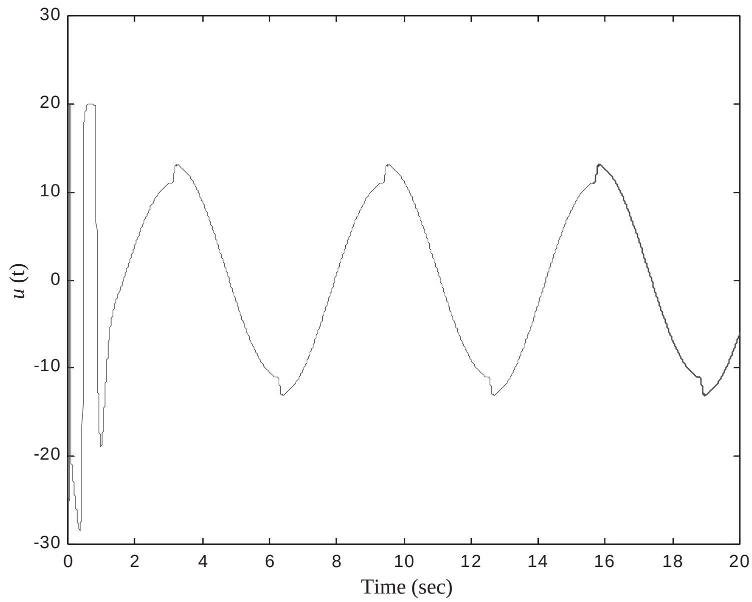


Fig. 5. Control input $u$ of Case 1 using $v$ in (24).

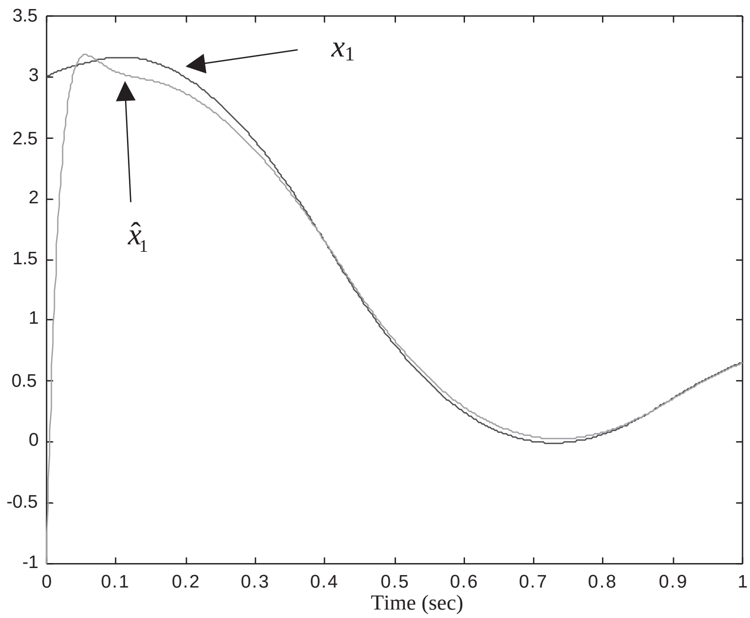


Fig. 6. Trajectories of the states $x_1$ and $\hat{x}_1$ of Case 2 using $v$ in (24).

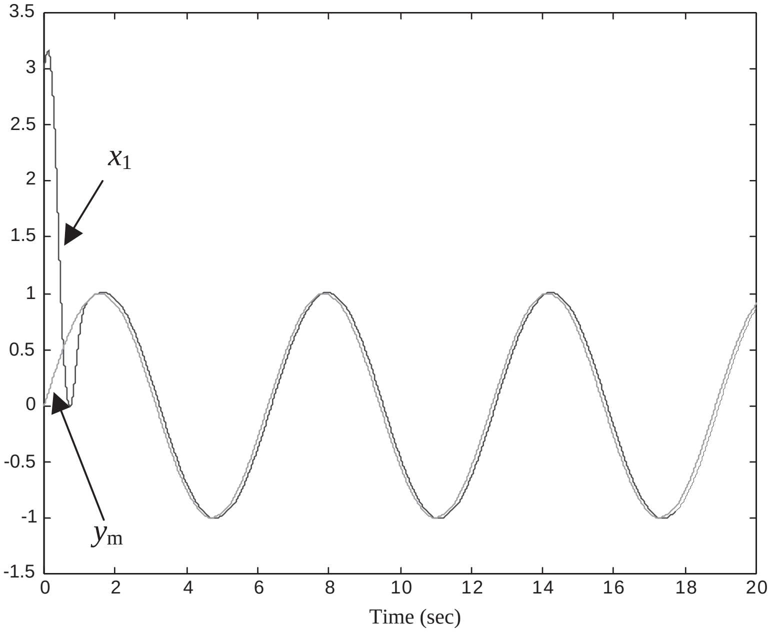


Fig. 7. Trajectories of the states $x_1$ and $y_m$ of Case 2 using $v$ in (24).

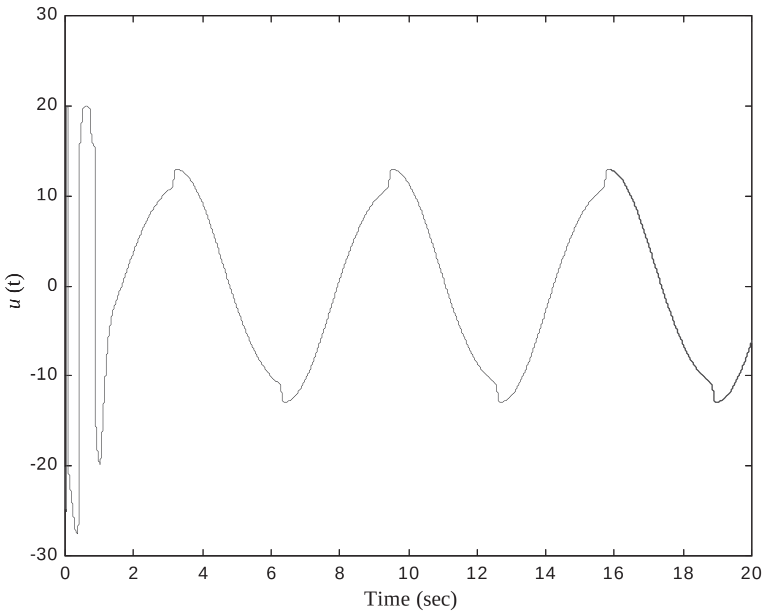


Fig. 8. Control input $u$ of Case 2 using $v$ in (24).

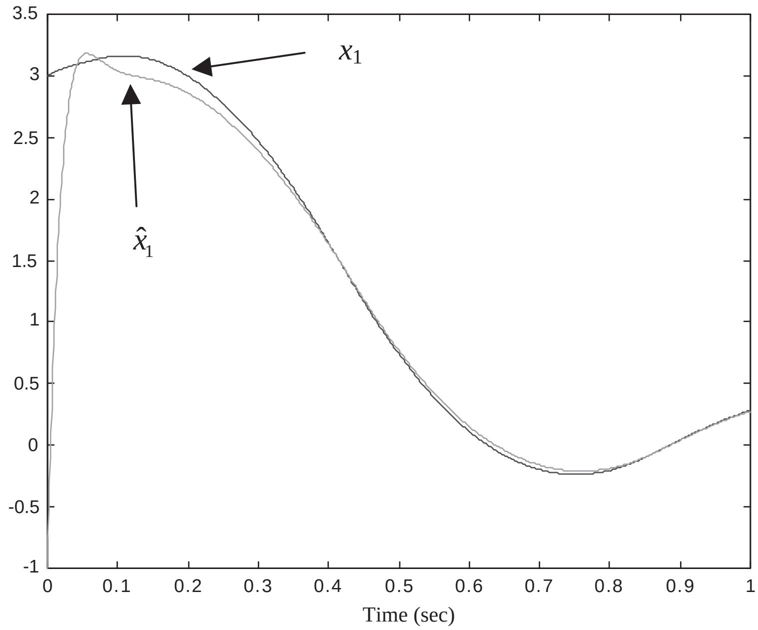


Fig. 9. Trajectories of the states $x_1$ and $\hat{x}_1$ of Case 3 using $v$ in (24).

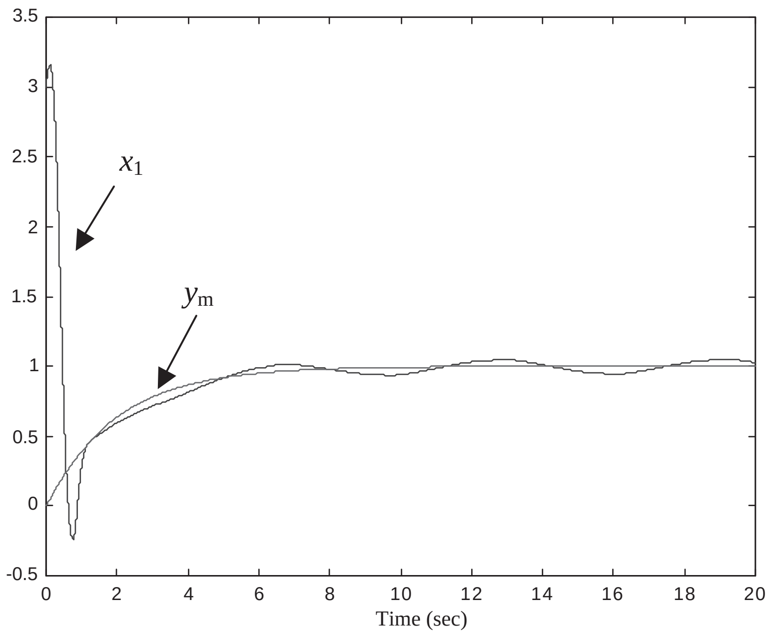


Fig. 10. Trajectories of the states $x_1$ and $y_m$ of Case 3 using $v$ in (24).

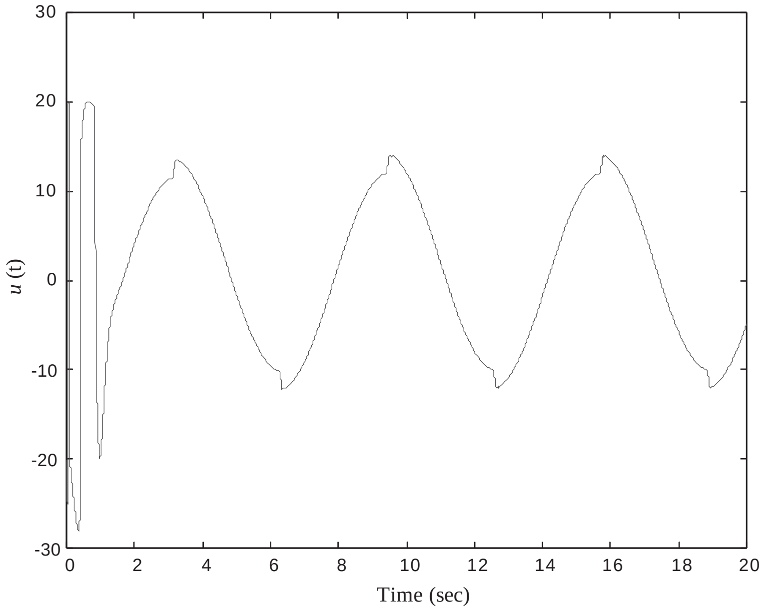


Fig. 11. Control input $u$ of Case 3 using $v$ in (24).

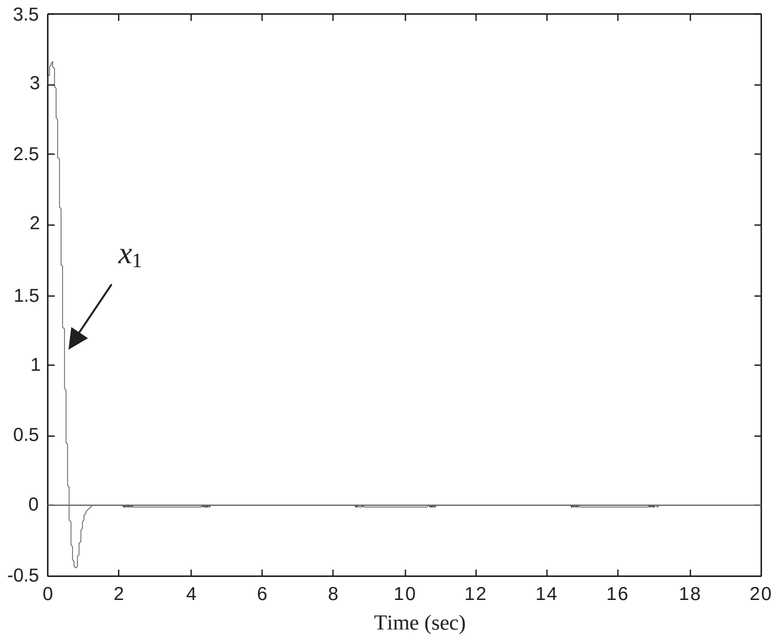


Fig. 12. Trajectories of the states $x_1$ and $y_m = 0$ of Case 1 using $v$ in (21).

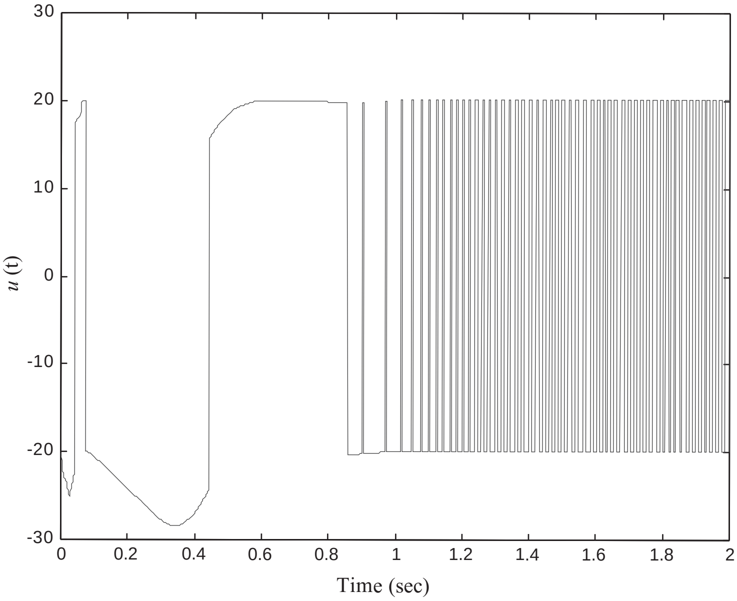


Fig. 13. Control input $u$ of Case 1 using $v$ in (21).

## 5. Conclusions

The direct adaptive fuzzy-neural output-feedback controller (DAFOC), which can be subject to on-line tuning for nonlinear systems, has been proposed in this paper. In designing the output feedback control law, no differentiation of system outputs is performed in order to avoid the noise amplification associated with numerical differentiation, and no knowledge on nonlinearities of the nonlinear systems is required. Also, preliminary off-line tuning of the weighting factors of the fuzzy-neural controller is no longer required. The overall adaptive scheme guarantees that all signals involved are bounded and the output of the closed-loop system asymptotically tracks the desired output trajectory. Moreover, the proposed design algorithm has been successfully applied to control the nonlinear Duffing forced oscillation system to track a reference trajectory. Simulation results have shown that the DAFOC performs good control and achieve desired performance.

## Acknowledgements

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## Appendix A.

**Proof of Theorem 1.** Consider the Lyapunov-like function candidate

$$V = \frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{e}} + \frac{1}{2\gamma}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}, \tag{A.1}$$

where $\mathbf{P}=\mathbf{P}^{\mathrm{T}}>0$. Differentiating (A.1) with respect to time and inserting (17) in the above equation yield

$$\dot{V}=\frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}}(\mathbf{A}_c^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{A}_c)\tilde{\mathbf{e}}+\tilde{\mathbf{e}}^{\mathrm{T}}\mathbf{P}\mathbf{B}_c[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\phi-v_f+w_f]+\frac{1}{\gamma}\dot{\tilde{\boldsymbol{\theta}}}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}. \tag{A.2}$$

Because $H(s)L(s)$ is SPR, there exists $\mathbf{P}=\mathbf{P}^{\mathrm{T}}>0$ such that

$$\mathbf{A}_c^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{A}_c=-\mathbf{Q},$$
$$\mathbf{P}\mathbf{B}_c=\mathbf{C}_c, \tag{A.3}$$

where $\mathbf{Q}=\mathbf{Q}^{\mathrm{T}}>0$. By using (A.3), (A.2) becomes

$$\dot{V}=-\frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}}\mathbf{Q}\tilde{\mathbf{e}}+\tilde{e}_1[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\phi\cdot-v_f+w_f]+\frac{1}{\gamma}\dot{\tilde{\boldsymbol{\theta}}}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}. \tag{A.4}$$

By using Assumptions 2–3, (21) and the fact $\lambda_{\min}(\mathbf{Q})\|\tilde{\mathbf{e}}\|^2\geqslant\lambda_{\min}(\mathbf{Q})|\tilde{e}_1|^2$, where $\lambda_{\min}(\mathbf{Q})>0$, we have

$$\dot{V}\leqslant-\frac{1}{2}\lambda_{\min}(\mathbf{Q})|\tilde{e}_1|^2+\tilde{e}_1\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\phi+\frac{1}{\gamma}\dot{\tilde{\boldsymbol{\theta}}}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}. \tag{A.5}$$

Inserting (18) in (A.5) and after some manipulation yields

$$\dot{V}\leqslant-\frac{1}{2}\lambda_{\min}(\mathbf{Q})|\tilde{e}_1|^2. \tag{A.6}$$

Eqs. (21) and (A.6) only guarantee that $\tilde{e}_1(t)\in L_\infty$ and $\tilde{\mathbf{e}}(t)\in L_\infty$, but do not guarantee the convergence. Because all variables in the right-hand side of (17) are bounded, $\dot{\tilde{e}}_1(t)$ is bounded, i.e., $\dot{\tilde{e}}_1(t)\in L_\infty$. Integrating both side of (A.6) and after some manipulation yields

$$\int_0^\infty|\tilde{e}_1(t)|^2\,\mathrm{d}t\leqslant\frac{V(0)-V(\infty)}{(1/2)\lambda_{\min}(\mathbf{Q})}. \tag{A.7}$$

Since the right side of (A.7) is bounded, so $\tilde{e}_1(t)\in L_2$. Using Barbalat's lemma [28], we have $\lim_{t\to\infty}|\tilde{e}_1(t)|=0$. This completes the proof. □

## Appendix B.

**Proof of Theorem 2.** First, from Theorem 1, we have $\lim_{t\to\infty}|\tilde{e}_1(t)|=0$. Next, consider Eq. (14). Define $\bar{u}=g\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\varphi(\hat{\mathbf{e}})-gv+w-d$. Because $\mathbf{A}-\mathbf{K}_o\mathbf{C}^{\mathrm{T}}$ is a Hurwitz matrix, and $\bar{u}$ is bounded from Lemma 2 and under Assumptions 1–3, we have

$$\|\tilde{\mathbf{e}}(t)\|\leqslant\lambda_0\mathrm{e}^{-\alpha_0 t}\|\tilde{\mathbf{e}}(0)\|+\frac{\|\mathbf{B}\|\lambda_0}{\sqrt{2\alpha_0-\delta}}\|\bar{u}_t\|_{2\delta} \tag{B.1}$$

according to Lemma 1. Therefore $\tilde{\mathbf{e}}(t) \in L_\infty$. By using the state observer (22), we obtain the dynamic system

$$\dot{\hat{\mathbf{e}}} = (\mathbf{A} - \mathbf{B}\mathbf{K}_c^{\mathrm{T}})\hat{\mathbf{e}} + \mathbf{K}_{\mathrm{o}}\mathbf{C}^{\mathrm{T}}\tilde{\mathbf{e}}$$

$$\hat{e}_1 = \mathbf{C}^{\mathrm{T}}\hat{\mathbf{e}}. \tag{B.2}$$

Similarly, because $\mathbf{A}-\mathbf{B}\mathbf{K}_c^{\mathrm{T}}$ is a Hurwitz matrix and $\tilde{\mathbf{e}}(t)$ is bounded, $\hat{\mathbf{e}}(t)$ is bounded. From $\tilde{\mathbf{e}}=\mathbf{e}-\hat{\mathbf{e}}$, it follows that $e_1, \mathbf{e} \in L_\infty$ and $e_1(t) \to 0$ as $t \to \infty$. From $\hat{\mathbf{e}}, \mathbf{e} \in L_\infty$, it follows that $\mathbf{x}, \hat{\mathbf{x}} \in L_\infty$. The boundedness of $y(t)$ follows that of $e_1(t)$ and $y_m(t)$. This completes the proof. □

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