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Determination of Dynamic Elastic Constants of Transversely Isotropic Rocks Using a Single Cylindrical Specimen

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A new experimental procedure and its associated interpretation theories are proposed to determine the five dynamic elastic constants of a transversely isotropic rock. Using Christoffel's equations, the elastic constants can be uniquely determined based on ultrasonic wave velocity measurements on a single cylindrical specimen. The new method requires that the orientation of planes of elastic symmetry be parallel to its longitudinal axis. The wave velocity measuring devices (i.e. transducers) can be mounted on the sides or ends of the specimen. The new method has been implemented in the laboratory on a transversely isotropic rock, argillite. Wave velocity measurements were taken on specimens with and without a tensile load applied at the ends. Consistent results were obtained in both cases. This paper describes details of the analytical background and the proposed experimental procedure. Available test results are presented to demonstrate the efficacy of the new method. © 1997 *Elsevier Science Ltd*

INTRODUCTION

Stiffness, a fourth order tensor C_{ijkl} that relates stress σ_{ij} to strain ϵ_{kl} is an important parameter in the analysis of rocks under stress. In general, for the fourth order tensor C_{ijkl} there are $3^4 = 81$ constants. Since σ_{ij} and ϵ_{kl} are symmetrical, thus C_{ijkl} has the following symmetry conditions:

$$C_{ijkl} = C_{jikl} = C_{ijlk} = C_{jilk} \quad (1)$$

Hence, the total number of independent constants is reduced to 36. Through considerations of the existence of an elastic potential (strain energy per volume), the number of independent constants is reduced from 36 to 21. That is, if we know these 21 constants, we know all 81 constants. A rock having 21 independent constants in C_{ijkl} is called generally anisotropic [1]. If C_{ijkl}, σ_{ij} and ϵ_{kl} are defined in a Cartesian coordinate system (X, Y, Z), these tensors are related in a matrix form (i.e. the generalized Hooke's law) as:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} \quad (2)$$

where σ_x, σ_y, σ_z and ϵ_x, ϵ_y, ϵ_z are normal stresses and normal strains in the X, Y, Z directions; τ_{yz}, τ_{xz}, τ_{xy} and γ_{yz}, γ_{xz}, γ_{xy} are shear stresses and shear strains on the YZ, XZ, XY coordinate planes; C_{11}, $C_{12} \ldots C_{66}$ are material parameters that relate σ_{ij} to ϵ_{kl}.

The constants of C_{ijkl} are referred to as elastic constants. The 21 independent elastic constants can be divided into five groups according to their physical meanings as follows [2]:

Group I: C_{11}, C_{22}, C_{33}, relate normal strains to normal stresses.

Group II: C_{44}, C_{55}, C_{66}, relate shear strains to shear stresses.

Group III: C_{12}, C_{13} and C_{23}, relate normal strains to normal stresses in different directions (i.e. ϵ_y to σ_x and ϵ_z to σ_y).

Group IV: C_{45}, C_{46}, C_{56}, relate shear strains to shear stresses in different directions (i.e. γ_{xz} to τ_{yz} and γ_{xy} to τ_{yz}).

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Group V: C_{14}, C_{15}, ... C_{35}, C_{36}, these are coupling factors between normal stresses and shear strains.

Determination of such parameters are, unfortunately, complicated by the often anisotropic nature of rocks. Anisotropy is common for foliated metamorphic rocks (e.g. argillite, slate, schist, phyllite, and gneiss), stratified sedimentary rocks (e.g. shale, sandstone, and coal), and rocks cut by one or several sets of regularly spaced joints. The engineering analysis of rock deformation under a given loading condition often involves the use of elastic theories that assume rock as a linear elastic continuum. For such analysis to be valid, it is imperative to consider the rock anisotropy.

The number of elastic constants can be reduced if the elastic material has some form of symmetry. The rock anisotropy is typically distinguished according to the number and orientation of planes of elastic symmetry. In most cases, an anisotropic rock can be modeled as orthotropic or transversely isotropic materials. An orthotropic rock contains three orthogonal planes of elastic symmetry and these planes have the same orientation throughout the rock. In this case, the elastic constants of group IV and V are reduced to zero, and thus only nine independent elastic constants remain. Transverse isotropy implies that, at each point in the rock, there exists an axis of symmetry of rotation, and the rock has isotropic properties in a plane normal to this axis. For a rock that is transversely isotropic, only five out of the nine remaining elastic constants are independent. If all directions in a rock are elastically equivalent, only two elastic constants are needed to describe its deformability.

Inversely, strains can be expressed in terms of stresses in the following tensor form:

$$(\sigma) = [a_{ij}](\epsilon) \quad (3)$$

where a_{ij} is defined as the elastic compliance of an elastic material. Similar to the elastic constants, there are 21, 9, 5, and 2 components of elastic compliances for a generally anisotropic, orthotropic, transversely isotropic, and isotropic rock, respectively. These elastic compliances are directly related to the engineering elastic constants, such as Young's modulus (E) and Poisson's ratio (v) for an isotropic rock. The relation between elastic compliances and engineering elastic constants for a transversely isotropic rock is given in the Appendix.

These elastic constants are usually assessed by static or dynamic tests in the field or laboratory [3]. The static elastic constants are suitable for conventional rock mechanics analysis. For rocks subjected to transient dynamic loading conditions, the dynamic elastic constants are required. Also, the dynamic elastic constants are frequently used in assessing the degree of fractures in rocks.

The borehole jack or pressure plate are popular means of measuring rock stiffness on tunnel walls [4–6]. However, these applications are limited to isotropic rocks. It is possible to use borehole jacking in conjunction with seismic methods to determine the five elastic constants in a transversely isotropic rock [7]. The application of such field methods is rare because they are expensive, time consuming, and unreliable. Static and dynamic laboratory tests are frequently conducted on rock specimens to determine the elastic constants.

Ultrasonic wave velocity measurements are commonly used to determine the dynamic elastic constants for rocks in the laboratory. This method could require the use of multiple rock specimens [8–11], depending on the nature of anisotropy or the number of elastic constants involved. Alternatively, a single specimen with a special shape, such as an 18-faced polyhedron or a sphere, could be used to perform the ultrasonic tests [12, 13]. Most of the anisotropic rocks are either orthotropic or transversely isotropic. Three or four cylindrical specimens [14–16] with different orientations of elastic symmetry planes have been used in the past to determine the elastic constants using wave velocity measurements. The accuracy of the test results depends on the homogeneity of rock specimens, repeatability of the testing techniques, among others. Hence, the less the number of test specimens the better it is. Although the number of a specially shaped specimen can be restricted to one as previously mentioned, the sample preparation is difficult and these specimens can not be used for other types of rock mechanics tests. Jones and Wang [17] proposed a testing method to determine the five dynamic elastic constants using a cylindrical specimen with planes of transverse isotropy perpendicular to the longitudinal axis. Their method involves a series of end to end shear and compression wave velocity measurements on the specimens. A single cylindrical specimen would suffice in Jones and Wang's method in case of transverse isotropy, if compression wave velocities at 45° from the planes of elastic symmetry are measured. However, no test results using a single specimen were presented in their paper. This is apparently because end to end, 45° measurements are difficult to conduct, especially when loading is applied at the ends.

In this paper, a practical method of conducting and interpreting ultrasonic tests is proposed to determine the dynamic elastic constants of a transversely isotropic rock. The new method uses a single cylindrical specimen with planes of transverse isotropy parallel to the longitudinal axis. The ultrasonic sensors can be placed on the side of the specimen, thus allow end loading to be applied while conducting 45° compressive velocity measurements. To demonstrate the advantages of the new method, a series of tests were performed on an argillite specimen that was transversely isotropic. The ultrasonic wave velocities were measured under direct tension, and loading free conditions. This paper briefly reviews the principles of wave propagation in a transversely isotropic body. Two types of ultrasonic wave velocity measurement techniques are then presented. Finally, test results on the argillite rock specimen, using both techniques, are presented and discussed.

WAVE PROPAGATION IN A TRANSVERSELY ISOTROPIC BODY

A general form for the constitutive relationship of an anisotropic medium [equation (2)] can be written in a tensor form as:

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} \tag{4}$$

Where C_{ijkl} represents the elastic stiffness. Incorporating the equation of motion into equation (4), wave propagation in an anisotropic space can be derived using Christoffel's equation [18] as follows:

$$C_{ijkl}n_k n_l \Delta_i - \rho V^2 \Delta i = 0 \tag{5}$$

If we set $\Gamma_{ij} = C_{ijkl}n_k n_l$, then Christoffel's equation becomes:

$$\Gamma_{ij}\Delta_j - \rho V^2 \Delta_i = 0 \tag{6}$$

where Γ_{ij} is called Christoffel's tensor and is symmetrical; n_k and n_l are the components of a unit vector that defines the direction of wave propagation; Δ_i is the polarization of the wave; V is the magnitude of wave velocity; and ρ is the unit weight of the medium.

Solving Christoffel's equation is an eigenvalue problem. The solution is obtained from the determinant of the coefficients of equation (6):

$$\det(\Gamma_{ij} - \rho V_k^2 \delta_{ij}) = 0 \tag{7}$$

ρV_i^2 are the three eigenvalues of the corresponding eigenvector Δ_i of tensor Γ_{ij}. The solutions include three orthogonal polarization directions at any given point on a wavefront. The wave velocity in these three directions may be different in an anisotropic medium. Depending on the direction of propagation and polarization (direction of particle motion) of a wavefront, three wave types may be identified (i.e. one compression and two shear wave types). The equation also implies that the velocity at a point depends on the orientation of the planes of elastic symmetry and the elastic compliance of the medium. Thus, the elastic constants (i.e. components of C_{ijkl}) of an anisotropic rock can be calculated from Christoffel's equation if V_i through the medium is known. The constants thus calculated are generally said to be dynamic, as distinguished from the static constants obtained from compression or tension tests. For a general anisotropic material, to calculate the 21 dynamic elastic constants using equation (6), 21 or more wave velocities of different propagation and polarization directions are required.

For a material that is transversely isotropic, consider Z to be the rotation axis of elastic symmetry, X and Y axes in the plane of transverse isotropy, equation (2) becomes:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} \tag{8}$$

$C_{66} = (C_{11} - C_{12})/2$ and thus, only five independent elastic constants C_{11}, C_{33}, C_{12} (or C_{66}), C_{13}, and C_{44} exist in C_{ijkl}. These components are directly related to a set of engineering elastic constants E, E', ν, ν', and G' (see the Appendix) where:

- E, E' are Young's moduli in the plane of transverse isotropy and in a direction normal to it, respectively,
- ν, ν' are Poisson's ratios characterizing the lateral strain response in the plane of transverse isotropy to a stress acting parallel and normal to it, respectively, and,
- G' is the shear modulus in planes normal to the plane of transverse isotropy.

For a plane wave in the XZ or YZ planes, equations of body wave velocities in terms of the five constants are derived from equation (7) as follows:

$$V_{s1}, \theta_Z(x, z) = V_{s1}, \theta_Z(y, z) = \sqrt{\frac{C_{44}\cos^2\theta_Z + C_{66}\sin^2\theta_Z}{\rho}} \tag{9}$$

$$V_P, \theta_Z(X, Z) = V_P, \theta_Z(y, Z) = \left\{\frac{C_{11}\sin^2\theta_Z + C_{33}\cos^2\theta_Z + C_{44} + \Delta}{2\rho}\right\}^{1/2} \tag{10}$$

$$V_{s2}, \theta_Z(X, Z) = V_{s2}, \theta_Z(y, Z) = \left\{\frac{C_{11}\sin^2\theta_Z + C_{33}\cos^2\theta_Z + C_{44} - \Delta}{2\rho}\right\}^{1/2} \tag{11}$$

$$\Delta = \{[(C_{11} - C_{44})\sin^2\theta_Z - (C_{33} - C_{44})\cos^2\theta_Z]^2 + 4(C_{13} + C_{44})^2 \sin^2\theta_Z \cos^2\theta_Z\}^{1/2}$$

where

- θ_x, θ_y, and θ_z are angles between the direction of the plane wave on the coordinate planes XY, YZ, and XZ and the coordinate axes X, Y, and Z, respectively, as shown in Fig. 1.
- V_p is the velocity of compression wave; V_{s1} is the shear wave velocity polarized on the plane of wave propagation; and V_{s2} is the shear wave velocity polarized perpendicular to the plane of wave propagation.

The velocities in the XY plane are expressed only in terms of three independent constants, i.e. C_{11}, C_{44}, and C_{12} (or C_{66}) as follows:

$$V_P, \theta_X(x, y) = \sqrt{\frac{C_{11}}{\rho}}; \quad V_{S1}, \theta_X(x, y) = \sqrt{\frac{C_{44}}{\rho}}; \quad V_{S2}, \theta_X(x, y) = \sqrt{\frac{C_{66}}{\rho}} \tag{12}$$

Since the plane XY is isotropic, the velocities in the plane depend only on the elastic constants and ρ.

For waves propagating parallel to the three coordinate axes, the wave velocities in terms of dynamic elastic constants and ρ are:

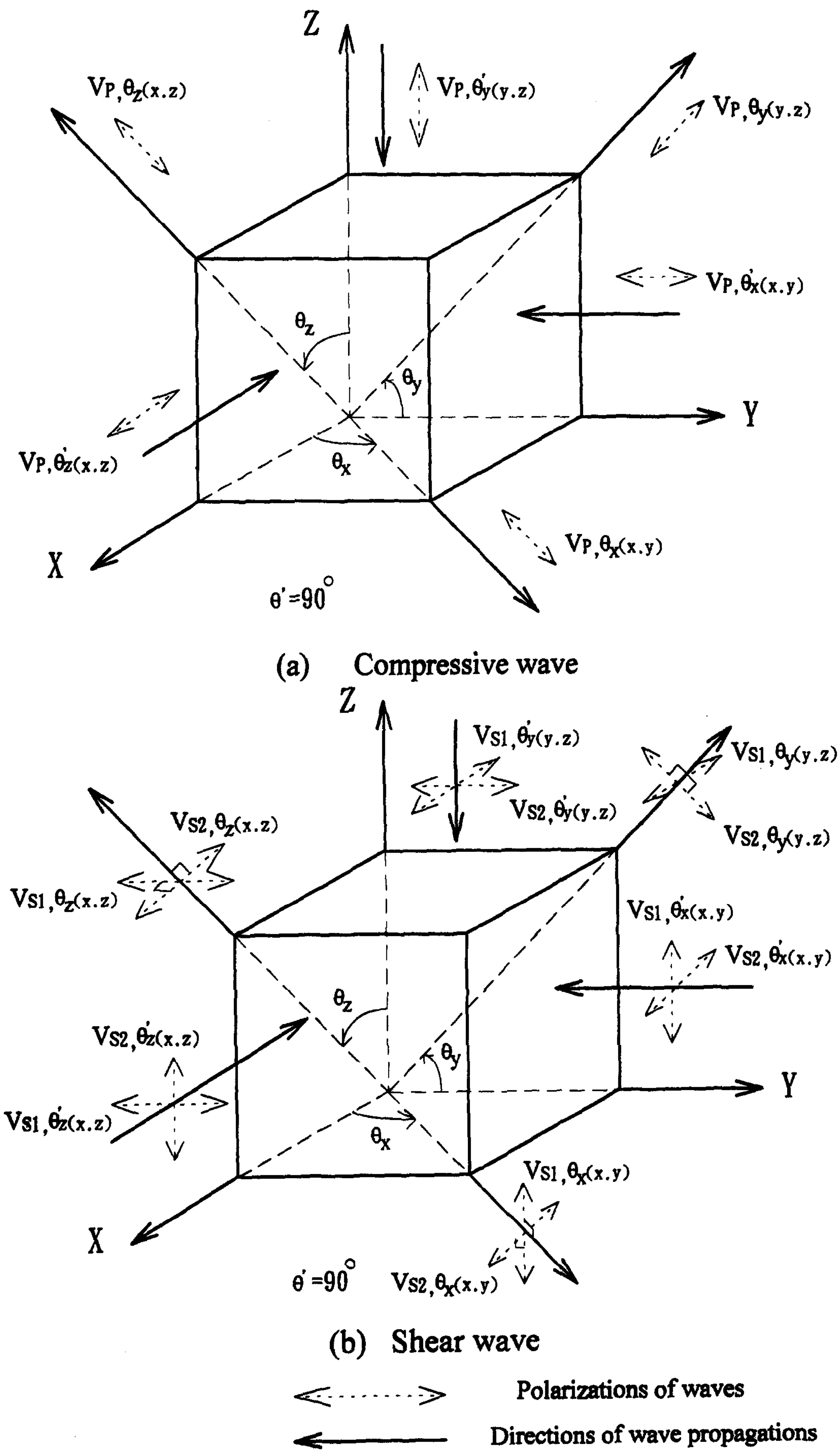

Fig. 1. Wave types in an anisotropic medium.

$$V_{\mathrm{P}}, \theta_x(x, y) = V_{\mathrm{P}}, \theta_z(x, z) = \sqrt{\frac{C_{11}}{\rho}}; \quad V_{\mathrm{P}}, \theta_z(y, z) = \sqrt{\frac{C_{33}}{\rho}} \quad (13)$$

$$V_{\mathrm{S1}}, \theta_x(x, y) = V_{\mathrm{S2}}, \theta_z(x, z) = V_{\mathrm{S1}}, \theta_z(y, z) = V_{\mathrm{S2}}, \theta_z(y, z) = \sqrt{\frac{C_{44}}{\rho}} \quad (14)$$

$$V_{\mathrm{S2}}, \theta_x(x, y) = V_{\mathrm{S1}}, \theta_z(x, z) = \sqrt{\frac{C_{66}}{\rho}} \quad (15)$$

where $\theta_x = \theta_z = 90°$.

Only four constants can be calculated if velocity measurements are performed on the three coordinate planes. The value of C_{13} can be calculated from equation (10) using a compression wave velocity measurement with an inclined angle (θ_Z) to the axis perpendicular to the plane of transverse isotropy (Z axis), or a plane involving Z axis (for example XZ or YZ planes).

LABORATORY MEASUREMENTS OF ELASTIC CONSTANTS ON TRANSVERSELY ISOTROPIC ROCK

The wave velocities, as shown in equations (9)–(11), are directly related to the elastic constants. Theoretically, the five elastic constants can be uniquely determined with five wave velocity measurements, if the direction of these waves are strategically selected. Wave velocity measurements are conducted using a pair of ultrasonic transducers. The wave direction is controlled by the relative locations of the transducer pair on the specimen surface. Figure 2 depicts possible locations of ultrasonic transducers and their relationship with the plane of symmetry. Two alternatives of transducer set ups and their interpretation of test results are proposed as follows:

Transducers mounted on the sides of a cylindrical specimen

This set up takes advantage of the fact that coordinate plane XY is on the plane of elastic symmetry, and YZ is parallel to the cross section of specimen. For waves propagating in YZ plane, equations (9)–(11) can be directly applied to solve the five elastic constants. Pairs of ultrasonic transducers are located at different vertical locations on a YZ plane. The velocities calculated from equations (9)–(11) are expressed in terms of the five elastic constants and θ_Z which is calculated from the relative locations of the two transducers. Hence, constants C_{12}, C_{44}, and C_{11} can be obtained from measuring V_{s1} with different θ_Z. With two additional measurements of V_{P} or V_{s2} at different θ_Z, the remaining constants are calculated.

Equations (10) and (11) imply that a velocity measurement on the YZ plane but with a θ_Z between 0 and 90° is required to obtain C_{13}. Theoretically, the five elastic constants can be calculated with five or more measurements of V_{P} or V_{s2} with different θ_Z angle. For example, a set of measurements at $\theta_Z = 15$, 30, 45, 60, and 75°, or at $\theta_Z = 10$, 20, 30, 40, 50, 60, 70, and 80° are all acceptable locations. When the number of measurements exceeds five, a statistical technique, such as a non-linear least square analysis, should be used to optimize a set of constants. However, wave velocity measurements with θ_Z between 0 and 90° are more susceptible to errors, and thus are not recommended to replace those along the axes of symmetry. Based on the previous experience and theoretical considerations, recommended transducer arrangements to obtain the five dynamic elastic constants are shown as Fig. 2. Figure 2 depicts V_{s1} and V_p measurements at $\theta_Z = 0°$ and at $\theta_Z = 90°$, and an extra V_p measurement at $\theta_Z = 45°$.

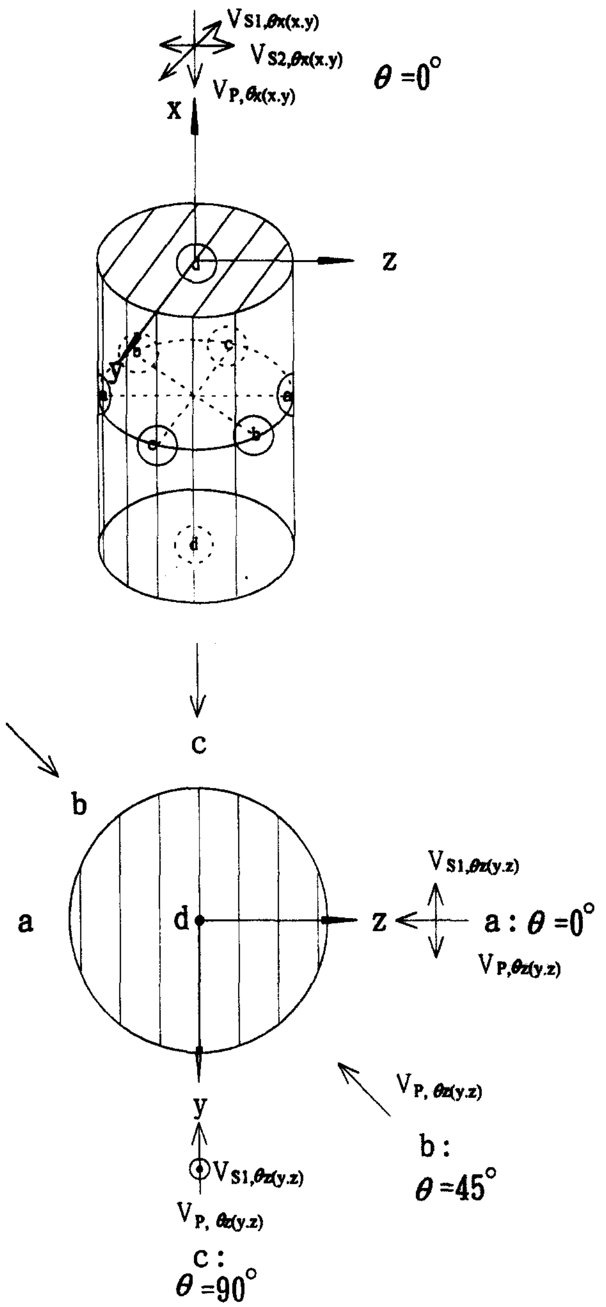

Fig. 2. Transducer locations to measure elastic constants of a transversely isotropic rock.

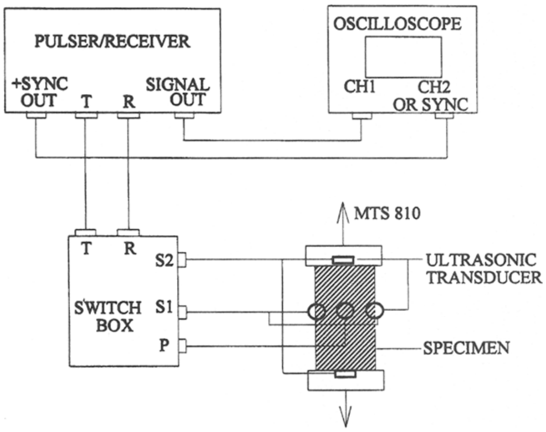

Fig. 3. Ultrasonic wave velocity measuring system.

The resulting dynamic elastic constants expressed in terms of measured velocities are:

$$C_{11} = \rho V_p^2, \theta_z^*(y, z);$$

$$C_{33} = \rho V_p^2, \theta'_z(y, z);$$

$$C_{44} = \rho V_{s1}^2, \theta'_z(y, z);$$

$$C_{66} = \rho V_{s1}^2, \theta_z^*(y, z);$$

$$C_{13} = [(2\rho V_p^2, \theta''_z(y, z) - 0.5(C_{11} + C_{33}) - C_{44})^2 - 0.25(C_{11} - C_{33})^2]^{0.5} - C_{44}; \quad (16)$$

where $\theta_z^* = 90°$, $\theta'_z = 0°$, and $\theta''_z = 45°$.

Transducers mounted on the sides and ends of a cylindrical specimen

Because of better contact conditions, transducers mounted on a flat surface (i.e. ends of a cylinder) are inherently better than on a curved surface (i.e. side of a cylinder). An alternative method, involving transducers mounted on ends of a cylindrical specimen, for part of the measurements, is proposed. In this method, V_P, V_{s1}, and V_{s2} from transducers on the ends of cylinder (i.e. $\theta_X = 0$) are used to determine C_{11}, C_{44}, and C_{66}. Constants C_{13} and C_{33} are calculated from the side to side measurements of V_P with $\theta_Z = 45$ and 0 as previously described. The dynamic elastic constants expressed in terms of the measured velocities are:

$$C_{11} = \rho V_p^2, \theta'_x(x, y); \quad C_{44} = \rho V_{s1}^2, \theta'_x(x, y);$$

$$C_{66} = \rho V_{s2}^2, \theta'_x(x, y); \quad C_{33} = \rho V_p^2, \theta'_z(y, z)$$

$$C_{13} = [(2\rho V_p^2, \theta''_z(y, z) - 0.5(C_{11} + C_3) - C_{44})^2 - 0.25(C_{11} - C_{33})^2]^{0.5} - C_{44}; \quad (17)$$

where $\theta'_x = \theta'_z = 0°$ and $\theta''_z = 45°$.

For the case presented in this paper, only two velocities V_p and V_{s1} (or V_{s2}) can be measured by the pair of transducers due to the limitation of their capabilities. Hence, three velocity measurements on the sides and two on the ends of the specimen are proposed. The arrangement of ultrasonic transducers is depicted in Fig. 2. In this case, the dynamic elastic constants can be expressed as similar to equation (17), except that C_{44} is replaced by:

$$C_{44} = \rho V_{s1}^2, \theta'_z(y, z); \quad (18)$$

where $\theta'_z = 0°$.

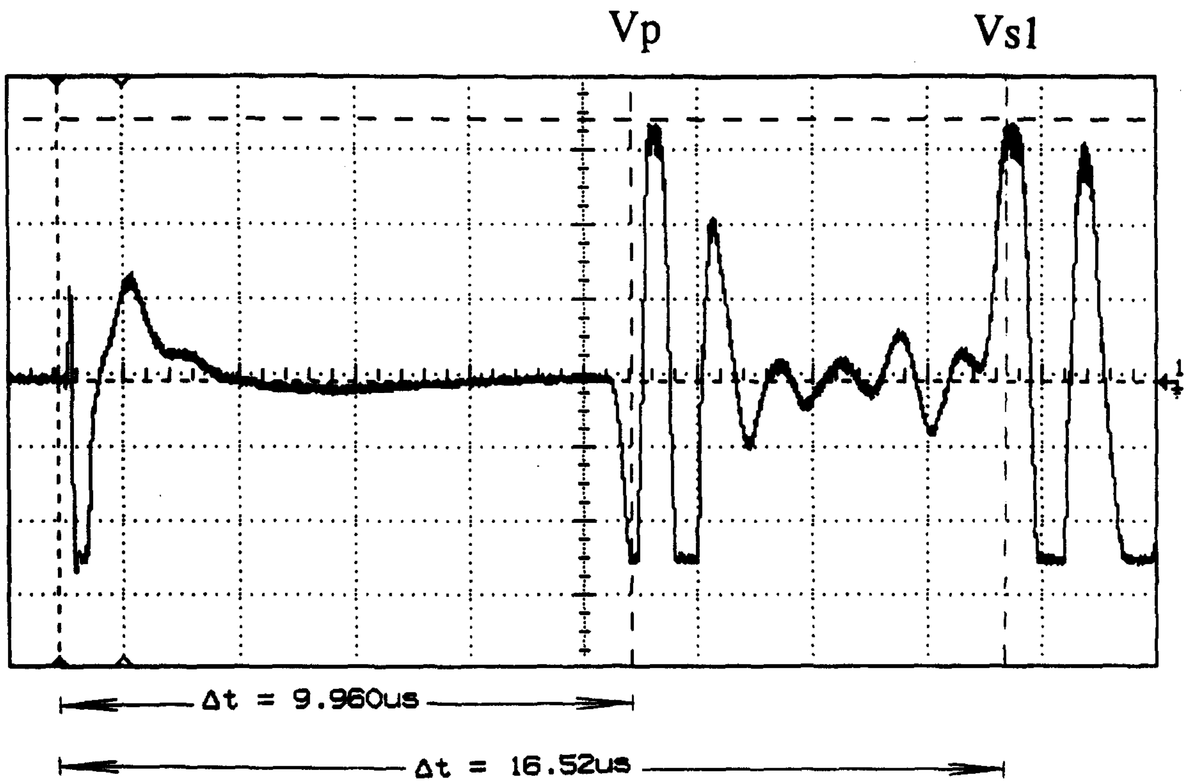

Fig. 4. A typical wave record.

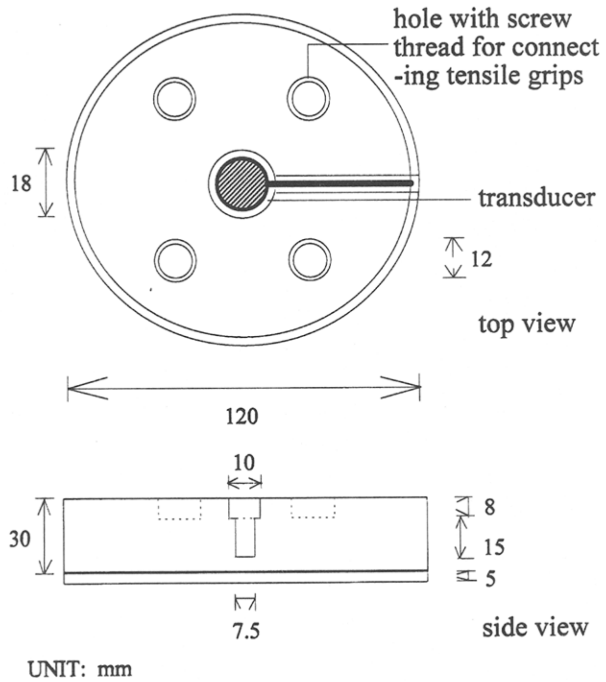

Fig. 5. Transducer housing cap.

LABORATORY DEMONSTRATION

In order to verify and demonstrate the proposed methods described in this paper, a series of ultrasonic tests were conducted on cylindrical specimens of a transversely isotropic rock. In the first part of the tests, wave velocity measurements were performed on eight specimens with no load applied. In the second part of the tests, wave velocity measurements were made when the specimen was under a direct tension loading condition. Details of the rock specimens, test set up, and results are described in the following sections.

The rock specimens

NX sized core specimens with a length to diameter ratio of 2.5 were prepared with core axis parallel to foliation planes from block samples of argillite, a metamorphic rock. The argillite was formed from slight metamorphism of shale or silty shale. Given a moderately increased degree of metamorphism, it could transform into slate. Argillite is often encountered in mountainous areas of Taiwan. The argillite block samples used in this study were taken from a tunnel in northern Taiwan. The core samples were prepared following the procedure suggested by ISRM [19]. The microscopy studies on the rock specimens revealed that the argillite consisted of 45% quartz and 55% other silicate minerals (Illite, chlorite, . . .). Clear foliation planes are well-developed by the recrystallization of clay minerals. The foliation planes can be considered as planes of elastic symmetry of an anisotropic material and the argillite considered as a transversely isotropic material. The grain size of argillite is between that of silt and clay. The argillite has a dry unit weight of 26.3–27.2 kN/m^3, specific gravity of 2.71–2.75, and porosity of 0.014–0.018.

Laboratory experimental program and set up

The elastic constants were determined based on ultrasonic wave velocity measurements in rock specimens. The wave velocity measurement set up is depicted in Fig. 3. The ultrasonic testing system includes a pulser/receiver (Parametrics 5058 PR), 10 mm diameter ultrasonic transducers capable of generating/receiving 1 MHz shear waves (Parametrics V101-RM) and compressive waves (Parametrics V153-RM), and an oscilloscope (HP 54600A). V101-RM can generate and receive both compressive and shear waves. The ultrasonic system is used to measure the wave travel time. Knowing the distance between the generating and receiving ultrasonic transducers, the wave velocity is then determined. The accuracy of velocity measurements is directly related to the reliability of observed wave travel time. Figure 4 shows a typical wave form generated/received from a pair of V101-RM transducers. The arrivals of the compressive and shear waves are clearly distinguishable. Possible errors in selecting the wave arrival time, and hence the wave travel time, as depicted in Fig. 4 are minimal.

In the first part of the tests, wave velocity measurements were performed on eight specimens (numbered 1–8) with no load applied. As depicted in Fig. 2, five sets of velocity measurements were conducted with transducers mounted on the sides of the specimens only.

In the second part of the tests, specimen No. 7 of the first part was used for wave velocity measurements.

Table 1. Velocity measurements under loading free conditions

Sample no.	$V_P, \theta_z'(y,z)$ (m/sec)	$V_{S1}, \theta_z'(y,z)$ (m/sec)	$V_P, \theta_z^*(y,z)$ (m/sec)	$V_{S1}, \theta_z^*(y,z)$ (m/sec)	$V_P, \theta_z''(y,z)$ (m/sec)
1	4742	2874	5319	3236	4955
2	4697	2849	5303	3283	4931
3	4730	2915	5387	3331	4977
4	4621	2709	5357	3277	4834
5	4560	2745	5351	3254	4806
6	4551	2670	5363	3247	4754
7	4565	2892	5318	3266	4803
8	4499	2844	5273	3395	4725
mean	4621	2812	5334	3286	4848
Coefficients of variation (%)	2.0	3.2	0.7	1.6	2.0

($\theta_z' = 0°$, $\theta_z^* = 90°$, $\theta_z'' = 45°$).

The transducers were located on ends and sides of the specimen. Wave velocity measurements were made when the specimen was under different levels of tension applied at the ends. In addition, strain gauges were attached to the surface of the specimen to provide reference measurements of axial and transverse strains during loading.

A material testing machine (MTS 810) with a capacity of 250 kN capacity was used to apply tension on the specimen in Part 2 tests. Readers are referred to [9] for details of the tensile test set up and the associated gripping device. A pair of metal caps were used to house the ultrasonic transducers as shown in Fig. 5. The transducers are connected to the pulser/receiver through a switch box that allows velocity readings to be taken sequentially. The transducers may be damaged due to the sudden impact when specimens fail. The average tensile strength of argillite specimens with foliation parallel to the loading direction is 12.0 MPa [9]. Hence, the maximum applied stress (6 MPa) is selected to be less than half of the expected tensile strength. Tension was applied in 6 steps of 1 MPa. For each increment, the tensile stress was kept constant for 1 min.

Test results

The results of Part 1, including velocity measurements and calculated engineering elastic constants, are shown in Tables 1 and 2. The results indicate that the measured velocities and elastic constants are rather consistent among the eight specimens, with coefficient of variation less than 6%, except for v and v'. The values of E/E', v/v', and G/G' are between 1 and 2. These ratios are within reasonable thermal dynamic constraints of an anisotropic material [20,21]. Hence, the measured dynamic elastic constants can be considered reasonable and the test method valid.

Figure 6 shows the velocity measurements on specimen No. 7 at different tensile stresses from Part 2 tests. The result indicates that compression wave velocity changes little with tensile stress. The shear wave velocity increases slightly with the tensile stress after 2 MPa. The total increase of the shear wave velocity at 6 MPa is 0.5%. The results imply that the dynamic elastic constants are approximately constant when the applied stress is low. As the tensile stress increases, the dynamic

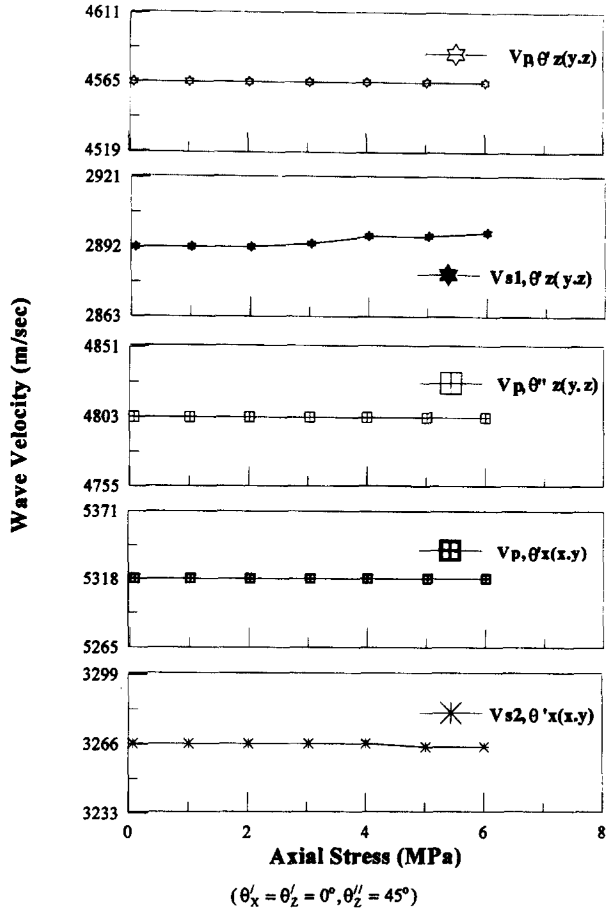

Fig. 6. Wave velocities (specimen No. 7) under tensile stress.

elastic constants begin to be moderately sensitive to the change of stress. This means that the magnitude of stress within the specimen and the presence of microcracking can influence the elastic constants (Fig. 7). Hence, the elastic constant measurements can be used to evaluate the degree of fracture in the specimen during loading.

The E and v values from wave velocity measurements (part of dynamic elastic constants) are consistent with those deduced from strain gage readings (see Fig. 8) which are static parameters. The static elastic constants, $E = 65.2$ GPa and $v = 0.21$, are tangential values at the tensile stress of 5 MPa. These two sets of values agree within 5%, although the dynamic E and v values are slightly higher. Again, the results indicate the test method and its interpretation are valid.

Table 2. Results of calculated dynamic engineering elastic constants of argillite under loading free conditions

Sample no.	E (GPa)	E' (GPa)	G' (GPa)	v	v'
1	66.58	52.49	21.89	0.20	0.19
2	66.69	50.95	21.51	0.17	0.20
3	69.53	53.07	22.52	0.18	0.18
4	68.02	50.25	19.45	0.20	0.18
5	68.20	50.08	19.97	0.22	0.16
6	68.53	50.37	18.90	0.23	0.15
7	69.04	52.42	22.16	0.22	0.12
8	70.13	51.18	21.43	0.15	0.12
mean	68.34	51.35	20.98	0.196	0.163
Coefficients of variation (%)	1.8	2.2	6.0	14	18

CONCLUSION

A new experimental method has been proposed for the determination of dynamic elastic constants for a transversely isotropic rock. The interpretation of test results follows the theories of wave propagation in a transversely isotropic elastic medium. Available tests have demonstrated that this method can be used for rock specimens with or without an axial load applied through the ends of the specimen. A single cylindrical specimen would suffice in the new method to obtain the five elastic constants.

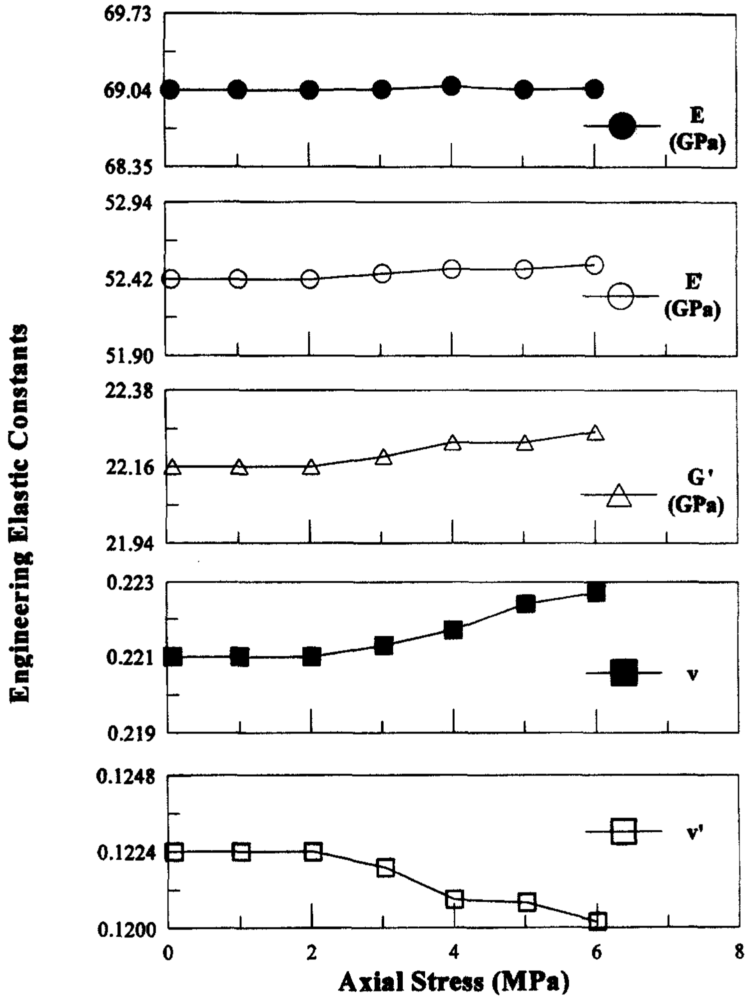

Fig. 7. Calculated dynamic engineering elastic constants (specimen No. 7) under tensile stress.

An important requirement for the new method is that the planes of transverse isotropy have to be parallel to the longitudinal axis of the specimen. Unfortunately, the orientation of the planes of elastic symmetry is not easily to be controlled when taking rock cores in the field. As a result, the proposed method may only be applicable to specimens specifically prepared from a block sample. An obvious desirable improvement is to extend the theories and test procedure to include the cases of inclined planes of transverse isotropy. In that case, the method can be practically applied to any transversely isotropic rock cores directly from the field.

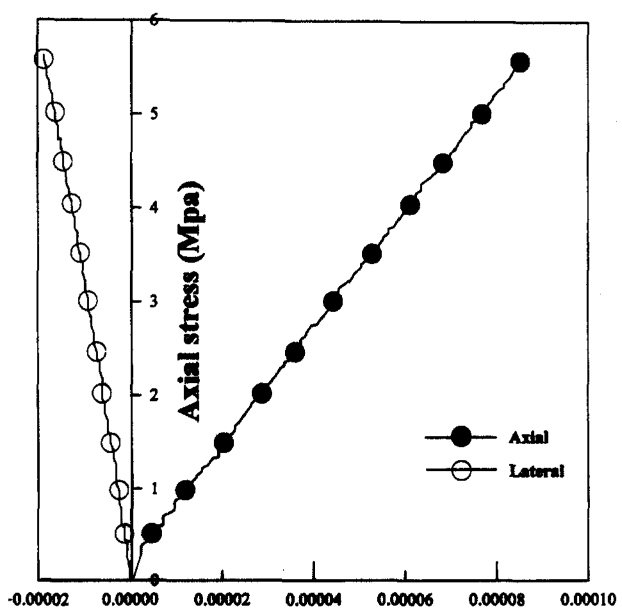

Fig. 8. Stress–strain curves of specimen No. 7 under tensile stress.

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APPENDIX

The relation between elastic compliances and engineering elastic constants of a transversely isotropic material is expressed as follows:

$$a = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{12} & a_{11} & a_{13} & 0 & 0 & 0 \\ a_{13} & a_{13} & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{E} & -\frac{v}{E} & -\frac{v'}{E'} & 0 & 0 & 0 \\ -\frac{v}{E} & +\frac{1}{E} & -\frac{v'}{E'} & 0 & 0 & 0 \\ -\frac{v'}{E'} & -\frac{v'}{E'} & \frac{1}{E'} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G'} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G'} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix}$$

Based on the general Hooke's law for a transversely isotropic material, the elastic complainces can be expressed in terms of the elastic constants:

$$a_{11} = \frac{C_{11}C_{33} - C_{13}^2}{(C_{11} - C_{12})(C_{11}C_{33} + C_{12}C_{33} - 2C_{13}^2)}$$

$$a_{33} = \frac{C_{11} + C_{12}}{(C_{11}C_{33} + C_{12}C_{33} - 2C_{13}^2)}$$

$$a_{12} = \frac{C_{13}^2 - C_{12}C_{33}}{(C_{11} - C_{12})(C_{11}C_{33} + C_{12}C_{33} - 2C_{13}^2)}$$

$$a_{13} = \frac{-C_{13}}{(C_{11}C_{33} + C_{12}C_{33} - 2C_{13}^2)}$$

$$a_{44} = \frac{1}{C_{44}}$$

$$a_{66} = \frac{1}{C_{66}}$$

Then, the engineering elastic constants are easy to obtain as:

$$E = \frac{1}{a_{11}}, \quad E' = \frac{1}{a_{33}}, \quad G' = \frac{1}{a_{44}},$$

$$G = \frac{1}{a_{66}}, \quad v = -\frac{a_{12}}{a_{11}}, \quad v' = -\frac{a_{13}}{a_{33}}$$