# Strictly Nonblocking Three-Stage Clos Networks With Some Rearrangeable Multicast Capability

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***Abstract*—Hwang and Lin introduced a new nonblocking requirement for 2-cast traffic which imposes different requirements on different types of coexisting calls. The requirement is strictly nonblocking for point-to-point calls among the 2-cast traffic, and is rearrangeable for genuine 2-cast calls. We generalize the 2-cast calls to multicast calls and give a sufficient condition for such networks when the number of multicast calls is upper bounded.**

***Index Terms*—Genuine multicast call, multicast call, rearrangeable, strictly nonblocking, three-stage Clos network.**

## I. Introduction

CONSIDER a three-stage Clos network $C(n_1, n_2, r_1, r_2, m)$ where the input stage consists of $r_1$ $n_1 \times m$ crossbars, the middle stage $m$ $r_1 \times r_2$ crossbars, the output stage $r_2$ $m \times n_2$ crossbars, and there exists one link between every pair of switches between two adjacent stages (see Fig. 1).

The inlets of the input switches are the *inputs* of the network and the outlets of the output switches are the *outputs* of the network. A network is called *strictly nonblocking* if any pair of idle input and output can be connected regardless of the existing connections of other pairs in the network (all paths must be link disjoint). A network is called *rearrangeable* if any set of disjoint pairs of inputs and outputs can be simultaneously connected. If the calls come sequentially, rearrangeability means we can disconnect all existing connections and reroute them together with the new call simultaneously.

Besides the point-to-point traffic as mentioned above, there is also multicast traffic where each input can request connection to many outputs. The multicast traffic is called *$f$-cast* if, at most, $f$ outputs can be requested in each connection. Note that the $f$-cast traffic can include point-to-point calls. We will call a multicast call with more than one output a *genuine multicast call*.

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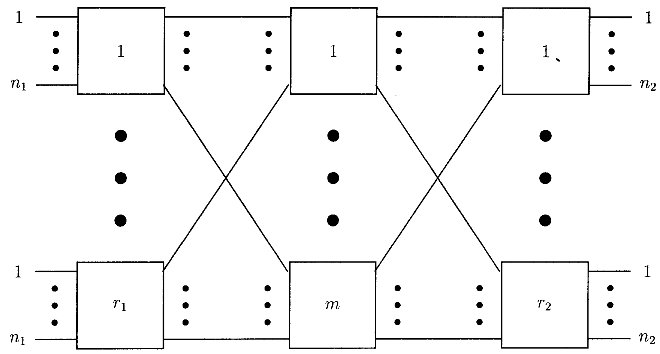


Fig. 1. $C(n_1, n_2, r_1, r_2, m)$.

Hwang and Lin [6] consider a three-stage Clos network in the situation where the predominant use is point-to-point traffic, but occasionally the network is requested to carry some multicast traffic. In particular, they conjectured the following.

*Conjecture 1:* $C(n, r, 2n) = C(n, n, r, r, 2n)$ is rearrangeable for 2-cast calls and strictly nonblocking for the point-to-point calls among them.

It is well known [1] that $C(n_1, n_2, r_1, r_2, m)$ is strictly nonblocking if $m \geq n_1 + n_2 - 1$. It is also well known [7] that $C(n_1, n_2, r_1, r_2, m)$ is $f$-cast rearrangeable if $m \geq fn_1$. Therefore, $C(n, r, 2n)$ is indeed strictly nonblocking if all traffic is point-to-point, and rearrangeable if all traffic is 2-cast. However, this does not imply that when the traffic contains both point-to-point calls and genuine 2-cast calls, $C(n, r, 2n)$ can still be strictly nonblocking for each point-to-point call and rearrangeable for each genuine 2-cast call. This is because the routing for the "strictly nonblocking" part assumes each connection, point-to-point or 2-cast, to use only one outlet of the involved input switch, while the routing for the "rearrangeable" part assumes each genuine 2-cast call to use two such outlets. Thus, the two routings conflict with each other in how to route a 2-cast call. To preserve the strict nonblockingness for point-to-point calls, we must find a new routing for genuine 2-cast calls in which the input switches do not fan out a connection (with the fringe benefit that the input switches do not need fan-out capability).

In this letter, we prove a result related to *Conjecture 1*. It is a special case of *Conjecture 1* in the sense that the number of multicast calls is restricted. Note that this special case fits the intended application when multicast calls are occasional. Our result is more general than *Conjecture 1* in the sense that we deal with the more general $C(n_1, n_2, r_1, r_2, m)$, we require a smaller $m$, and we handle multicast, not just 2-cast, calls.

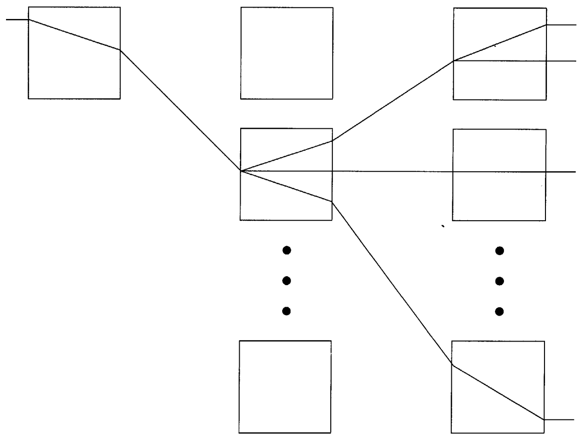

Fig. 2. Connection of a multicast call.

## II. The Main Result

Let $M$ denote the number of multicast calls in the network, including those already connected and the newly arrived.

*Theorem 1:* $C(n_1, n_2, r_1, r_2, m)$ with $M \leq m$ is rearrangeable for multicast calls and strictly nonblocking for the point-to-point calls among them if and only if $m \geq \min\{n_1 + n_2 - 1, n_1 r_1, n_2 r_2\}$.

*Proof:* "if." Since $n_1 r_1$ and $n_2 r_2$ are trivially sufficient for nonblocking, we show that $n_1 + n_2 - 1$ is also enough. We first prove the "rearrangeability" part. When a multicast call arrives, we disconnect all existing calls, and then route all multicast calls, at most, $m$ of them, each to a distinct middle switch. Using the fan-out property of a crossbar, the connection splits $f$ ways in that middle switch to go to the $f$ destination output switches. An output switch containing $g$ destinations will have $g$ splits within that switch (see Fig. 2). Note that each multicast call consumes one outlink of the involved input switch, and one inlink of each involved output switch. Next we connect the point-to-point call. Suppose we are connecting a call from input switch $I$ to output switch $O$. Due to the way we route the multicast calls, regardless of whether the other $n_1 - 1$ inlets of $I$ are connected, and whether these connections are point-to-point or multicast, at most, $n_1 - 1$ outlinks of $I$ are consumed. Similarly, at most, $n_2 - 1$ inlinks of $O$ are consumed. Since

$$(n_1 - 1) + (n_2 - 1) < n_1 + n_2 - 1$$

there must exist a middle switch with open links to both $I$ and $O$, and can be used to carry the $(I, O)$ call. Next we prove the "strict nonblockingness" part. Note that the above argument to route a point-to-point call is valid regardless of how previous point-to-point calls are connected. Therefore, that argument can also be used to prove that the point-to-point calls are strictly nonblocking.

"only if." Since $m \geq \min\{n_1 + n_2 - 1, n_1 r_1, n_2 r_2\}$ is necessary for strict nonblockingness for the special case when all calls are point-to-point, it is also necessary for the general multicast traffic. ∎

The $M \leq m$ restriction applies only when $\min\{n_1 + n_2 - 1, n_1 r_1, n_2 r_2\} = n_1 + n_2 - 1$. We next show that this restriction can be dropped if $n_1$ is much larger than $n_2$.

*Corollary 1:* $C(n_1, n_2, r_1, r_2, n_1 + n_2 - 1)$ is rearrangeable for multicast calls and strictly nonblocking for the point-to-point calls among them, if $n_1 \geq k n_2$ for some integer $k$, and $r_2 \leq k + 3$, except for $n_1 = n_2$.

*Proof:*

$$\begin{aligned} M &\leq \left\lfloor \frac{r_2 n_2}{2} \right\rfloor \leq \left\lfloor \frac{(k+3) n_2}{2} \right\rfloor \\ &\leq \left\lfloor \frac{n_1 + 3 n_2}{2} \right\rfloor \leq n_1 + n_2 - 1. \end{aligned}$$

∎

*Corollary 2:* $C(n, r, 2n)$ is rearrangeable for multicast calls and strictly nonblocking for the point-to-point calls among them if $r \leq 4$.

*Proof:*

$$M \leq \frac{rn}{2} \leq 2n.$$

∎

From the proof of *Theorem 1*, we know that if there is a way to route all multicast calls by using, at most, $n_1 + n_2 - 1$ middle switches, then we can properly route the remaining point-to-point calls with strictly nonblocking properties. In [5], Hwang and Liaw have shown that $C(n_1, n_2, r_1, r_2, n_1 + f(n_2 - 1))$ is strictly nonblocking for $f$-cast. From their argument, we know that $k + f(n_2 - 1)$ middle switches are enough to route $f$-cast calls, with each input switch having, at most, $k$ multicast requests. These results suggest the following theorem to replace the bound on the total number of multicast calls by the bound on multicast calls in each input switch.

*Theorem 2:* $C(n_1, n_2, r_1, r_2, \max\{k + f(n_2 - 1), n_1 + n_2 - 1\})$ is rearrangeable for multicast calls and strictly nonblocking for the point-to-point calls among them if each call can have, at most, $f$ outputs, and each input switch has, at most, $k$ genuine multicast requests.

We can take advantage of *Theorem 2* to design a three-stage Clos network with $n_1$ much larger than $n_2$ so that more genuine multicast calls can be accomodated. For example, for $f = 2$, then by *Theorem 2*, $n_1 + n_2 - 1$ middle switches are enough for the mixed traffic with each input switch having, at most, $n_1 - n_2 + 1$ 2-cast calls. So, the total number of 2-cast calls allowed can be much larger than $n_1 + n_2 - 1$, as given by *Theorem 1*.

## III. Some Related Literature

Some special cases of *Conjecture 1* have been proved. For example, Du and Ngo [3] proved for $n_2 = 2$ (except $n_1 = 3$) and $n_2 = 3$. They also formulated a graph-theoretic conjecture based on *Conjecture 1*.

*Conjecture 2:* Consider a graph $H$ with maximum degree $n_2$. Let $L(H)$ be the line graph of $H$. Divide all vertices of $L(H)$ into disjoint groups of size, at most, $n_1$. Connect all vertices in each group into a clique. If $n_2 \leq n_1$, then the resulting graph is $(n_2 + n_1)$ colorable.

Here, $H$ is the graph with output switches as vertices and a link $(u, v)$ represents a 2-cast call involving output switches $u$ and $v$.

*Conjecture 2* extends another conjecture by Du *et al.* [2] which was eventually proved by Fleischner and Stiebitz [4].

*Conjecture 3:* A graph consisting of a $3n$ cycle and $n$ disjoint triangles is $n$ colorable.

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