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Determining flight frequencies on an airline network with demand–supply interactions

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Abstract

This paper determines flight frequencies on an airline network with demand–supply interactions between passenger demand and flight frequencies. The model consists of two submodels, a passenger airline flight choice model and an airline flight frequency programming model. The demand–supply interactions relevant to determining flight frequency on an airline's network are analyzed by integrating these two submodels. The necessary condition for the convergence of the demand–supply interaction is discussed. An example demonstrates the feasibility of applying the proposed models. The results are more accurate than those obtained without considering demand–supply interactions, and the models provide ways to consider demand–supply interactions well in advance to determine flight frequencies on an airline network.

Keywords: Flight frequency programming; Airline network; Passenger demand; Demand–supply interactions

1. Introduction

One of the most important problems airlines encounter is determining flight frequencies and aircraft types on individual routes of their air service networks (Jaillet et al., 1996; Hsu and Wen, 2000, 2002). These decisions affect the cost and quality of airline passenger flight services. Passengers usually choose airlines based on their service quality and airfares, so airlines' decisions on frequencies and types of aircraft further affect airlines' passenger demand and revenues.

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A profit-maximizing airline must trade-off the cost of providing service and the revenue generated by that service, when determining flight frequencies and airfares (Lederer, 1993).

Passengers select flights offered by different airlines based on airfares, flight frequencies, the availability of non-stop flights, travel and layover times, and the airlines' reputations and frequent-flyer (FFlyer) programs. Airlines strive to maximize profits, increase load factors and maximize market share (Teodorovic and Krcmar-Nozic, 1989; Teodorovic et al., 1994). Determining flight frequencies can be a complex problem (Teodorovic and Krcmar-Nozic, 1989; Hsu and Wen, 2000). Airlines may use high flight frequencies or non-stop flights to attract passengers, but excess frequencies or non-stop flights may entail diseconomies of scale. Conversely, airlines may consolidate passenger flows from several city-pair routes, combining these individual routes into hub-and-spoke networks, using larger aircraft and offering fewer non-stop flights. Airlines may realize economies of flow concentration and perhaps reduce operating costs, thereby lowering average basic airfares and attracting more price-elastic passengers. However, these strategies may also increase passenger schedule delays, and thus lose time-sensitive passengers. Significant interaction between demand and supply necessitates the use of a system analysis approach to formulate flight frequencies.

Determining flight frequencies on an airline network is fundamental to an airline's operational planning. This paper demonstrated how demand–supply interactions might be considered well in advance to solve the flight frequency programming problem for an airline network. The results provide a decision-support tool that determines flight frequencies and basic airfares; estimates passenger demand and profits; and analyzes their interactions for airlines. Notably, airlines can use this demand–supply interaction process to evaluate the effect of their flight frequency plans on passenger demand.

This paper utilizes an equilibrium approach and integrates demand–supply interactions into a flight frequency programming problem. Unlike the logit-based models found in the literature, this paper develops an analytical model of passengers' airline flight choices for estimating airline-route market shares. Passengers' airline flight choices are assumed to depend on perceived line-haul travel time costs, schedule delay costs and airfares. The model treats different individuals' values of time as random variables. The model also accounts for the long-term customer benefits of FFlyer membership on individual passengers' choices. The idea behind the proposed model is similar to the concept of the random coefficient probit model. As indicated by Hausman and Wise (1978), random specification allows covariance between alternatives and thus solves the problem of the independence of irrelevant alternatives (IIA) associated with the logit models. This paper aggregates individual passengers' airline flight choices to estimate market shares for an airline's routes by integrating the joint probability density functions of the value of time and FFlyer-membership valuation. The total origin–destination (OD) demands are endogenous and functions of socioeconomic variables and the flight frequencies served by all airlines, determined by network programming.

This paper has three features: (1) The demand submodel estimates the market shares for all routes of an airline network and predicts market-sizes for all OD-pairs; (2) the supply submodel determines optimal flight frequencies and basic airfares on individual routes of an airline network by maximizing the airline's total profit; and (3) the demand–supply interactions are analyzed by integrating demand and supply submodels, and using an algorithm to solve the interaction problem. The rest of this paper is organized as follows. Section 2 reviews the literature on airline operational planning. Section 3 develops the demand and supply submodels. Section 4 presents an iterative algorithm to solve the flight frequency programming problem with demand–supply interactions, and analyzes the convergence conditions of the demand–supply interactions. Section 5 provides an example that illustrates the proposed model's effectiveness. Section 6 draws conclusions.

2. Literature review

Previous studies of airline operational planning have focused mainly on network modeling and hub-location problems for hub-and-spoke airline networks (e.g., O'Kelly, 1986, 1987; Campbell, 1994, 1996; Aykin, 1995; Jaillet et al., 1996). These studies are concerned with location–allocation *p*-hub median problems. Other studies of airline network models have addressed the fleet assignment problem (e.g., Abara, 1989; Subramanian et al., 1994; Larke et al., 1996; Barnhart et al., 1998) and the crew scheduling problem (e.g., Etschmaier and Mathaisel, 1985; Hoffman and Padberg, 1993; Stojkovic et al., 1998). The research generally uses deterministic integer programming and investigates model improvements and algorithms.

The literature on airline networks has mostly addressed the shapes of airline networks, the determination of flight frequency, and the choice of aircraft (e.g., Swan, 1979; Teodorovic, 1983, 1986; Teodorovic and Krcmar-Nozic, 1989; Teodorovic et al., 1994; Hsu and Wen, 2000). Flight frequency programming problems involve a single airline network and are constructed using mathematical programming. The passenger demand pattern is assumed to be exogenous, and demand is assumed to be inelastic, although passenger demand may be elastic to flight frequency in a competitive environment.

Studies of economic competition in air transportation have integrated competitive decisions made by airlines and passenger route choices into one model (e.g., Kanafani and Ghobrial, 1985; Hansen and Kanafani, 1988; Hansen, 1990; Dobson and Lederer, 1993; Adler, 2001). Early studies considered schedule rivalry in a system with fixed OD demand and airfares, and considered such rivalry apart from network modeling (Kanafani and Ghobrial, 1985; Hansen and Kanafani, 1988; Hansen, 1990). Kanafani and Ghobrial (1985) developed an equilibrium model, which integrated a passenger route choice model to assign OD flows, and a frequency assignment model to assign link service frequencies to service links to minimize schedule delay. Hansen and Kanafani (1988) developed a system of models that predicted passenger flows caused by various airline gateway hubbing strategies to analyze airline gateway hubbing. Hansen (1990) modeled frequency competition in a multiple hub setting as a non-cooperative Nash game between profit-seeking airlines. The decision variables of each airline (that is, each player of the game) are a set of service frequencies between a given OD-pair in a hub-dominated environment. Hansen (1990) assumed fixed airfares, implying inelastic demand with respect to price and service, and considered only non-stop and one-stop services.

Recent work has addressed competitive airline network modeling and scheduling in a hub-and-spoke system with exogenous OD demand (Dobson and Lederer, 1993; Adler, 2001). Dobson and Lederer (1993) developed a three-level hierarchical process to study the competitive choice of flight schedules and airfares by airlines in a pure hub-and-spoke system (with a single hub). Their model determines optimal route airfares that satisfy capacity constraints and passenger route choices as well as a set of routes, and flights that maximize profits. Adler (2001) proposed a two-stage Nash-game-theoretical model to evaluate an airline hub-and-spoke network in a competitive environment. His work used an integer linear program to generate potential networks and a non-linear programming model to determine maximize airline profits.

Kanafani and Ghobrial (1985), Hansen and Kanafani (1988), Hansen (1990), Dobson and Lederer (1993), Pels et al. (2000) and Adler (2001) constructed demand models for various routes as logit-based functions of frequency, service quality and route prices. Demand functions have been formulated as multinomial logit (MNL) models (Kanafani and Ghobrial, 1985; Hansen and Kanafani, 1988; Dobson and Lederer, 1993; Adler, 2001). However, the IIA property of MNL models prohibits covariance between the utilities of pairs of alternatives (McFadden, 1974; Ben-Akiva and Lerman, 1985), A nested MNL model relaxes the zero covariance restriction of the MNL model, but does impose equal covariance among all alternatives in a common nest and zero covariance otherwise (Ben-Akiva and Lerman, 1985). Hansen (1990) applied the elemental-aggregate alternative hierarchy concept proposed by Ben-Akiva and Lerman (1985) to define the flight alternatives chosen by passengers as elemental alternatives and the airline flight services chosen as aggregate alternatives. Hansen modeled the market share of competing airlines as a logit model with aggregate alternatives. Pels et al. (2000) developed an airport–airline choice model that was based on a nested MNL model to examine airport and airline competition in an area with multiple airports. Adler (2001) incorporated the airline market-share model into the logit model and restricted the model to enable business and non-business travelers to select airlines in an OD market.

3. Model formulation

The flight frequency programming problem for an airline network is typically defined as follows. "Given the capacities and operating costs of various aircraft types, determine flight frequencies for each route in an airline network that satisfy demands and minimize the total transportation costs (Teodorovic et al., 1994; Jaillet et al., 1996; Hsu and Wen, 2000, 2002)." The model developed herein is a profit-maximizing flight frequency program for an airline network. It determines the optimal flight frequencies and basic airfares on individual routes by considering demand–supply interactions.

3.1. Demand submodel: passengers' airline flight choice model

The decision rule applied to modeling individual passengers' airline flight choices assumes a rational decision maker who minimizes his/her generalized travel costs. The flights offered by various airlines present passengers with a three-dimensional choice context. A range of attributes, representing an airline's level of service, the convenience of the schedules of alternative flights, and fare levels, can be used to characterize each dimension of choice (Proussaloglou and Koppelman, 1999). The level of service provided by an airline is characterized by the overall quality of its service, its reputation and its safety record. The level of service affects each individual passenger's perceived value of the time spent flying. The convenience of a flight to an individual passenger is usually quantified by schedule delays (Swan, 1979; Kanafani and Ghobrial, 1982; Teodorovic and

Krcmar-Nozic, 1989; Proussaloglou and Koppelman, 1999). Moreover, FFlyer program membership is often taken into account to reflect its loyalty-ensuring influence on passengers' airline choices. Previous studies have verified that FFlyer programs significantly affect upon airline/route choice (Toh and Hu, 1988, 1990; Morrison and Winston, 1989; Nako, 1992; Proussaloglou and Koppelman, 1999). FFlyer programs award passengers mileage whenever they fly with a particular airline. The passengers' accumulated mileage credits then gain them travel awards such as free tickets, free tickets with purchased ticket redemption, upgrades or tour package discounts. Thus, the rewards for passengers' participation in one particular airline's FFlyer program membership can be seen as discounts from the original airfares. Passengers' benefits from various levels of FFlyer membership are considered when formulating the actual paid airfares associated with each flight of an airline. The demand submodel incorporates an analytical passenger airline flight choice model to estimate airline-route market shares.

Let t^x_{rsp} denote the average line-haul travel time (block time) spent flying route p on airline x's aircraft between OD-pair r–s, where superscript x indicates the airline; $x = 0$ refers to the object airline. Let sd^x_{rsp} denote the schedule delay time arising from the difference between the time a passenger desires to travel and the time he or she can travel, due to the existing flight frequencies offered by airline x. Following the definition of Swan (1979), Kanafani and Ghobrial (1982) and Teodorovic and Krcmar-Nozic (1989), the schedule delay per passenger on airline x's route p between OD-pair r–s is defined as one quarter of the average headway, that is

$$\mathrm{sd}^x_{rsp} = \frac{\overline{T}}{4N^x_{rsp}}, \tag{1}$$

where N^x_{rsp} represents the flight frequency of airline x on its route p between OD-pair r–s, and $\overline{T}$ is the average operating time of the (origin) airport over a specific period of analysis. As mentioned above, the schedule delay concept has been used to calculate an index of the convenience of flight service (Proussaloglou and Koppelman, 1999). Thus, an individual passenger's total perceived cost of time on a trip via airline x's route p is $\tau_{rs}(t^x_{rsp} + v^{\mathrm{sd}}\mathrm{sd}^x_{rsp})$, where τ_{rs} is the perceived value of travel time, which is a unit time-cost transformation that reflects the perceived monetary cost of time spent on a flight between OD-pair r–s; v^{sd} represents a multiplier that reflects the value of schedule delay, and $\tau_{rs}v^{\mathrm{sd}}$ is the value of time for schedule delay. This paper assumes that the perceived value of travel time, τ_{rs}, varies (i.e., it is a random variable) from individual to individual in light of socioeconomic variations, such as income and trip purposes, and among passengers traveling between OD-pair r–s.

Let tp^x_{rsp} denote the basic list airfare of airline x on its route p between OD-pair r–s. Basic airfare level is the backbone of the airfare structure in that it applies to all passengers at all times and is the basis for all other airfare levels (Wells, 1993). For simplicity, yield management issues are not considered here, and the setting of airfares is assumed to involve only determining the basic airfares. Consider passengers' participation in airlines' FFlyer programs. Their different levels of FFlyer membership gain them various discounts on particular routes.

Let Φ^x_{rs} denote the route-airfare-discount factor due to passenger membership of airline x's FFlyer program when a passenger travels via airline x between OD-pair r–s, where $0 \leqslant \Phi^x_{rs} \leqslant 1$. Suppose that Φ^x_{rs} is a random variable, and Φ^x_{rs} varies among passengers with different levels of FFlyer membership in different airlines. Moreover, Φ^x_{rs} also reflects how airline x's quality of service and reputation impact passengers' choices. An individual passenger's generalized travel cost of flying airline x's route p between OD-pair r–s, G^x_{rsp}, includes route-airfare, perceived costs of travel time and schedule delay time, and can be expressed as

$$G^x_{rsp} = (1 - \Phi^x_{rs})\mathrm{tp}^x_{rsp} + \tau_{rs}(t^x_{rsp} + v^{\mathrm{sd}}\mathrm{sd}^x_{rsp}). \tag{2}$$

This paper assumes that passengers are well informed about airlines' route-airfares, tp^x_{rsp}, travel times, t^x_{rsp}, and schedule delays, sd^x_{rsp}. The optimal choice of an individual passenger among airline flight alternatives is assumed to involve minimizing generalized travel cost.

Previous studies found that FFlyers were less sensitive to higher airfares than non-FFlyers (Toh and Hu, 1990; Nako, 1992). These studies also presented evidence that business travelers were more likely to be FFlyers than were pleasure travelers (Toh and Hu, 1990; Nako, 1992; Proussaloglou and Koppelman, 1999). Furuichi and Koppelman (1994) found that the value of travel time for business travelers is higher than that of pleasure travelers. These papers support the conjecture that the higher level of FFlyer membership that passengers participate in, the higher their values of travel time. The model treats τ_{rs} and Φ^x_{rs} as two correlated random variables, and allows covariance between τ_{rs} and Φ^x_{rs} in any generalized travel cost function G^x_{rsp}. The covariance of τ_{rs} and Φ^x_{rs} is expected to be positive. Although the study ignores the difference in fare classes, the positive correlation implies that passengers with a higher value of time can enjoy higher FFlyer discounts, and are more likely to choose a higher fare class with fewer restrictions since they can enjoy more utility than that actually paid for.

Some assumptions and simplifications were made for mathematical tractability. For the object airline ($x = 0$), the probability distribution of τ_{rs} and Φ^0_{rs} across passenger populations is specified here as a joint probability density function, $f(\tau_{rs}, \Phi^0_{rs})$, where τ_{rs} and Φ^0_{rs} are jointly continuous. However, the joint distribution of τ_{rs} and Φ^x_{rs} must still be derived for other competing airlines x ($\forall x \neq 0$); in most instances, such a derivation is formidable. To simplify the analysis, passenger populations are assumed to be divisible into groups with similar realizations of τ_{rs} and Φ^x_{rs} for any competing airline x ($\forall x \neq 0$). Let τ_{rsi} denote the value of travel time for group i ($i = 1, \ldots, g_1$), and Φ^x_{rsj} denote the value of airfare-discounts due to FFlyer membership for group j ($j = 1, \ldots, g_2$), where g_1 and g_2 are the number of realizations of τ_{rs} and Φ^x_{rs}, respectively. Notably, the distributions of τ_{rsi} and Φ^x_{rsj} are set as discrete approximations to continuous distributions of τ_{rs} and Φ^x_{rs}, respectively. Passenger populations can then be divided into $g_1 \times g_2$ groups with $\{(\tau_{rsi}, \Phi^x_{rsj}), \forall i, j\}$. Let $p(i, j)$ represent the percentage of the passenger population in group i–j associated with τ_{rsi} and Φ^x_{rsj}. Then, for competing airlines x ($\forall x \neq 0$), the joint probability of τ_{rs} and Φ^x_{rs} is given by $\Pr\{\tau_{rs} = \tau_{rsi}, \Phi^x_{rs} = \Phi^x_{rsj}\} = p(i, j), \forall i, j$. For a given group i–j of passengers associated with τ_{rsi} and Φ^x_{rsj}, the generalized travel cost of flying on airline x's route p ($\forall x \neq 0$) between OD-pair r–s is $G^x_{rsp(i,j)} = (1 - \Phi^x_{rsj})\mathrm{tp}^x_{rsp} + \tau_{rsi}(t^x_{rsp} + v^{\mathrm{sd}}\mathrm{sd}^x_{rsp})$.

Accordingly, the probability with which a passenger chooses the object airline's route p to travel between OD-pair r–s, is given by

$$\Pr\{G^0_{rsp} \leqslant G^x_{rsp(i,j)}, \forall x \neq 0, \forall i, j\} = \sum_i \sum_j \Pr\{G^0_{rsp} \leqslant \min_{x,p,\forall x\neq 0}(G^x_{rsp(i,j)})\} p(i, j). \tag{3}$$

Define $G^{*}_{rs(i,j)} = \min_{x,p,\forall x\neq 0}(G^{x}_{rsp(i,j)})$, $\forall i,j$, then $\Pr\{G^{0}_{rsp} \leqslant G^{x}_{rsp(i,j)}, \forall x \neq 0, \forall i,j\}$ can be expressed as:

$$\Pr\{G^{0}_{rsp} \leqslant G^{*}_{rs(i,j)}, \forall i,j\} = \sum_{i}\sum_{j}\Pr\{G^{0}_{rsp} \leqslant G^{*}_{rs(i,j)}\}p(i,j)$$
$$= \sum_{i}\sum_{j}\left[\int\int_{(1-\Phi^{0}_{rs})\mathrm{tp}^{0}_{rsp}+\tau_{rs}(t^{0}_{rsp}+v^{\mathrm{sd}}\mathrm{sd}^{0}_{rsp}) \leqslant G^{*}_{rs(i,j)}} f(\tau_{rs},\Phi^{0}_{rs})\,\mathrm{d}\tau_{rs}\,\mathrm{d}\Phi^{0}_{rs}\right]p(i,j). \tag{4}$$

If the object airline provides two or more routes between OD-pair r–s, and the generalized travel cost of flying on the object airline's route k ($\forall k \neq p$) is G^{0}_{rsk}, then the object airline's route p will be selected by a passenger if and only if $G^{0}_{rsp} \leqslant G^{0}_{rsk}$, $\forall k \neq p$, and $G^{0}_{rsp} \leqslant G^{x}_{rsp(i,j)}$, $\forall x \neq 0, \forall i,j$. The market share MS^{0}_{rsp} of the object airline's route p between OD-pair r–s can then be defined as

$$\mathrm{MS}^{0}_{rsp} = \Pr\{G^{0}_{rsp} \leqslant G^{0}_{rsk}, \forall k \neq p\} \times \Pr\{G^{0}_{rsp} \leqslant G^{x}_{rsp(i,j)}, \forall x \neq 0, \forall i,j\}$$
$$= \prod_{\substack{k\\ \forall k\neq p}} \Pr\{G^{0}_{rsp} \leqslant G^{0}_{rsk}\} \times \left[\sum_{i}\sum_{j}\Pr\{G^{0}_{rsp} \leqslant G^{*}_{rs(i,j)}\}p(i,j)\right]$$
$$= \prod_{\substack{k\\ \forall k\neq p}} \left[\int\int_{\tau_{rs} \geqslant (1-\Phi^{0}_{rs})(\mathrm{tp}^{0}_{rsp}-\mathrm{tp}^{0}_{rsk})/[(t^{0}_{rsk}-t^{0}_{rsp})+v^{\mathrm{sd}}(\mathrm{sd}^{0}_{rsk}-\mathrm{sd}^{0}_{rsp})]} f(\tau_{rs},\Phi^{0}_{rs})\,\mathrm{d}\tau_{rs}\,\mathrm{d}\Phi^{0}_{rs}\right]$$
$$\times \sum_{i}\sum_{j}\left[\int\int_{(1-\Phi^{0}_{rs})\mathrm{tp}^{0}_{rsp}+\tau_{rs}(t^{0}_{rsp}+v^{\mathrm{sd}}\mathrm{sd}^{0}_{rsp}) \leqslant G^{*}_{rs(i,j)}} f(\tau_{rs},\Phi^{0}_{rs})\,\mathrm{d}\tau_{rs}\,\mathrm{d}\Phi^{0}_{rs}\right]p(i,j). \tag{5}$$

Let F_{rs} denote the total demand (market-size) on OD-pair r–s during a specific period. F_{rs} is assumed to be a function of socioeconomic and airline supply attributes. This paper uses the grey systematic model (Deng, 1989), $\mathrm{GM}(1,N)$, to construct a polyfactor model for estimating OD-pair market-size. Thus, F_{rs} can be expressed by

$$F_{rs} = \mathrm{GM}_{(1,N)}(X_{rs}, N_{rs}), \tag{6}$$

where X_{rs} are socioeconomic variables (e.g., per capita GNP and per capita income); N_{rs} is the total flight frequency between OD-pair r–s, given by $N_{rs} = \sum_{x}\sum_{p} N^{x}_{rsp}$, and $\mathrm{GM}_{(1,N)}(X_{rs}, N_{rs})$ [1] represents a $\mathrm{GM}(1,N)$ model with variables X_{rs} and N_{rs}. $\mathrm{GM}(1,N)$ is a polyfactor model (Deng, 1989; Hsu and Wen, 1998, 2000, 2002); herein, $\mathrm{GM}(1,N)$ models are developed to predict all OD-pair market-sizes. Hsu and Wen (1998) presented grey models (GM) for predicting total passenger traffic and country-pair passenger traffic in the trans-Pacific market. Hsu and Wen (2000, 2002) then used these models to predict route flows, which were used as input parameters to design an airline network. The grey systematic model in this paper can be used to examine the effects of socioeconomic variables and total flight frequencies on passenger demand for individual OD-pairs

[1] Appendix A presents formulations of grey systematic models that are used to predict OD-pair market-sizes. For detailed descriptions of methods in building $\mathrm{GM}(1,N)$, refer to Hsu and Wen (2002).

in an airline network. Such a model for predicting demand incorporates the effects of uncertain socioeconomic variables and airline flight frequencies, fully accounting for the dynamic aspects of demand changes.

Finally, the total number of passengers f^0_{rsp} carried by the object airline on its route p between OD-pair r–s can then be estimated as

$$f^0_{rsp} = F_{rs}\mathrm{MS}^0_{rsp}. \tag{7}$$

The total number of passengers carried by the object airline between OD-pair r–s, f^0_{rs}, is then $f^0_{rs} = \sum_p f^0_{rsp}$.

3.2. Supply submodel: airline flight frequency programming model

The modeling of the flight frequency programming problem for an airline network herein follows that of Teodorovic et al. (1994), and Hsu and Wen (2000, 2002). Consider the object airline network, $\boldsymbol{G}(\boldsymbol{N}, \boldsymbol{A})$, where $\boldsymbol{N}$ and $\boldsymbol{A}$ represent, respectively, the set of nodes and set of links in graph $\boldsymbol{G}$. Let $\boldsymbol{R}$ ($\boldsymbol{R} \subseteq \boldsymbol{N}$) denote the set of origin cites, and $\boldsymbol{S}$ ($\boldsymbol{S} \subseteq \boldsymbol{N}$) denote the set of destination cities in graph $\boldsymbol{G}$, where $\boldsymbol{R} \cap \boldsymbol{S} \neq \emptyset$. Any given OD-pair r–s is connected by a set of routes P_{rs} ($r \in \boldsymbol{R}, s \in \boldsymbol{S}$) through the network. An airline's fleet that serves international routes normally contains many aircraft of various sizes. Accordingly, the main decision variables of airline network modeling are assumed to be the flight frequencies on individual routes served by various types of aircraft in the object airline network (Hsu and Wen, 2000, 2002). Let N^0_{rspq} denote the flight frequencies served by the object airline's (airline '0') type q aircraft between OD-pair r–s along route p ($p \in P_{rs}$). Restated, if N^0_{rsp} represents the total flight frequencies of all aircraft used by the object airline on its route p between OD-pair r–s, then, $N^0_{rsp} = \sum_q N^0_{rspq}$. Let N^0_{rs} represent the total flight frequencies served by the object airline between OD-pair r–s; that is, $N^0_{rs} = \sum_p N^0_{rsp}$.

Let Y^0_{aq} denote the flight frequencies served by the object airline's type q aircraft on link a ($a \in \boldsymbol{A}$). The term is the sum of the flight frequencies on all object airline's routes containing link a served by type q aircraft. That is

$$Y^0_{aq} = \sum_{r,s} \sum_{p} \delta^{r,s}_{a,p,q} N^0_{rspq}, \tag{8}$$

where $\delta^{r,s}_{a,p,q}$ is the indicator variable,

$$\delta^{r,s}_{a,p,q} = \begin{cases} 1, & \text{if link } a \text{ is part of route } p \text{ served by type } q \text{ aircraft from city } r \text{ to city } s; \\ 0, & \text{otherwise.} \end{cases}$$

The total flight frequency on link a of the object airline, Y^0_a, can now be expressed as $Y^0_a = \sum_q Y^0_{aq} = \sum_q \sum_{r,s} \sum_p \delta^{r,s}_{a,p,q} N^0_{rspq}$. The number of passengers, f^0_{rsp}, carried by the object airline on its route p between OD-pair r–s can be estimated using the aforementioned market-share model, as Eq. (7). Letting f^0_a denote the link flow on link a allows f^0_a to be the sum of the flows on all routes of the object airline's network passing through that link, i.e., f^0_a may be expressed as

$$f^0_a = \sum_{r,s} \sum_{p} \delta'^{r,s}_{a,p} f^0_{rsp}, \tag{9}$$

where $\delta'^{r,s}_{a,p}$ is the indicator variable, and

$$\delta'^{r,s}_{a,p} = \begin{cases} 1, & \text{if link } a \text{ is part of route } p \text{ from city } r \text{ to city } s; \\ 0, & \text{otherwise.} \end{cases}$$

In airline network modeling, two-way OD passenger flows are assumed to be symmetric, an assumption commonly made in practice by airlines when designing their networks. Related studies have made a similar assumption, including Jaillet et al. (1996) and Hsu and Wen (2000, 2002).

Airline operating costs are normally divided into direct operating costs and indirect operating costs. Direct operating costs are all expenses associated with the operation of the aircraft, including all flying costs, maintenance and aircraft depreciation. Indirect operating costs are expenses related to passengers rather than aircraft. Let C^{T}_a denote the total airline operating cost for link a, i.e.,

$$C^{\mathrm{T}}_a(Y^0_{aq}) = \sum_q C^{\mathrm{D}}_{aq}(Y^0_{aq}) + C^{\mathrm{I}}_a(Y^0_{aq}), \tag{10}$$

where C^{D}_{aq} is the direct operating cost in US dollars of type q aircraft for flights over link a with stage length d_a, and C^{I}_a is the total indirect operating cost in US dollars for link a. Cost functions $C^{\mathrm{D}}_{aq}(Y^0_{aq})$ and $C^{\mathrm{I}}_a(Y^0_{aq})$ are formulated as

$$C^{\mathrm{D}}_{aq}(Y^0_{aq}) = (\alpha_q + \beta_q d_a)Y^0_{aq}, \tag{11}$$

$$C^{\mathrm{I}}_a(Y^0_{aq}) = c_{\mathrm{h}} \sum_q n_q l_a Y^0_{aq}, \tag{12}$$

where d_a is the stage length of link a in miles; α_q and β_q are parameters specific to type q aircraft; c_{h} is the unit handling cost in US dollars per passenger; n_q is the number of available seats on type q aircraft, and l_a is the specified load factor for link a.

The total revenue of the object airline can be expressed as $\sum_{r,s}\sum_p (1 - E[\Phi^0_{rs}])\mathrm{tp}^0_{rsp} f^0_{rsp}$, where $E[\Phi^0_{rs}]$ is the expected value of the FFlyer airfare-discount factor Φ^0_{rs}. The flight frequency programming for the object airline's network derived by maximizing total profit π_0 can then be modeled as

$$\max_{Y^0_{aq}, N^0_{rspq}} \quad \pi_0 = \sum_{r,s}\sum_p (1 - E[\Phi^0_{rs}])\mathrm{tp}^0_{rsp} f^0_{rsp} - \sum_{a\in A} C^{\mathrm{T}}_a(Y^0_{aq}) \tag{13a}$$

$$\text{s.t.} \quad \sum_q n_q l_a Y^0_{aq} - \sum_{r,s}\sum_p \delta'^{r,s}_{a,p} f^0_{rsp} \geqslant 0 \quad \forall a \in \boldsymbol{A}, \tag{13b}$$

$$Y^0_{aq} = \sum_{r,s}\sum_p \delta^{r,s}_{a,p,q} N^0_{rspq} \quad \forall q, \quad \forall a \in \boldsymbol{A}, \tag{13c}$$

$$\sum_a t^0_{aq} Y^0_{aq} \leqslant u^0_q U^0_q \quad \forall q, \tag{13d}$$

$$f^0_{rsp} = F_{rs}\mathrm{MS}^0_{rsp} \quad \forall r, s, p, \tag{13e}$$

$$Y^0_{aq}, N^0_{rspq} \geqslant 0 \text{ and are integers.} \tag{13f}$$

Eq. (13a) yields the objective function that maximizes the total profit π_0 of the object airline network. Eq. (13b) indicates that the transportation capacities in terms of numbers of seats for each link must be equal to or greater than the numbers of passengers on all routes that include that link. Eq. (13c) specifies the relationship between link frequency and route frequency. Eq. (13d) states that total aircraft utilization must be equal to or less than the maximum possible utilization—where t^0_{aq} is the block time for the object airline's type q aircraft on link a; u^0_q is the maximum possible utilization; and U^0_q is the total number of type q aircraft in the object airline's fleet. Eq. (13e) defines the total number of passengers f^0_{rsp} carried by the object airline on its route p between OD-pair r–s. Finally, Eq. (13f) constrains variables Y^0_{aq} and N^0_{rspq} to be non-negative integers.

The flight frequency programming problem is usually considered apart from short-run yield management issues, during the global airline network planning phase. This paper assumes that the object airline selects basic route airfares at or above average operating cost on every route. A similar assumption can be found in Lederer (1993), stating that such pricing behavior is expected on routes served by an airline, otherwise airlines could raise its airfares and increase profits. As in Eq. (10), the total cost to the object airline of operating route p between OD-pair r–s, C^{T}_{rsp}, can be expressed as

$$C^{\mathrm{T}}_{rsp}(N^0_{rspq}) = \sum_q (\alpha_q + \beta_q d_{rsp}) N^0_{rspq} + c_{\mathrm{h}} \sum_q n_q l_p N^0_{rspq}, \tag{14}$$

where d_{rsp} is the stage length of route p between OD-pair r–s; l_p is the specified load factor on route p between OD-pair r–s; and if $N^0_{rsp} = \sum_q N^0_{rspq} = 0$, then $C^{\mathrm{T}}_{rsp} = 0$. The basic airfare per passenger, tp^0_{rsp}, on route p of stage length d_{rsp} can then be determined using

$$\mathrm{tp}^0_{rsp} = (1 + \bar{r}^0_{rsp}) \frac{C^{\mathrm{T}}_{rsp}}{\sum_q n_q l_p N^0_{rspq}}, \tag{15}$$

where $\bar{r}^0_{rsp}$ is the profit margin specified by the object airline for route p between OD-pair r–s; and if $N^0_{rsp} = 0$, then both $C^{\mathrm{T}}_{rsp} = 0$ and $\mathrm{tp}^0_{rsp} = 0$. Consequently, tp^0_{rsp} in the objective function, Eq. (13a), can be replaced with Eq. (15), i.e., $(1 + \bar{r}^0_{rsp}) C^{\mathrm{T}}_{rsp} / \sum_q n_q l_p N^0_{rspq}$, and then the flight frequencies and basic airfares on individual routes can be determined from Eqs. (13a)–(13f).

In aircraft utilization/frequency decisions made in flight frequency programming, airlines may either select a strategy of using larger aircraft and fewer flights or one of using smaller aircraft and more flights to fulfill the given demand. The former strategy, while reducing the airline's unit operating cost and basic airfare, increases the schedule delays to passengers; the latter increases the basic airfare, but reduces the delays. Notably, both basic airfare and schedule delay influence passenger choice and demand.

4. Demand–supply interaction

The demand–supply interaction between passenger demand and flight frequencies on an airline network are analyzed here by integrating the demand and supply submodels described above. In the demand submodel, market shares on the object airline's routes between each OD-pair are estimated by aggregating passengers' airline flight choices, since passenger choices are functions of perceived travel time costs, schedule delay costs and airfares (Eqs. (1)–(5)). The grey systematic model (Eq. (6) and Eqs. (A.1)–(A.5) in Appendix A) predicts market-sizes for individual OD-pairs

(OD demand), giving forecasts of socioeconomic variables and total flight frequencies. The traffic flows of the object airline on all routes between OD-pair r–s, f^0_{rsp}, $\forall p$, are then estimated using the demand submodel (Eq. (7)) and used as input parameters to the airline flight frequency programming submodel (Eqs. (13a)–(13f)). The route flight frequencies and airfares determined by the airline flight frequency programming submodel influence both passenger choices and estimated market shares. The changes in total flight frequencies on individual OD-pairs further affect the total market-size.

This paper studies the above demand–supply interactions between passenger demand and flight frequencies on an airline network using an iterative algorithm that incorporates demand and supply submodels. First, all airlines' (object airline and its competitors) flight frequencies and airfares are initialized using present values. Eqs. (A.1)–(A.5) initially predict the OD demand for all OD-pairs for a future planning period by giving the forecasts of the socioeconomic variables for that period and the initial flight frequencies between each OD-pair. The market shares for all routes on the object airline network are then estimated by applying the market-share submodel; then traffic flows are calculated for all object airline's routes using Eq. (7). Next, the object airline's flight frequencies and airfares on individual routes are determined by airline flight frequency programming to maximize total profit (Eqs. (13a)–(13f)). Other competing airlines use a similar approach (Eqs. (3)–(5)) to estimate their market shares; their flight frequencies and airfares are also determined by maximizing their profits simultaneously against the flight frequencies and airfares of the object airline. The aforementioned steps conclude the first "round" of interaction; this process is repeated for many more rounds. The process continues until the demand–supply equilibrium is reached. Game-theoretical papers have modeled similar processes (e.g., Kanafani and Ghobrial, 1985; Hansen and Kanafani, 1988; Hansen, 1990; Dobson and Lederer, 1993; Adler, 2001).

In this paper, the demand–supply equilibrium is reached when the demand–supply interaction converges. When the object airline's profit is assessed to have converged, its route flight frequencies and airfares are determined; when the OD market-sizes are also unchanging, the competitors' profit-maximizing decisions on frequencies and airfares in the OD market will not change in response to the object airline's determined flight frequencies and airfares. In such a case, passengers' choices do not change in response to the flight frequencies and airfares determined by airlines, and the demand–supply interaction converges.

The demand–supply interaction is assumed to involve simplistic competitive interactions in which each competing airline only responds to changes in the object airline's flight frequencies and basic airfares. However, decisions regarding pricing strategies can be made in a short time-frame, while decisions regarding flight frequency changes may involve a longer time horizon. Accordingly, this paper assumes that the setting of airfares is confined to determining basic airfares rather than making decisions on pricing strategies during the determination of flight frequency. Notably, airline competition focuses on the changes in the flight frequencies among competing airlines rather than on price competition. In this model, not all competing airlines will necessarily respond similarly to the object airline's behavior. [2] In a more complex analysis of competitive interactions

[2] For example, if the object airline decreases flight frequencies, thereby shrinking its market shares, competing airlines may increase their flight frequencies due to the growth in their market shares.

among all airlines, the above interaction process may be applied to all competing airline networks, and then be further reconstructed by combination with game-theoretical models to analyze the equilibria of the competitive interactions among airlines.

Changes in airline profits, OD passenger demand and OD flight frequencies are calculated after each round to assess the properties of convergence of the demand–supply interaction. Based on the definition in Hansen (1990), the measure of relative change in airline profits, π_0, is calculated as

$$\mathrm{RC}(\pi_0) = \frac{|\pi_0^i - \pi_0^{i-1}|}{0.5(\pi_0^i + \pi_0^{i-1})}, \tag{16}$$

where $\mathrm{RC}(\pi_0)$ measures the relative change in π_0; π_0^i is the value of π_0 after i rounds, and the superscript i indicates variable/function values after i rounds, as in N_{rspq}^{0i}, tp_{rsp}^{0i}, N_{rs}^{0i}, N_{rsp}^{xi}, tp_{rsp}^{xi}, MS_{rsp}^{0i}, F_{rs}^i and f_{rsp}^{0i}. Similarly, $\mathrm{RC}(N_{rs}^0)$ and $\mathrm{RC}(f_{rs}^0)$ are, respectively, measures of relative change in N_{rs}^0 and f_{rs}^0. This paper proposes an algorithm that includes an iterative scheme involving demand and supply submodels to solve flight frequency programming problems for an airline network with demand–supply interactions.

Step 1. Input the initial values of N_{rspq}^0, tp_{rsp}^0, $\forall r,s,p,q$, N_{rsp}^x, tp_{rsp}^x, $\forall x \neq 0$, $\forall r,s,p$, the initially estimated OD-pair market-sizes (using Eqs. (A.1)–(A.5)), F_{rs}, $\forall r,s$, and other exogenous parameters.
Step 2. In the ith round, input N_{rspq}^{0i-1}, tp_{rsp}^{0i-1}, $\forall r,s,p,q$, N_{rsp}^{xi-1}, tp_{rsp}^{xi-1}, $\forall x \neq 0$, $\forall r,s,p$, to estimate the market share, MS_{rsp}^{0i}, $\forall r,s,p$, of the object airline network using the demand submodel (Eqs. (3)–(5)); F_{rs}^i, $\forall r,s$, using Eqs. (A.1)–(A.5) and Eq. (6), and f_{rsp}^{0i}, $\forall r,s,p$, using Eq. (7).
Step 3. Use the supply submodel (Eqs. (13a)–(13f), (14) and (15)) to determine the flight frequencies, N_{rspq}^{0i}, $\forall r,s,p,q$, the objective function value π_0^i, and the route airfares, tp_{rsp}^{0i}, $\forall r,s,p$, of the object airline network. Competing airlines' flight frequencies, N_{rsp}^{xi}, and airfares, tp_{rsp}^{xi}, are determined using a similar approach (Eqs. (13a)–(13f)) in maximizing their profits simultaneously.
Step 4. If $\mathrm{RC}(N_{rs}^0) < \varepsilon_1$, $\forall r,s$ and $\mathrm{RC}(\pi_0) < \varepsilon_2$ (ε_1 and ε_2 are small numbers), then STOP. Otherwise, $i = i + 1$, and return to Step 2.

The flight frequency elasticity of demand is also considered. Let $\hat{e}_{rs}$ denote the arc elasticity for the flight frequency elasticity of OD passenger demand between OD-pair r–s, i.e.,

$$\hat{e}_{rs} \cong \frac{\mathrm{RC}(f_{rs}^0)}{\mathrm{RC}(N_{rs}^0)}. \tag{17}$$

Herein, one desirable condition for the convergence of the demand–supply interaction is that the flight frequency elasticity of OD passenger demand, $\hat{e}_{rs}$, $\forall r,s$, is less than one. Pels et al. (2000) showed analytically that if the demand frequency elasticity is less than one, then a demand–supply equilibrium exists and is unique. Cohas et al. (1995) also presented empirical evidence concerning a demand frequency elasticity below one. Statistical estimates for the OD market model obtained by Cohas et al. (1995), involving New York-New Jersey, the San Francisco Bay Area, and Washington-Baltimore OD-pair systems, suggested that the elasticity of OD market share with respect to flight frequency was less than one.

This convergence condition can be illustrated by considering $\hat{e}_{rs}$ greater than one. Then, if the airline increases its flight frequency between a certain OD-pair by 1%, its OD passenger demand changes by more than 1%. In that case, the airline will increase its flight frequency to satisfy the increase in passenger demand—if the increase in flight frequency can be feasibly included in the airline's flight frequency programming. The increase in flight frequency again causes a greater-than-proportional increase in OD passenger demand. The demand–supply interaction between this OD-pair therefore cannot converge, since each increase in OD flight frequency further increases OD passenger demand. Consequently, $\hat{e}_{rs}$ must be smaller than one. If $\hat{e}_{rs} < 1$, then $\mathrm{RC}(f^0_{rs}) < \mathrm{RC}(N^0_{rs})$. If $\mathrm{RC}(N^0_{rs}) < \varepsilon_1$, then $\mathrm{RC}(f^0_{rs}) < \mathrm{RC}(N^0_{rs}) < \varepsilon_1$; and both N^0_{rs} and f^0_{rs} can be seen to remain constant from the $(i-1)$th round to the ith round; thus, the demand–supply interaction for OD-pair r–s converges.

5. Example

This section presents an application of the proposed models. The object airline is China Airlines (CI) of Taiwan, and the proposed models are applied to a simplified version of CI's international network. The aim of the example is to determine CI's route flight frequencies on its network for the year 2004. For simplicity, only 10 cities (nodes $\in \boldsymbol{N}$) in eight countries [3] were selected from all the cities currently served by CI, and 14 wide-body aircraft [4] were assumed to serve these 10 cities. Traffic between these selected OD-pairs represents a major part of the traffic carried by CI.

Historic annual data (years 1995–2001) were used for Taiwan-resident departures and foreign-visitor arrivals (annual country-pair/city-pair traffic among the nine OD-pairs), and annual gross national product per capita of the countries were used as socioeconomic variables. The annual total flight frequencies between OD-pairs were used to build the grey systematic model (Eq. (6)). Eqs. (A.1)–(A.5) in Appendix A were used to estimate the grey systematic models and forecast the OD-pair passenger traffic for the year 2004. Given base values of the cost-function-related parameters are used to solve the flight frequency programming problem for CI's network. [5]

In this example, the assumptions of Proussaloglou and Koppelman (1999) and Nako (1992) are made. The passenger population is assumed to be divisible into two groups, business $(i = 1)$ and non-business $(i = 2)$ travelers, distinguished by their values of time. Passengers are also assumed to fall into four groups, non-FFlyer $(j = 1)$, basic member $(j = 2)$, low-frequency active member $(j = 3)$, and high-frequency active member $(j = 3)$, according to their participation in FFlyer programs (Proussaloglou and Koppelman, 1999). For simplicity, the values and distributions of τ_{rs} and Φ^x_{rs} are assumed not to vary across different OD-pairs in this example. Values of travel

[3] The nine OD-pairs selected are Taipei (TPE)–Hong Kong (HKG), –Tokyo (TYO), –Bangkok (BKK), –Singapore (SIN), –Kuala Lumpur (KUL), –Los Angeles (LAX), –San Francisco (SFO), –New York (NYC) and –Amsterdam (AMS).

[4] The assumed fleet includes eight Boeing 747-400s (394 seats) and six Airbus 300s (265 seats).

[5] Since some of CI's operating cost data were unavailable, operating cost data reported in Kane (1990) were used instead. Aircraft characteristic data found in CI's fleet facts and those reported in Horonjeff and McKelvey (1994) were used to estimate flight times and airport times. The load factor for each route was taken as the average load factor on routes operated by CI during 2001.

time, τ_i, for business ($i = 1$) and non-business ($i = 2$) travelers are assumed by appropriately adjusting the values of travel time obtained by Furuichi and Koppelman (1994), i.e., $\tau_1 =$ US$21.39/h and $\tau_2 =$ US$20.05/h. The mean value of travel time is assumed to be $20.545/h with a standard deviation of $0.619/h; these values are also taken from Furuichi and Koppelman (1994). The value $v^{sd} = 1.3$ was also assumed, based upon slight adjustments in the ratios of the mean line-haul travel time to the mean access time found in Furuichi and Koppelman (1994).

Empirical results in previous studies (Nako, 1992; Proussaloglou and Koppelman, 1999) were applied to specify FFlyer airfare-discount factors, Φ_j^x, for various airlines. [6] Table 1 lists the airline-specific FFlyer airfare-discounts associated with basic membership ($j = 2$), low-frequency active membership ($j = 3$), and high-frequency active membership ($j = 3$), for business ($i = 1$) and non-business ($i = 2$) travelers. The joint probabilities, $p(i, j)$, associated with business and non-business travelers, non-FFlyers, basic members, low-frequency active members, and high-frequency active members were set based on actual data and data of Toh and Hu (1988), [7] and are found in Table 2.

For simplicity, the joint distribution of two normally distributed variables (Greene, 1993) was used to define the joint distribution of τ_0 and Φ_0 for the object airline, CI. Following the formulation by Greene (1993), the joint p.d.f. is

$$
\begin{aligned}
f(\tau^0, \Phi^0) &= \frac{1}{2\pi\sigma_\tau\sigma_\Phi\sqrt{1-\rho^2}} e^{-(z_\tau^2 + z_\Phi^2 - 2\rho z_\tau z_\Phi)/(2(1-\rho^2))}, \\
z_\tau &= \frac{\tau^0 - \mu_\tau}{\sigma_\tau}, \\
z_\Phi &= \frac{\Phi^0 - \mu_\Phi}{\sigma_\Phi},
\end{aligned}
\tag{18}
$$

where parameters μ_τ, σ_τ, μ_Φ, and σ_Φ are the sample means and sample standard deviations of τ_0 and Φ_0, respectively, and the additional parameter, ρ, is the correlation between τ_0 and Φ_0, given by $\rho = \mathrm{Cov}(\tau_0, \Phi_0)/\sigma_\tau\sigma_\Phi$. From Tables 1 and 2, the sample mean of FFlyer airfare-discount factors for all passengers traveling with CI can be calculated, yielding $\mu_\Phi = 0.08$; the sample standard deviation, σ_Φ, is 0.043. In this example, $\rho = 0.5346$.

The accuracy of the market-share submodel was verified by comparing its results with the actual market shares for all OD-pairs. The present flight frequencies and average basic airfares of each airline in the year 2000 were used as input data to the market-share submodel. Market shares on each of CI's routes between OD-pairs were then estimated using Eqs. (3)–(5), and the esti-

[6] Proussaloglou and Koppelman (1999) estimated the values (in US$) of premiums for different FFlyers that business and non-business travelers are willing to pay to travel with an airline in whose program they participate. Premiums can be seen as perceived discounts on airfares for different FFlyers. Here, the premiums are divided by the flight airfares to approximate the FFlyer airfare-discount factor, Φ_j^x. The airline-specific FFlyer airfare-discounts are obtained by appropriately adjusting the results of the airline-specific FFlyer effects estimated by Nako (1992).

[7] According to data on Taiwan-resident departures and foreign-visitor arrivals in 2000, 36.75% of travelers visited for business reasons and 63.25% for non-business reasons. From the sample of FFlyers in Toh and Hu (1988), the proportion of FFlyer program members was assumed to be 39% FFlyers for basic members, 34% for low-frequency active members, 17% for high-frequency active members, and 10% for non-FFlyers.

Table 1
Airline-specific FFlyer airfare-discounts associated with non-FFlyers and various FFlyer program memberships for business and non-business travelers

Airlines	FFlyer program membership			
	Non-FFlyer	Basic	Low-frequency active	High-frequency active
Business travelers				
UA	0	0.1382	0.1960	0.2333
KL	0	0.1075	0.1653	0.2026
SQ	0	0.1021	0.1599	0.1972
CX	0	0.0946	0.1524	0.1896
TG	0	0.0946	0.1524	0.1896
EG	0	0.0816	0.1394	0.1767
CI	0	0.0816	0.1394	0.1767
BR	0	0.0816	0.1394	0.1767
MH	0	0.0648	0.1225	0.1598
Non-business travelers				
UA	0	0.1122	0.1327	0.1476
KL	0	0.0815	0.1020	0.1169
SQ	0	0.0761	0.0966	0.1115
CX	0	0.0685	0.0890	0.1039
TG	0	0.0685	0.0890	0.1039
EG	0	0.0555	0.0760	0.0909
CI	0	0.0555	0.0760	0.0909
BR	0	0.0555	0.0760	0.0909
MH	0	0.0387	0.0592	0.0741

Table 2
Joint probability distribution $p(i, j)$

	$p(i)$		
$p(j)$	Business	Non-business	
Non-FFlyer	0.03675	0.06325	0.1
Basic	0.143325	0.246675	0.39
Low-frequency active	0.12495	0.21505	0.34
High-frequency active	0.062475	0.107525	0.17
	0.3675	0.6325	

mation percentage errors of the model were calculated. Table 3 presents the results of the market-share submodel and the estimation percentage errors. The table shows that the model slightly underestimated the market shares for most of these test OD-pairs. This error may be due to the neglecting of some factors (such as fare classes, terminal and on-board service amenities, and other unobserved attributes) in the market-share submodel. However, the percentage errors averaged about 9.83%, implying that the proposed market-share submodel is sufficiently accurate to estimate the airlines' route market shares.

In this example, present flight frequencies and airfares for airlines in the year 2000 were used as initial inputs to the model. Table 4 lists the initial values of N^0_{rspq}, tp^0_{rsp}, $\forall r, s, p, q$, and N^x_{rsp}, tp^x_{rsp},

Table 3
Market shares and percentage errors for OD-pairs

OD-pairs	Model	Actual[a]	Error (%)
TPE–HKG	27.694%	31.005%	10.679
TPE–TYO	31.790%	37.621%	15.498
TPE–BKK	24.198%	26.554%	8.873
TPE–SIN	23.631%	26.146%	9.618
TPE–KUL	30.874%	28.335%	8.962
TPE–LAX	37.368%	35.896%	4.102
TPE–SFO	23.654%	30.698%	22.944
TPE–NYC	28.950%	29.379%	1.460
TPE–AMS	35.781%	33.654%	6.322
Average percentage error			9.829

[a] Source: Civil Aeronautics Administration, M.O.T.C., ROC (2000).

$\forall x \neq 0$, $\forall r, s, p$, and the initially predicted OD-pair market-size in year 2004, F_{rs}. LINGO [8] was used to run the IP-based flight frequency programming problem, and the demand–supply interactions were then determined using the proposed algorithm in which the stop criteria were set to $\mathrm{RC}(N_{rs}^0) < 0.01$, $\forall r, s$ and $\mathrm{RC}(\pi_0) < 0.01$. The demand–supply interaction for the object airline network converged after six rounds. Table 5 lists the determined route flight frequencies, the objective function values, the basic airfares, the estimated market shares and the OD-pair market-sizes for each round. Table 5(a) presents the measures of relative change in airline profits and route flight frequencies, $\mathrm{RC}(\pi_0)$, and $\mathrm{RC}(N_{rs}^0)$, respectively, in each round. Except on routes TPE–HKG, –BKK and –LAX, the demand–supply interactions converged soon after two or three rounds. The flight frequency elasticity of demand for all routes of the object airline network was also examined, and route flight frequency elasticity was smaller than one in every round. This finding confirmed that the demand–supply interactions converged.

The sensitivity of the result of this model to the value of τ (the value of travel time) is of interest, so a sensitivity analysis was performed such that the mean values of travel time for all and business travelers were varied, while other parameters were held constant. Table 6 presents the flight frequencies, basic airfares, market shares for route TPE–HKG, and the total profits with respect to variations in the value of travel time, τ, under the convergence of demand–supply interaction. As the mean values of travel time for all and business travelers increase, the object airline should increase flight frequencies, thereby reducing passenger schedule delays to attract time-sensitive passengers, while raising basic airfares. This strategy also slightly increases the market share of the object airline. Consequently, the total profits increased as both the airfares and the market shares increased.

As stated, previous studies have considered flight frequency programming problems apart from demand–supply interactions, assuming passenger demand to be exogenous. Consequently, the flight frequencies determined with and without demand–supply interactions were compared. In addition to the aforementioned example, flight frequency data in 1998 were used as initial inputs

[8] LINGO is a linear, non-linear and integer programming solver with a mathematical modeling language; it is a software product available from LINDO systems, Inc.

Table 4
Airline flight frequencies and basic airfares, and estimated market-sizes on individual OD-pairs

OD-pairs	Estimated market-sizes (annual traffic)[a]	Airlines	Weekly flight frequencies (flights/week)[a]	Basic airfares (US$)[a]
TPE–HKG	2,684,676	CI	64	205.882
		BR	21	201.238
		CX	52	204.108
		EG	7	170.075
		SQ	3	182.147
		TG	14	173.375
TPE–TYO	953,999	CI	21	208.978
		EG	22	211.789
		SQ	3	198.142
		CX	7	213.622
TPE–BKK	714,591	CI	21	229.924
		BR	22	235.949
		TG	21	226.405
		KL	7	203.287
TPE–SIN	310,684	CI	7	227.554
		BR	7	246.904
		SQ	8	247.678
TPE–KUL	290,101	CI	8	206.333
		BR	7	206.778
		MH	9	225.000
TPE–LAX	617,091	CI	13	446.717
		BR	14	455.050
		MH	4	397.475
		SQ	7	429.356
		UA	7	434.898
TPE–SFO	333,037	CI	7	414.898
		BR	10	473.367
		UA	7	439.898
TPE–NYC	130,447	CI	6	476.894
		BR	7	498.568
		UA	7	489.898
TPE–AMS	148,525	CI	6	528.363
		BR	3	596.212
		KL	7	569.409

[a] Note: one direction.

to the model, including demand–supply interactions, to determine flight frequencies on CI's network for the year 2001. For comparison, the supply submodel—a flight frequency programming, without demand–supply interactions—was applied to determine flight frequencies on CI's network in the same year, using exogenous 2001 OD demands and 1998 CI's market shares. Table 7 presents the flight frequencies and market shares obtained from the model in the

Table 5
Round-by-round results for the object airline network

Routes	Weekly flight frequencies (flights/week) (one direction)							
	Initial	Aircraft	Round of interaction					
			1st	2nd	3rd	4th	5th	6th
(a) Route flight frequencies and objective function values								
TPE–HKG	64	B747-400	1	0	0	0	0	1
		A300	62	64	63	64	63	62
		Relative changes:	0.0157	0.0157	0.0157	0.0157	0.0157	0
TPE–TYO	21	B747-400	0	0	0	0	0	0
		A300	26	29	29	29	29	29
		Relative changes:	0.2128	0.1091	0	0	0	0
TPE–BKK	21	B747-400	4	5	2	2	2	2
		A300	12	10	14	14	14	14
		Relative changes:	0.2703	0.0645	0.0645	0	0	0
TPE–SIN	7	B747-400	6	6	6	6	6	6
		A300	0	0	0	0	0	0
		Relative changes:	0.1538	0	0	0	0	0
TPE–KUL	8	B747-400	7	7	7	7	7	7
		A300	0	0	0	0	0	0
		Relative changes:	0.1333	0	0	0	0	0
TPE–LAX	13	B747-400	15	16	17	17	17	17
		Relative changes:	0.1429	0.0645	0.0606	0	0	0
TPE–SFO	7	B747-400	6	6	6	6	6	6
		Relative changes:	0.1538	0	0	0	0	0
TPE–TYO–NYC	6	B747-400	6	6	6	6	6	6
		Relative changes:	0	0	0	0	0	0
TPE–BKK–AMS	6	B747-400	7	7	7	7	7	7
		Relative changes:	0.1538	0	0	0	0	0
		Objective function values (US$):	1,635,607	1,659,859	1,636,009	1,665,827	1,705,034	1,716,382
		Relative changes:		0.0147	0.0145	0.0181	0.0233	0.0066

Table 5 (*continued*)

Routes	Weekly flight frequencies (flights/week) (one direction)						
	Initial	Round of interaction					
		1st	2nd	3rd	4th	5th	6th
(b) Route basic airfares, market shares and estimated OD-pair market-sizes							
Routes							
Route basic airfares (US$) (one direction)							
TPE–HKG	205.882	204.091	205.739	204.292	205.739	204.292	204.091
TPE–TYO	208.978	220.536	224.889	224.889	224.889	224.889	224.889
TPE–BKK	229.924	204.966	203.369	207.592	207.592	207.592	207.592
TPE–SIN	227.554	205.171	205.171	205.171	205.171	205.171	205.171
TPE–KUL	206.333	205.206	205.206	205.206	205.206	205.206	205.206
TPE–LAX	446.717	449.543	449.543	449.543	449.543	449.543	449.543
TPE–SFO	414.898	393.492	393.492	393.492	393.492	393.492	393.492
TPE–TYO–NYC	476.894	490.858	490.858	490.858	490.858	490.858	490.858
TPE–BKK–AMS	528.363	513.918	513.918	513.918	513.918	513.918	513.918
Routes							
Route market shares (one direction)							
TPE–HKG	27.872%	26.810%	26.463%	26.794%	26.500%	26.795%	26.810%
TPE–TYO	31.790%	35.788%	35.534%	35.534%	35.534%	35.534%	35.534%
TPE–BKK	24.198%	23.585%	23.597%	23.565%	23.565%	23.565%	23.565%
TPE–SIN	23.631%	24.547%	24.547%	24.547%	24.547%	24.547%	24.547%
TPE–KUL	30.874%	30.425%	30.425%	30.425%	30.425%	30.425%	30.425%
TPE–LAX	37.368%	39.417%	39.660%	39.013%	38.585%	38.585%	38.585%
TPE–SFO	23.654%	22.899%	22.899%	22.899%	22.899%	22.899%	22.899%
TPE–TYO–NYC	28.950%	21.549%	21.549%	21.549%	21.549%	21.549%	21.549%
TPE–BKK–AMS	35.781%	31.242%	31.242%	31.242%	31.242%	31.242%	31.242%
OD-pairs							
Estimated market-sizes (annual traffic) (one direction)							
TPE–HKG	2,684,676	2,804,903	2,818,775	2,807,793	2,804,903	2,793,343	2,793,343
TPE–TYO	953,999	952,183	951,094	951,094	951,094	951,094	951,094
TPE–BKK	714,591	704,534	684,419	684,419	684,419	684,419	684,419
TPE–SIN	310,684	294,818	294,818	294,818	294,818	294,818	294,818
TPE–KUL	290,101	283,129	283,129	283,129	283,129	283,129	283,129
TPE–LAX	617,091	648,803	680,516	697,593	726,866	726,866	726,866
TPE–SFO	333,037	321,548	321,548	321,548	321,548	321,548	321,548
TPE–TYO–NYC	130,447	130,447	130,447	130,447	130,447	130,447	130,447
TPE–AMS	148,525	157,779	157,779	157,779	157,779	157,779	157,779

demand–supply convergent state in 2001, and the solutions obtained using flight frequency programming without demand–supply interactions for 2001 using 1998 market shares. Table 7 also lists CI's actual flight frequencies and market shares by route in 2001. The table indicates that in the demand–supply convergent state for 2001, the market-share results were accurate and the route flight frequencies obtained from the models were reasonable.

Except on route TPE–BKK, the flight frequencies determined with demand–supply convergence on all routes more closely approximated actual flight frequencies than those results obtained

Table 6
Model results for TPE–HKG with various value of travel time

Mean values of travel time for all travelers ($/h)	Mean values of travel time for business travelers ($/h)	TPE–HKG				Total profits (US$)
		Weekly flight frequencies (flights/week)[a]		Basic airfares (US$)[a]	Market shares (%)[a]	
20.545	21.39	B747-400	1	204.091	26.810	1,716,382
		A300	62			
21.331	23.53	B747-400	1	204.095	27.089	1,729,186
		A300	63			
2.117	25.67	B747-400	1	204.098	27.546	1,751,524
		A300	64			
22.903	27.81	B747-400	1	204.101	27.988	1,771,859
		A300	65			
23.689	29.95	B747-400	0	204.292	28.263	1,788,031
		A300	67			
24.475	32.09	B747-400	0	204.288	28.667	1,807,210
		A300	68			

[a] Note: one direction.

without demand–supply interactions using 1998 market shares. However, demand–supply converged solutions may depend on the initially estimated market shares. The initially estimated market shares on route TPE–HKG were underestimated; therefore, the flight frequencies determined with demand–supply convergence were lower than the actual flight frequencies. On routes TPE–BKK, –SIN and –KUL flown by Boeing 747-400 and Airbus 300 aircraft, the flight frequencies determined from the models with demand–supply convergence were lower than the actual flight frequencies on those routes. CI's current timetable also shows that half or two-thirds of the present flights on routes TPE–BKK, –SIN and –KUL are flown by smaller types aircraft, such as Boeing 737–800s, MD11s or Airbus 340s. Hence, the determined flight frequencies of larger aircraft on those routes were lower than the actual frequencies but the results were reasonable. Conversely, the flight frequencies on route TPE–TYO, flown by Airbus 300s, determined from the models with demand–supply convergence, exceeded those actually flown by Boeing 747-400s. Although the determined flight frequencies on route TPE–TYO with demand–supply convergence seemed less accurate than the results obtained without demand–supply interactions, the determined numbers of available seats obtained in the former case were more accurate than those obtained in the latter case.

6. Conclusions

This paper developed a model for determining optimal flight frequencies and basic airfares on airline network routes, taking into account demand–supply interactions. The model proposed

Table 7
Comparisons of flight frequencies with and without demand–supply interactions

Routes	Market shares (%)		Determined flight frequencies (flights/week) (one direction)			Actual flight frequencies (flights/week)[a]	
	Model with demand–supply convergence	Actual[a]	Model with demand–supply convergence		Model without demand–supply interactions[b]		
			Aircraft	Freq.	Freq.	Aircraft	Freq.
TPE–HKG	27.442	31.353%	B747-400 A300	0 63	1 68	A340/A300/ B737-800/ B747-400	66
TPE–TYO	35.145	36.261	B747-400 A300	1 26	1 22	B747-400	22
TPE–BKK	23.584	31.017	B747-400 A300	11 2	7 6	MD11/A340/ B737-800/ B747-400	23
TPE–SIN	23.585	23.532	B747-400 A300	5 0	4 0	A300	6
TPE–KUL	25.919	23.544	B747-400 A300	6 0	4 0	B737-800/A300	6
TPE–LAX	39.229	38.490	B747-400	14	10	B747-400	13
TPE–SFO	22.899	32.449	B747-400	6	6	B747-400	7
TPE–NYC	21.549	29.571	B747-400	6	6	A340	7
TPE–BKK–AMS	31.243	31.835	B747-400	7	7	B747-400	7

[a] Source: Civil Aeronautics Administration, M.O.T.C., ROC (2001).
[b] Note: only flight frequency programming without demand–supply interactions using 1998 CI's market shares.

herein consists of two submodels, a passenger airline flight choice model and an airline flight frequency programming model. Unlike the logit-based models found in the literature, the demand submodel treats values of travel time and FFlyer airfare-discount factors as correlated random variables with covariance. The market shares are estimated by aggregating individual passengers' airline flight choices. The traffic flows for all routes estimated from the demand submodel are used as input parameters to the airline flight frequency programming submodel. Airline flight frequency with demand–supply interactions is determined using an algorithm that iteratively integrates the demand and supply submodels.

The model is applied to a simplified version of CI's network that includes 10 selected cities. The accuracy of the market-share submodel is reasonable since the average estimated percentage error is 9.83%. The results of demand–supply interaction converged after six rounds. When the flight frequency elasticity associated with all city-pair routes was smaller than one, the demand–supply interaction converged. Moreover, the determined flight frequencies obtained with demand–supply convergence were more accurate than those obtained without demand–supply interactions. The results of this example were shown to be reasonable by comparing the solutions obtained using the proposed models with CI's actual flight frequencies.

This paper has demonstrated how demand–supply interactions might be considered well in advance in solving a flight frequency programming problem for an airline network. Consequently, the results of this paper not only verify that the flight frequency programming model for an airline network with demand–supply interaction is practicable, but also provides a decision-support tool that determines flight frequencies and basic airfares, estimates passenger demand and profits, and analyzes their interactions for airlines.

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Appendix A. Grey systematic model for forecasting OD-pair market-sizes

Formulation of the $\mathrm{GM}(1,N)$ model is briefly described below. Assume an original historic series of annual traffic flows between a given OD-pair r–s (i.e., OD-pair market-sizes), $F_{rs}^{(0)}$, is $F_{rs}^{(0)} = (F_{rs}^{(0)}(1), \ldots, F_{rs}^{(0)}(n))$, where n denotes the number of years observed. Accumulated generating operations (AGOs), an important feature of GM, focus largely on reducing data randomness. The AGO formation of $F_{rs}^{(0)}$ is $F_{rs}^{(1)} = (F_{rs}^{(1)}(1), \ldots, F_{rs}^{(1)}(n))$, where $F_{rs}^{(1)}(k) = \sum_{t=1}^{k} F_{rs}^{(0)}(t)$, $k = 2, 3, \ldots, n$ and $F_{rs}^{(1)}(1) = F_{rs}^{(0)}(1)$. Assume that $X_{1_{rs}}, X_{2_{rs}}, \ldots, X_{N-2_{rs}}$ are socioeconomic variables (e.g., per capita GNP, per capita income), and N_{rs} is the total flight frequencies between OD-pair r–s (i.e., $N_{rs} = \sum_x \sum_p N_{rsp}^x$) for polyfactor $\mathrm{GM}(1,N)$ models. The original series of these variables, $X_{1_{rs}}, X_{2_{rs}}, \ldots, X_{N-2_{rs}}$, and N_{rs} are, respectively, $X_{1_{rs}}^{(0)} = (X_{1_{rs}}^{(0)}(1), X_{1_{rs}}^{(0)}(2), \ldots, X_{1_{rs}}^{(0)}(n))$, $X_{2_{rs}}^{(0)} = (X_{2_{rs}}^{(0)}(1), X_{2_{rs}}^{(0)}(2), \ldots, X_{2_{rs}}^{(0)}(n)), \ldots, X_{N-2_{rs}}^{(0)} = (X_{N-2_{rs}}^{(0)}(1), X_{N-2_{rs}}^{(0)}(2), \ldots, X_{N-2_{rs}}^{(0)}(n))$, $N_{rs} = (N_{rs}(1), N_{rs}(2), \ldots, N_{rs}(n))$; and $X_{1_{rs}}^{(1)}, X_{2_{rs}}^{(1)}, \ldots, X_{N-2_{rs}}^{(1)}$, and $N_{rs}^{(1)}$ are their respective AGO-series; where $N_{rs}^{(1)}(k) = \sum_{t=1}^{k-1} N_{rs}(t) + N_{rs}(k)$, and $N_{rs}(k)$ is the total flight frequencies between OD-pair r–s for the planning year k. The $\mathrm{GM}(1,N)$ model can be constructed by formulating a group of differential equations for $F_{rs}^{(1)}$ and $X_{1_{rs}}^{(1)}$, $X_{2_{rs}}^{(1)}, \ldots, X_{N-2_{rs}}^{(1)}$, $N_{rs}^{(1)}$. That is,

$$
\begin{cases}
\frac{\mathrm{d}F_{rs}^{(1)}}{\mathrm{d}t} = -aF_{rs}^{(1)} + b_1 X_{1_{rs}}^{(1)} + b_2 X_{2_{rs}}^{(1)} + \cdots + b_{N-2} X_{N-2_{rs}}^{(1)} + b_{N-1} N_{rs}^{(1)}, \\
\frac{\mathrm{d}X_{1_{rs}}^{(1)}}{\mathrm{d}t} = -a_1 X_{1_{rs}}^{(1)} + u_1, \\
\frac{\mathrm{d}X_{2_{rs}}^{(1)}}{\mathrm{d}t} = -a_2 X_{2_{rs}}^{(1)} + u_2, \\
\vdots \\
\frac{\mathrm{d}X_{N-2_{rs}}^{(1)}}{\mathrm{d}t} = -a_{N-2} X_{N-2_{rs}}^{(1)} + u_{N-2}, \\
\frac{\mathrm{d}N_{rs}^{(1)}}{\mathrm{d}t} = -a_{N-1} N_{rs}^{(1)} + u_{N-1}.
\end{cases}
\tag{A.1}
$$

In Eq. (A.1), the parameters, a, b_i, a_i, u_i, $i = 1, 2, \ldots, N-1$, can be determined by applying the least-squares method. The first-order differential equation for the AGO-series of each of socioeconomic variable, $X_{1_{rs}}^{(1)}, X_{2_{rs}}^{(1)}, \ldots, X_{N-2_{rs}}^{(1)}$, are GM(1,1) models that can be formulated as

$$\frac{\mathrm{d}X_{i_{rs}}^{(1)}}{\mathrm{d}k} + a_i X_{i_{rs}}^{(1)} = u_i, \quad i = 1, 2, \ldots, N-2. \tag{A.2}$$

The forecasting functions of $X_{i_{rs}}^{(1)}$ can then be obtained from Eq. (A.2) as follows:

$$\widehat{X}_{i_{rs}}^{(1)}(k) = \left(X_{i_{rs}}^{(0)}(1) - \frac{\hat{u}_i}{\hat{a}_i}\right)\mathrm{e}^{-\hat{a}_i(k-1)} + \frac{\hat{u}_i}{\hat{a}_i}, \quad i = 1, 2, \ldots, N-2;\ k = 2, 3, \ldots \tag{A.3}$$

The grey systematic model (Eq. (A.1)) expresses a dynamic relationship between socioeconomic variables, $X_{i_{rs}}^{(1)}$, $i = 1, 2, \ldots, N-2$, total flight frequency $N_{rs}^{(1)}$, and the OD-pair market-size, F_{rs}. Then, the forecasted value of $F_{rs}^{(1)}(k)$ can be obtained by combining all forecasted socioeconomic variables, $\widehat{X}_{i_{rs}}^{(1)}$, $i = 1, 2, \ldots, N-2$, and total flight frequency, $N_{rs}^{(1)}$, as follows:

$$\begin{aligned}\widehat{F}_{rs}^{(1)}(k) = {} & \left[F_{rs}^{(0)}(1) - \frac{1}{\hat{a}}\left(\hat{b}_1\widehat{X}_{1_{rs}}^{(1)}(k) + \hat{b}_2\widehat{X}_{2_{rs}}^{(1)}(k) + \cdots + \hat{b}_{N-1}N_{rs}^{(1)}(k)\right)\right]\mathrm{e}^{-\hat{a}(k-1)} \\ & + \frac{1}{\hat{a}}\left(\hat{b}_1\widehat{X}_{1_{rs}}^{(1)}(k) + \hat{b}_2\widehat{X}_{2_{rs}}^{(1)}(k) + \cdots + \hat{b}_{N-1}N_{rs}^{(1)}(k)\right), \quad k = 1, 2, \ldots\end{aligned} \tag{A.4}$$

$$\widehat{F}_{rs}^{(0)}(k) = \widehat{F}_{rs}^{(1)}(k) - \widehat{F}_{rs}^{(1)}(k-1), \tag{A.5}$$

where $\widehat{F}_{rs}^{(0)}(1) = F_{rs}^{(0)}(1)$.

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