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Lower confidence bounds with sample size information for C_{pm} applied to production yield assurance

W. L. PEARN^{†*} and MING-HUNG SHU[‡]

The process capability index C_{pm} , sometimes called the Taguchi index, has been proposed to the manufacturing industry as providing numerical measures on process performance. A lower confidence bound estimates the minimum process capability, conveying critical information regarding product quality, which is essential to quality assurance. The sample size determination is directly related to the cost of the data collection plan. The purpose of this paper is to provide explicit formulas with efficient algorithms to obtain the lower confidence bounds and sample sizes required for specified precision of the estimation on C_{pm} using the maximum likelihood estimator (MLE) of C_{pm} . We also provide tables for the engineers/practitioners to use for their in-plant applications. A real-world example taken from a microelectronics manufacturing process is investigated to illustrate the applicability of the proposed approach. The implementation of existing statistical theory for capability assessment bridges the gap between the theoretical development and factory applications.

1. Introduction

The loss-based process capability index C_{pm} , sometimes called the Taguchi index, has been proposed to the manufacturing industry to measure process performance. The index measures the ability of the process to cluster around the target, which reflects the degrees of process targeting (centring). The index C_{pm} incorporates the variation of production items with respect to the target value and the specification limits preset in the factory (see Hsiang and Taguchi 1985, Chan *et al.* 1988, Kotz and Johnson 1993, Kotz and Lovelace 1998). The index C_{pm} is defined in the following:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation, and T is the target value setting of the midpoint of the specification limits ($T = m = (USL + LSL)/2$). The capability index C_{pm} is not primarily designed to provide an exact measure on the number of conforming items, i.e. the process yield. However, C_{pm} considers the process departure $(\mu - T)^2$ (rather than 6σ alone) in the denominator of the

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C_{pm}	PPM	C_{pm}	PPM
0.95	4371.923	1.50	6.795
1.00	2699.796	1.55	3.319
1.05	1632.705	1.60	1.587
1.10	966.848	1.65	0.742
1.15	560.587	1.70	0.340
1.20	318.217	1.75	0.152
1.25	176.835	1.80	0.067
1.30	96.193	1.85	0.029
1.35	51.218	1.90	0.012
1.40	26.691	1.95	0.005
1.45	13.614	2.00	0.002

Table 1. Various values of C_{pm} and the maximum possible non-conformities (in PPM).

definition to reflect the degrees of process targeting (Hsiang and Taguchi 1985, Chan *et al.* 1988). We note that $\sigma^2 + (\mu - T)^2 = E[(X - T)^2]$ which is the major part of the denominator of C_{pm} . Since $E[(X - T)^2]$ is the expected loss, where the loss of a characteristic X missing the target is often assumed to be well approximated by the symmetric squared error loss function, $loss(X) = k(X - T)^2$, the capability index C_{pm} has been referred to as a loss-based index.

In general the process mean μ and the process standard deviation σ are unknown. However, in practice, μ and σ can be estimated using the sample data. We then consider the natural estimator of the index C_{pm} . In order to calculate the estimator, however, sample data must be collected, and a great degree of uncertainty may be introduced into capability assessments due to sampling errors. The approach by simply looking at the calculated values of the estimated indices and then making a conclusion on whether the given process is capable, is highly unreliable as the sampling errors have been ignored. A reliable approach for estimating the true value of C_{pm} is to construct the lower confidence bound. The lower confidence bound is not only essential to production yield assurance, but can also be used in capability testing for decision making. In fact, Rucinski (1996) showed that $Yield \leq 2\Phi(3C_{pm}) - 1$, or the fraction of non conformities ($2\Phi(-3C_{pm})$). Table 1 displays various values of $C_{pm} = 0.95(0.05)2.00$, and the corresponding maximum possible non-conformities (in PPM). For example, if a process has capability with $C_{pm} \geq 1.25$, then the production yield would be at least 99.982%.

2. Estimation of C_{pm}

The index C_{pm} can be rewritten as the following:

$$C_{pm} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}},$$

where $d = (USL - LSL)/2$ is half the length of the specification interval. Chan *et al.* (1988) and Boyles (1991) proposed the following two estimators of C_{pm} ,

$$\tilde{C}_{pm} = \frac{d}{3\sqrt{S^2 + (\bar{X} - T)^2}}, \quad \hat{C}_{pm} = \frac{d}{3\sqrt{S_n^2 + (\bar{X} - T)^2}},$$

where $\bar{X} = \sum_{i=1}^n X_i/n$, $S^2 = \sum_{i=1}^n (X_i - \bar{X})^2/(n - 1)$ and $S_n^2 = \sum_{i=1}^n (X_i - \bar{X})^2/n$. In fact, the two estimators, \hat{C}_{pm} and \hat{C}_{pm} , are asymptotical equivalent. We note that \bar{X} and S_n^2 are the MLEs of μ and σ^2 , respectively. Hence, the estimated index \hat{C}_{pm} is the MLE of C_{pm} . Furthermore, the term $S_n^2 + (\bar{X} - T)^2$ in the denominator of \hat{C}_{pm} is the uniformly minimum variance unbiased estimator (UMVUE) of the term $\sigma^2 + (\mu - T)^2$ in the denominator of C_{pm} , where $S_n^2 + (\bar{X} - T)^2 = \sum_{i=1}^n (X_i - T)^2/n$ and $\sigma^2 + (\mu - T)^2 = E[(X - T)^2]$. Therefore, it is reasonable, for reliability purpose, that we use the estimator \hat{C}_{pm} to evaluate process performance. Under the assumption of normality, Kotz and Johnson (1993) obtained the r th moment, and calculated the first two moments, the mean, and the variance of \hat{C}_{pm} . Boyles (1991) and Pearn *et al.* (1992) showed that \hat{C}_{pm} is distributed as:

$$\hat{C}_{pm} \sim \frac{USL - LSL}{6\sigma} \sqrt{\frac{n}{\chi_{n,\lambda}^2}},$$

which can be alternatively expressed as

$$\hat{C}_{pm} \sim C_{pm} \sqrt{1 + \frac{\lambda}{n}} \sqrt{\frac{n}{\chi_{n,\lambda}^2}}, \tag{1}$$

where $\chi_{n,\lambda}^2$ denotes the non-central Chi-square distribution with n degrees of freedom and non-centrality parameter $\lambda = n\xi^2$ where $\xi = (\mu - T)/\sigma$. Several methods have been proposed for constructing approximate lower confidence bounds of C_{pm} in the literature. Marcucci and Beazley (1988) proposed using the ordinary Chi-square distribution to approximate the non-central Chi-square distribution to obtain the following approximate lower confidence bound, denoted as $C_{pm}^{L(MB)}$,

$$C_{pm}^{L(MB)} \cong \sqrt{\frac{\chi_n^2(1 - \gamma)}{n}} \hat{C}_{pm}, \tag{2}$$

where $\chi_n^2(1 - \gamma)$ is the $(1 - \gamma)$ th percentile of the ordinary central Chi-square with n degrees of freedom. Thus, the approximate sample size needed for given estimation precision $R_{pm}^{(MB)}$ can be obtained as (Marcucci and Beazley 1988):

$$n \cong \frac{\chi_n^2(1 - \gamma)}{R_{pm}^{(MB)^2}}, \quad \text{where } R_{pm}^{(MB)} = C_{pm}^{L(MB)} / \hat{C}_{pm}. \tag{3}$$

Boyles (1991) used a moment approximation to the non-central Chi-square distribution, $\chi_{n,\lambda}^2$, to obtain the approximate lower confidence bound. It has been shown that the distribution of

$$\frac{S_n^2 + (\bar{X} - T)^2}{\sigma^2 + (\mu - T)^2} = \frac{\hat{\tau}^2 \chi_v^2}{\tau^2 v}, \quad \text{where } v = \frac{n(1 + \xi^2)^2}{(1 + 2\xi^2)}. \tag{4}$$

Since $C_{pm}/\hat{C}_{pm} = \hat{\tau}/\tau$, an approximate $100\gamma\%$ lower confidence bound for C_{pm} can be found as

$$\hat{C}_{pm} \sqrt{\frac{\chi_v^2(1 - \gamma)}{v}}. \tag{5'}$$

In practice, Boyles (1991) recommended obtaining the approximate lower confidence bound $C_{pm}^{L(Bo)}$ as

$$C_{pm}^{L(Bo)} \cong \hat{C}_{pm} \sqrt{\frac{\chi_{\hat{v}}^2(1-\gamma)}{\hat{v}}}, \quad \text{where } \hat{v} = \frac{n(1+\hat{\xi}^2)^2}{(1+2\hat{\xi}^2)}, \quad \hat{\xi} = \frac{(\bar{X}-T)}{S}. \quad (5)$$

Thus, the approximate sample size required for given estimation precision $R_{pm}^{(Bo)}$ can be obtained by solving \hat{v} , where

$$\hat{v} \cong \frac{\chi_{\hat{v}}^2(1-\gamma)}{R_{pm}^{(Bo)^2}}. \quad (6)$$

On the other hand, based on equation (1), the $100\gamma\%$ lower confidence bound $C_{pm}^{L(ZH)}$ can be found as (Zimmer and Hubele 1997):

$$C_{pm}^{L(ZH)} = \hat{C}_{pm} \sqrt{\frac{\chi_{n,\lambda}^2(1-\gamma)}{n+\lambda}}. \quad (7)$$

Thus, the sample size required with a given estimation precision $R_{pm}^{L(ZH)}$ can be obtained as:

$$n = \frac{\chi_{n,\lambda}^2(1-\gamma)}{R_{pm}^{(ZH)^2}} - \lambda \quad (8)$$

Using equations (1) and (7), Zimmer and Hubele (1997) and Zimmer *et al.* (2001) presented graphical procedures and tables of $\sqrt{n/\chi_{n,\lambda}^2(1-\gamma)}$ to obtain confidence intervals and sample sizes required of C_{pm} , where the parameter $(\mu - T)/\sigma$ is assumed to be a known constant. It is noted that when $(\mu - T)/\sigma = 0$, the lower confidence bounds (7) (Zimmer *et al.* 2001) are the same as the approximate lower confidence bounds obtained in (2) by Marcucci and Beazley (1988) and (5) by Boyles (1991). Moreover, when $(\mu - T)/\sigma = 0$, equation (8) for the sample sizes would be identical to those obtained from approximate equations (3) and (6). In this paper, we first provide an explicit form of the cumulative distribution function of the maximal likelihood estimator (MLE) of C_{pm} , which can be expressed in terms of a mixture of the Chi-square distribution and the normal distribution. We then develop an efficient algorithm to compute the lower confidence bounds to determine the sample sizes required for specified precision of the estimation on C_{pm} . We also provide tables for the engineers/practitioners to use in their factory applications. A real-world example on the simultaneous sampling analogue-to-digital converters, taken from a microelectronics-component manufacturing factory, is investigated to illustrate the applicability of the algorithm.

3. Cumulative Distribution Function of Estimated C_{pm}

From a demonstrably stable process (under statistical control), the MLE of C_{pm} , $\hat{C}_{pm} = d/(3\sqrt{S_n^2 + (\bar{X}-T)^2})$. To derive the cumulative distribution function (CDF) and the probability density function (PDF) of \hat{C}_{pm} , we define

- (1) $D = n^{1/2} d/\sigma$,
- (2) $K = nS_n^2/\sigma^2$, which is distributed as χ_{n-1}^2 ,

- (3) $Z = n^{1/2}(\bar{X} - T)/\sigma$, which is distributed as $N(\delta, 1)$, where $\delta = n^{1/2}(\mu - T)/\sigma$,
- (4) $Y = Z^2$, then, the PDF of Y can be expressed as:

$$f_Y(y) = \frac{1}{2\sqrt{y}} (f_Z(-\sqrt{y}) + f_Z(\sqrt{y})), \quad \text{for } y > 0. \tag{9}$$

We note that the estimator \hat{C}_{pm} can be rewritten as:

$$\begin{aligned} \hat{C}_{pm} &= \frac{D}{3\sqrt{K+Y}}. \\ F_{\hat{C}_{pm}}(x) &= 1 - \text{P}\left\{\frac{D}{3\sqrt{K+Y}} > x\right\} = 1 - \int_0^\infty \text{P}\left\{\sqrt{K+Y} < \frac{D}{3x} \mid Y = y\right\} f_Y(y) dy \\ &= 1 - \int_0^{D^2/(9x^2)} \text{P}\{K < D^2/(9x^2) - y\} f_Y(y) dy, \quad x > 0. \end{aligned} \tag{10}$$

The last equality in (10) holds since $\text{P}\{K < D^2/(9x^2) - y\} = 0$, for $y > D^2/(9x^2)$. Hence, we have

$$F_{\hat{C}_{pm}}(x) = 1 - \int_0^{D^2/(9x^2)} F_K(D^2/(9x^2) - y) f_Y(y) dy. \tag{11}$$

Using the representation in (9), we may obtain

$$F_{\hat{C}_{pm}}(x) = 1 - \int_0^{D^2/(9x^2)} F_K(D^2/(9x^2) - y) \frac{1}{2\sqrt{y}} (f_Z(-\sqrt{y}) + f_Z(\sqrt{y})) dy, \tag{12}$$

$$f_{\hat{C}_{pm}}(x) = \int_0^{D^2/(9x^2)} f_K(D^2/(9x^2) - y) \frac{D^2}{9x^3\sqrt{y}} (f_Z(-\sqrt{y}) + f_Z(\sqrt{y})) dy. \tag{13}$$

Changing the variable $g = (3x/D)^2 y$ in integral (13), we can obtain the PDF of \hat{C}_{pm} , as follows:

$$f_{\hat{C}_{pm}}(x) = \frac{D^3}{27x^4} \int_0^1 \frac{1}{\sqrt{g}} f_K\left(\frac{D^2(1-g)}{9x^2}\right) \left\{ f_Z\left(-\frac{D\sqrt{g}}{3x}\right) + f_Z\left(\frac{D\sqrt{g}}{3x}\right) \right\} dt, \quad x > 0. \tag{14}$$

We note that the statistic Z^2 follows a non-central chi-square distribution with one degree of freedom and non-centrality parameter δ^2 . Chen (1998) defined the distribution of Y as a weighted non-central chi-square distribution with one degree of freedom and non-centrality parameter δ^2 under the assumption of normality. Chen (1998) also derived the PDF of Y , in an alternative form of equation (9), as the following, where $\lambda = \delta^2$ and Y_j is distributed as χ_{1+j}^2 .

$$f_Y(y) = \frac{e^{-\lambda/2}}{2\sqrt{\pi}} \sum_{j=0}^\infty \left\{ \frac{(\sqrt{2}\delta)^j}{j!} \Gamma\left(\frac{1+j}{2}\right) \left((-1)^j f_{Y_j}(y) + f_{Y_j}(y) \right) \right\}, \quad y > 0. \tag{15}$$

Using the expression for the PDF of Y in equation (11), we may obtain

$$\begin{aligned} F_{\hat{C}_{pm}}(x) &= 1 - \frac{e^{-\lambda/2}}{2\sqrt{\pi}} \sum_{j=0}^\infty \frac{(\sqrt{2}\delta)^j}{j!} \Gamma\left(\frac{1+j}{2}\right) \\ &\quad \times \int_0^{D^2/(9x^2)} F_K(D^2/(9x^2) - y) \left((-1)^j f_{Y_j}(y) + f_{Y_j}(y) \right) dy \end{aligned} \tag{16}$$

Therefore, the PDF of \hat{C}_{pm} , in an alternative form of (14), can be expressed as

$$f_{\hat{C}_{pm}}(x) = \frac{e^{-\lambda/2}}{2\sqrt{\pi}} \sum_{j=0}^{\infty} \frac{(\sqrt{2}\delta)^j}{j!} \Gamma\left(\frac{1+j}{2}\right) \times \int_0^{D^2/(9x^2)} f_K(D^2/(9x^2) - y) 2D^2/(9x^3) dy. \tag{17}$$

Changing variable with $g = (3x/D)^2 y$ in equation (17), we can obtain the PDF of \hat{C}_{pm} , as expressed in equation (18),

$$f_{\hat{C}_{pm}}(x) = \frac{2^{1-n/2} D^n x^{-(n+1)}}{3^n \Gamma((n-1)/2)} \times \frac{e^{-\lambda/2}}{2\sqrt{\pi}} \exp\left(\frac{-D^2}{18x^2}\right) \times \sum_{j=0}^{\infty} \frac{1}{j!} \left(\frac{\delta D}{3x}\right)^j \times \sum_{i=1}^2 (-1)^{ij} \int_0^1 (1-g)^{(n-3)/2} g^{(j-1)/2} dg, x > 0. \\ = \frac{2^{1-n/2} D^n}{3^n x^{n+1}} \exp\left(-\frac{\lambda}{2} - \frac{D^2}{18x^2}\right) \sum_{j=0}^{\infty} \left\{ \left(\frac{\lambda D^2}{36x^2}\right)^j / (j! \Gamma\left(\frac{n}{2} + j\right)) \right\}, x > 0, \tag{18}$$

which is identical to the PDF of \hat{C}_{pm} obtained in Vännman and Kotz (1995). From equation (12), changing the variable with $y = t^2$, the cumulative distribution function of \hat{C}_{pm} can be expressed in terms of a mixture of the Chi-square distribution and the normal distribution:

$$F_{\hat{C}_{pm}}(x) = 1 - \int_0^{b\sqrt{n}/(3x)} F_K\left(\frac{b^2 n}{9x^2} - t^2\right) [\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})] dt,$$

for $x > 0$, where $b = d/\sigma$, $F_K(\cdot)$ is the cumulative distribution function of the ordinary central Chi-square distribution χ_{n-1}^2 , and $\phi(\cdot)$ is the PDF of the standard normal distribution $N(0, 1)$. The framework of the derivation can be found in Vännman and Kotz (1995). Figures 1(a)–(d) display the PDF and CDF plots of \hat{C}_{pm} for $\xi = 0$ and 1, $b = 3$, $d = 2$, with various sample sizes $n = 10, 20$ and 50. From figures 1(a)–(d) we observe that for $n = 10$ the distributions are skew and have large spread. As n increases, the spread decreases and so does the skewness. For $n > 50$, the estimate \hat{C}_{pm} is approximately unbiased.

4. Lower confidence bounds on C_{pm}

We note that when $C_{pm} = C$, the term $b = d/\sigma$ can be expressed as $b = 3C\sqrt{1 + \xi^2}$. Thus, the index C_{pm} may be expressed as a function of the distribution characteristic parameter .

$$C_{pm} = \frac{d}{3\sigma\sqrt{1 + \xi^2}} = \frac{d/\sigma}{3\sqrt{1 + \xi^2}}.$$

Hence, given the sample of size n , the confidence level γ , the estimated value, \hat{C}_{pm} , and the parameter ξ , the lower confidence bounds (denoted as $C_{pm}^{L(PS)}$) can be obtained using a numerical integration technique with iterations, to solve the following (equation (19)). In practice, the parameter ξ is unknown, but it can be calculated from the sample data as $\hat{\xi} = (\bar{X} - T)/S$. It should be noted, in particular, that equation (19) is an even function of ξ . Thus, for both $\xi = \xi_0$ and $\xi = -\xi_0$ we may obtain the same

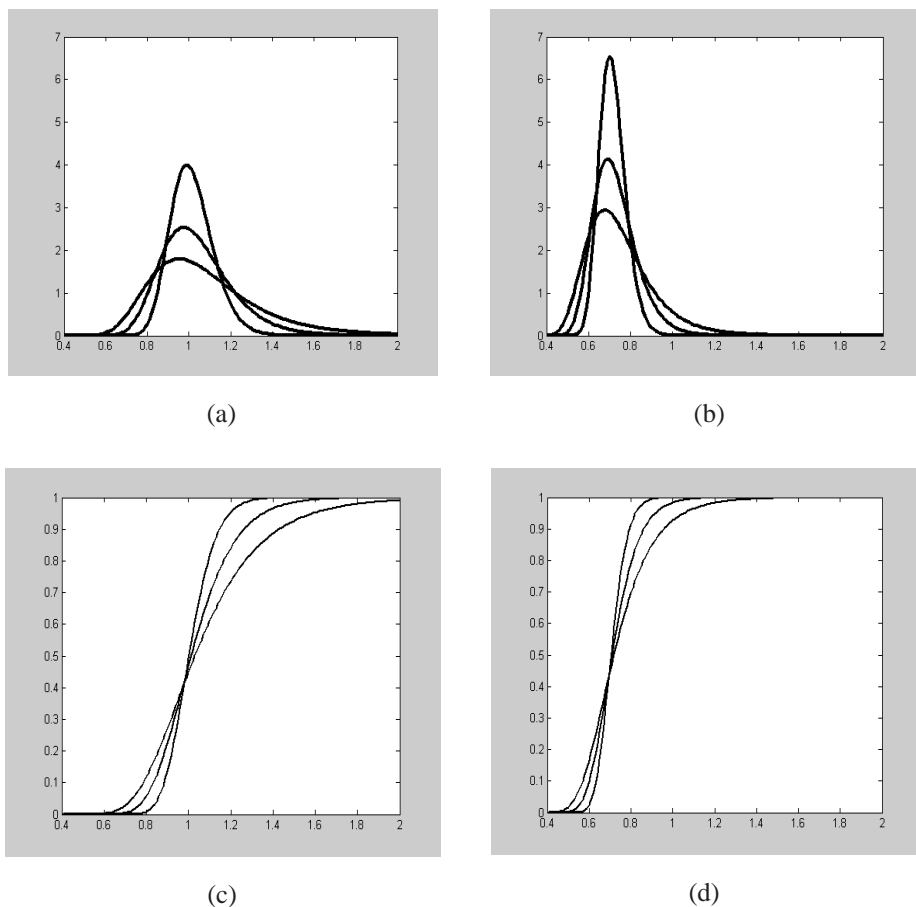


Figure 1. (a) PDF plots of \hat{C}_{pm} with $\xi = 0, b = 3, d = 2,$ and $n = 10, 20, 50$ (bottom to top); (b) PDF plots of \hat{C}_{pm} with $\xi = 1.0, b = 3, d = 2,$ and $n = 10, 20, 50$ (bottom to top); (c) CDF plots of \hat{C}_{pm} with $\xi = 0, b = 3, d = 2,$ and $n = 10, 20, 50$ (bottom to top); (d) CDF plots of \hat{C}_{pm} with $\xi = 1.0, b = 3, d = 2,$ and $n = 10, 20, 50$ (bottom to top).

lower confidence bound $C_{pm}^{L(PS)}$.

$$\int_0^{b\sqrt{n}/(3\hat{C}_{pm})} F_K\left(\frac{b^2n}{9\hat{C}_{pm}^2} - t^2\right) [\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})] dt = 1 - \gamma. \quad (19)$$

Using equation (19), we may compute the lower confidence bounds, $C_{pm}^{L(PS)}$. An algorithm using the technique called the direct search is developed. An efficient Matlab program (available on request) executing the algorithm is also developed, which incorporates three auxiliary functions for evaluating $C_{pm}^{L(PS)}$, including (a) the cumulative distribution function of the chi-square $\chi_{n-1}^2, F_K(\cdot)$, (b) the probability density function of the standard normal distribution $N(0,1), \phi(\cdot)$, and (c) the function of numerical integration using the recursive adaptive Simpson quadrature – ‘quad’. The Matlab program requires no more than 10 CPU seconds for all sample sizes, to obtain the lower bound values for all sample sizes investigated.

4.1. Lower confidence bounds $C_{pm}^{L(PS)}$ and parameter ξ

Since the process parameters μ and σ are unknown, then the distribution characteristic parameter, $\xi = (\mu - T) / \sigma$ is also unknown, and has to be estimated in real applications, naturally by substituting μ and σ by the sample mean \bar{X} and the sample standard deviation S . Such an approach introduces additional sampling errors from estimating ξ in finding the lower confidence bounds, and would certainly make our approach (and of course including all the existing methods) less reliable. Consequently, the conclusions made would result in less production yield assurance to the factories, and provide less quality protection to the customers. To eliminate the need for further estimating the distribution characteristic parameter ξ , we examine the behaviour of the lower confidence bounds C^L against the parameter ξ , using the following approximation formula, in equation (20), proposed by Wilson and Hilferty (1931). Note that the approximation is very accurate for a wide range of percentile points, even with sample sizes as small as 10.

$$\chi_v^2(1 - \gamma) = v \left[Z(1 - \gamma) \sqrt{\frac{2}{9v}} + 1 - \frac{2}{9v} \right]^3 \tag{20}$$

Substituting equations (20) and (4) into equation (5'), we may obtain

$$C^L = \hat{C}_{pm} \left[Z(1 - \gamma) \sqrt{\frac{2(1 + 2\xi^2)}{9n(1 + \xi^2)^2}} + 1 - \frac{2(1 + 2\xi^2)}{9n(1 + \xi^2)^2} \right]^{3/2}$$

Define the function

$f(\xi) = \hat{C}_{pm} [Z(1 - \gamma) \sqrt{2(1 + 2\xi^2)/9n(1 + \xi^2)^2} + 1 - ((2(1 + 2\xi^2)/ 9n(1 + \xi^2)^2)]^{3/2}$, then

$$f'(\xi) = \frac{-4\xi^3 \hat{C}_{pm}}{3n(1 + \xi^2)^3} \left[Z(1 - \gamma) \sqrt{\frac{2(1 + 2\xi^2)}{9n(1 + \xi^2)^2}} + 1 - \frac{2(1 + 2\xi^2)}{9n(1 + \xi^2)^2} \right]^{1/2} \times \left[\frac{Z(1 - \gamma)}{2} \left(\frac{2(1 + 2\xi^2)}{9n(1 + \xi^2)^2} \right)^{-1/2} - 1 \right] \tag{21}$$

It is easy to verify that $f'(0) = 0$, $f''(\xi) > 0$ when $\xi > 0$, and $f''(\xi) > 0$ when $\xi < 0$ (even function). So, the function $f(\xi)$ takes its absolute minimum at $\xi = 0$. To demonstrate that this result is valid for a non-central Chi-square distribution, we perform extensive calculations to obtain the lower confidence bounds $C_{pm}^{L(PS)}$ for $\xi = 0(0.05)3.00$, $n = 10(5)200$, $\hat{C}_{pm} = 0.7(0.1)3.0$, and confidence level $\gamma = 0.95$. Note that of the parameter values we investigated, $\xi = 0(0.05)3.00$ cover a sufficiently wide range of applications with process capability $C_{pm} \geq 0$. The results indicate that (i) the lower confidence bound $C_{pm}^{L(PS)}$ is decreasing in ξ , and is increasing in n , (ii) the lower confidence bound $C_{pm}^{L(PS)}$ obtains its minimum at $\xi = 0$ in all cases. Figures 2–7 plot the curves of the lower confidence bound, $C_{pm}^{L(PS)}$, versus the parameter ξ for $\hat{C}_{pm} = 0.7, 0.9, 1.2, 2.0, 2.5, 3.0$, respectively, with confidence level $\gamma = 0.95$. For bottom curve 1, sample size $n = 30$; for bottom curve 2, sample size $n = 50$; for bottom curve 3, sample size $n = 70$; for top curve 3, sample size $n = 100$;

for top curve 2, sample size $n = 150$; for top curve 1, sample size $n = 200$. Hence, for practical purpose we may solve equation (19) with $\xi = 0$ to obtain the required lower confidence bounds for given \hat{C}_{pm} , n and γ , without having to estimate further the parameter ξ . Thus, the level of confidence γ can be ensured, and the decisions made based on such approach are indeed more reliable.

We analytically justify (equations (20), (21)), and numerically demonstrate in figures 2–7 that (i) the lower bound function $f(\xi)$ indeed takes its absolute minimum

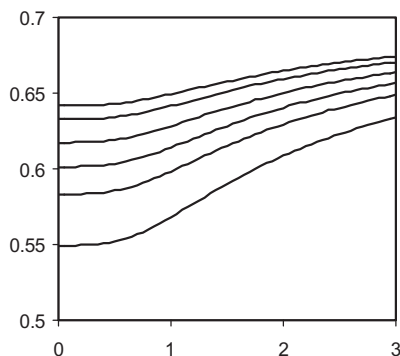


Figure 2. Plots of $C_{pm}^{L(PS)}$ versus $|\xi|$ for $\hat{C}_{pm} = 0.7$, $\gamma = 0.95$ and $n = 30, 50, 70, 100, 150, 200$.

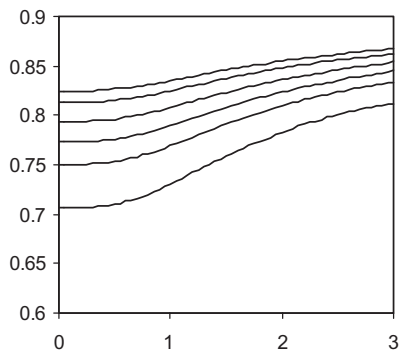


Figure 3. Plots of $C_{pm}^{L(PS)}$ versus $|\xi|$ for $\hat{C}_{pm} = 0.9$, $\gamma = 0.95$ and $n = 30, 50, 70, 100, 150, 200$.

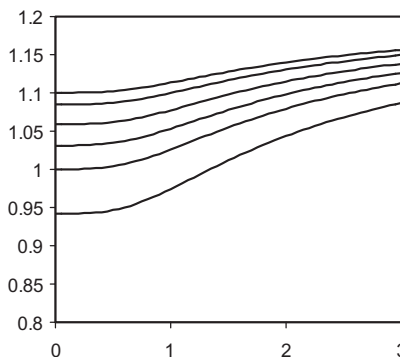


Figure 4. Plots of $C_{pm}^{L(PS)}$ versus $|\xi|$ for $\hat{C}_{pm} = 1.2$, $\gamma = 0.95$ and $n = 30, 50, 70, 100, 150, 200$.

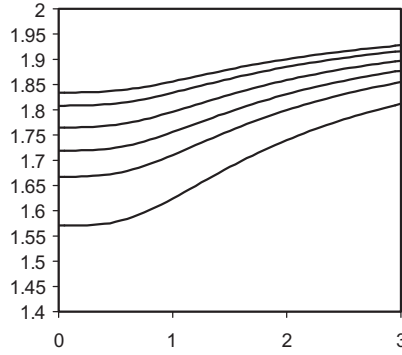


Figure 5. Plots of $C_{pm}^{L(PS)}$ versus $|\xi|$ for $\hat{C}_{pm} = 2.0$, $\gamma = 0.95$ and $n = 30, 50, 70, 100, 150, 200$.

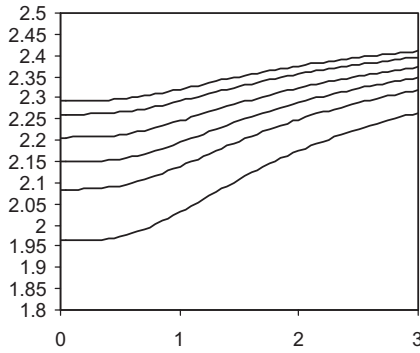


Figure 6. Plots of $C_{pm}^{L(PS)}$ versus $|\xi|$ for $\hat{C}_{pm} = 2.5$, $\gamma = 0.95$ and $n = 30, 50, 70, 100, 150, 200$.

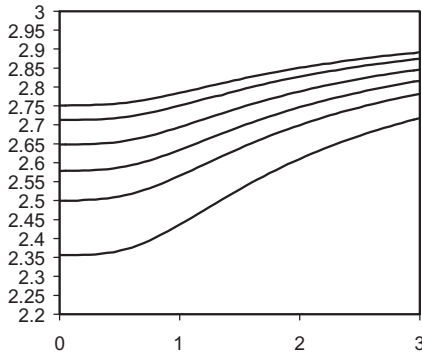


Figure 7. Plots of $C_{pm}^{L(PS)}$ versus $|\xi|$ for $\hat{C}_{pm} = 3.0$, $\gamma = 0.95$ and $n = 30, 50, 70, 100, 150, 200$.

at $\xi = 0$, (ii) the lower bound obtained by setting the unknown parameter $\xi = 0$ is indeed maximal among those which guarantee that the level of confidence is at least γ , and (iii) the lower bound cannot be improved unless the value of the unknown parameter $\xi = 0$ is given. Tables 2(a)–(b) tabulate the lower confidence bound values, $C_{pm}^{L(PS)}$, for $\hat{C}_{pm} = 0.7(0.1)3.0$, $n = 10(5)200$, and $\gamma = 0.95$. For example, if $\hat{C}_{pm} = 1.5$ for $n = 100$, then the lower confidence bound $C_{pm}^{L(PS)} = 1.324$, and so we may conclude that $C_{pm} > 1.324$, with 95% confidence. We remark here that if $\lambda = 0$

n	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
5	0.335	0.383	0.431	0.475	0.525	0.575	0.623	0.670	0.718	0.766	0.814	0.862
10	0.439	0.500	0.565	0.625	0.691	0.750	0.816	0.875	0.942	1.000	1.068	1.125
15	0.487	0.557	0.625	0.696	0.765	0.835	0.905	0.974	1.044	1.114	1.183	1.250
20	0.515	0.589	0.663	0.737	0.810	0.884	0.958	1.031	1.105	1.179	1.250	1.325
25	0.535	0.611	0.688	0.764	0.841	0.917	0.994	1.070	1.147	1.223	1.300	1.375
30	0.549	0.628	0.706	0.785	0.864	0.942	1.021	1.099	1.178	1.256	1.335	1.413
35	0.560	0.640	0.720	0.800	0.881	0.961	1.041	1.121	1.200	1.281	1.361	1.441
40	0.569	0.651	0.732	0.813	0.895	0.976	1.058	1.139	1.220	1.302	1.383	1.465
45	0.577	0.659	0.742	0.824	0.907	0.989	1.072	1.154	1.237	1.319	1.401	1.484
50	0.583	0.666	0.750	0.833	0.917	1.000	1.083	1.167	1.250	1.333	1.417	1.500
55	0.589	0.673	0.757	0.841	0.925	1.009	1.093	1.178	1.262	1.346	1.430	1.514
60	0.593	0.678	0.763	0.848	0.933	1.017	1.102	1.187	1.272	1.357	1.442	1.526
65	0.598	0.683	0.768	0.854	0.939	1.025	1.110	1.196	1.281	1.366	1.452	1.537
70	0.601	0.687	0.773	0.859	0.945	1.031	1.117	1.203	1.289	1.375	1.461	1.547
75	0.605	0.691	0.778	0.864	0.950	1.037	1.123	1.210	1.296	1.383	1.469	1.556
80	0.608	0.695	0.781	0.868	0.955	1.042	1.129	1.216	1.303	1.390	1.476	1.563
85	0.610	0.698	0.785	0.872	0.959	1.047	1.134	1.221	1.309	1.396	1.483	1.570
90	0.613	0.701	0.788	0.876	0.963	1.051	1.139	1.226	1.314	1.402	1.489	1.577
95	0.615	0.703	0.791	0.879	0.967	1.055	1.143	1.231	1.319	1.407	1.495	1.583
100	0.617	0.706	0.794	0.882	0.970	1.059	1.147	1.235	1.324	1.412	1.500	1.588
105	0.619	0.708	0.797	0.885	0.974	1.062	1.151	1.239	1.328	1.421	1.505	1.594
110	0.621	0.710	0.799	0.888	0.977	1.065	1.154	1.243	1.332	1.425	1.509	1.598
115	0.623	0.712	0.801	0.890	0.979	1.068	1.157	1.246	1.336	1.428	1.514	1.603
120	0.625	0.714	0.803	0.893	0.982	1.071	1.160	1.250	1.339	1.432	1.518	1.607
125	0.626	0.716	0.805	0.895	0.984	1.074	1.163	1.253	1.342	1.435	1.521	1.611
130	0.628	0.717	0.807	0.897	0.986	1.076	1.166	1.256	1.345	1.438	1.525	1.615
135	0.629	0.719	0.809	0.899	0.989	1.078	1.168	1.258	1.348	1.441	1.528	1.618
140	0.630	0.720	0.810	0.900	0.991	1.081	1.171	1.261	1.351	1.444	1.531	1.621
145	0.631	0.722	0.812	0.902	0.992	1.083	1.173	1.263	1.354	1.446	1.534	1.624
150	0.633	0.723	0.81	0.904	0.994	1.085	1.175	1.266	1.356	1.449	1.537	1.627
155	0.634	0.724	0.815	0.905	0.996	1.087	1.177	1.268	1.358	1.451	1.540	1.630
160	0.635	0.725	0.816	0.907	0.998	1.088	1.179	1.270	1.361	1.454	1.542	1.633
165	0.636	0.727	0.817	0.908	0.999	1.090	1.181	1.272	1.363	1.456	1.545	1.635
170	0.637	0.728	0.819	0.910	1.001	1.092	1.183	1.274	1.365	1.458	1.547	1.638
175	0.638	0.729	0.820	0.911	1.002	1.093	1.184	1.275	1.367	1.460	1.549	1.640
180	0.638	0.730	0.821	0.912	1.004	1.095	1.186	1.277	1.369	1.462	1.551	1.642
185	0.639	0.731	0.822	0.913	1.005	1.096	1.188	1.279	1.370	1.44	1.553	1.645
190	0.640	0.732	0.823	0.915	1.006	1.098	1.189	1.281	1.372	1.464	1.555	1.647
195	0.641	0.732	0.824	0.916	1.007	1.099	1.191	1.282	1.374	1.465	1.557	1.649
200	0.642	0.733	0.825	0.917	1.008	1.100	1.192	1.284	1.375	1.467	1.559	1.651

Table 2(a). Lower confidence bounds $C_{pm}^{L(PS)}$ of C_{pm} for $\hat{C}_{pm} = 0.7(0.1)1.8$, $n = 5(5)200$,

(or $\xi = 0$) in the formula $C_{pm}^{L(ZH)}$ obtained in Zimmer and Hubele (1997), then we obtain the same lower bound.

5. Sample size determination for C_{pm}

The sample size determination is important, as it directly relates to the cost of the data collection plan. From equation (19), it can be rewritten as

$$\int_0^{R_{pm}^{(PS)} \sqrt{n(1+\xi^2)}} F_K \left(R_{pm}^{(PS)^2} n(1+\xi^2) - t^2 \right) [\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})] dt = 1 - \gamma. \quad (22)$$

n	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
5	0.900	0.950	1.000	1.050	1.100	1.150	1.198	1.246	1.294	1.341	1.375	1.425
10	1.193	1.250	1.319	1.375	1.445	1.500	1.570	1.625	1.696	1.750	1.822	1.875
15	1.322	1.392	1.462	1.531	1.600	1.671	1.740	1.810	1.875	1.949	2.019	2.088
20	1.400	1.474	1.547	1.621	1.695	1.768	1.842	1.916	1.990	2.063	2.137	2.211
25	1.450	1.529	1.606	1.682	1.759	1.835	1.912	1.988	2.065	2.141	2.218	2.294
30	1.492	1.571	1.649	1.725	1.806	1.885	1.963	2.042	2.120	2.199	2.275	2.356
35	1.521	1.600	1.682	1.762	1.842	1.922	2.000	2.082	2.162	2.242	2.322	2.400
40	1.546	1.627	1.709	1.790	1.872	1.953	2.034	2.116	2.197	2.279	2.360	2.441
45	1.566	1.649	1.731	1.814	1.896	1.979	2.061	2.144	2.225	2.309	2.391	2.474
50	1.584	1.667	1.750	1.834	1.917	2.000	2.084	2.167	2.250	2.334	2.417	2.500
55	1.598	1.683	1.767	1.850	1.935	2.019	2.103	2.187	2.272	2.356	2.440	2.524
60	1.611	1.696	1.781	1.866	1.950	2.035	2.120	2.205	2.290	2.375	2.460	2.544
65	1.623	1.708	1.794	1.879	1.964	2.050	2.135	2.221	2.306	2.392	2.475	2.562
70	1.633	1.719	1.805	1.891	1.977	2.063	2.149	2.235	2.321	2.407	2.492	2.578
75	1.642	1.728	1.815	1.900	1.988	2.074	2.161	2.247	2.334	2.420	2.506	2.593
80	1.650	1.737	1.824	1.911	1.998	2.085	2.171	2.258	2.345	2.432	2.519	2.606
85	1.658	1.745	1.832	1.919	2.007	2.094	2.181	2.269	2.356	2.443	2.530	2.618
90	1.665	1.752	1.840	1.927	2.015	2.103	2.190	2.278	2.366	2.453	2.541	2.629
95	1.671	1.759	1.847	1.935	2.023	2.111	2.199	2.287	2.375	2.463	2.550	2.638
100	1.677	1.765	1.853	1.941	2.030	2.118	2.206	2.295	2.383	2.471	2.559	2.648
105	1.682	1.771	1.859	1.948	2.036	2.125	2.213	2.302	2.391	2.479	2.568	2.656
110	1.687	1.776	1.865	1.954	2.042	2.131	2.220	2.309	2.398	2.487	2.575	2.664
115	1.692	1.781	1.870	1.959	2.048	2.137	2.226	2.315	2.404	2.493	2.583	2.672
120	1.696	1.786	1.875	1.964	2.053	2.143	2.232	2.321	2.411	2.500	2.589	2.679
125	1.700	1.790	1.879	1.969	2.058	2.148	2.237	2.327	2.416	2.506	2.596	2.685
130	1.704	1.794	1.884	1.973	2.063	2.153	2.243	2.332	2.422	2.512	2.601	2.691
135	1.708	1.798	1.888	1.978	2.068	2.157	2.247	2.337	2.427	2.517	2.607	2.697
140	1.711	1.801	1.892	1.982	2.072	2.162	2.252	2.342	2.432	2.522	2.612	2.702
145	1.715	1.805	1.895	1.985	2.075	2.166	2.256	2.347	2.437	2.527	2.617	2.708
150	1.718	1.808	1.899	1.989	2.080	2.170	2.260	2.350	2.441	2.532	2.622	2.713
155	1.721	1.811	1.902	1.993	2.083	2.174	2.264	2.355	2.446	2.536	2.627	2.717
160	1.724	1.814	1.905	1.996	2.087	2.177	2.268	2.359	2.450	2.540	2.631	2.722
165	1.726	1.817	1.908	1.999	2.090	2.181	2.272	2.362	2.453	2.544	2.635	2.725
170	1.729	1.820	1.911	2.002	2.093	2.184	2.275	2.366	2.457	2.548	2.639	2.730
175	1.731	1.823	1.914	2.005	2.096	2.187	2.278	2.369	2.461	2.552	2.643	2.734
180	1.734	1.825	1.916	2.008	2.099	2.190	2.281	2.373	2.464	2.555	2.646	2.738
185	1.736	1.827	1.919	2.010	2.102	2.193	2.284	2.375	2.467	2.559	2.650	2.741
190	1.738	1.830	1.921	2.013	2.104	2.196	2.287	2.379	2.470	2.562	2.653	2.745
195	1.740	1.832	1.923	2.015	2.107	2.198	2.290	2.382	2.473	2.565	2.656	2.748
200	1.742	1.834	1.926	2.017	2.109	2.201	2.293	2.384	2.476	2.568	2.660	2.751

Table 2(b). Lower confidence bounds $C_{pm}^{L(PS)}$ of C_{pm} for $\hat{C}_{pm} = 1.9(0.1)3.0$, $n = 5(5)200$, $\gamma = 0.95$.

Hence, given the desired estimation precision $R_{pm}^{(PS)}$, the confidence level γ , and the parameter ξ , the sample size can be obtained using the numerical integration technique with iterations, to solve equation (22). It should be noted, in particular, that equation (22) is an even function of ξ . Thus, for both $\xi = \xi_0$ and $\xi = -\xi_0$ we may obtain the same sample size n .

5.1. Sample size n and parameter ξ

Since the process parameters μ and σ are unknown, then the distribution characteristic parameter, $\xi = (\mu - T)/\sigma$ is also unknown. To eliminate the

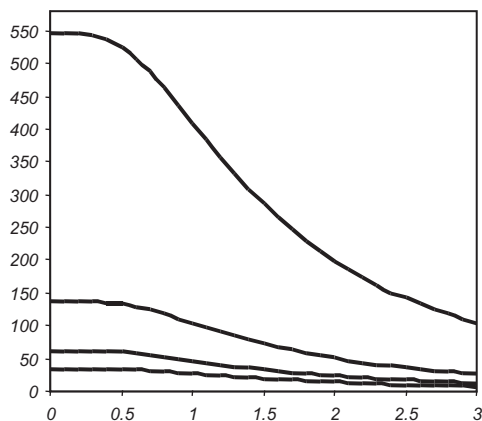


Figure 8. Plots of n versus $|\xi|$ for $R_{pm}^{(PS)} = 0.80, 0.85, 0.9, 0.95$ (from bottom to top).

need for further estimating the distribution characteristic parameter ξ , we examine the behaviour of the sample size n against the parameter ξ . We perform extensive calculations to obtain the sample sizes n for $\xi = 0(0.1)3.00$, estimation precision $R_{pm}^{(PS)} = 0.75(0.05)0.95$, and confidence level $\gamma = 0.90, 0.95, 0.975$ and 0.99 . Figure 8 plots the curves of the required sample size, n , versus the parameter ξ for $R_{pm}^{(PS)} = 0.8(0.05)0.95$, with confidence level $\gamma = 0.95$. In figure 8, for bottom curve 1, $R_{pm}^{(PS)} = 0.8$, for bottom curve 2, $R_{pm}^{(PS)} = 0.85$, for top curve 2, $R_{pm}^{(PS)} = 0.9$, for top curve 1, $R_{pm}^{(PS)} = 0.95$. The results indicate that (i) the sample size n is decreasing in ξ , and is increasing in $R_{pm}^{(PS)}$, (ii) the sample sizes n obtains its maximum at $\xi = 0$ in all cases. Hence, for practical purposes we may solve (22) with $\xi = 0$ to obtain the required sample sizes n for given $R_{pm}^{(PS)}$ and γ , without having to estimate further the parameter ξ . Thus, the level of confidence γ can be ensured, and the decisions made based on such an approach are indeed more reliable. In fact, Franklin (1999) indicated (without justifications) that one can provide the largest sample size with $\xi = 0$, which coincides with our results. We remark that to maintain the confidence level no less than γ , the required sample size n obtained using the proposed approach cannot be reduced.

5.2. Computing the required sample size

To compute the sample size required n , we develop a Matlab program (available on request). The program reads the desired estimation precision $R_{pm}^{(PS)}$ and the confidence level γ , and outputs with the sample size n (always rounding up if n is not an integer) and the actual estimation precision R_γ . Table 3 displays the sample size n required for $R_\gamma \geq R_{pm}^{(PS)}$ with $R_{pm}^{(PS)} = 0.75(0.01)0.95$ and $\gamma = 0.9, 0.95, 0.975$ and 0.99 . For example, if $R_{pm}^{(PS)}$ is set to 0.89 , then with $\gamma = 0.95$ the sample size needed is $n = 114$. We conclude that a minimum sample size of $n = 114$ is required to be 95% certain that the true C_{pm} is no less than $R_\gamma = 89.03\%$ of the sample estimate \hat{C}_{pm} . Thus, if the sample estimate $\hat{C}_{pm} = 1.3$, then the true value of C_{pm} is no less than $1.3 \times 89.03\% = 1.157$, with 95% confidence.

6. Capability measure for analogue-to-digital converters

The type of analogue-to-digital converter we investigated is a CMOS, low-power device with 10-bit, and taking 6 mega samples per second (MSPS), which is a

R_{pm}	$\gamma = 0.90$		$\gamma = 0.95$		$\gamma = 0.975$		$\gamma = 0.99$	
	n	R_γ	n	R_γ	n	R_γ	n	R_γ
0.75	15	0.7549	23	0.7545	31	0.7522	42	0.7505
0.76	16	0.7629	25	0.7645	34	0.7633	46	0.7613
0.77	17	0.7702	27	0.7735	37	0.7730	50	0.7708
0.78	19	0.7629	29	0.7815	40	0.7816	55	0.7813
0.79	21	0.7941	32	0.7921	44	0.7917	60	0.7905
0.80	23	0.8035	35	0.8012	48	0.8005	66	0.8001
0.81	25	0.8118	39	0.8117	53	0.8101	74	0.8110
0.82	28	0.8225	43	0.8207	60	0.8215	82	0.8203
0.83	31	0.8316	48	0.8304	67	0.8310	92	0.8302
0.84	35	0.8418	54	0.8402	75	0.8403	104	0.8403
0.85	39	0.8503	62	0.8509	86	0.8508	119	0.8505
0.86	45	0.8609	71	0.8608	98	0.8602	136	0.8601
0.87	52	0.8709	82	0.8705	114	0.8703	158	0.8701
0.88	61	0.8810	96	0.8804	134	0.8804	186	0.8802
0.89	72	0.8907	114	0.8903	159	0.8902	222	0.8903
0.90	86	0.9002	138	0.9004	193	0.9003	268	0.9001
0.91	106	0.9103	170	0.9103	238	0.9102	332	0.9102
0.92	133	0.9201	214	0.9201	301	0.9202	420	0.9201
0.93	173	0.9301	279	0.9301	393	0.9301	549	0.9301
0.94	235	0.9401	380	0.9401	534	0.9401	748	0.9401
0.95	337	0.9501	546	0.9501	769	0.9501	1078	0.9500

Table 3. Sample size n required for $R_\gamma \geq R_{pm}^{(PS)}$, with $R_{pm}^{(PS)} = 0.75(0.01)0.95$, and $\gamma = 0.90, 0.95, 0.975, 0.99$.

simultaneous sampling analogue-to-digital converter (SSADC). The characteristics of SSADC in speed, resolution, bandwidth, and single-supply operation make it suitable for applications in radar signal detecting, imaging, high-speed acquisition, and communications. A multi-stage pipelined architecture with output error correction logic provides no missing codes over the full operating temperature range. Internal control registers are used to program the SSADC into the desired mode. The SSADC product consists of four analogue inputs, which are sampled simultaneously. These inputs can be selected individually and configured to single-ended or differential inputs. An integrated 16 word deep FIFO allows the storage of data in order to improve data transfers to the processor.

Internal reference voltages for the SSADC (1.5 V and 3.5 V) are provided. The SSADC has a built-in reference, which provides the reference voltages for the SSADC. The VREFP is set to 3.5 V and the VREFM is set to 1.5 V. An external reference can also be used through two reference input pins, REFP and REFM, if the reference source is programmed as external. The voltage levels applied to these pins establish the upper and lower limits of the analogue inputs to produce a full-scale and zero-scale reading respectively. This SSADC product is available in 32-pin DA (TSSOP) package and its functional block diagram, as depicted in figure 9. The precision of the reference voltage is an essential product characteristic, which has a significant impact on the SSADC quality. For the reference voltage of this product, the upper specification limit, USL , is set to 3.7 V, and the lower specification limit, LSL , is set to 3.3 V. Sample data collected from 120 analogue-to-digital converters are displayed in table 4. A histogram and the normal probability plot of the 120

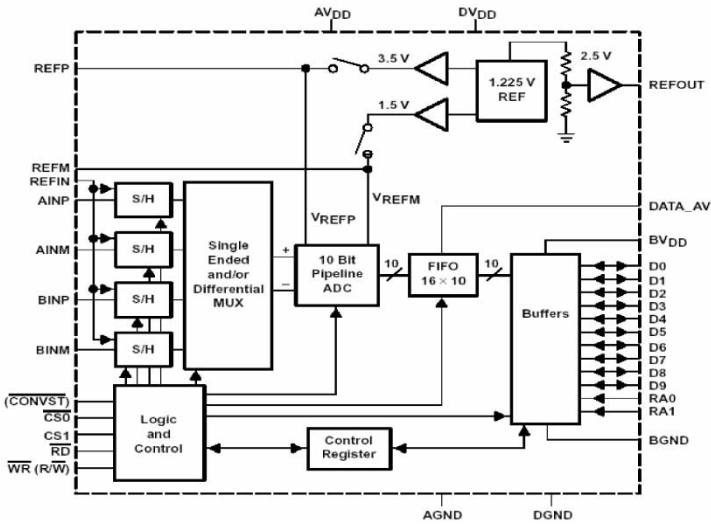
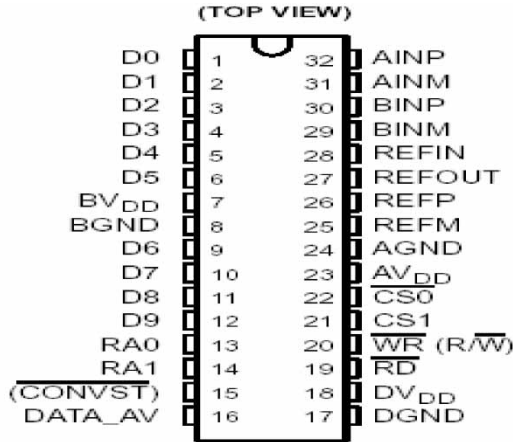


Figure 9. The SSADC and the functional block diagram.

3.54	3.49	3.51	3.53	3.54	3.51	3.61	3.53	3.57	3.55
3.56	3.58	3.50	3.59	3.59	3.54	3.55	3.45	3.45	3.52
3.48	3.54	3.52	3.51	3.57	3.56	3.60	3.51	3.50	3.55
3.51	3.59	3.56	3.48	3.47	3.61	3.52	3.55	3.50	3.56
3.52	3.58	3.50	3.55	3.53	3.48	3.48	3.52	3.49	3.53
3.48	3.50	3.48	3.49	3.50	3.49	3.52	3.58	3.52	3.57
3.48	3.48	3.53	3.53	3.49	3.57	3.58	3.50	3.59	3.62
3.57	3.53	3.53	3.50	3.48	3.51	3.49	3.44	3.53	3.55
3.53	3.52	3.55	3.55	3.54	3.47	3.56	3.48	3.58	3.53
3.50	3.50	3.51	3.55	3.51	3.54	3.51	3.57	3.50	3.57
3.56	3.55	3.52	3.52	3.53	3.59	3.55	3.53	3.53	3.53
3.54	3.59	3.51	3.45	3.52	3.56	3.49	3.55	3.50	3.49

Table 4. The 120 sample observations.

observation data both show that the sample data appears to be normal. The Shapiro–Wilk test is also used to check whether the sample data follow the normal distribution. The statistic W is found to be 0.9855 with p -value 0.2287. Thus, we conclude that the sample data can be regarded as taken from a normal process (or a near normal process).

6.1. Capability calculation and data analysis

6.1.1. LCB applications

In order to obtain the lower confidence bound on $C_{pm}^{L(PS)}$, we execute the Matlab program. The program reads the sample data file, and the input of sample size $n = 120$, $LSL = 3.3$ V, $USL = 3.7$ V, target value $T = 3.5$ V, and confidence level $\gamma = 0.95$, then outputs with the sample mean, $\bar{X} = 3.528$, the sample standard deviation $S_n = 0.0377$, the estimator $\hat{C}_{pm} = 1.415$, and the lower confidence bound $C_{pm}^{L(PS)} = 1.263$. We therefore conclude that the true value of the process capability C_{pm} is no less than 1.263 with 95% level of confidence. We note that the conclusions made here have used the particular value of $\xi = 0$ in finding the lower confidence bound, thus the confidence level is ensured to be no less than 0.95 (or the Type I error is no greater than 0.05). We thus can ensure that the production yield is 99.984%, and the number of the non-conformities is less than 156 PPM.

6.2. Sample size information applications

For situations where a specified sample data collection plan must be implemented due to limitations on production resource—say one is only allowed to take a sample of $n \leq 150$ —we could first check table 3 to obtain the appropriate sample size, $n = 138$, under the column of confidence level $\gamma = 0.95$. The corresponding (actual) estimation precision is $R_\gamma = 0.9004$. Thus, if the sample measurements of the 138 SSADCs yield an estimate $\hat{C}_{pm} = 1.62$, then we may conclude that the true value of C_{pm} is no less than $1.62 \times 90.04\% = 1.46$, with 95% confidence.

7. Control chart applications

7.1. Multiple control chart samples application

Many of the existing simultaneous sampling analogue-to-digital converters integrated circuit (SSADC IC) manufacturing factories implemented a daily-based production control plan for monitoring/controlling process stability. A routine-basis data collection procedure is executed to run \bar{X} and S control charts (for moderate sample sizes). The past ‘in control’ data consist of multiple samples of m_s groups, with variable sample size $n_i = (x_{i1}, x_{i2}, \dots, x_{in_i})$ and $N = \sum_{i=1}^{m_s} n_i$. The control chart samples are then analysed to compute the manufacturing capability. Thus, manufacturing information regarding the product quality characteristic is derived from multiple samples in this case, rather than based on one single sample. Under the assumption that these samples are taken from the normal distribution $N(\mu, \sigma^2)$, we consider the following estimators of process mean and process standard deviation,

$$\bar{X}_i = \frac{\sum_{j=1}^{n_i} x_{ij}}{n_i} \text{ and}$$

$$S_i = \sqrt{\frac{\sum_{j=1}^{n_i} (x_{ij} - \bar{X}_i)^2}{n_i - 1}},$$

for the i th sample mean and the sample standard deviation, respectively. Then, $\bar{X} = \sum_{i=1}^{m_s} \bar{X}_i / m_s$ and $S_p^2 = \sum_{i=1}^{m_s} (n_i - 1)S_i^2 / \sum_{i=1}^{m_s} (n_i - 1)$ are used for calculating the manufacturing capability C_{pm} . For cases with multiple samples, two estimators of C_{pm} can be considered, $\hat{C}_{pm}^{M(Zh)}$ and $\hat{C}_{pm}^{M(PS)}$, as shown below. The estimator $\hat{C}_{pm}^{M(Zh)}$ is proposed in Zhang (2001). Unfortunately, the exact distribution of $\hat{C}_{pm}^{M(PS)}$ is difficult to obtain. The exact distribution of the estimator proposed here, $\hat{C}_{pm}^{M(PS)}$, can be found as a function of the non-central Chi-square distribution, which is analytically tractable (Pearn *et al.* 2003). Consequently, the critical values, lower confidence bounds, sample size determination, and the manufacturing capability calculations can be performed using the same technique for cases with one single sample.

$$\hat{C}_{pm}^{M(Zh)} = \frac{USL - LSL}{6\sqrt{\left((N-1)S_p^2 + N(\bar{X} - T)^2\right)/N}}$$

$$\hat{C}_{pm}^{M(PS)} = \frac{USL - LSL}{6\sqrt{\frac{\sum_{i=1}^{m_s} (n_i - 1)S_i^2 + N(\bar{X} - T)^2}{N}}}$$

7.2. MPPAC control chart application

Furthermore, for SSADC IC manufacturing factories having a group of processes to be monitored and controlled, it would be effective to use the MPPAC (Multi-Process Performance Analysis Chart). The MPPAC can be used to illustrate and analyse the manufacturing capability for multiple processes, which conveys critical information regarding the departure of the process mean from the target value, process variability, capability levels, and provides a guideline of directions for capability improvement. Singhal (1991) introduced the C_{pk} MPPAC for monitoring multiple processes. Pearn and Chen (1997) proposed a modification to the C_{pk} MPPAC combining the more advanced capability index C_{pm} and C_{pmk} to identify the problems causing the processes to fail to centre around the target. Pearn *et al.* (2002) developed the MPPAC based on the incapability index C_{pp} . Using the same technique, the C_{pm} MPPAC can be developed to monitor the capability for multiple SSADC IC manufacturing processes. Using the C_{pm} MPPAC, practitioners/engineers can simultaneously analyse the performance of multiple processes based on one single chart. The C_{pm} MPPAC also prioritizes the order of the processes which the quality improvement effort should focus on, for either moving the process mean closer to the target value or reducing the process variation. The developed confidence lower bounds can then be applied to the C_{pm} MPPAC to ensure the accuracy of the MPPAC for given sample sizes.

8. Conclusions

In this paper, we considered the problem of finding the lower confidence bound and sample sizes required for specified precision of the estimation for the C_{pm} . The lower confidence bounds present a measure on the minimum capability of the process based on the sample data. The sample size determination is directly related to the cost of the data collection plan. We used the explicit form of the CDF of the

estimator \hat{C}_{pm} expressed in terms of a mixture of the Chi-square distribution and the normal distribution, to calculate the lower confidence bounds and sample sizes required for specified precision of the estimation. We investigated the behaviour of the lower confidence bound values and sample sizes required for specified precision of the estimation versus the process characteristic parameter $\xi = (\mu - T)/\sigma$. The results indicated that the maximal lower confidence bound obtaining its minimal value at $\xi = 0$, and the minimal sample sizes required for specified precision of the estimation can be found by setting $\xi = 0$. The proposed approach ensures that the risk of making incorrect decisions will be no greater than the preset Type I error $1 - \gamma$. We also provided tables for the engineers/practitioners to use for their in-plant applications. A real-world example on analogue-to-digital converters, taken from a microelectronics manufacturing process is investigated, to illustrate the applicability of our approach. The implementation of the existing complicated statistical theory for capability assessment bridges the gap between the theoretical development and the factory applications.

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