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Improved technique for measuring refractive index and thickness of a transparent plate

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Abstract

The phase difference between s- and p-polarizations of a circularly polarized heterodyne light beam reflected from a transparent plate is measured. The measured data is substituted into the specially derived equations and the refractive index can be calculated. Next, the variations of phase difference between s- and p-polarizations due to the wavelength shift and the extraction of the plate in a modified Michelson interferometer are measured. Then, its thickness can be calculated based on the measured value of refractive index, the variations of phase difference, and the specified value of wavelength shift.

Keywords: Refractive index; Thickness; Phase measurement; Heterodyne interferometry

1. Introduction

A transparent plate such as glass plate is often used in optics and semiconductor industries. Its refractive index and its thickness are very important optical parameters, especially in the design and fabrication of optical components and optoelectronic devices. Its thickness is always extremely larger than the wavelength of the light beam. So it is difficult to measure the thickness with conventional interferometric methods. Some optical methods, such as low-coherence interferometry [1–3] and confocal microscopy [4,5], are proposed to measure the thickness of a transparent plate. However, they deliver not the geometry thickness, but the optical thickness, which is the product of the geometrical thickness and the refractive index. The associated optical thickness can be evaluated from the direct readouts of the division marks of the high-resolution stage. In addition, its numerical aperture affects the measurement resolution of the confocal microscopy. On the other hand, some other methods [6–9] are reported for measuring the refractive index. Almost all of them are related to the measurement of light intensity variations. Consequently, the stability of the light source, the scattering light, the internal reflection, and other factors influence the accuracy of measurements and thus the resolution of the results is decreased.

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Although several other papers [10–14] are presented for the measurement of refractive index and the thickness of a transparent plate, they also need a high-resolution stage to displace the test plate, due to the introduction of the low-coherence interferometry or the confocal microscopy.

In this paper, an improved optical technique for measuring the refractive index and the thickness of a transparent plate is presented by using the heterodyne interferometry [15–17] and the two-wavelength interferometry [18]. Firstly, the phase difference between s- and p-polarizations of a circularly polarized heterodyne light beam reflected from a transparent plate is measured. The measured data is substituted into specially derived equations and the refractive index can be calculated. Secondly, the variations of phase difference between s- and p-polarizations due to the wavelength shift and the extraction of the plate in a modified Michelson interferometer [19] are measured. Then, its thickness can be calculated based on the measured value of refractive index, the variation of phase difference, and the specified value of wavelength shift. Consequently, the measurements of the thickness and refractive index can be operated precisely only with this single measurement system. This technique has merits such as a simple optical setup, easy operation, and rapid measurement. And the validity of this method is demonstrated.

2. Principle

The schematic diagram of this method is shown in Fig. 1. For convenience, the $+z$-axis is chosen to be along the light propagation direction and the x-axis is along the direction perpendicular to the paper plane. A light coming from a heterodyne light source [20] has an angular frequency difference ω

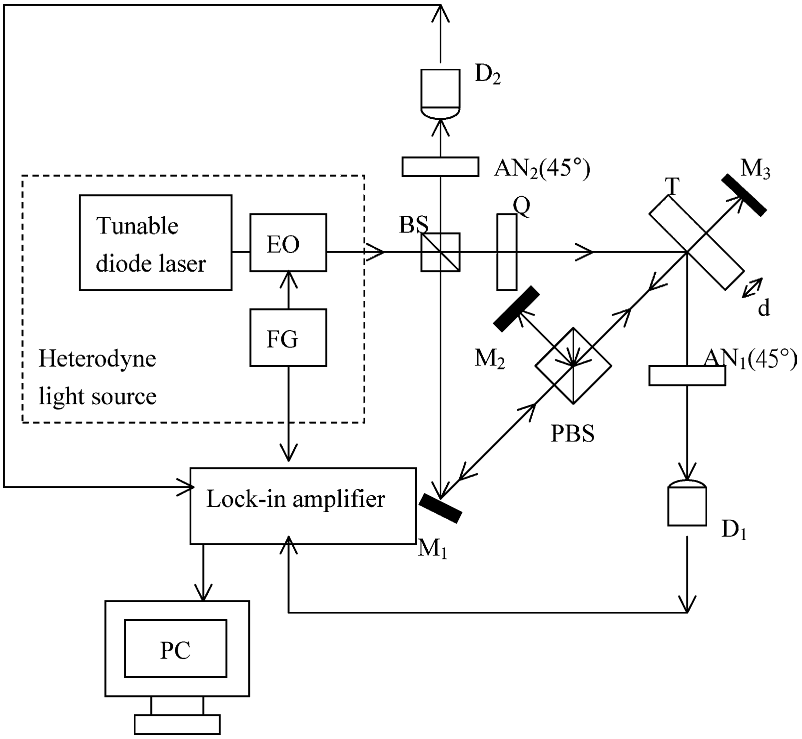

Fig. 1. Schematic diagram for measuring the phase differences: EO, electro-optic modulator; BS, beam splitter; Q, quarter-wave plate; T, test plate; M, mirror; PBS, polarizing beam splitter; AN, analyzer; D, detector; FG, function generation; and PC, personal computer.

between s- and p-polarizations, and its Jones vector [21] can be written as

$$E = \begin{pmatrix} E_x \exp(\mathrm{i}\omega t/2) \\ E_y \exp(-\mathrm{i}\omega t/2) \end{pmatrix}. \tag{1}$$

It is incident on a beam-splitter BS, and is divided into two parts: the transmitted light and the reflected light. The transmitted light passes through a quarter-wave plate Q. If the fast axis of Q is located at 45° relative to the x-axis, then the Jones vector of the light can be written as

$$\begin{aligned} E_i = Q(0^\circ) \cdot E &= \begin{pmatrix} \cos(\frac{\omega t}{2}) \\ -\sin(\frac{\omega t}{2}) \end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix} 1 \\ \mathrm{i} \end{pmatrix} \exp\left(\mathrm{i}\frac{\omega t}{2}\right) + \frac{1}{2}\begin{pmatrix} 1 \\ -\mathrm{i} \end{pmatrix} \exp\left(-\mathrm{i}\frac{\omega t}{2}\right). \end{aligned} \tag{2}$$

From Eq. (2), we can see that the right- and left-circular polarizations have frequency shifts $\omega/2$ and $-\omega/2$, respectively. Thus there is an angular frequency difference ω between them. The light, at the central wavelength λ_1 of the heterodyne light source, is incident at θ on the test plate T with refractive index n_1. The light reflected from this test plate passes through an analyzer AN_1 and enters a photodetector D_1. If the transmission axis of AN_1 is located at 45° with respect to the x-axis, then the Jones vector of the light arriving at D_1 is

$$E_{11} = \left[r_\mathrm{p} \cos\frac{\omega t}{2} - r_\mathrm{s} \sin\frac{\omega t}{2}\right] \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}, \tag{3}$$

where r_p and r_s are the reflection coefficients for p- and s-polarizations, respectively. According to the Fresnel's equations [22], we have

$$r_\mathrm{p} = \frac{\cos[\sin^{-1}(\sin\theta/n_1)] - n_1 \cos\theta}{\cos[\sin^{-1}(\sin\theta/n_1)] + n_1 \cos\theta}, \tag{4a}$$

and

$$r_\mathrm{s} = \frac{\cos\theta - n_1 \cos[\sin^{-1}(\sin\theta/n_1)]}{\cos\theta + n_1 \cos[\sin^{-1}(\sin\theta/n_1)]}, \tag{4b}$$

respectively. Hence, the intensity measured by D_1 is

$$I_{11} = I_0[1 + \gamma \cos(\omega t + \phi_{11})]. \tag{5}$$

Here, the average intensity I_0, the visibility γ, and the phase difference ϕ_{11} are given as

$$I_0 = \frac{1}{4}\left(\left|r_\mathrm{p}\right|^2 + \left|r_s\right|^2\right), \tag{6a}$$

$$\gamma = \frac{\sqrt{A^2 + B^2}}{I_0}, \tag{6b}$$

and

$$\begin{aligned} \phi_{11} &= \tan^{-1}\left(\frac{B}{A}\right) \\ &= \tan^{-1}\left[\frac{r_\mathrm{p} r_\mathrm{s}}{1/2(\left|r_\mathrm{p}\right|^2 - \left|r_\mathrm{s}\right|^2)}\right]. \end{aligned} \tag{6c}$$

Here I_{11} is the first test signal. Besides, the modulated electronic signal of the heterodyne light source is filtered and becomes the reference signal. It has the form as

$$I_r = \frac{1}{2}[1 + \cos(\omega t)]. \tag{7}$$

Both of these two sinusoidal signals I_{11} and I_r are sent to a lock-in amplifier, and then ϕ_{11} can be obtained. Moreover, it is easily seen from Eqs. (4)–(7) that the refractive index n_1 depends on ϕ_{11}. Consequently, n_1 can be calculated with the measurement of the phase difference ϕ_{11} under the condition in which θ is specified.

On the other hand, the reflected light coming from BS is reflected again by a mirror M_1 and enters a modified Michelson interferometer [19]. It consists of a polarization beam-splitter PBS, two mirrors M_2 and M_3, an analyzer AN_2, and a photodetector D_2. The test plate T is just located in one arm and the light beam passes through it perpendicularly. In the interferometer, PBS divides the light into two beams. The paths of these two beams are $\mathrm{PBS} \to \mathrm{M}_2 \to \mathrm{PBS} \to \mathrm{M}_1 \to \mathrm{BS} \to \mathrm{AN}_2 \to \mathrm{D}_2$ (for the reflected s-polarization light) and $\mathrm{PBS} \to \mathrm{T} \to \mathrm{M}_3 \to \mathrm{T} \to \mathrm{PBS} \to \mathrm{M}_1 \to \mathrm{BS} \to \mathrm{AN}_2 \to \mathrm{D}_2$(for the transmitted p-polarization light). If the transmission axis of AN_2 is 45° to the x-axis, then Jones vectors of p- and s-polarizations arriving at D_2 are

$$E_\mathrm{p} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \mathrm{e}^{\mathrm{i}((\omega t/2)+\phi_{21})}, \tag{8a}$$

and

$$E_s = \frac{1}{2\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}e^{i(-\omega t/2)}, \tag{8b}$$

respectively. ϕ_{21} is the phase difference due to the optical path difference and it can be expressed as

$$\phi_{21} = \frac{4\pi(n_1 d + d_0)}{\lambda_1}, \tag{9}$$

where d is the thickness of the glass plate, and d_0 is the length difference between the two arms except the thickness d in the interferometer. Hence the intensity measured by D_2 is

$$I_{21} = \left|E_p + E_s\right|^2 = \frac{1}{2}[1 + \cos(\omega t + \phi_{21})]. \tag{10}$$

Here I_{21} is the second test signal. As these two sinusoidal signals I_{21} and I_r are sent to a lock-in amplifier, and then ϕ_{21} can be obtained as previously.

If the central wavelength of the heterodyne light source is changed to λ_2, then the test signals I_{11} and I_{21} still have the forms of Eqs. (5) and (10) but with the phase differences ϕ_{12} and ϕ_{22}, respectively. The refractive index n_2 of T at wavelength λ_2 can be obtained with the measurement of ϕ_{12} by using the same technique to estimate n_1. The variation of phase difference of the second test signal due to the wavelength variation $\Delta\lambda$,

$$\begin{aligned}\Psi_1 &= \phi_{22} - \phi_{21}\\ &= \frac{4\pi(n_2 d + d_0)}{\lambda_2} - \frac{4\pi(n_1 d + d_0)}{\lambda_1}\\ &= \frac{4\pi[d(n_2\lambda_1 - n_1\lambda_2) - d_0\Delta\lambda]}{\lambda_1\lambda_2},\end{aligned} \tag{11}$$

can be obtained, where $\Delta\lambda$ equals $\lambda_2 - \lambda_1$. Next the test plate T is removed from the interferometer. The second test signal also still has the form of Eq. (10) but with another phase difference ϕ_{20}. Hence, the phase difference variation of the second test signal after the test plate has been removed can be obtained similarly, and it can be written as

$$\Psi_2 = \phi_{20} - \phi_{21} = \frac{-4\pi(d + d_0)\Delta\lambda}{\lambda_1\lambda_2}. \tag{12}$$

Consequently, we have

$$\Psi = \Psi_2 - \Psi_1 = \frac{4\pi d(n_1\lambda_2 - n_2\lambda_1 - \Delta\lambda)}{\lambda_1\lambda_2}. \tag{13}$$

It is obvious from Eq. (13) that the thickness d can be calculated with the measurement of Ψ under the conditions in which n_1, n_2, λ_1, and λ_2 are specified.

3. Experiments and results

In order to show the feasibility of this method, we measured the thickness and the refractive indices of one BK7 plate, one fused silica plate, and one BASF2 plate at 25 °C. The heterodyne light source consisting of a tunable diode laser (Model 6304, New Focus) and an electro-optic modulator EO driven by a function generator FG was used, as shown in Fig. 1. The frequency difference between p- and s-polarizations was 1 kHz. A lock-in amplifier with resolution 0.01° (Model SR850, Stanford Research System) was used to measure the phase difference. And a personal computer was employed to record and analyze the data. The experimental conditions $\theta = 45°$, $\lambda_1 = 632.80$ nm, $\lambda_2 = 632.81$ nm, and $\Delta\lambda = 0.01$ nm were used, and the results are listed in Table 1. Because $\Delta\lambda$ is so small, the measured results of n_1 and n_2 are equal

Table 1
Measured results and their associated reference data

	BK7	Fused silica	BASF2
n	1.5151	1.4498	1.6607
n_{ref}	1.51509[a]	1.45[b]	1.6606[c]
d (mm)	1.3201	2.9299	5.0368
d_{ref} (mm)	1.31	2.93	5.03
$\lvert\Delta n\rvert$	3.77×10^{-4}	3.16×10^{-4}	5.27×10^{-4}
$\lvert\Delta d\rvert$ (μm)	0.66	0.68	0.62

Note 1. Superscripts a, b, and c represent the reference data coming from Coring Ltd., Scott Ltd., and Ohara Ltd., respectively.
Note 2. The temperatures are fixed at 25 °C during the measurement procedure.

[23]. We used the symbol n to represent them in the Table. In addition, the thickness of these test plates was measured with a conventional micrometer. Their associated results are also added to Table 1 for comparison. It is clear that they show good agreement.

4. Discussion

Let Δn_i, $\Delta\phi_{1i}$, Δd, and $\Delta\Psi$ be the errors in n_i, ϕ_{1i}, d, Ψ, respectively, where i is either 1 or 2. From Eqs. (3)–(6) we get

$$|\Delta n_i| = \frac{(B+C)\times|\Delta\phi_{1i}|}{A}; \quad (i = 1 \text{ or } 2), \tag{14a}$$

where

$$A = -8(1+n_i^2)\cos\theta\cos[\sin^{-1}(\sin\theta/n_i)] \times \{\cos 2\theta - \cos[2\sin^{-1}(\sin\theta/n_i)]\}, \tag{14b}$$

$$B = 2(-1+n_i^2)^2\cos 2\theta + n_i^2\cos 4\theta + \cos 2[\theta - \sin^{-1}(\sin\theta/n_i)] + 2(1-n_i^2+n_i^4+\cos[2\sin^{-1}(\sin\theta/n_i)]) + \cos 2[\theta + \sin^{-1}(\sin\theta/n_i)], \tag{14c}$$

and

$$C = n_i^2\{2[-2+n_i^2+(-4+n_i^2)\cos 2\theta] \times \cos[2\sin^{-1}(\sin\theta/n_i)] + \cos[4\sin^{-1}(\sin\theta/n_i)]\}. \tag{14d}$$

In our experiments, Eq. (13) can be rewritten as

$$\Psi = \frac{4\pi d(n-1)\Delta\lambda}{\lambda_1\lambda_2} = \frac{4\pi d(n-1)}{\lambda_{eq}}, \tag{15}$$

where λ_{eq} is the equivalent wavelength. In addition, from Eq. (15) we also get

$$|\Delta d| = \frac{\lambda_{eq}}{4\pi(n-1)}|\Delta\Psi|. \tag{16}$$

Angular resolution of the lock-in amplifier, second harmonic error, and polarization-mixing errors are the factors that may influence the accuracy in the phase difference errors in this method. So the total phase difference errors of $|\Delta\phi_{11}|$ and $|\Delta\Psi|$ can be decreased to 0.03° [24] in our experiments. Substituting the conditions $|\Delta\phi_{1i}| = |\Delta\Psi| = 0.03°$ and $\Delta\lambda = 0.01$ nm into Eqs. (14) and (16), the measurement errors of each plate are calculated and listed in Table 1. In this table, $|\Delta n|$ is used to represent $|\Delta n_1|$ and $|\Delta n_2|$ And we also obtain $\lambda_{eq} \cong 40$ mm.

To avoid phase wrapping [25], it is necessary to let the data of Ψ be smaller than π. So our experimental conditions are suitable for the transparent plate with the thickness smaller than $\lambda_{eq}/4(n-1)$, which is about 20 mm. From Eqs. (15) and (16), it can be seen that as λ_{eq} increases, the measurable range of the thickness becomes wider and the associated measurement resolution decreases.

The phase difference between s- and p-polarizations of the light reflected from the surface of the test plate is measured to evaluate the refractive index and the thickness in this method. So, surface contamination will distort the phase difference, the associated errors will introduce to the measured results. In industry, a transparent plate must be cleaned and its homogeneity tested with a conventional Twyman–Green interferometer or Mach–Zehnder interferometer before it is fabricated. Hence, the test plate should be done with the same procedures as in industrial fabrication before it is tested with this method. If not, we would obtain the refractive index of the surface material on the test plate and its nominal thickness.

Moreover, both the refractive index and the thickness of the test plate depend on the ambient temperature. Here, BK7 is taken as an example. Its temperature coefficient of refractive index [26] is about 2.89×10^{-6}/°C in the range 20–40 °C, and its coefficient of linear expansion [26] is 7.1×10^{-6}/°C in the range 30–70 °C. If the shift in the ambient temperature is 10 °C in our experiments, then the induced errors in the measurements of refractive index and thickness are 2.89×10^{-5} and 0.094 μm, respectively. Both of them are so small compared with the associated measurement errors listed in Table 1 that they may be neglected under our experimental conditions. Other test plates give similar results. In our experiments, the ambient temperature was kept at 25 °C.

5. Conclusion

Based on the heterodyne interferometry and the two-wavelength interferometry, an improved technique for measuring the refractive index and the thickness of a transparent plate has been presented. These two optical parameters can be estimated in just one optical configuration. This technique has merits such as a simple optical setup, easy operation, and rapid measurement. Its validity has been demonstrated. It is suitable for a transparent plate with the thickness smaller than 20 mm in our experiments.

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