Collision Analysis for a Multi-Bluetooth Picocells Environment

Ting-Yu Lin and Yu-Chee Tseng

***Abstract*—Operating in the unlicensed 2.4-GHz ISM band, a Bluetooth piconet will inevitably encounter the interference problem from other piconets. With a special channel model and packet formats, one research issue is how to predict the packet collision effect in a multi-piconet environment. In an earlier work by El-Hoiydi in 2001, this problem is studied, but the result is still very limited, due to the assumptions that packets must be single-slot ones and that time slots of each piconet must be fully occupied by packets. A more general analysis is presented in this work by eliminating these constraints.**

***Index Terms*—Bluetooth, collision analysis, frequency hopping (FH), piconet, Wireless Personal-Area Network (WPAN).**

I. Introduction

As a PROMISING Wireless Personal-Area Network (WPAN) technology, Bluetooths are expected to be used in many applications, such as wireless earphones, keyboards, wireless access points, etc., [3]. Operating in the unlicensed 2.4-GHz ISM band, multiple Bluetooth piconets are likely to coexist in a physical environment. With a frequency-hopping (FH) radio and without coordination among piconets, transmissions from different piconets will inevitably encounter the collision problem. In a previous work [1], the author investigates the co-channel interference between Bluetooth piconets and derives an upper bound on packet error rate. The analysis in [1] has two limitations. First, all packets are assumed to be single-slot ones. Second, it is assumed that each piconet is fully loaded, in the sense that packets are sent in a back-to-back manner. These constraints greatly limit the applicability of the result in [1].

Also focusing on the same problem, this paper derives a more general analysis model where all packet types (1-slot, 3-slot, and 5-slot) can coexist in the network, and the system is not necessarily fully-loaded. The latter is achieved by modeling idle slots as individual single slots with *no* traffic load. So the result greatly relax the constraints in [1].

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II. Problem Statement

Bluetooth is a master-driven, time-division duplex (TDD), FH wireless radio system [2]. The smallest networking unit is a *piconet*, which consists of one master and no more than seven active slaves. Each picocell channel is represented by a pseudo-random hopping sequence comprised of 79 or 23 frequencies. In Bluetooth, the hopping sequence is determined by the master's ID and clock value. The channel is divided into time slots, each corresponding to one random frequency. In the following discussion, we assume 79 frequencies.

In each piconet, the master and slaves take turns to exchange packets. While the master only transmits in even-numbered slots, slaves must reply in odd-numbered slots. Three packet sizes are available: 1-slot, 3-slot, and 5-slot. For a multislot packet, its frequency is fully determined by the first slot and remains unchanged throughout.

We consider N piconets coexisting in a physically closed environment. Since no coordination is possible between piconets, each piconet has $N-1$ potential competitors. In any time instance, if two piconets transmit with the same frequency, the corresponding two packets are considered damaged. Our goal is to derive an analytic model to evaluate the impact of collisions in such a multi-piconet environment.

We assume a uniform traffic in each piconet, and let λ_1, λ_3, and λ_5 be the arrival rates of 1-, 3-, and 5-slot packets per slot, respectively, to a piconet. Note that for a multi-slot packet, only the header slot counts as arrival. It is easy to see that $\lambda_1 + 3\lambda_3 + 5\lambda_5 \leq 1$. Further, we can regard the remaining vacant slots as "dummy" single-slot packets. Thus, the arrival rate of such dummy (1-slot) packets is $\lambda_0 = 1 - (\lambda_1 + 3\lambda_3 + 5\lambda_5)$.

III. Collision Analysis in a Multi-Piconet Environment

Let us consider a piconet X and another competitor piconet Y, which is regarded as the unique source of interference to X. With the interference from Y, we first derive the success probability $P_S(i)$ of i-slot packets in X, where $i = 1, 3, 5$. We start by introducing the concept of "slot delimiter." Consider any slot in X. One or two slot delimiters in Y may cross X's slot. However, since we are considering continuous probability, the possibility of two crossing slot delimiters can be ignored, and thus we will deal with one crossing delimiter in the rest of the discussion. For example, for a 1-slot packet in X, it succeeds only if there is no interference from the two slots before and after the delimiter, so the success probability of X's packet could be 1, 78/79, or $(78/79)^2$, depending on whether Y transmits or not. Below, we denote the constant factor 78/79 by P_0.

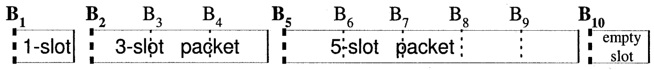


Fig. 1. Illustration of slot delimiters.

Next, we further elaborate on slot delimiters. Depending on what packet(s) is divided by it, a delimiter is classified into ten types (refer to Fig. 1):

- B_1, B_2, B_5: the beginning of a 1-, 3-, and 5-slot packet, respectively.
- B_3, B_4: the beginnings of the second and third slots of a 3-slot packet, respectively.
- B_6, B_7, B_8, B_9: the beginnings of the second, third, fourth, and fifth slots of a 5-slot packet, respectively.
- B_{10}: the beginning of a dummy slot.

It is easy to see that the rate of B_1 is λ_1 per slot; the rate of each of B_2, B_3, and B_4 is λ_3; the rate of each of B_5, B_6, B_7, B_8, and B_9 is λ_5; and the rate of B_{10} is λ_0. For ease of presentation, we denote the arrival rate of B_j by $\lambda(B_j)$, $j = 1 \ldots 10$. Given any B_j, we also define $g(j)$ to be the number of slots that follows delimiter B_j and belongs to the same packet. For example, $g(1) = 1$, $g(3) = 2$, $g(7) = 3$, and $g(10) = 1$.

Intuitively, when a packet in X is crossed by a delimiter of type $B_1/B_2/B_5$ in Y, there may exist two packets (of different frequencies) in both sides of the delimiter in Y which are potential sources of interference to X's packet. On the other hand, when the delimiter is of the other types, the interference source reduces to one.

Next, we formulate the success probability $P_S(i)$ of an i-slot packet, $i = 1, 3, 5$, in X, given the interference source Y. Toward this goal, we first introduce another probability function.

Definition 1: Given any i-slot packet in piconet X and any interference source piconet Y, define $L(k), k < i$, to be the probability that the packet of X experiences no interference from Y starting from the delimiter of Y crossing the $(i - k + 1)$-th slot of the packet to the end of the packet, under the condition that the aforementioned delimiter is of type $B_1/B_2/B_5/B_{10}$. For $k \leq 0$ (in which case the above definition is not applicable), $L(k) = 1$.

Intuitively, $L(k)$ is the success probability of the last k slots of X's packet excluding the part before the first delimiter of Y crossing these k slots, given the delimiter type constraint. With this definition, we can find $P_S(i)$ by repeatedly cutting off some slots from the head of X's packet, until there is no remaining slot. Specifically, we establish $P_S(i)$ by $L()$ as follows:

$$P_S(i) = \sum_{j=1}^{10} \lambda(B_j) \cdot f(j) \cdot L(i - g(j)) \qquad (1)$$

where

$$f(j) = \begin{cases} (1 - \lambda_0) \cdot P_0^2 + \lambda_0 \cdot P_0, & \text{if } j = 1, 2, 5 \\ (1 - \lambda_0) \cdot P_0 + \lambda_0, & \text{if } j = 10 \\ P_0, & \text{otherwise.} \end{cases}$$

In the equation, we consider each type $B_j, j = 1 \ldots 10$, of the first delimiter in Y crossing X's packet. The corresponding probability is $\lambda(B_j)$. Function $f(j)$ gives the probability that the packet(s) of Y on both sides of the first delimiter B_j does (do) not interfere with X's packet. It remains to consider the success probability of the last $i - g(j)$ slots of X's packet, excluding the part before the first delimiter of Y crossing these $i - g(j)$ slots (which must be of delimiter type $B_1/B_2/B_5/B_{10}$). This is reflected by the last factor $L(i - g(j))$.

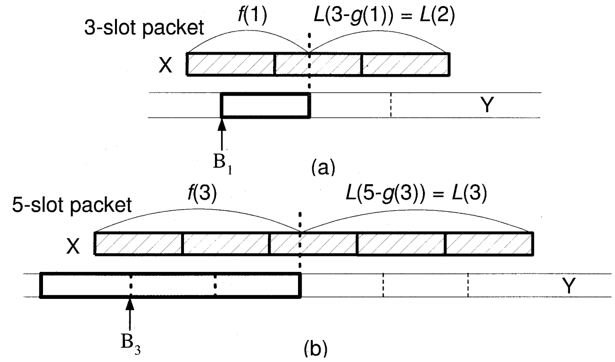


Fig. 2. Analysis of success probabilities for (a) 3-slot and (b) 5-slot packets.

For example, Fig. 2(a) illustrates a 3-slot packet in X. The first delimiter in Y crossing the 3-slot packet is of type B_1. The success probability of the first part in X is $f(1) = (1 - \lambda_0) \cdot P_0^2 + \lambda_0 \cdot P_0$. Intuitively, if the packet of Y before the delimiter B_1 is a dummy packet (of probability λ_0), the success probability is simply P_0; otherwise, there are two packets which are potential interference sources, and the success probability is P_0^2. Then we can move on to consider the success probability of the remaining part of X after the second delimiter in Y, which is given by $L(2)$. Another example of a 5-slot packet is shown in Fig. 2(b). The first delimiter in Y crossing the 5-slot packet is of type B_3. So the success probability from the beginning of the packet up to the third delimiter in Y crossing the packet is $f(3)$. For the remaining part, the success probability is $L(3)$. So the success probability of the 5-slot packet is $f(3) \cdot L(3)$

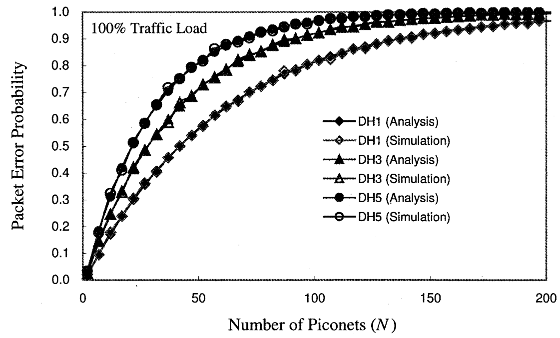
The remaining part of X's packet covered by $L(k)$ must start with a delimiter in Y of a restricted type of $B_1/B_2/B_5/B_{10}$. Since the packet in Y after the delimiter must be a complete packet, it can be solved recursively as follows $(k > 0)$:

$$\begin{aligned} L(k) = &\frac{\lambda_0}{\lambda_0 + \lambda_1 + \lambda_3 + \lambda_5} \cdot L(k - g(10)) \\ &+ \frac{\lambda_1}{\lambda_0 + \lambda_1 + \lambda_3 + \lambda_5} \cdot P_0 \cdot L(k - g(1)) \\ &+ \frac{\lambda_3}{\lambda_0 + \lambda_1 + \lambda_3 + \lambda_5} \cdot P_0 \cdot L(k - g(2)) \\ &+ \frac{\lambda_5}{\lambda_0 + \lambda_1 + \lambda_3 + \lambda_5} \cdot P_0 \cdot L(k - g(5)). \end{aligned} \qquad (2)$$

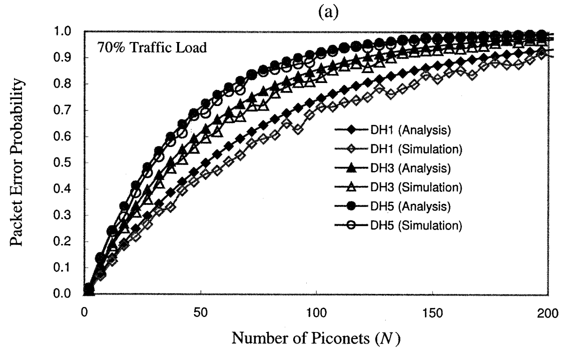
In each term, the first part is the probability of the corresponding packet type in Y. As to the boundary conditions, $L(k) = 1$, for $k \leq 0$.

Next, we consider an N-piconet environment. For each piconet X, there are $N - 1$ piconets each serving as an interference source. Since these interferences are uncoordinated and independent, the success probability of an i-slot packet in X can be written as $P_S(i)^{N-1}$. So the network throughput of X is:

$$\begin{aligned} T = &\lambda_1 \cdot P_S(1)^{N-1} \cdot R_1 + 3 \cdot \lambda_3 \cdot P_S(3)^{N-1} \cdot R_3 \\ &+ 5 \cdot \lambda_5 \cdot P_S(5)^{N-1} \cdot R_5, \end{aligned} \qquad (3)$$

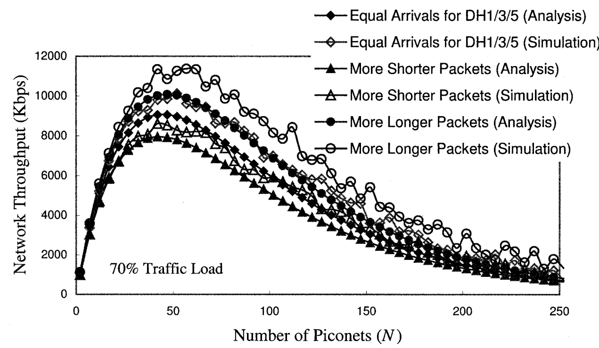


(a)

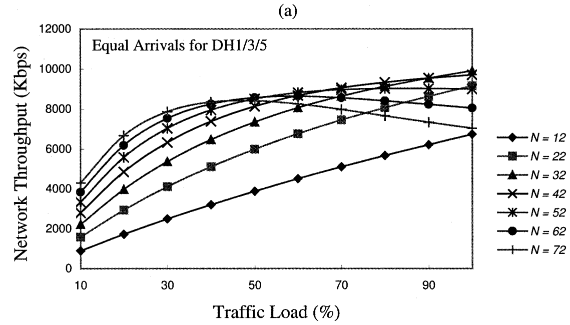


(b)

Fig. 3. Packet error probabilities under traffic loads of: (a) 100% and (b) 70%.



(a)



(b)

Fig. 4. Network throughput: (a) under 70% traffic load, and (b) against traffic loads for various network sizes.

where R_1, R_3, and R_5 are the data rates (bits/slot) of 1-, 3-, and 5-slot packets, respectively (for example, if DH1/DH3/DH5 are used, $R_1 = 216$, $R_3 = 488$, and $R_5 = 542.4$). The aggregate network throughput of N piconets is $N \times T$.

IV. Simulation and Analysis Results

From the above discussion, it can be seen that the result in [1] is in fact a special case of our analysis when $\lambda_1 = 1$ and $\lambda_3 = \lambda_5 = 0$.

To verify our analysis, simulations are conducted. We investigate DH1/3/5 packets. Assuming $\lambda_1 = \lambda_3 = \lambda_5$, we inject traffic loads of 100% and 70% (the percentage of busy slots) to reflect heavy and medium loads, respectively. That is, $\lambda_1 + 3\lambda_3 + 5\lambda_5 = 1$ and 0.7. Fig. 3 plots the error probabilities of DH1/3/5 packets under different numbers of picocells. The packet error probability increases as the traffic load or the number of piconets grows. Small packets (DH1) suffer less collisions than large ones (DH5) due to shorter transmission durations. However, larger packets are much more bandwidth-efficient than smaller ones (e.g., a DH5 carries 542.4/216 times more bits per slot than a DH1 does). This observation leads us to conduct the next experiment by using network throughput as the metric.

Here we evaluate aggregate network throughput (i.e., $N \times T$). We show the case of 70% traffic load. In addition to equating $\lambda_1 = \lambda_3 = \lambda_5$, we also set λ_1:λ_3:λ_5 as 3:2:1 to reflect the case of more shorter packets, and as 1:2:3 to reflect the case of more longer packets. The results are shown in Fig. 4(a). The aggregate throughput saturates at a certain point as the number of piconets increases, and then drops sharply. Different from the earlier observation, the figure reveals that longer packets are more preferable in terms of throughput because the collision problem can be compensated by the benefit of bandwidth efficiency.

Finally, Fig. 4(b) plots the throughput against traffic loads by fixing the value of N. It indicates that throughput goes up steadily as traffic load increases when $N \leq 32$. However, for larger N's, throughputs saturate at certain points, due to more serious collisions. The results suggest that at most 42 piconets can be placed in a physical area.

References

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