CHINESE JOURNAL OF PHYSICS VOL. 41 , NO. 4 AUGUST 2003

Revisited Reparameterization Invariance of HQET

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(Received June 7, 2002)

The heavy quark effective theory is invariant under reparameterization. The specific form of the reparameterization transformation is not unique; and it is closely related to the effective theory. The theory invariant under Luke and Manohar's reparameterization transformation is derived.

PACS numbers: 11.15.Pg

I. INTRODUCTION

The heavy quark effective theory (HQET) [1] is a useful tool in studies of heavy quark systems. In the infinite limit of the heavy quark mass, $M_Q \to \infty$, there exists a QCD heavy flavor-spin symmetry. The relevant degrees of freedom of the HQET are those fields with scales much lower than M_Q. The contributions from the fields with scales greater than M_Q appear to be mass correction terms. Fields with scales higher than two times M_Q can be integrated out by employing the equation of motion (EOM) method [2] or the functional integration (FI) method [3]. The HQET theories derived from these two methods are equivalent. For convenience, we denote these theories as EOM-HQET. An investigation by Das [4] shows that the EOM-HQET Lagrangian contains Hermitian as well as non-Hermitian mass correction terms. For a Hermitian theory, the non-Hermitian terms need to be regularized. In the next section, we shall give a detailed discussion of the physics of the non-Hermitian terms.

The heavy quark momentum can be separated into two parts, the heavy quark velocity part and the residual momentum part. A change of parameterization of the heavy quark velocity and the residual momentum would lead to the same effective theory. This implies that the coefficients of the mass correction terms in the HQET Lagrangian can be fixed by means of reparameterization [5]. There are two versions of the reparameterization transformation: Luke and Manohar's and Chen's [6]. As indicated by Chen, the application of Luke and Manohar's transformation to the EOM-HQET Lagrangian is only valid for correction terms not higher than the second order. On the other hand, the EOM-HQET Lagrangian is invariant under Chen's transformation [6]. However, this does not mean that Luke and Manohar's transformation is incorrect. Because there exist field redefinitions for the effective field, the HQET Lagrangian and the reparameterization transformation are not unique. As shown in [10], there are an infinite number of equivalent theories. As found in [7], Luke and Manohar's transformation is equivalent to Chen's transformation up to a field redefinition. This implies that the invariant Lagrangian under Luke and Manohar's transformation should be different from the Lagrangian invariant under Chen's transformation. The purpose of this paper is to derive the Lagrangian invariant under Luke and Manohar's transformation. The removal of non-Hermitian terms then plays an important role in our approach.

The remainder of this paper is organized as follows. Section II is devoted to figuring out the physical meanings of the non-Hermitian terms. In Section III we derive a Hermitian HQET Lagrangian from QCD. In Section IV, we show that the derived Hermitian HQET Lagrangian is invariant under Luke and Manohar's transformation. The last section contains a discussion and conclusions. The mass correction terms up to $O(1/M_Q^3)$ are enumerated in the Appendix.

II. THE NON-HERMITIAN TERMS

Because the non-Hermitian terms in the EOM-HQET play an important role in our derivation, it is better to have a closer look at their physical meaning. We begin with a simple example: the non-relativistic reduction of the Hamiltonian of an electron interacting with static electromagnetic fields. The equation of motion for an electron under the static Coulomb potential reads

$$i\frac{\partial\psi}{\partial t} = [\vec{\alpha}\cdot\vec{\pi} + \beta m + eV]\psi \ , \tag{1}$$

where V represents the Coulomb potential, m denotes the electron mass, ψ is the electron wave function, $(\vec{\alpha})_i = \gamma^0\gamma^i$ with $i = 1, 2, 3$, $\beta = \gamma^0$ and $\vec{\pi} = -i\vec{\nabla}$. In the non-relativistic limit $E \sim m + \vec{p}^2/2m$, it is convenient to recast ψ into its large and small components, ϕ and χ, in the form

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix} \ . \tag{2}$$

In this way, Eq. (1) can be rewritten as two coupled equations

$$\begin{aligned} i\frac{\partial\phi}{\partial t} &= \vec{\sigma}\cdot\vec{\pi}\chi + eV\phi + m\phi \ , \\ i\frac{\partial\chi}{\partial t} &= \vec{\sigma}\cdot\vec{\pi}\phi + eV\chi - m\chi \ , \end{aligned} \tag{3}$$

where we have employed the Dirac matrix representation

$$\beta = (\begin{matrix} I & 0 \\ 0 & -I \end{matrix}) \ , \ \alpha = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \ . \tag{4}$$

As the electron propagates over time, the contribution from the potential term is smeared out in the weak field limit $m \gg V$. To avoid this, one can transform the components ϕ and χ into other ones, Φ and X, which are slowly varying functions of time. These two new components are related to the old ones through the following relations

$$\Phi = e^{imt}\phi \ , \ X = e^{imt}\chi \ . \tag{5}$$

The equations of motion for Φ and X are as follows:

$$\begin{aligned} i\frac{\partial\Phi}{\partial t} &= \vec{\sigma}\cdot\vec{\pi}X + eV\Phi \ , \\ i\frac{\partial X}{\partial t} &= \vec{\sigma}\cdot\vec{\pi}\Phi + eVX - 2mX. \end{aligned} \tag{6}$$

Since $eV \ll 2m$, we can expand X in terms of $1/m$:

$$\begin{aligned} X &= \frac{1}{2m+\pi^0}\vec{\sigma}\cdot\vec{\pi}\Phi \\ &\approx [\frac{\vec{\sigma}\cdot\vec{\pi}}{2m} - \frac{\pi^0\vec{\sigma}\cdot\vec{\pi}}{4m^2} + \cdots]\Phi \ , \end{aligned} \tag{7}$$

and substitute the expanded X into the first equation of (6) to obtain

$$i\frac{\partial\Phi}{\partial t} = eV\Phi + \frac{(\vec{\sigma}\cdot\vec{\pi})^2}{2m}\Phi - \frac{e}{4m^2}\{[V(\vec{\sigma}\cdot\vec{\pi})^2 + \vec{\sigma}\cdot\vec{\pi}V]\vec{\sigma}\cdot\vec{\pi}\}\Phi \ , \tag{8}$$

where

$$[\vec{\sigma}\cdot\vec{\pi}V]\vec{\sigma}\cdot\vec{\pi} = \vec{\pi}V\cdot\vec{\pi} + i\vec{\sigma}\cdot(\vec{\pi}V\times\vec{\pi}) \ . \tag{9}$$

Rewriting (8) as the Hamiltonian gives

$$H = \Phi^\dagger eV\Phi + \Phi^\dagger\frac{(\vec{\sigma}\cdot\vec{\pi})^2}{2m}\Phi - \Phi^\dagger\frac{e}{4m^2}\{V(\vec{\sigma}\cdot\vec{\pi})^2 + [\vec{\sigma}\cdot\vec{\pi}V]\vec{\sigma}\cdot\vec{\pi}\}\Phi \ . \tag{10}$$

Note that the Darwin term (the last term) in the above Hamiltonian,

$$O_D = \frac{e}{4m^2}\Phi^\dagger\vec{\pi}V\cdot\vec{\pi}\Phi\,, \tag{11}$$

is **non-Hermitian**:

$$O_D^\dagger = \frac{e}{4m^2}(\vec{\pi}\Phi^\dagger\cdot\vec{\pi}V)\Phi \neq O_D \ . \tag{12}$$

One can add up $O_D^\dagger$ and O_D, and then divide their sum by 2 to obtain the average. By performing an integration by parts for the average, with the surface terms being neglected, we can derive a Hermitian Darwin term [8]:

$$O_D^R = \frac{e}{8m^2}\Phi^\dagger[(\vec{\pi})^2V]\Phi \ . \tag{13}$$

One should note that the above regularization method is only valid for leading order terms. This is because the regularization for the leading order terms also affects the higher order terms. Alternatively, one can make use of a renormalized Φ, with the expression [8]

$$\Phi_{NR} = (1 + \frac{(\vec{\pi})^2}{8m^2} + \cdots)\Phi\,, \tag{14}$$

to obtain the regularized Darwin term O_D^R. The regularized Darwin term O_D^R is, in fact, the second spatial variations of V due to the jittery motion of the electron with the Compton wavelength $\delta\vec{r} \sim 1/m$. To see this connection, we may expand the potential $V(\vec{r}+\delta\vec{r})$ with respect to $V(r)$:

$$< V(\vec{r}+\delta\vec{r}) > \approx < V(r) > + \frac{1}{6m^2} < (\vec{\pi})^2 V > , \tag{15}$$

where the operator with a bracket means that the integration of the operator is convoluted with the electron wave functions. The integration of the first order of variation of the operator vanishes, since the electron wave functions are assumed to be spherically symmetric.

The above example exhibits the physics of the non-Hermitian terms. The non-Hermitian terms in the EOM-HQET Lagrangian are similar. The equation of motion for the heavy quark field ψ is

$$(i\not{D} - M_Q)\psi = 0 , \tag{16}$$

where M_Q denotes the heavy quark mass and $i\not{D}$ is the covariant derivative $i\not{D} = i\not{\partial} - g\not{A}^a T^a$. At energies much lower than M_Q, ψ is no longer a good variable for describing the relevant physics. One needs to employ a field redefinition $Q(x) = \exp(iM_Q v \cdot x)\psi(x)$ to remove the large phase factor $M_Q v$ from the wave function. The variable v represents the heavy quark velocity. Rewriting (16) in terms of $Q(x)$ yields

$$(i\not{D} - 2M_Q\frac{(1-\not{v})}{2})Q = 0 . \tag{17}$$

By imposing the condition $v^2 = 1$, one can separate Q into its large and small components, h and H:

$$Q = \frac{1+\not{v}}{2}Q + \frac{1-\not{v}}{2}Q \equiv h + H . \tag{18}$$

Substituting (18) into (17) and multiplying $(1-\not{v})/2$ from the left of (17) yields

$$H = \frac{1}{2M_Q + iD_\parallel}(i\not{D}_\perp)h , \tag{19}$$

with $D_\parallel = v \cdot D$ and $\not{D}_\perp = \not{D} - \not{v}D_\parallel$. Using (18) and (19) leads to

$$Q = [1 + \frac{1}{2M_Q + iD_\parallel} i\not{D}_\perp]h . \tag{20}$$

Substituting (20) into (17) and expanding it up to $O(1/M_Q^2)$ one then arrives at

$$\begin{aligned} iD_\parallel h \; = \; & [-\frac{1}{2M_Q}[-D_\perp^2 + \frac{1}{2}\sigma \cdot G] - \frac{1}{4M_Q^2}[i\sigma_{\alpha\beta}v_\lambda G^{\alpha\lambda}D_\perp^\beta \\ & + iD_\parallel \sigma_{\alpha\beta}G^{\alpha\beta} - iD_\parallel D_\perp^2 + v_\alpha G^{\alpha\beta}D_\beta^\perp] + O(\frac{1}{M_Q^3})]h , \end{aligned} \tag{21}$$

where $\gamma^\mu\gamma^\nu = g^{\mu\nu} + i\sigma^{\mu\nu}$ and $[iD^\mu, iD^\nu] = -iG^{\mu\nu}$ have been used. Note that the Darwin term (the last term in the second line of (21)) is non-Hermitian. Following a procedure similar to that employed in the previous QED calculation, we can regularize the Darwin term by using the renormalized large components h':

$$h' = (1 + \frac{1}{8M_Q^2} i\not{D}_\perp^2 + \dots)h \ . \tag{22}$$

The equation of motion for h' takes the form

$$\begin{aligned} iD_\parallel h' &= [-\frac{1}{2M_Q}[-D_\perp^2 + \frac{1}{2}\sigma \cdot G] \\ &\quad + \frac{1}{8M_Q^2}[i\sigma_{\alpha\beta}v_\lambda\{D_\perp^\alpha, G^{\beta\lambda}\} + v_\alpha[D_\beta^\perp, G^{\alpha\beta}]] + O(\frac{1}{M_Q^3})]h' \ . \end{aligned} \tag{23}$$

The Darwin term in the second line of (23) corresponds to the relativistic effects of Zitterbewegung from the jittery motion of the heavy quarks with Compton wavelength $\lambda_Q \approx 1/M_Q$. This implies that the large component h still contains frequency modes with scales being larger than M_Q. These frequency modes should be integrated out for the low energy effective theory. In summary, we see that the mass corrections receive two kinds of contributions: the first kind comes from frequency modes with scales higher than $2M_Q$, while the second kind is from the frequency modes with scales between M_Q and $2M_Q$. Only both kinds of contribution together can result in a Hermitian theory. The result of integrating out the frequency modes with scales higher than M_Q leads to the renormalized heavy quark field (22). This means that the renormalized field h' contains only the frequency modes with scales less than M_Q and is responsible for the low energy physics. A systematic method, which can derive a Hermitian Lagrangian as well as the relevant effective field, is very useful in theory and phenomenology. To develop this method is the main purpose of this paper.

To reveal the eligibility of the unrenormalized large components h, we discuss two examples in the following. The first example we encounter is the spin sum of the large component h in the free theory, in which the heavy quark is a free particle. From the definition (20), the spin sum has the expression

$$\sum_\lambda h(\lambda)\overline{h}(\lambda) = \frac{1+\not{v}}{2}\sum_\lambda Q(\lambda)\overline{Q}(\lambda)\frac{1+\not{v}}{2} \ , \tag{24}$$

where λ denotes the spin indices of the summed spinors. The spin sum over Q is equal to

$$\sum_\lambda Q(\lambda)\overline{Q}(\lambda) = \frac{1+\not{v}}{2} + \frac{\not{k}}{2M_Q} \ , \tag{25}$$

where k means the residual momentum whose magnitude is much smaller than M_Q. Substituting (25) into (24) yields

$$\sum_\lambda h(\lambda)\overline{h}(\lambda) = \frac{1+\not{v}}{2}(1 - \frac{\not{k}^2}{4M_Q^2}) \, . \tag{26}$$

It is noted that the spin sum of the effective HQET spinor h_v is equal to

$$\sum_\lambda h_v(\lambda)\overline{h}_v(\lambda) = \frac{1+\not{v}}{2} \ . \tag{27}$$

This example shows that the propagator of h differs from that of h_v,

$$S_{h_v} = \frac{i}{v\cdot k + i\epsilon}\frac{1+\not{v}}{2} \ , \tag{28}$$

by a factor $(1-\frac{\not{k}^2}{4M_Q^2})$, which is just twice the inverse of the renormalization factor defined in (22). For the second example, we would like to take the matrix representation of the heavy quark spinor. Let u_Q denote a free full heavy quark spinor with energy E_Q, mass M_Q, and spatial momentum $\vec{k}$. u_Q can be expressed in terms of its rest frame spinor as

$$u_Q = \begin{pmatrix} \sqrt{\frac{E_Q+M_Q}{2M_Q}} & \phi^{(\alpha)} \\ \frac{\vec{\sigma}\cdot\vec{k}}{\sqrt{2M_Q(M_Q+E_Q)}} & \phi^{(\alpha)} \end{pmatrix} \ , \tag{29}$$

where

$$\phi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ , \ \phi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ , \tag{30}$$

denote the rest frame spinors. In the static approximation, we can expand E_Q to

$$E_Q = \sqrt{M_Q^2 + \vec{k}^2} \approx M_Q - \frac{\not{k}_\perp^2}{2M_Q} \ , \tag{31}$$

where $\vec{k}^2 = -\not{k}_\perp^2$ and $k_\perp = (0,\vec{k})$ have been used. Under this approximation, the full spinor u_Q becomes

$$u_Q = \sqrt{1-\frac{\not{k}_\perp^2}{4M_Q^2}} \begin{pmatrix} \phi^{(\alpha)} \\ \frac{\not{k}_\perp}{2M_Q-\not{k}_\perp}\phi^{(\alpha)} \end{pmatrix} \ . \tag{32}$$

From (32), we can identify the large components h and small components H of u_Q

$$\begin{aligned} h &= \sqrt{1-\frac{\not{k}_\perp^2}{4M_Q^2}}\phi \ , \\ H &= \frac{\not{k}_\perp}{2M_Q-\not{k}_\perp} h \ . \end{aligned} \tag{33}$$

Spinors ϕ and u_Q are well normalized, $\phi^\dagger\phi = \overline{u}_Q u_Q = 1$, while the large components h have an incorrect normalization $\overline{h}h = 1-(\not{k}_\perp/2M_Q)^2$ as pointed out in [9].

Finally, we emphasize that, from the equations of motion for Q (17), we can directly derive the relation between h and H as

$$H = [\frac{1}{2M_Q - i\not{D}} i\not{D}]h \,, \tag{34}$$

and the on-shell condition for Q is

$$[iD_{\parallel} + \frac{(i\not{D})^2}{2M_Q}]Q = 0 \,. \tag{35}$$

III. CONSTRUCTION OF AN EFFECTIVE THEORY

III-1. Derivation of the Effective Field

One can match QCD to HQET at the scale of the heavy quark mass, M_Q, by requiring that the 1PI Green's functions of the two theories describe the same physics. The simplest way to achieve this is to set the external quarks to be on shell [7]. In momentum space, the LSZ reduction formula for a heavy quark fermion is expressed as

$$\begin{aligned} S(P_Q, \dots) &= \frac{-i}{\sqrt{Z_Q}} \overline{u_Q}(P_Q/M_Q) \frac{(\not{P}_Q - M_Q)}{2M_Q} \cdots \int dx e^{iP_Q \cdot x} < 0|T[\psi(x)\dots]|0 > |_{P_Q^2 = M_Q^2} \\ &= \frac{-i}{\sqrt{Z_Q}} \overline{Q}(v + k/M_Q)(\frac{\not{k}}{2M_Q} - \Lambda_v^-) \dots \\ &\quad \int dx e^{ik\cdot x} < 0|T[Q(x)\dots]|0 > |_{v\cdot k = -k^2/2M_Q} \,, \end{aligned} \tag{36}$$

where $Q(x) = \exp{(iM_Q v \cdot x)}\psi(x)$ and $\psi(x)$ denotes the heavy quark field. In this paper, we would like to develop the projection operator method to derive the matching between the effective spinors Q and h_v. The field Q denotes that the heavy quark carries the momentum P_Q, with expression $P_Q = M_Q v + k$, while the field h_v represents the heavy quark carrying momentum k with respect to a constant moving frame with velocity v. Both spinors are equivalent variables for low energy physics. However, in the limit $M_Q \to \infty$, or at an energy scale much lower than M_Q, h_v is a more appropriate variable.

Using the projection operator approach, we specify the state of Q by means of a positive energy projection operator

$$\Lambda^+ = \frac{(1+\not{v})}{2} + \frac{\not{k}}{2M_Q} \tag{37}$$

which is defined to select the spinor Q which has the momentum just equal to $M_Q v + k$,

$$\Lambda^+ Q = Q \,. \tag{38}$$

Equation (38) is equivalent to the equation of motion $[i\not{D} - M_Q(1-\not{v})]Q(x) = 0$. Being a projection operator, Λ^+ obeys the identity

$$(\Lambda^+)^2 = \Lambda^+ \,, \tag{39}$$

which implies the on-shell condition:

$$[k_\parallel + \frac{\not{k}^2}{2M_Q}]Q = 0\,, \tag{40}$$

with $k_\parallel = v\cdot k$. The inverse operator of Λ^+ is the negative energy projection operator

$$\Lambda^- = \frac{(1-\not{v})}{2} - \frac{\not{k}}{2M_Q}\,, \tag{41}$$

defined by the identity

$$\Lambda^+ + \Lambda^- = 1\ . \tag{42}$$

In order to derive h_v, which respects the physics in the limit $M_Q \to \infty$, we define the relevant projectors for h_v

$$\Lambda_v^\pm = \frac{1\pm\not{v}}{2} \equiv \lim_{M_Q\to\infty} \Lambda^\pm\ . \tag{43}$$

Note that operators Λ_v^+ (Λ_v^-) are the infinite mass limit of the energy projection operators $\Lambda^+(\Lambda^-)$. $\Lambda_v^\pm$ satisfy the identities

$$(\Lambda_v^\pm)^2 = \Lambda_v^\pm\ . \tag{44}$$

Via $1 = \Lambda_v^+ + \Lambda_v^-$, we recast Q to be

$$Q = \Lambda_v^+ Q + \Lambda_v^- Q = h + H\ . \tag{45}$$

From (38) and (45) we arrive at

$$H = [\frac{1}{2M_Q - \not{k}}\not{k}]h\ , \tag{46}$$

and

$$Q = [\frac{1}{1-\frac{\not{k}}{2M_Q}}]h\ . \tag{47}$$

In the literature, people always stop at this point to identify h as h_v. As pointed out in the last Section, h is not identical to h_v. It is natural to assume that h_v is the infinite mass limit $M_Q \to \infty$ of h and the two spinors are proportional to each other. The first assumption comes from the definition for the effective spinor, $h_v \equiv \lim_{M_Q\to\infty} Q$, and the second one is based on the fact that both h and h_v are projected out by Λ_v^+. In this way, we argue that $h = [1+\omega]h_v$, with the ansatz $\overline{\omega} = \omega$ and $\not{v}\omega = \omega$. To derive ω, we note the identities

$$\Lambda^+ \ = \ \sum Q\overline{Q}\ , \tag{48}$$

$$\Lambda_v^+ \ = \ \sum h_v \overline{h}_v\ , \tag{49}$$

where summations over the spin indices of the spinors are implied. Equations (48) and (49) hold, if and only if, $\overline{Q}Q = \overline{h}_v h_v = 1$. Equation (49) is due to the definition: the limit $M_Q \to \infty$ of (48). Substituting (47) and $h = [1+\omega]h_v$ into (48) and using (49), we obtain the equation for ω:

$$\omega^2 + 2\omega + \frac{1+\not{v}}{2} = (1 - \frac{\not{k}}{2M_Q})(\frac{1+\not{v}}{2} + \frac{\not{k}}{2M_Q})(1 - \frac{\not{k}}{2M_Q}) \ . \tag{50}$$

With the help of the on-shell condition (40), (50) is recast as

$$(\omega)^2 + 2\omega + (\frac{\not{k}}{2M_Q})^2(\frac{1+\not{v}}{2}) = 0 \ . \tag{51}$$

The above equation is easily solved leading to the solution

$$\omega = -1 + \sqrt{1+T} \, , \tag{52}$$

where $T = -(\frac{\not{k}}{2M_Q})^2(\frac{1+\not{v}}{2})$. We then obtain

$$h = \sqrt{1 - (\frac{\not{k}}{2M_Q})^2(\frac{1+\not{v}}{2})} h_v \ . \tag{53}$$

The relation between h and h_v is consistent with that relation between h and h' found in the last Section.

Combining (47) and (53) leads to

$$Q = \sqrt{\frac{1+\not{k}/(2M_Q)}{1-\not{k}/(2M_Q)}} \Lambda_v h_v \equiv \Lambda(w = v + k/M_Q, v) h_v \ . \tag{54}$$

Note that (54) is just the Lorentz transformation between two spinors with relative velocity k/M_Q. The transformation operator $\Lambda(w = v + k/M_Q, v)$ is identical to the Lorentz boost in the spinor representation [5]:

$$\tilde{\Lambda}(w, v) = \frac{1 + \not{w}\not{v}}{\sqrt{2(1 + v \cdot w)}} \ . \tag{55}$$

In the presence of interactions, the Lorentz boost interpretation for (54) is no longer correct. The reverse transformation from h_v into Q can be derived in a similar way. The result is

$$h_v = \sqrt{\frac{1-\not{k}/(2M_Q)}{1+\not{k}/(2M_Q)}} \Lambda^+ Q \ . \tag{56}$$

Transforming (54) into coordinate space by the replacements $\not{k} \to i\not{D}$ results in the relation

$$Q(x) = \sqrt{\frac{1+i\not{D}/(2M_Q)}{1-i\not{D}/(2M_Q)}} \Lambda_v^+ h_v(x). \tag{57}$$

The field $Q(x)$ is consistent with the field derived by Luke and Manohar [5]:

$$\Psi_v(x) = \tilde{\Lambda}(v + iD/M_Q, v) h_v(x) \ . \tag{58}$$

By employing (54) and (57), we arrive at the matching between Q and h_v at the scale equal to M_Q,

$$S(k, \dots) = \frac{-i}{\sqrt{Z_Q}} \overline{h_v}(v) (\frac{\not{k}}{2M_Q} - \frac{1-\not{v}}{2}) \dots \int dx e^{ik\cdot x} \langle 0|T[h_v(x)\dots]|0\rangle|_{v\cdot k = -\frac{k^2}{2M_Q}} \ , \tag{59}$$

which is different from (41) in [10] by a factor of $\sqrt{\tilde{Z}(k)} = \sqrt{(1-\not{k}^2/(4M_Q^2)}$. This is because the effective field h_v^{KO} employed in [10] is the unrenormalized large components $h_v^{KO} = h = \sqrt{\tilde{Z}(k)} h_v$.

By matching QCD and HQET at 2PI and quark-gluon-quark interaction Green functions, we can derive the HQET Lagrangian:

$$\begin{aligned} L &= \overline{\psi}(i\not{D} - M_Q)\psi \\ &= \overline{Q}(i\not{D} - 2M_Q\Lambda_v^-)Q \\ &= \overline{h_v}\Lambda_v^+ \sqrt{\frac{1+i\not{D}/(2M_Q)}{1-i\not{D}/(2M_Q)}} (i\not{D} - 2M_Q\Lambda_v^-) \sqrt{\frac{1+i\not{D}/(2M_Q)}{1-i\not{D}/(2M_Q)}} \Lambda_v^+ h_v \ . \end{aligned} \tag{60}$$

Note that the HQET Lagrangian is Hermitian. This fact can be verified by explicitly performing the operations of Hermitian conjugation and integration by parts. We only show the particle part of the Lagrangian. The antiparticle part of the Lagrangian can be derived in a similar way.

IV. THE VELOCITY REPARAMETERIZATION TRANSFORMATION

IV-1. Field Transformation

The heavy quark momentum P_Q is independent of which parameterization is employed, $P_Q = M_Q v + k$ or $P_Q = M_Q v' + k'$, where v, v', k and k' denote different variables, $v \neq v'$ and $k \neq k'$. It was found that the HQET Lagrangian should be invariant under the reparameterization $v \to v'$ and $k \to k'$. We show the following theorem: If v and v' are related to each other as $v' = v + \delta v$, with $(v')^2 = v^2 = 1$ and $v \cdot \delta v + (\delta v)^2/2 = 0$, then $h_{v'}$ is related to h_v as

$$h_{v'} = \sqrt{\frac{1+\delta\not{v}/2}{1-\delta\not{v}/2}} \Lambda_v^+ h_v \tag{61}$$

and

$$h_{v'}(x) = e^{iM_Q \delta v\cdot x} \sqrt{\frac{1+\delta\not{v}/2}{1-\delta\not{v}/2}} \Lambda_v^+ h_v(x) \ . \tag{62}$$

Furthermore, if $M_Q\delta v = k - k'$, $Q(P_Q = M_Q v' + k')$ equals $Q(P_Q = M_Q v + k)$, then

$$Q(P_Q = M_Q v' + k')(x) = e^{iM_Q\delta v\cdot x} Q(P_Q = M_Q v + k)(x) \ . \tag{63}$$

The proof of this theorem is straightforward. Since $v' = v+\delta v$ and $(v')^2 = v^2 = 1$, velocities v' and v have corresponding energy projectors $(1+\not{v}')/2$ and $(1+\not{v})/2$, which will project effective fields $h_{v'}$ and h_v, respectively. By replacing δv with k/M_Q in the transformation (54), we thus derive the transformation from $h_{v'}$ to h_v. The proof of the equality between $Q(P_Q = M_Q v' + k')$ and $Q(P_Q = M_Q v + k)$ is also trivial by noting that

$$\begin{aligned} h_{v'} &= \sqrt{\frac{1+\delta\not{v}/2}{1-\delta\not{v}/2}}\Lambda_v^+ h_v \ , & (64)\\ Q(P_Q = M_Q v + k) &= \sqrt{\frac{1+\not{k}/(2M_Q)}{1-\not{k}/(2M_Q)}}\Lambda_v^+ h_v \ , & (65)\\ Q(P_Q = M_Q v' + k') &= \sqrt{\frac{1+\not{k}'/(2M_Q)}{1-\not{k}'/(2M_Q)}}\Lambda_{v'}^+ h_{v'} \ , & (66) \end{aligned}$$

and $M_Q\delta v = k - k'$. It leads to the following identity

$$\begin{aligned} Q(P_Q = M_Q v' + k') &= \sqrt{\frac{1+\not{k}'/(2M_Q)}{1-\not{k}'/(2M_Q)}}\Lambda_{v'}^+ h_{v'} \\ &= \sqrt{\frac{1+\not{k}'/(2M_Q)}{1-\not{k}'/(2M_Q)}}\sqrt{\frac{1+\delta\not{v}/2}{1-\delta\not{v}/2}}\Lambda_v^+ h_v \\ &= \sqrt{\frac{1+\not{k}/(2M_Q)}{1-\not{k}/(2M_Q)}}\Lambda_v^+ h_v \\ &= Q(P_Q = M_Q v + k) \ . \end{aligned} \tag{67}$$

The proof is completed.

The most important property of this transformation is the association of successive transformations. If we denote the transformation from h_v into $h_{v'=v+\delta v}$ by $h_{v'} = L(v,v')h_v$, then we have $L(v,v'') = L(v',v'')L(v,v')$. We show this explicitly below. The successive transformations $v \to v' = v + \delta v_1$ followed by $v' \to v'' = v' + \delta v_2 = v + \delta v_1 + \delta v_2$, would have the effective field transformations

$$h_v \to h_{v'} = \sqrt{\frac{1+\delta\not{v}_1/2}{1-\delta\not{v}_1/2}}\frac{1+\not{v}}{2}h_v \tag{68}$$

and

$$\begin{aligned} h_{v'} \to h_{v''} &= \sqrt{\frac{1+\delta\not{v}_2/2}{1-\delta\not{v}_2/2}}\frac{1+\not{v}'}{2}h_{v'} & (69)\\ &= \sqrt{\frac{1+(\delta\not{v}_1+\delta\not{v}_2)/2}{1-(\delta\not{v}_1+\delta\not{v}_2)/2}}\frac{1+\not{v}}{2}h_v \ . & (70) \end{aligned}$$

IV-2. Reparameterization Invariance

We now show that the reparameterization invariance is trivial and manifest for the Lagrangian (60). Using the previous theorem, it is also straightforward to prove that the effective Lagrangian in terms of Q is invariant under the transformations $v \to v' = v + \delta v$, $M_Q \delta v = k - k'$, and $Q(M_Q v' + k')(x) = \exp(iM_Q \delta v \cdot x) Q(M_Q v + k)(x)$:

$$\begin{aligned} L &= \overline{Q}(P_Q = M_Q v + k)(i\not{D} - 2M_Q \Lambda_v^-) Q(P_Q = M_Q v + k) \\ &= \overline{Q}(P_Q = M_Q v' + k')(i\not{D} - 2M_Q \Lambda_{v'}^-) Q(P_Q = M_Q v' + k') \ . \end{aligned} \tag{71}$$

It is also trivial to prove the invariance of the effective Lagrangian in terms of h_v under the same transformation:

$$\begin{aligned} L &= \overline{Q}(P_Q = M_Q v' + k')(i\not{D} - 2M_Q \Lambda_{v'}^-) Q(P_Q = M_Q v' + k') \\ &= \overline{h_{v'}} \Lambda_{v'}^+ \sqrt{\frac{1 + i\not{D}/(2M_Q)}{1 - i\not{D}/(2M_Q)}} (i\not{D} - 2M_Q \Lambda_{v'}^-) \sqrt{\frac{1 + i\not{D}/(2M_Q)}{1 - i\not{D}/(2M_Q)}} \Lambda_{v'}^+ h_{v'} \\ &= \overline{h_v} \Lambda_v^+ \sqrt{\frac{1 + \delta\not{v}/2}{1 - \delta\not{v}/2}} e^{-iM_Q \delta v \cdot x} \sqrt{\frac{1 + i\not{D}/(2M_Q)}{1 - i\not{D}/(2M_Q)}} (i\not{D} - 2M_Q \Lambda_{v'}^-) \\ &\quad \sqrt{\frac{1 + i\not{D}/(2M_Q)}{1 - i\not{D}/(2M_Q)}} e^{iM_Q \delta v \cdot x} \sqrt{\frac{1 + \delta\not{v}/2}{1 - \delta\not{v}/2}} \Lambda_v^+ h_v \\ &= \overline{h_v} \Lambda_v^+ \sqrt{\frac{1 + i\not{D}/(2M_Q)}{1 - i\not{D}/(2M_Q)}} (i\not{D} - 2M_Q \Lambda_v^-) \sqrt{\frac{1 + i\not{D}/(2M_Q)}{1 - i\not{D}/(2M_Q)}} \Lambda_v^+ h_v \\ &= \overline{Q}(P_Q = M_Q v + k)(i\not{D} - 2M_Q \Lambda_v^-) Q(P_Q = M_Q v + k) \, , \end{aligned} \tag{72}$$

where we has used

$$e^{-iM_Q \delta v \cdot x} \sqrt{\frac{1 + i\not{D}/(2M_Q)}{1 - i\not{D}/(2M_Q)}} e^{iM_Q \delta v \cdot x} \sqrt{\frac{1 + \delta\not{v}/2}{1 - \delta\not{v}/2}} = \sqrt{\frac{1 + i\not{D}/(2M_Q)}{1 - i\not{D}/(2M_Q)}} \ . \tag{73}$$

V. DISCUSSIONS AND CONCLUSIONS

For comparison, we discuss different versions of the reparameterization transformation for EOM-HQET theories. The reparameterization transformation proposed by Luke and Manohar [5] for a spinor h_v is defined as follows

$$h_v(x) \to h^{\mathrm{LM}}_{v'=v+\delta v}(x) = e^{iM_Q \delta v \cdot x} \Lambda_{LM}(v', \hat{u}) \Lambda_{LM}(v, \hat{u})^{-1} h_v(x) \ . \tag{74}$$

The transformation operator $\Lambda_{LM}(v', \hat{u})$ has the form

$$\Lambda_{LM}(v', \hat{u}) = \frac{1 + \not{v}' \hat{\not{u}}}{\sqrt{2(1 + v' \cdot \hat{u})}} \, , \tag{75}$$

with

$$\hat{u} = \frac{v^\mu + \frac{iD^\mu}{M_Q}}{\sqrt{1 + \frac{2iv\cdot D}{M_Q} - \frac{D^2}{M_Q^2}}} \ . \tag{76}$$

Note that v, v' and $\hat{u}$ are unit vectors. They all satisfy $v^2 = (v')^2 = \hat{u}^2 = 1$. It should be noted that Luke and Manohar's transformation (74) is equivalent to the transformation (62). This implies that our reparameterization transformation, shown in last section, is identical to Luke and Manohar's transformation. Chen's version of the reparameterization transformation is defined as

$$h^{\rm Ch}_{v'=v+\delta v}(x) = e^{iM_Q \delta v\cdot x} \Lambda_{Ch}(v', v) h_v(x) \, , \tag{77}$$

where the operator $\Lambda_{Ch}(v', v)$ is

$$\Lambda_{Ch}(v', v) = \frac{1 + \not{v} + \delta\not{v}}{2}[1 + \frac{1}{2M_Q + iv\cdot D} i\not{D}_\perp] \ . \tag{78}$$

The above transformations are proposed for the Lagrangian

$$L = \overline{h_v}[iv\cdot D + i\not{D}_\perp \frac{1}{2M_Q + iv\cdot D} i\not{D}_\perp] h_v \ . \tag{79}$$

As shown in [7], the differences between Luke and Manohar's transformation and Chen's transformation is at least of order $O(1/M_Q^2)$. However, as we showed in the previous sections, the Lagrangian which is invariant under Luke and Manohar's transformation should be the Lagrangian defined in (60).

In summary, we have regularized the non-Hermitian terms in EOM-HQET [1–3] to all orders in $O(1/M_Q)$. We have shown that the large components of the heavy quark field should be renormalized with respect to the low energy physics. In terms of the renormalized large components, the Lagrangian (60) is Hermitian and invariant under Luke and Manohar's transformation. We have only considered the tree level cases. It is interesting to see whether the same method can be applied to the higher order in α_s cases.

Acknowledgments

This research was partly supported in part by the National Science Council of the R.O.C. under the Grant No. NSC89-2811-M-009-0024.

APPENDIX A: MASS EXPANSION LAGRANGIAN

We discuss the mass expansion of the HQET Lagrangian defined in (60). The HQET Lagrangian L is expanded into mass correction terms in the form

$$L = \sum_{n=0}^{\infty} \frac{L^{(n)}}{(2M_Q)^n} \, , \tag{A1}$$

where the first leading terms $L^{(n)}, n = 0, 1, 2, 3$ are enumerated as follows:

$$L^{(0)} = \overline{h}_v iD_{\|} h_v \ , \tag{A2}$$

$$L^{(1)} = \overline{h}_v \left[-D_{\|}^2 - D^2 + \frac{1}{2}\sigma_{\alpha\beta}G^{\alpha\beta} \right] h_v \ , \tag{A3}$$

$$L^{(2)} = \overline{h}_v \left[-2iD_{\|}^3 + \frac{1}{2}(v_\alpha[D_\beta, G^{\alpha\beta}] + i\sigma_{\alpha\beta}v_\lambda\{D^\beta, G^{\lambda\alpha}\}) \right] h_v \ , \tag{A4}$$

$$\begin{aligned} L^{(3)} = \overline{h}_v \Big[& D^2(D^2 + D_{\|}^2) + \frac{1}{2}G^2 + \frac{1}{2}\sigma \cdot G D_{\|}^2 - \{D^2, \sigma \cdot G\} \\ & +\sigma_{\alpha\beta} \left(D_\lambda\{D^\beta, G^{\lambda\alpha}\} + [D^\beta, G^{\lambda\alpha}]D_\lambda - iG^{\lambda\alpha}G_\lambda^{\ \beta} \right) \\ & -\frac{i}{4}\gamma_5\epsilon_{\alpha\beta\lambda\rho}G^{\alpha\beta}G^{\lambda\rho} \Big] h_v \ . \end{aligned} \tag{A5}$$

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