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Analytical functions for the optimization of second-harmonic generation and parametric generation by focused Gaussian beams

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ABSTRACT An analytical function involving four parameters is proposed to express the second-harmonic generation efficiency as well as the parametric generation gain coefficient in the Boyd–Kleinman theory. The analytical function clearly reveals the dependence of conversion efficiency on the focusing parameter and the walk-off parameter. Moreover, the optimum focusing parameter and its corresponding maximum efficiency are explicitly given in the analytical function, leading to a straightforward evaluation of a given crystal performance.

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1 Introduction

The demand for various laser sources covering the spectrum from ultraviolet to infrared is always increasing in many areas. The processes of second-harmonic generation (SHG) and parametric generation (PG) are often employed for optical wavelength conversion [1–4]. Thus, a study of optimization in SHG and PG is important.

In most applications, either the crystal length or the focusing parameter is optimized to maximize the generation efficiency. The optimum value for the crystal length or the focusing parameter for focused Gaussian beams is usually evaluated on the basis of the classical theory of Boyd and Kleinman (BK) [5]. However, it is necessary to perform a two-dimensional (2D) integral plus an optimization procedure on the mismatch parameter in the calculation of the BK theory. In view of the complexity, a simple analytical function for the BK theory is desired and practically useful.

In this work, we propose a simple analytical function involving four parameters to straightforwardly obtain the predictions of the BK theory. The analytical function expresses the dependence of conversion efficiency on the focusing parameter and the walk-off parameter for the processes of SHG and PG. Furthermore, the optimum focusing parameter and its corresponding maximum efficiency are explicitly given in the analytical function; therefore, the evaluation of a given crystal performance becomes extremely straightforward.

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2 Boyd–Kleinman theory

Boyd and Kleinman presented a general theory for the dependence of the SHG power and the PG threshold power on the focusing parameter in a nonlinear uniaxial crystal. Here a brief description is given for completeness. In terms of the function $h\,(\sigma, B, \xi)$, the SHG power is given by

$$\eta = P_2/P_1 = \left[\left(2\omega_1{}^2 d_{\text{eff}}{}^2\right) / \left(\pi n_1^2 n_2 \varepsilon_0 c^3\right)\right] P_1 l k_1 h\,(\sigma, B, \xi)\ , \tag{1}$$

where P_1 and P_2 are the powers of the fundamental beam and the second-harmonic beam, respectively; ω_1 is the fundamental laser frequency; d_{eff} is the effective nonlinear coefficient; l is the crystal length; k_1 is the fundamental wavevector in the crystal, n_1 and n_2 are the refractive indices for the fundamental beam and the second-harmonic beam, respectively; c is the light velocity in vacuum; and ε_0 is the vacuum permittivity. The function $h\,(\sigma, B, \xi)$ is given by

$$h(\sigma, B, \xi) = \frac{1}{4\xi} \int_{-\xi}^{\xi} \int_{-\xi}^{\xi} \frac{e^{i\sigma(\tau-\tau')} e^{-B^2(\tau-\tau')^2/\xi}}{(1+i\tau)\,(1-i\tau')} d\tau d\tau', \tag{2}$$

and the parameters are $\xi = l/b$, $B = \left(\varrho\sqrt{lk_1}\right)/2$, and $\sigma = b\Delta k/2$. Here $b = \varpi_0^2 k_1$ is the confocal parameter, ϖ_0 is the beam waist, $\Delta k = 2k_1 - k_2$ is the phase mismatch, and ϱ is the walk-off angle. Since the SHG efficiency is usually optimized with respect to the mismatch parameter σ, it is more practically useful to define the function

$$h_{\text{m}}\,(B, \xi) = \max\left[h\,(\sigma, B, \xi)\right]_\sigma\ . \tag{3}$$

Similar to the expression of the SHG efficiency, a general function $\bar{h}\,(\sigma, B, \xi)$ is defined such that the PG gain coefficient is given by

$$G = \left[\left(2\omega_0^2 d_{\text{eff}}^2\right) / \left(\pi n_0^2 n_3 \varepsilon_0 c^3\right)\right] P_3 l k_0 \left(1-\delta^2\right)^2 \bar{h}\,(\sigma, B, \xi)\ , \tag{4}$$

where ω_0 is the degeneracy frequency, P_3 is the pump power, n_0 is defined as $2n_0 = n_1 + n_2$, k_0 is defined as $k_0 = n_0\omega_0/c$, and $\left(1-\delta^2\right)^2$ is the degeneracy factor defined by $\omega_1 = \omega_0\,(1-\delta)$, $\omega_2 = \omega_0\,(1+\delta)$, and $\omega_1\omega_2 = \omega_0^2\left(1-\delta^2\right)$. The function $\bar{h}\,(\sigma, B, \xi)$ is given by

$$\bar{h}\,(\sigma, B, \xi) = \frac{1}{4\xi}\int_{-\xi}^{\xi}\int_{-\xi}^{\xi}\frac{e^{i\sigma(\tau-\tau')}e^{-B^2\tau^2/\xi}}{(1+i\tau)\,(1-i\tau')}d\tau d\tau'. \quad (5)$$

As defined in (3), the function $\bar{h}\,(\sigma, B, \xi)$ optimized with respect to the mismatch parameter σ is given by

$$\bar{h}_m\,(B, \xi) = \max\left[\bar{h}\,(\sigma, B, \xi)\right]_\sigma . \quad (6)$$

It can be found from (1)–(6) that the calculation of the SHG efficiency or the PG gain coefficient includes a 2D integral plus an optimization procedure on the mismatch parameter σ. Therefore, it is practically useful to develop an analytical representation for the functions $h_m\,(B, \xi)$ and $\bar{h}_m\,(B, \xi)$.

3 Analytical functions for BK theory

The calculated result reveals that the dependence of the function $h_m\,(B, \xi)$ on the focusing parameter ξ is similar to the Lorentzian line-shape function. To increase the fitting accuracy, we propose

$$h_m\,(B, \xi) = \frac{h_{mm}\,(B)\,\gamma\,(B)\,\xi}{|\xi-\xi_m\,(B)|^{n(B)}+\gamma\,(B)\,\xi} \quad (7)$$

for the SHG optimization. Here $\xi_m\,(B)$ is the optimum focusing parameter and $h_{mm}\,(B)$ represents the optimized SHG, i.e. $h_{mm}\,(B) = h_m\,[B, \xi\,(B)]$. Moreover, $\gamma\,(B)$ is the damping coefficient to describe the sensitivity of the function $h_m\,(B, \xi)$ on the focusing parameter ξ. Finally, the parameter $n\,(B)$ is used to increase the fitting accuracy; its value is usually in the range of 1.83–1.91. We have fitted (7) to the numerical results of the BK theory to find $\xi_m\,(B)$, $h_{mm}\,(B)$, $\gamma\,(B)$, and $n\,(B)$ that are given by

$$\xi_m(B) = \frac{2.84+1.39B^2}{1+0.1B+B^2},$$

$$h_{mm}(B) = \frac{1.068}{1-0.7\sqrt{B}+1.62B},$$

$$n(B) = \frac{1.91+1.83B}{1+B},$$

$$\gamma(B) = \frac{[\xi_m(0)]^{n(0)}}{h_{mm}(0)}e^{-B} + 13\left(1-e^{-B/3}\right). \quad (8)$$

Figure 1 shows a comparison of our results with the corresponding numerical data for the function $h_m\,(B, \xi)$ as a function of focusing parameter ξ for several values of walk-off parameter B. Good agreement is found for all cases.

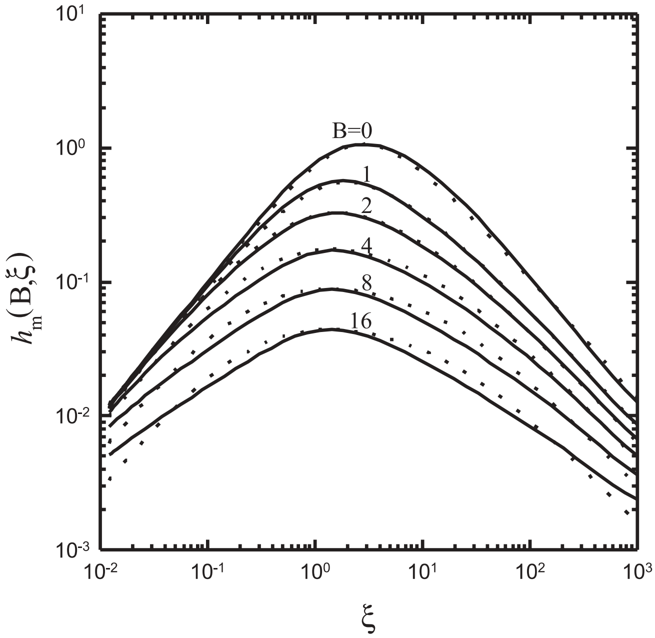

FIGURE 1 Comparison between the numerical calculations with (2) and (3) (*solid lines*) and analytical results with (7) and (8) (*dashed lines*) for the function $h_m\,(B, \xi)$ of the BK theory, for several values of the walk-off parameter B

The same functional form in (7) is also used to represent the function $\bar{h}_m\,(B, \xi)$:

$$\bar{h}_m\,(B, \xi) = \frac{\bar{h}_{mm}\,(B)\,\bar{\gamma}\,(B)\,\xi}{\left|\xi-\bar{\xi}_m\,(B)\right|^{\bar{n}(B)}+\bar{\gamma}\,(B)\,\xi}. \quad (9)$$

With (9) fitting the numerical results, we obtain

$$\bar{\xi}_m(B) = \frac{2.84+0.4B}{1+B},$$

$$\bar{h}_{mm}(B) = \frac{1.068}{1+1.3B^2},$$

$$\bar{n}(B) = \frac{1.91+1.88B^2}{1+0.06B+B^2},$$

$$\bar{\gamma}(B) = \frac{\left[\bar{\xi}_m(0)\right]^{\bar{n}(0)}}{\bar{h}_{mm}(0)}e^{-1.75B} + 3.5B + 0.6B^2. \quad (10)$$

Figure 2 shows a comparison of our results with the corresponding numerical data for the function $\bar{h}_m\,(B, \xi)$ as a function of focusing parameter ξ for several values of walk-off parameter B. Again, the agreement is so close that only minute differences can be seen.

One useful property of the present analytical function is in terms of the optimum focusing parameter and the optimum conversion efficiency. This property makes the present function extremely straightforward in the optimum analysis. As shown in Fig. 3, no obvious difference can be seen between the numerical calculations with (2)–(5) and analytical results with (8) and (10) for the optimum SHG $h_{mm}\,(B)$ and

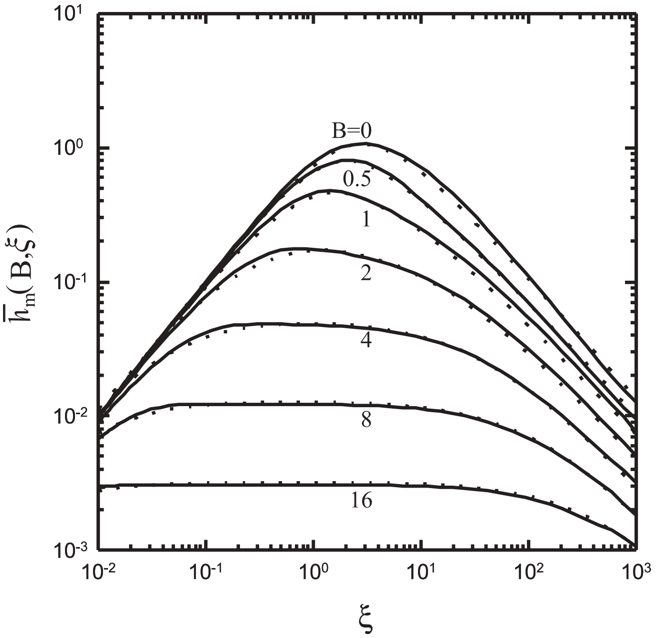

FIGURE 2 Comparison between the numerical calculations with (5) and (6) (*solid lines*) and analytical results with (7) and (8) (*dashed lines*) for the function $\bar{h}_{\mathrm{m}}(B,\xi)$ of the BK theory, for several values of the walk-off parameter B

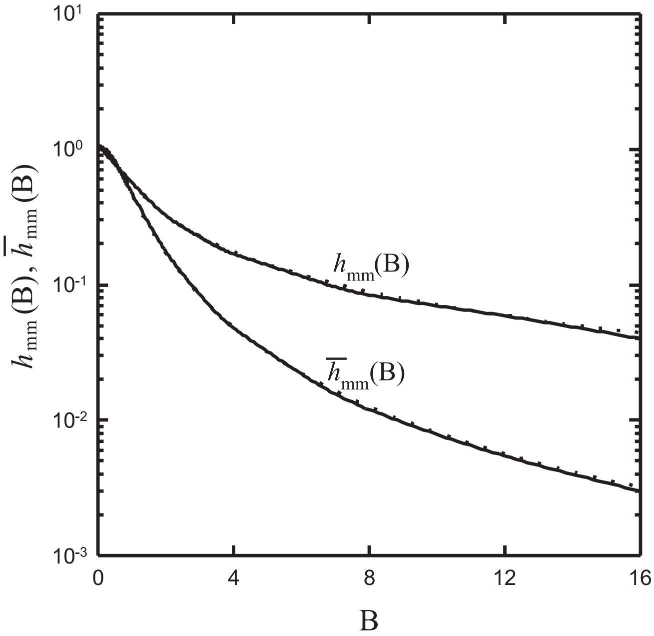

FIGURE 3 Comparison between the numerical calculations with (2)–(6) (*solid lines*) and analytical results with (8) and (10) (*dashed lines*) for the optimum SHG $h_{\mathrm{mm}}(B)$ and the optimum PG $\bar{h}_{\mathrm{mm}}(B)$

the optimum PG $\bar{h}_{\mathrm{mm}}(B)$. Finally, note that, in the absence of walk-off, the analytical functions in (7) and (9) have the asymptotic behaviors

$$h_{\mathrm{m}}(0,\xi) = \bar{h}_{\mathrm{m}}(0,\xi) \rightarrow \xi \qquad (\xi \rightarrow 0)\,. \tag{11}$$

With this asymptotic behavior, the SHG efficiency and the PG gain coefficient are given by a plane-wave expression [6].

4 Conclusion

In conclusion, we have constructed simple analytical expressions for the functions $h_{\mathrm{m}}(B,\xi)$ and $\bar{h}_{\mathrm{m}}(B,\xi)$ of the BK theory. Since the optimum focusing parameters $\xi_{\mathrm{m}}(B)$ and $\bar{\xi}_{\mathrm{m}}(B)$ and their corresponding maximum efficiencies $h_{\mathrm{mm}}(B)$ and $\bar{h}_{\mathrm{mm}}(B)$ are explicitly given, the present analytical functions lead to an extreme simplification of the evaluation of a given crystal performance.

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