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Manufacturing capability control for multiple power-distribution switch processes based on modified C_{pk} MPPAC

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Abstract

A modification of the multiple process performance analysis chart based on process capability index C_{pk}, called modified C_{pk} MPPAC, has been developed for controlling product quality/reliability of a group of multiple manufacturing processes. The modified C_{pk} MPPAC conveys critical information of each individual process regarding process accuracy and process precision from one single chart, which is an effective tool for controlling product quality/reliability for multiple processes. Existing MPPAC charts never considered sampling errors hence the capability information provided from those charts is often unreliable and misleading. In this paper, we develop an efficient algorithm to compute the lower confidence bounds of C_{pk}. The lower confidence bound presents the minimum true capability of the process, which is essential to product reliability assurance. We apply the lower confidence bounds to the modified C_{pk} MPPAC to provide reliable simultaneous capability control for multiple processes. A case involving multiple processes manufacturing power-distribution switch (PDS) is investigated. The modified C_{pk} MPPAC incorporating with the lower confidence bound is applied to the capability control of multiple PDS processes.

1. Introduction

Process capability indices, including C_p, C_{pk}, C_{pm}, and C_{pmk} [2,7,12], have been proposed in the manufacturing industry to provide numerical measures on whether a process is capable of reproducing items meeting the quality/reliability requirement preset in the factory. These indices have been defined as:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma},$$

$$C_{pk} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right\},$$

$$C_{pm} = \frac{\text{USL} - \text{LSL}}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

$$C_{pmk} = \min\left\{\frac{\text{USL} - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - \text{LSL}}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\},$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation, and T is the target value. Statistical process control charts have been widely used for monitoring and controlling individual factory manufacture processes on a routine basis. Those charts are essential tools for product reliability control and

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improvement. In the multiple manufacturing lines environment where a group of processes need to be controlled, it could be difficult and time consuming for factory engineers or supervisors to analyze each individual chart to evaluate overall factory performance of process control activities. Singhal [24,25] introduced an MPPAC using process capability index C_{pk}, for controlling product reliability of a group of multiple processes regarding process accuracy and process precision on a single chart. Process accuracy reflects the departure of process mean from the target value, and process precision reflects overall process variability. Pearn and Chen [15] proposed a modification to the C_{pk} MPPAC combining the more-advanced process capability indices, C_{pm} or C_{pmk}, to identify the problems causing the processes failing to center around the target. Pearn et al. [19] introduced the MPPAC based on the incapability index. Chen et al. [3] extended the MPPAC for controlling product reliability with multiple characteristics where the manufacturing tolerances could be symmetric or asymmetric.

Existing research works in developing and applying those MPPAC control charts, however, never considered sampling errors. Therefore, product reliability information provided from those charts is often unreliable and misleading (more unreliable products than what is expected). In current practice of implementing those charts, practitioners simply plot the estimated index values on the chart then make conclusions on whether processes meet the capability requirement and directions need to be taken for further capability improvement. Their approach is highly unreliable since the estimated index values are random variables and sampling errors are ignored. A reliable approach is to first convert the estimated index values to the lower confidence bounds then plot the corresponding lower confidence bounds on the C_{pk} MPPAC. The lower confidence bound not only gives us a clue on the minimal actual performance of the process which is tightly related to the fractions of nonconforming units (unreliable products), but is also useful in making decisions for capability testing.

Construction of the exact lower confidence bounds on C_{pk} is complicated since the distribution of $\widehat{C}_{pk}$ involves the joint distribution of two non-central t-distributed random variables, or alternatively, the joint distribution of the folded normal and the chi-square random variables, with an unknown process parameter even when the samples are given [12]. Numerous methods for obtaining approximate confidence bounds of C_{pk} have been proposed, including Bissell [1], Zhang et al. [28], Porter and Oakland [21,22], Nagata and Nagahata [11], Tang et al. [26] and many others. A different approach was taken by Chou et al. [4] and Levinson [9] who derived the exact lower confidence bounds by working with a bivariate non-central t-distribution. However, an impractical assumption made in their work was that the two sample estimates, $\widehat{C}_{pu} = (\text{USL} - \overline{X})/3S$, and $\widehat{C}_{pl} = (\overline{X} - \text{LSL})/3S$ must be the same. Several authors including Franklin and Wasserman [5], Kushler and Hurley [8], and Rodridguez [23] have commented that such lower confidence bounds on C_{pk} are rather conservative when $\widehat{C}_{pu} = \widehat{C}_{pl}$ is not satisfied, noting that the probability $\text{p}(\widehat{C}_{pu} = \widehat{C}_{pl}) = 0$. Other investigations on the estimation of C_{pk} include Pearn and Chen [13,14,16,17], and Pearn et al. [18]. In this paper, we overcome the difficulty by first obtaining an explicit form of the cumulative distribution function of the sampling distribution of C_{pk}. We then apply direct integration techniques over the cumulative distribution function to obtain the lower confidence bounds of C_{pk}. A Matlab computer program is developed for accurate computation of the lower confidence bounds. The behavior of the lower confidence bound against the distribution characteristic parameter, $\xi = (\mu - m)/\sigma$, is investigated. Exact lower confidence bound ensuring type I error of estimating true C_{pk} no greater than the preset value, $1 - \gamma$, are obtained and used to construct the C_{pk} MPPAC for multiple power-distribution switch (PDS) processes capability control.

2. Power-distribution switch

Consider the following case taken from a manufacturing factory making various types of PDS. The family of PDS is made for applications where heavy capacitive loads and short circuits are likely to be encountered. These devices are around 33 and 80 mΩ N-channel MOSFET high-side power switches. The functional block diagram of a single 33 mΩ PDS is displayed in Fig. 1. The switch is controlled by a logic enable compatible with 5-V logic and 3-V logic. Gate drive is provided with an internal charge pump designed to control the power-switch rise times and fall times to minimize current surges during switching. The charge pump requires no external components and allows operation from supplies as low as 2.7 V. When the output load exceeds the current-limit threshold or a short is present, the switch

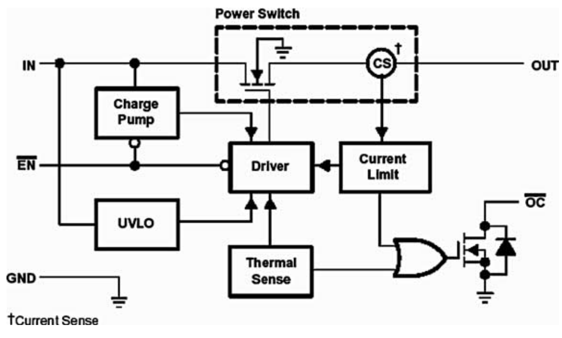


Fig. 1. The functional block diagram of a single 33 mΩ PDS.

limits the output current to a safe level by switching into a constant-current mode, pulling the over-current logic output low. When continuous heavy overloads and short circuits increase the power dissipation in the switch, causing the junction temperature to rise, a thermal protection circuit shuts off the switch to prevent damage. Recovery from a thermal shutdown is automatic once the device has cooled sufficiently. Internal circuitry ensures the switch remains off until valid input voltage is present. Short-circuit current threshold characteristic of the PDS process is essential for product reliability performance, which has significant impact to product quality/reliability. Eight manufacturing lines need to be controlled and monitored at the same time in the factory making different types of PDS. There are eight manufacturing lines in the factory, which need to be simultaneously investigated and the short-circuit current threshold characteristic is of two-sided specification, using C_{pk} MPPAC for this typical multiple processes environment is appropriate for product reliability control and improvement.

2.1. Manufacturing capability and PDS product reliability

The index C_{pk} is yield-based which provides a lower bound on the process yield; that is, $2\Phi(3C_{pk}) - 1 \leqslant \text{Yield} \leqslant \Phi(3C_{pk})$. A manufacturing process is said to be inadequate if $C_{pk} < 1.00$; it indicates that the process is not adequate with respective to the manufacturing tolerances, the process variation σ^2 needs to be reduced (often using design of experiments). The fraction of unreliable PDS products for such process exceeds 2700 parts per million (ppm). A manufacturing process is said to be capable if $1.00 \leqslant C_{pk} < 1.33$; it indicates that caution needs to be taken regarding the process consistency and some process control is required (usually using R or S control charts). The fraction of unreliable PDS products for such process is within 66–2700 ppm. A manufacturing process is said to be satisfactory if $1.33 \leqslant C_{pk} < 1.67$; it indicates that process consistency is satisfactory, material substitution may be allowed, and no stringent precision control is required. The fraction of unreliable PDS products for such process is within 0.54–66 ppm. A manufacturing process is said to be excellent if $1.67 \leqslant C_{pk} < 2.00$; it indicates that process precision exceeds satisfactory. The fraction of unreliable PDS products for such process is within 0.002–0.54 ppm. Finally, a manufacturing process is said to be super if $C_{pk} \geqslant 2.00$. The fraction of unreliable PDS products for such process is less than 0.002 ppm. Table 1 summarizes the above five capability requirements for the PDS processes, the corresponding C_{pk} values, and fractions of non-conformities (NC in ppm). Some minimum capability requirements have been recommended in the manufacturing industry [10], for specific process types, which must run under some more designated stringent quality conditions. For existing manufacturing processes, the capability must be no less than 1.33, and for new manufacturing processes, the capability must be no less than 1.50. For existing manufacturing processes on safety, strength, or critical parameters (such as manufacturing soft drinks or chemical solution bottled with glass containers), the capability must be no less than 1.50, and for new manufacturing processes on safety, strength, or critical parameters, the capability must be no less than 1.67.

Table 1
Some commonly used capability requirements and the process conditions

Condition	C_{pk} values	ppm
Inadequate	$C_{pk} < 1.00$	NC > 2700
Capable	$1.00 \leqslant C_{pk} < 1.33$	NC < 2700
Satisfactory	$1.33 \leqslant C_{pk} < 1.67$	NC < 66
Excellent	$1.67 \leqslant C_{pk} < 2.00$	NC < 0.54
Super	$2.00 \leqslant C_{pk}$	NC < 0.002

3. The modified C_{pk} MPPAC

Singhal [24] developed the C_{pk} MPPAC for controlling and monitoring multiple processes, which sets the priorities among multiple processes for capability improvement and indicate if reducing the variability, or the departure of the process mean should be the focus of improvement. The C_{pk} MPPAC provides an easy way to process improvement by comparing the locations on the chart of the processes before and after the improvement effort. The modified C_{pk} MPPAC is introduced by Pearn and Chen [15], which incorporates the capability zones setting from using the more-advanced capability index C_{pm}. The modified C_{pk} MPPAC is shown in Fig. 2. Four contours for $C_{pk} = 1.00$, 1.33, 1.67, and 2.00 represent different categories of process conditions as summarized in Table 1. The narrow lines form capability zones using C_{pm} measures. On the modified C_{pk} MPPAC, we note that:

(a) The parallel line and perpendicular line through the plotted point intersecting the vertical axis (y-axis) and horizontal axis (x-axis) at points represented $C_{pu} = (\text{USL} - \mu)/(3\sigma)$ and $C_{pl} = (\mu - \text{LSL})/(3\sigma)$, respectively.
(b) The 45° target line represents the points where the process mean equal to the target ($\mu = T = m$) and the values of C_{pu} and C_{pl} are equal.
(c) For the points fall below the target line, $C_{pk} = C_{pl}$. On the other hand, for the points fall above the target line, $C_{pk} = C_{pu} < C_{pl}$.
(d) For the points inside the area to the right of the 45° target line, represents processes where the process

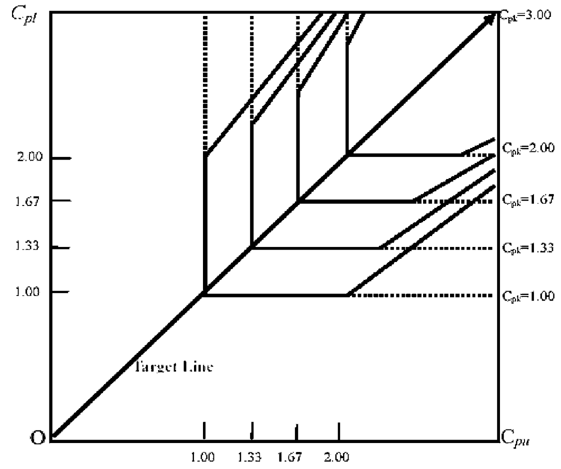


Fig. 2. The modified C_{pk} MPPAC.

mean is towards the lower specification limit (process mean is lower than target value). On the other hand, for the points inside the area to the left of the 45° target line represents processes where the process mean is towards the upper specification limit (process mean is higher than target value).

(e) The origin point represents a process with $C_{pu} = C_{pl} = 0$ which means that the standard deviation of the process is infinite. As the distance from origin of the projection of the plotted point on the target line increases, the variability of the corresponding process decreases.

In general, we never know the true values of the process parameters μ and σ^2 as well as C_{pk}. Hence, these parameters need to be estimated and sampling error of the index C_{pk} needs to be considered for product reliability purpose. In Section 4, sampling distribution of C_{pk} is obtained to compute the lower confidence bound on C_{pk}.

4. Sampling distribution of C_{pk}

Utilizing the identity $\min\{x, y\} = (x + y)/2 - |x - y|/2$, the index C_{pk} can be alternatively written as:

$$C_{pk} = \frac{d - |\mu - m|}{3\sigma},$$

where $d = (\mathrm{USL} - \mathrm{LSL})/2$ is half of the length of the specification interval, $m = (\mathrm{USL} + \mathrm{LSL})/2$ is the midpoint between the lower and the upper specification limits. The natural estimator $\widehat{C}_{pk}$ is obtained by replacing the process mean μ and the process standard deviation σ by their conventional estimators $\overline{X}$ and S, which may be obtained from a process that is demonstrably stable (under statistical control)

$$\widehat{C}_{pk} = \frac{d - |\overline{X} - m|}{3S} = \left\{1 - \left|\frac{\overline{X} - m}{d}\right|\right\}\widehat{C}_p,$$

where $\widehat{C}_p$ is distributed as $(n-1)^{1/2} C_p \chi_{n-1}^{-1}$, and $n^{1/2}|\overline{X} - m|/\sigma$ is distributed as the folded normal distribution with parameter $n^{1/2}|\mu - m|/\sigma$. Thus, $\widehat{C}_{pk}$ is a convolution of χ_{n-1}^{-1} and the folded normal distribution [12]. The probability density function of $\widehat{C}_{pk}$ can be obtained as [18], where $D = (n-1)^{1/2} d/\sigma$, $a = [(n-1)/n]^{1/2}$. A brief derivation of the probability density function of $\widehat{C}_{pk}$ is included in Appendix A

$$f_{\hat{C}_{pk}}(x) = \begin{cases} 4A_n \sum_{\ell=0}^{\infty} P_\ell(\lambda) B_\ell \times \frac{D^{n+2\ell}}{a^{2\ell+1}} \int_0^{\infty} (1 - xz)^{2\ell} z^{n-1} \\ \times \exp\left\{-\frac{D^2}{18a^2}\left(a^2 z^2 + 9(1 - xz)^2\right)\right\} \mathrm{d}z, & x \leqslant 0, \\ 4A_n \sum_{\ell=0}^{\infty} P_\ell(\lambda) B_\ell \times \frac{D^{n+2\ell}}{a^{2\ell+\ell}} \int_0^{\frac{1}{x}} (1 - xz)^{2\ell} z^{n-1} \\ \times \exp\left\{-\frac{D^2}{18a^2}\left(a^2 z^2 + 9(1 - xz)^2\right)\right\} \mathrm{d}z, & x > 0, \end{cases}$$

$$P_\ell(\lambda) = \frac{\mathrm{e}^{-(\lambda/2)} (\lambda/2)^\ell}{\ell!}, \quad A_n = \frac{1}{3^{n-1} 2^{n/2} \Gamma((n-1)/2)},$$

$$B_\ell = \frac{1}{2^\ell \Gamma((2\ell + 1)/2)}.$$

Using the integration technique similar to that presented in [27], we may obtain the following exact form of the cumulative distribution function of $\widehat{C}_{pk}$, under the assumption of normality.

4.1. Cumulative distribution function

The cumulative distribution function of $\widehat{C}_{pk}$ is expressed in terms of a mixture of the chi-square distribution and the normal distribution, for $x > 0$, where $b = d/\sigma$, $\xi = (\mu - m)/\sigma$, $G(\cdot)$ is the cumulative distribution function of the chi-square distribution χ_{n-1}^2, and $\phi(\cdot)$ is the probability density function of the standard normal distribution $N(0, 1)$

$$F_{\hat{C}_{pk}}(x) = 1 - \int_0^{b\sqrt{n}} G\left(\frac{(n-1)(b\sqrt{n} - t)^2}{9nx^2}\right) \times [\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})]\,\mathrm{d}t. \qquad (1)$$

5. Lower confidence bounds on C_{pk}

For processes with target value setting to the midpoint of the specification limits ($T = m$), the index may be rewritten as the following. We also note that when $C_{pk} = C$, $b = d/\sigma$ can be expressed as $b = 3C + |\xi|$.

Thus, the index C_{pk} may be expressed as a function of the characteristic parameter ξ

$$C_{pk} = \frac{d - |\mu - m|}{3\sigma} = \frac{d/\sigma - |\xi|}{3},$$

where $\xi = (\mu - m)/\sigma$.

Hence, given the sample of size n, the confidence level γ, the estimated value $\widehat{C}_{pk}$ and the parameter ξ, the lower confidence bounds C can be obtained using numerical integration technique with iterations, to solve the following Eq. (2). In practice, the parameter $\xi = (\mu - m)/\sigma$ is unknown, but it can be calculated from the sample data as $\hat{\xi} = (\overline{X} - m)/S$. It should be noted, particularly, that Eq. (2) is an even function of ξ. Thus, for both $\hat{\xi} = \hat{\xi}_0$ and $\hat{\xi} = -\hat{\xi}_0$ we may obtain the same lower confidence bound C

$$\int_0^{b\sqrt{n}} G\left(\frac{(n-1)(b\sqrt{n}-t)^2}{9n\widehat{C}_{pk}^2}\right) \times [\phi(t + \hat{\xi}\sqrt{n} + \phi(t - \hat{\xi}\sqrt{n})]\,\mathrm{d}t = 1 - \gamma. \quad (2)$$

5.1. Algorithm for the LCB

Using Eq. (2), we may compute the lower confidence bounds C. A Matlab program called the LCB is developed. Three auxiliary functions for evaluating C are included here, (a) the cumulative distribution function of the chi-square χ^2_{n-1}, $G(\cdot)$, (b) the probability density function of the standard normal distribution $N(0,1)$, $\phi(\cdot)$, and (c) the function of numerical integration computation using the recursive adaptive Simpson quardrature—"quad". The algorithm used is commonly known as the direct search method. We implement the algorithm, and develop the Matlab computer program (see Appendix A) to compute the minimal manufacturing capability

Step 1. Read the sample data $(X_1, X_2, \ldots, X_n)$, LSL, USL, T, and γ.
Step 2. Calculate $\overline{X}$, S, $\hat{\xi}$, and $\widehat{C}_{pk}$.
Step 3. Compute an initial guess for C.
Step 4. Find the lower confidence bound C on C_{pk}.
Step 5. Output the conclusive message, "The true value of the manufacturing capability C_{pk} is no less than the C with 100 γ % level of confidence".

5.2. Lower confidence bounds C and parameter ξ

Since the process parameters μ and σ are unknown, then the distribution characteristic parameter, $\xi = (\mu - m)/\sigma$ is also unknown, which has to be estimated in real applications, naturally by substituting μ and σ by the sample mean $\overline{X}$ and the sample standard deviation S. Such approach introduces additional sampling errors from estimating ξ in finding the lower confidence bounds, and certainly would make our approach (and of course including all the existing methods) less reliable. Consequently, any decisions made would provide less quality assurance to the customers. To eliminate the need for further estimating the distribution characteristic parameter $\xi = (\mu - m)/\sigma$, we examine the behavior of the lower confidence bound values C against the parameter $\xi = (\mu - m)/\sigma$.

We perform extensive calculations to obtain the lower confidence bound values C for $\xi = 0(0.05)3.00$, $n = 10(5)200$, $\widehat{C}_{pk} = 0.7(0.1)3.0$, and confidence level $\gamma = 0.95$. Note that the parameter values we investigated, $\xi = 0(0.05)3.00$, cover a wide range of applications with process capability $C_{pk} \geqslant 0$. It should be noted that in common practice negative values of C_{pk} are normally set to zero, indicating that process mean falls outside the manufacturing specification limits. The results indicate that (i) the lower confidence bound C is decreasing in ξ, and is increasing in n, (ii) the lower confidence bound C obtains its minimum at $\xi = 1.00$ in all cases, and stays at the same value for $\xi \geqslant 1.00$ for all C (with accuracy up to 10^{-4}). Furthermore, we observe that for $n > 30$, the lower confidence bound C reaches its minimum at $\xi = 0.50$ and stays at the same value for $\xi \geqslant 0.50$, and for $n \geqslant 100$, reaches its minimum at $\xi = 0.25$ (with accuracy up to 10^{-4}). Hence, for practical purpose we may solve Eq. (2) with $\xi = \xi = 1.00$ to obtain the required lower confidence bounds for given $\widehat{C}_{pk}$, n, and γ, without having to further estimate the parameter ξ. Thus, the level of confidence γ can be ensured, and the decisions made based on such approach are indeed more reliable. We note the above result is impossible to prove mathematically.

Fig. 3(a)–(f) plot the curves of the lower confidence bound, C, versus the parameter ξ for $\widehat{C}_{pk} = 0.7$, 0.9, 1.2, 2.0, 2.5, 3.0, respectively, with confidence level $\gamma = 0.95$. For bottom curve 1, sample size $n = 30$. For bottom curve 2, sample size $n = 50$, for bottom curve 3, sample size $n = 70$, for top curve 3, sample size $n = 100$; for top curve 2, sample size $n = 150$; for top curve 1, sample size $n = 200$. Table 2 (see Appendix A) tabulates the lower confidence bound, C, for $\widehat{C}_{pk} = 0.7(0.1)3.0$, $n = 5(5)200$, and $\gamma = 0.95$ with the process parameter ξ set to $\xi = 1.0$. For example, if $\widehat{C}_{pk} = 1.5$, then with $n = 100$ we find the lower confidence bound $C = 1.315$, and so the minimal manufacturing capability is no less than 1.315, i.e., $C_{pk} > 1.315$. Consequently, the manufacturing yield (fraction of reliable products) is no less than 99.992% and the fraction of non-conformities (unreliable products) is no greater than 79.80 ppm. We note that for other existing methods, either the confidence level γ cannot be assured (PDS product reliability assurance is uncertain), or the lower confidence bounds C are too conservative (C is too small in this case). Our approach provides best reliability assurance to the PDS products.

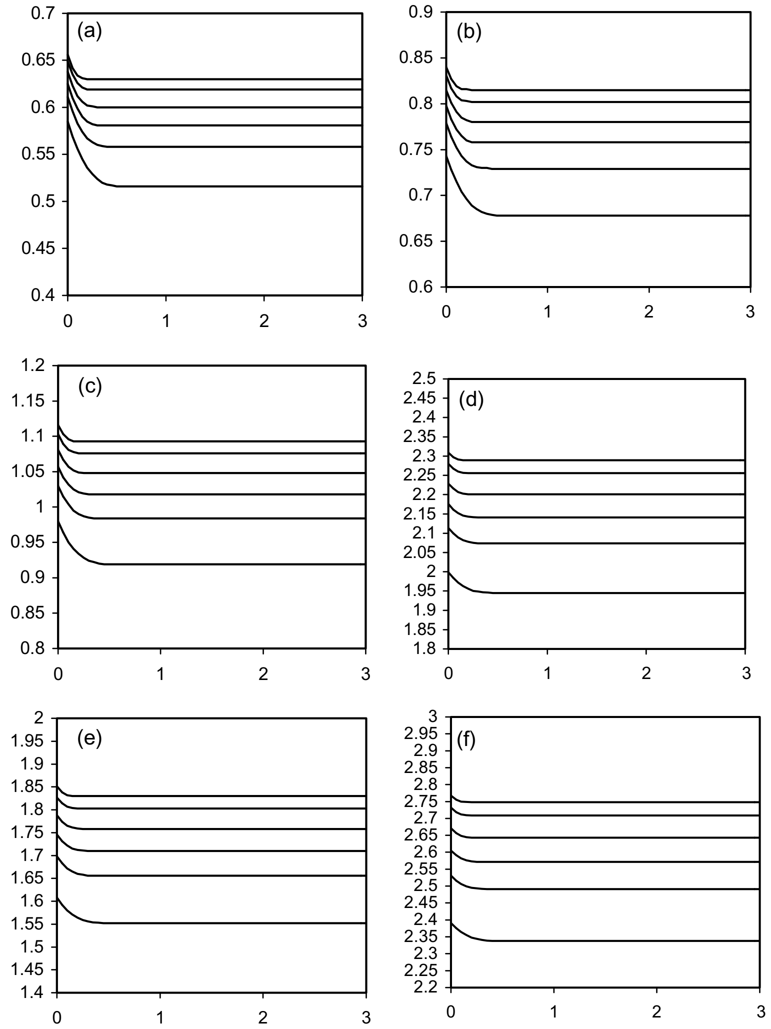


Fig. 3. Plots of C vs $|\xi|$ for (a) $\widehat{C}_{pk} = 0.7$, $n = 30, 50, 70, 100, 150, 200$ (bottom to top); (b) $\widehat{C}_{pk} = 0.9$, $n = 30, 50, 70, 100, 150, 200$ (bottom to top); (c) $\widehat{C}_{pk} = 1.2$, $n = 30, 50, 70, 100, 150, 200$ (bottom to top); (d) $\widehat{C}_{pk} = 2.0$, $n = 30, 50, 70, 100, 150, 200$ (bottom to top); (e) $\widehat{C}_{pk} = 2.5$, $n = 30, 50, 70, 100, 150, 200$ (bottom to top) and (f) $\widehat{C}_{pk} = 3.0$, $n = 30, 50, 70, 100, 150, 200$ (bottom to top).

We note that the lower confidence bound C calculated using the above proposed approach, is maximal (exact) which cannot be improved further.

6. Manufacturing capability computation and data analysis

We collected sample data of short-circuit current threshold from eight manufacturing processes making different kinds of PDS devices. One hundred observations from each PDS process are taken and calculated for the sample mean, sample standard deviation, and the estimate $\widehat{C}_{pk}$. The product codes, manufacturing specifications, the estimated index values, the lower confidence bounds, and the corresponding maximum nonconformities for each of the eight processes are tabulated in Tables 3 and 4. Fig. 4 plots the modified C_{pk} MPPAC for the eight processes based on the minimum true values tabulated in Table 4. We analyze these process points in Fig. 4 and obtain the following critical summary information of the capability condition for all processes.

(a) The plotted points E and H are not located within the contour of $C_{pk} = 1.00$. It indicates that the process has a very low capability. Since the points E and H are close to the 45° target line, both processes present that the process means are close to the target value, and the poor capabilities are mainly contrib-

Table 3
Manufacturing specifications of the eight PDS products

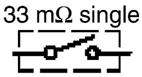
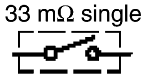
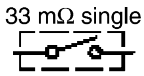
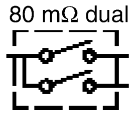
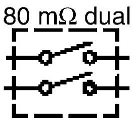
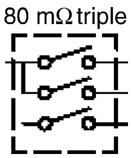
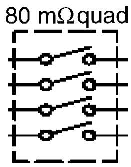
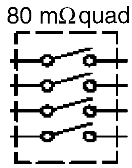
Code	A	B	C	D
Products	 33 mΩ single	 33 mΩ single	 33 mΩ single	 80 mΩ dual
T	1 A	1.2 A	500 mA	500 mA
USL	1.3 A	1.5 A	650 mA	600 mA
LSL	0.7 A	0.9 A	350 mA	400 mA
	E	F	G	H
Products	 80 mΩ dual	 80 mΩ triple	 80 mΩ quad	 80 mΩ quad
T	250 mA	550 mA	250 mA	250 mA
USL	320 mA	620 mA	310 mA	300 mA
LSL	180 mA	480 mA	190 mA	200 mA

Table 4
Calculated statistics, estimated C_{pk}, lower confidence bound, and the fractions of non-conformities (in ppm) of the eight PDS products

Code	A	B	C	D
$\overline{X}$	1.007153 A	1.25403 A	508.30 mA	483.76 mA
S	0.047687 A	0.04502 A	27.65 mA	17.18 mA
$(\mathrm{USL}-\overline{X})/(3S)$	2.047	1.821	1.708	1.625
$(\overline{X}-\mathrm{LSL})/(3S)$	2.147	2.621	1.908	1.625
$\widehat{C}_{pk}$	2.047	1.821	1.708	1.625
LCB	1.799	1.599	1.499	1.425
ppm	0.0678	1.61	6.89	19.11
	E	F	G	H
$\overline{X}$	252.09 mA	570.89 mA	231.21 mA	245.61 mA
S	27.91 mA	13.01 mA	10.02 mA	13.95 mA
$(\mathrm{USL}-\overline{X})/(3S)$	0.811	1.258	2.621	1.30
$(\overline{X}-\mathrm{LSL})/(3S)$	0.861	2.328	1.371	1.090
$\widehat{C}_{pk}$	0.811	1.258	1.371	1.090
LCB	0.7	1.099	1.2	0.949
ppm	35729	977.23	318.22	4413.3

uted by the significant process variation. Thus, immediate quality improvement actions must be taken for reducing the process variance for both processes.

(b) The plotted points G and F lie within the contour of $1.00 \leqslant C_{pk} < 1.33$. The point G lies inside the area which is to the right of the 45° target line represents processes where the process mean is towards the lower specification limit (process mean is lower than target value). On the other hand, the point F lies inside the area, which is to the left of the 45° target line represents processes where the process mean is towards the upper specification limit (process mean is higher than target value). Thus, quality improvement effort for these processes should be first focused on reducing their process departure from the target value T, then the reduction of the process variance.

(c) Process B and D lie inside the contours of $C_{pk} = 1.33 \leqslant C_{pk} < 1.67$. Both processes are considered performing well and no immediate improvement activities needed to be taken, but both processes obviously departure from the target value T. Therefore, both processes may be improved by simply reducing their process departure from the target value T.

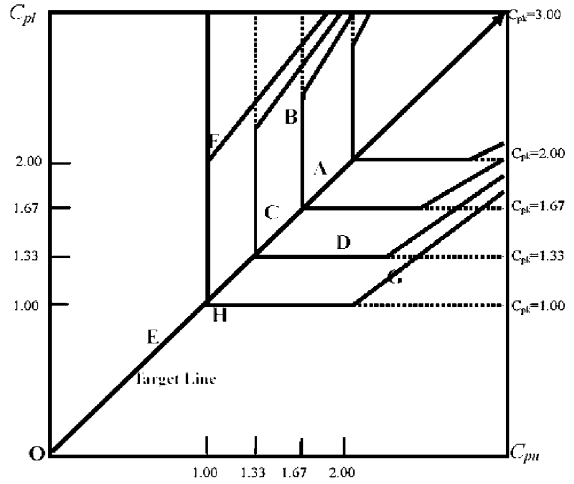


Fig. 4. The modified C_{pk} MPPAC groups for the eight PDS processes.

(d) The plotted points C and A lie inside the contours of $C_{pk} = 1.33$ and $C_{pk} = 1.67$, respectively. Capabilities of both processes are considered satisfactory and excellent. They have lower priorities in allocating quality improvement efforts than other processes.

Table 5 displays the manufacturing capabilities and capability groupings for the eight PDS processes using the estimated C_{pk} values (uncorrected) and the lower confidence bounds LCB (corrected) (with asterisks $*$ indicating incorrect groupings). The modified C_{pk} MPPAC for the eight processes based on the estimated C_{pk} index values (an approach widely used in current industrial applications) rather than using the lower confidence bounds, is displayed in Fig. 5. We note that such MPPAC obviously conveys unreliable information and is misleading, which should be avoided in real applications.

Table 5
Estimated and corrected (LCB) capabilities, and their groupings for the eight PDS processes

Code	Estimated C_{pk}	Grouping	LCB	Grouping
A	2.047	Super*	1.799	Excellent
B	1.821	Excellent*	1.599	Satisfactory
C	1.708	Excellent*	1.499	Satisfactory
D	1.625	Satisfactory	1.425	Satisfactory
E	0.811	Incapable	0.700	Incapable
F	1.258	Capable	1.099	Capable
G	1.371	Satisfactory*	1.200	Capable
H	1.090	Capable*	0.949	Incapable

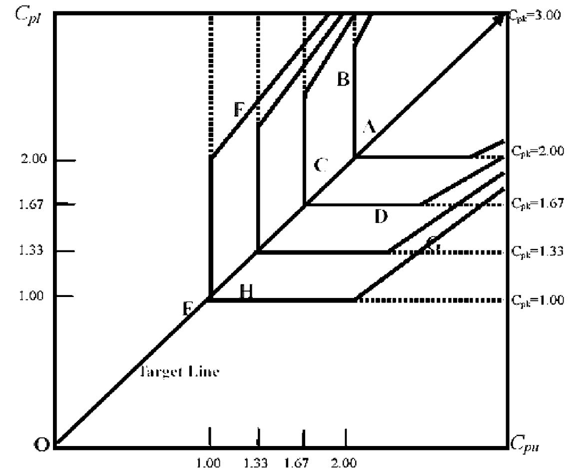


Fig. 5. The C_{pk} MPPAC based on $\widehat{C}_{pk}$.

7. Conclusions

Conventional investigations on manufacturing capability control ignore sampling errors. In this paper, we considered the sample errors by finding the exact lower confidence bound for the C_{pk}. The lower confidence bounds present a measure on the minimum capability of the process based on the sample data. Existing methods for computing the lower confidence bounds provided only approximate or rather conservative bounds. We investigated the behavior of the lower confidence bound versus the process characteristic parameter $\xi = (\mu - m)/\sigma$, which resulted that the lower confidence bound attains its minimal value at $\xi = 1.0$. The proposed decision making procedure ensures that the risk of making a wrong decision will be no greater than the preset Type I error $1 - \gamma$. The proposed modified C_{pk} MPPAC is useful for manufacturing capability control of a group of processes in a multiple process environment. The modified C_{pk} MPPAC prioritizes the order of the processes for further capability improvement effort should focus on, either to move the process mean closer to the target value or reduce the process variation. The developed lower confidence bounds can be used to construct accurate modified C_{pk} MPPAC providing information regarding the true capability, and fractions of non-conforming products. The modified C_{pk} MPPAC is applied to the PDS manufacturing process for controlling PDS product reliability.

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Appendix A. Derivation of the probability density function of $\widehat{C}_{pk}$

Let $X_1, \ldots, X_n$ be a random sample of size n from a normally distributed process $N(\mu, \sigma^2)$. From a process demonstrably stable (under statistical control), the natural estimator $\widehat{C}_{pk}$ is obtained by replacing the process mean μ and the process standard deviation σ by their conventional estimators $\overline{X}$ and S, which can be written as follows:

$$\widehat{C}_{pk} = \frac{d - |\overline{X} - m|}{3S}.$$

To derive the cumulative distribution function and the probability density function of $\widehat{C}_{pk}$, we define

(1) $D = (n-1)^{1/2} d/\sigma$,
(2) $a = [(n-1)/(n)]^{1/2}$,
(3) $K = (n-1)S^2/\sigma^2$, which is distributed as χ^2_{n-1},
(4) $Z = n^{1/2}(\overline{X} - m)/\sigma$, which is distributed as $N(\delta, 1)$, where $\delta = n^{1/2}(\mu - m)/\sigma$,
(5) $Y = Z^2$, which is distributed as the ordinary non-central chi-square distribution with one degree of freedom and non-centrality parameter δ, $\chi^2_1(\delta)$. Then, the probability density function of Y can be expressed as:

$$f_Y(y) = \frac{e^{-\lambda/2}}{2\sqrt{\pi}} \sum_{j=0}^{\infty} h_j(\lambda)\left[(-1)^j f_{Y_j}(y) + f_{Y_j}(y)\right] = \frac{e^{-\lambda/2}}{\sqrt{\pi}} \sum_{\lambda=0}^{\infty} h_{2\lambda}(\lambda) f_{Y_{2\lambda}}(y), \text{ for } y > 0,$$

where $\lambda = \delta^2$, $h_{2\lambda}(\lambda) = (2\lambda)^{\lambda}\Gamma[(1+2\lambda)/2]/(2\lambda!)$ and $Y_{2\lambda}$ is distributed as $\chi^2_{1+2\lambda}$.
We note that the estimator $\widehat{C}_{pk}$ can be rewritten as:

$$\widehat{C}_{pk} = \frac{D - a\sqrt{Y}}{3\sqrt{K}}.$$

[Case I]: For $x > 0$, the cumulative distribution function of $\widehat{C}_{pk}$ is

$$F_{\hat{C}_{pk}}(x) = P(\widehat{C}_{pk} \leqslant x) = P\left(\frac{D - a\sqrt{Y}}{3\sqrt{K}} \leqslant x\right) = 1 - \int_0^{\infty} P\left(\sqrt{K} \leqslant \frac{D - a\sqrt{Y}}{3x} | Y = y\right) f_Y(y)\,dy.$$

Since K is distributed as χ^2_{n-1}, then

$$P\left(\sqrt{K} \leqslant \frac{D - a\sqrt{y}}{3x}\right) = 0 \quad \text{for} \quad y > (D/a)^2 \quad \text{and} \quad x > 0.$$

Hence,

$$F_{\hat{C}_{pk}}(x) = 1 - \int_0^{(D/a)^2} p\left(\sqrt{K} \leqslant \frac{D - a\sqrt{y}}{3x}\right) f_Y(y)\,dy = 1 - \int_0^{(D/a)^2} F_K\left(\frac{(D - a\sqrt{y})^2}{9x^2}\right) f_Y(y)\,dy.$$

Substituting $f_Y(y)$, we obtain

$$F_{\hat{C}_{pk}}(x) = 1 - \frac{e^{-\lambda/2}}{\sqrt{\pi}} \times \sum_{\lambda=0}^{\infty} h_{2\lambda}(\lambda)\left(\int_0^{(D/a)^2} F_K\left(\frac{(D - a\sqrt{t})^2}{9x^2}\right) f_{Y_{2\lambda}}(t)\,dt\right).$$

[Case II]: Similarly, for $x < 0$, the cumulative distribution function of $\widehat{C}_{pk}$ is

$$F_{\hat{C}_{pk}}(x) = \frac{e^{-\lambda/2}}{\sqrt{\pi}} \times \sum_{\lambda=0}^{\infty} h_{2\lambda}(\lambda)\left(\int_{(D/a)^2}^{\infty} F_K\left(\frac{(D - a\sqrt{t})^2}{9x^2}\right) f_{Y_{2\lambda}}(t)\,dt\right).$$

[Case III]: For $x = 0$,

$$F_{\hat{C}_{pk}}(x) = 1 - \frac{e^{-\lambda/2}}{\sqrt{\pi}} \sum_{\lambda=0}^{\infty} h_{2\lambda}(\lambda)\left(\int_0^{(D/a)^2} f_{Y_{2\lambda}}(t)\,dt\right).$$

By Leibnitz's rule, taking the derivative of $F_{\hat{C}_{pk}}(x)$ with respect to x we have the probability density function of $\widehat{C}_{pk}$:

$$f_{\hat{C}_{pk}}(x) = \begin{cases} -2\frac{e^{-\lambda/2}}{\sqrt{\pi}} \sum_{\lambda=0}^{\infty} h_{2\lambda}(\lambda)\Big(\int_{(D/a)^2}^{\infty} f_K\left(\frac{(D-a\sqrt{t})^2}{9x^2}\right) \\ \quad \times \frac{(D-a\sqrt{t})^2}{9x^3} \times f_{Y_{2\lambda}}(t)\,dt\Big), \quad x < 0, \\ 0, \quad x = 0, \\ 2\frac{e^{-\lambda/2}}{\sqrt{\pi}} \sum_{\lambda=0}^{\infty} h_{2\lambda}(\lambda)\Big(\int_0^{(D/a)^2} f_K\left(\frac{(D-a\sqrt{t})^2}{9x^2}\right) \\ \quad \times \frac{(D-a\sqrt{t})^2}{9x^3} \times f_{Y_{2\lambda}}(t)\,dt\Big), \quad x > 0. \end{cases}$$

Changing the variable

$$z = \frac{D - a\sqrt{t}}{Dx}$$

in the above integral, we can rewrite $f_{\hat{C}_{pk}}(x)$ as:

$$f_{\hat{C}_{pk}}(x) = \begin{cases} 4\frac{e^{-\lambda/2}}{\sqrt{\pi}} \sum_{\lambda=0}^{\infty} h_{2\lambda}(\lambda)\Big(\int_0^{\infty} f_K\left(\frac{D^2z^2}{9}\right) f_{Y_{2\lambda}}\left(\frac{D^2(1-xz)^2}{a^2}\right) \\ \quad \times \frac{D^4(1-xz)z^2}{9a^2}\,dz\Big), \quad x < 0, \\ 0, \quad x = 0, \\ 4\frac{e^{-\lambda/2}}{\sqrt{\pi}} \sum_{\lambda=0}^{\infty} h_{2\lambda}(\lambda)\Big(\int_0^{1/x} f_K\left(\frac{D^2z^2}{9}\right) f_{Y_{2\lambda}}\left(\frac{D^2(1-xz)^2}{a^2}\right) \\ \quad \times \frac{D^4(1-xz)z^2}{9a^2}\,dz\Big), \quad x > 0. \end{cases}$$

Let

$$I_{2\lambda}(z) = f_K\left(\frac{D^2z^2}{9}\right) f_{Y_{2\lambda}}\left(\frac{D^2(1-xz)^2}{a^2}\right) \times \frac{D^4(1-xz)z^2}{9a^2}.$$

Then

$$f_{\hat{C}_{pk}}(x) = 4\frac{e^{-\lambda/2}}{\sqrt{\pi}}\sum_{\lambda=0}^{\infty} h_{2\lambda}(\lambda)\int_0^{\infty} I_{2\lambda}(z)\,dz \quad \text{for} \quad x \leqslant 0.$$

Further, for positive odd integer n. Hence,

$$h_{2\lambda}(\lambda) = \frac{(2\lambda)^{\lambda}}{(2\lambda)!}\Gamma\left(\frac{1+2\lambda}{2}\right) = \frac{(\lambda/2)^{\lambda}\sqrt{\pi}}{\lambda!}.$$

Thus,

$$f_{\hat{C}_{pk}}(x) = 4\sum_{\lambda=0}^{\infty} P_{\lambda}(\lambda)\int_0^{\infty} I_{2\lambda}(z)\,dz$$

for $x \leqslant 0$, where

$$P_{\lambda}(\lambda) = \frac{e^{-(\lambda/2)}(\lambda/2)^{\lambda}}{\lambda!}.$$

On the other hand, since K is distributed as χ^2_{n-1} and $Y_{2\lambda}$ is distributed as $\chi^2_{1+2\lambda}$, then

$$f_K\left(\frac{D^2z^2}{9}\right) = \frac{2^{-(n-1)/2}}{\Gamma((n-1)/2)}\left(\frac{D^2z^2}{9}\right)^{(n-3)/2} e^{-D^2z^2/18},$$

$$f_{Y_{2\lambda}}\left(\frac{D^2(1-xz)^2}{a^2}\right) = \frac{2^{-(2\lambda+1)/2}}{\Gamma((2\lambda+1)/2)}\left(\frac{D^2(1-xz)^2}{a^2}\right)^{(2\lambda-1)/2} e^{-D^2(1-xz)^2/(2a^2)}.$$

Hence,

$$I_{2\lambda}(z) = A_nB_{\lambda} \times \frac{D^{n+2\lambda}}{a^{2\lambda+1}}(1-xz)^{2\lambda}z^{n-1} \times \exp\left\{-\frac{D^2}{18a^2}(a^2z^2 + 9(1-xz)^2)\right\},$$

where

$$A_n = \frac{1}{3^{n-1}2^{n/2}\Gamma((n-1)/2)} \quad \text{and} \quad B_{\lambda} = \frac{1}{2^{\lambda}((2\lambda+1)/2)}.$$

Consequently, we have

$$f_{\hat{C}_{pk}}(x) = 4A_n\sum_{\lambda=0}^{\infty} P_{\lambda}(\lambda)B_{\lambda}\frac{D^{n+2\lambda}}{a^{2\lambda+1}}\int_0^{\infty}(1-xz)^{2\lambda}z^{n-1} \times \exp\left\{-\frac{D^2}{18a^2}(a^2z^2+9(1-xz)^2)\right\}dz \quad \text{for } x \leqslant 0.$$

Based on the similar derivation, we can obtain

$$f_{\hat{C}_{pk}}(x) = 4A_n\sum_{\lambda=0}^{\infty} P_{\lambda}(\lambda)B_{\lambda}\frac{D^{n+2\lambda}}{a^{2\lambda+1}}\int_0^{1/x}(1-xz)^{2\lambda}z^{n-1} \times \exp\left\{-\frac{D^2}{18a^2}(a^2z^2+9(1-xz)^2)\right\}dz \quad \text{for } x > 0.$$

Matlab Program for LCB

```
%-------------------------------------------
% Input the sample data (X1, X2, ..., Xn), LSL, USL, T, and γ.
%-------------------------------------------
clear global
[n1 usl lsl r1] = read('Enter values of sample size, upper specification limit,...
lower specification limit, confidence level:');
global b n epsilon ecpk
   n = n1;
   r = r1;
   [data(1:n,1)] = textread('PDS.dat','%f',n);
%-------------------------------------------
% Compute X̄, S, ξ̂, and Ĉpk.
%-------------------------------------------
   mdata = mean(data);
   stddata = std(data);
   epsilon = (mdata-T)/stddata;
   ecpk = (min(usl-mdata,mdata-lsl))/(3*stddata);
   fprintf('The Sample Mean is %g.\n',mdata);
   fprintf('The Sample Standard Deviation is %g.\n',stddata)
   fprintf('The Epsilon %g.\n',epsilon)
   fprintf('The Estimate of Cpk is %g.\n',ecpk)
%-------------------------------------------
% Compute a good initial value of C.
%-------------------------------------------
   b = 0;d = 0;
   c = 0.2:0.025:3;
for i = 1:1:113
   b = 0;d = 0;y = 0;b = 3*c(i)+abs(epsilon);
   d = b*sqrt(n);
   y = quad('cpk',0,d);
if (y-(1-r)) > 0 break
end; end
%-------------------------------------------
%Evaluate the lower confidence bound C on Cpk.
%-------------------------------------------
   c = 0.2+0.025*(i-1):-0.001:0.2;
for k = 1:(0.025*(i-1)*1000)+1
   b = 0;d = 0;y = 0;b = 3*c(k)+abs(epsilon);
   d = b*sqrt(n);
   y = quad('cpk',0,d);
if ((1-r)-y) > 0.0001 break
end; end
%-------------------------------------------
% Output the conclusive message, "The true value of the process
% capability Cpk is no less than C with 100 γ% level of confidence"
%-------------------------------------------
   fprintf('The true value of the process capability Cpk is no less than %g',c(k))
   fprintf('with %g',r)
```

```
    fprintf('level of confidence.')
    ppm = 1000000*2*normcdf(-3*c(k))
%-------------------------------------------
%Two function files included—read.m and cpk.m
%-------------------------------------------
function Q1 = cpk(t)
global n b epsilon ecpk
    Q1 = chi2cdf((((b*sqrt(n)-t).2̂)./(9*ecpk2̂))*((n-1)/
    n), ...
    n-1).*(normpdf((t+epsilon*sqrt(n)))+normpdf((t-
    epsilon*sqrt(n))));
function [a1, a2, a3, a4, a5] = read(labl)
if nargin = = 0, labl = '?'; end
    n = nargout;str = input(labl,'s');       str = ['[',str,']'];
    v = eval(str);L = length(v);
if L> = n, v = v(1:n);
    else, v = [v,zeros(1,n-L)]; end
for j = 1:nargout
    eval(['a', int2str(j),' = v(j);']); end
%-------------------------------------------
```

Table 2
Lower confidence bounds C of C_{pk} for $\widehat{C}_{pk} = 0.7(0.1)1.8$ (Panel A) and $\widehat{C}_{pk} = 1.9(0.1)3.0$ (Panel B), $n = 10(5)200$, $\gamma = 0.95$

n	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
Panel A												
10	0.371	0.438	0.503	0.568	0.632	0.696	0.759	0.822	0.885	0.948	1.010	1.072
15	0.435	0.508	0.581	0.653	0.724	0.795	0.865	0.936	1.006	1.075	1.146	1.215
20	0.472	0.549	0.626	0.702	0.777	0.852	0.927	1.001	1.076	1.150	1.224	1.298
25	0.497	0.577	0.656	0.735	0.813	0.890	0.968	1.045	1.123	1.200	1.277	1.353
30	0.516	0.597	0.678	0.759	0.839	0.918	0.998	1.077	1.157	1.236	1.315	1.394
35	0.530	0.613	0.695	0.777	0.859	0.940	1.021	1.102	1.183	1.264	1.344	1.425
40	0.541	0.625	0.709	0.792	0.875	0.957	1.040	1.122	1.204	1.286	1.368	1.450
45	0.550	0.635	0.720	0.804	0.888	0.972	1.055	1.138	1.222	1.305	1.388	1.471
50	0.558	0.644	0.729	0.814	0.899	0.984	1.068	1.152	1.236	1.320	1.404	1.488
55	0.565	0.651	0.737	0.823	0.909	0.994	1.079	1.164	1.249	1.334	1.418	1.503
60	0.571	0.658	0.745	0.831	0.917	1.003	1.089	1.174	1.260	1.345	1.430	1.516
65	0.576	0.664	0.751	0.838	0.924	1.011	1.097	1.183	1.269	1.355	1.441	1.527
70	0.581	0.669	0.756	0.844	0.931	1.018	1.105	1.191	1.278	1.364	1.451	1.537
75	0.585	0.673	0.761	0.849	0.937	1.024	1.111	1.198	1.286	1.373	1.459	1.546
80	0.588	0.677	0.766	0.854	0.942	1.030	1.117	1.202	1.292	1.380	1.467	1.555
85	0.592	0.681	0.770	0.858	0.947	1.035	1.123	1.211	1.299	1.387	1.474	1.562
90	0.595	0.684	0.774	0.862	0.951	1.040	1.128	1.216	1.305	1.393	1.481	1.569
95	0.598	0.687	0.777	0.866	0.955	1.044	1.133	1.221	1.310	1.398	1.487	1.575
100	0.600	0.690	0.780	0.870	0.959	1.048	1.137	1.226	1.315	1.403	1.492	1.581
105	0.603	0.693	0.783	0.873	0.962	1.052	1.141	1.230	1.319	1.408	1.497	1.586
110	0.605	0.696	0.786	0.876	0.965	1.055	1.145	1.234	1.323	1.413	1.502	1.591
115	0.607	0.698	0.788	0.878	0.968	1.058	1.148	1.238	1.327	1.417	1.506	1.596
120	0.609	0.700	0.791	0.881	0.971	1.061	1.151	1.241	1.331	1.421	1.511	1.600
125	0.611	0.702	0.793	0.883	0.974	1.064	1.154	1.244	1.335	1.424	1.514	1.604
130	0.613	0.704	0.795	0.886	0.976	1.067	1.157	1.248	1.338	1.426	1.518	1.608
135	0.614	0.706	0.797	0.888	0.979	1.069	1.160	1.250	1.341	1.431	1.521	1.612
140	0.616	0.707	0.799	0.890	0.981	1.072	1.162	1.253	1.344	1.434	1.525	1.615
145	0.617	0.709	0.801	0.892	0.983	1.074	1.165	1.256	1.346	1.437	1.528	1.618
150	0.619	0.711	0.802	0.894	0.985	1.076	1.167	1.258	1.349	1.440	1.531	1.621
155	0.620	0.712	0.804	0.895	0.987	1.078	1.169	1.260	1.352	1.443	1.534	1.624
160	0.621	0.713	0.805	0.897	0.989	1.080	1.171	1.263	1.354	1.445	1.536	1.627
165	0.623	0.715	0.807	0.899	0.990	1.082	1.173	1.265	1.356	1.447	1.539	1.630
170	0.624	0.716	0.808	0.900	0.992	1.084	1.175	1.267	1.358	1.450	1.541	1.632
175	0.625	0.717	0.810	0.902	0.994	1.085	1.177	1.269	1.360	1.452	1.543	1.635
180	0.626	0.718	0.811	0.903	0.995	1.087	1.179	1.271	1.362	1.454	1.546	1.637
185	0.627	0.720	0.812	0.904	0.997	1.089	1.181	1.272	1.364	1.456	1.548	1.639
190	0.628	0.721	0.813	0.906	0.998	1.090	1.182	1.274	1.366	1.458	1.550	1.642
195	0.629	0.722	0.814	0.907	0.999	1.091	1.184	1.276	1.368	1.460	1.552	1.644
200	0.630	0.723	0.815	0.908	1.000	1.093	1.185	1.277	1.369	1.462	1.554	1.646

(*continued on next page*)

Table 2 (*continued*)

n	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
Panel B												
10	1.134	1.196	1.258	1.320	1.381	1.443	1.505	1.566	1.628	1.689	1.750	1.812
15	1.285	1.354	1.424	1.493	1.562	1.631	1.700	1.770	1.839	1.908	1.975	2.046
20	1.372	1.446	1.519	1.593	1.667	1.740	1.814	1.887	1.961	2.034	2.107	2.181
25	1.430	1.507	1.583	1.660	1.737	1.813	1.890	1.966	2.042	2.119	2.195	2.271
30	1.473	1.552	1.630	1.709	1.788	1.866	1.945	2.023	2.102	2.181	2.259	2.337
35	1.506	1.586	1.666	1.747	1.827	1.907	1.988	2.068	2.148	2.228	2.308	2.388
40	1.532	1.614	1.695	1.777	1.859	1.940	2.022	2.103	2.185	2.266	2.348	2.429
45	1.553	1.636	1.719	1.802	1.885	1.967	2.050	2.132	2.215	2.298	2.380	2.463
50	1.572	1.655	1.739	1.823	1.906	1.990	2.074	2.157	2.241	2.324	2.408	2.491
55	1.587	1.672	1.756	1.841	1.925	2.010	2.094	2.178	2.263	2.347	2.431	2.515
60	1.601	1.686	1.771	1.856	1.942	2.027	2.112	2.197	2.282	2.367	2.452	2.536
65	1.613	1.699	1.784	1.870	1.956	2.042	2.127	2.213	2.298	2.384	2.470	2.555
70	1.624	1.710	1.796	1.882	1.969	2.055	2.141	2.227	2.313	2.399	2.485	2.572
75	1.633	1.720	1.807	1.893	1.980	2.067	2.153	2.240	2.327	2.413	2.500	2.586
80	1.642	1.729	1.816	1.903	1.990	2.078	2.165	2.252	2.339	2.426	2.513	2.600
85	1.650	1.737	1.825	1.912	2.000	2.087	2.175	2.262	2.350	2.437	2.525	2.612
90	1.657	1.745	1.833	1.921	2.008	2.096	2.184	2.272	2.360	2.448	2.535	2.623
95	1.663	1.752	1.840	1.928	2.016	2.105	2.193	2.281	2.369	2.457	2.545	2.633
100	1.669	1.758	1.847	1.935	2.024	2.112	2.201	2.289	2.377	2.466	2.554	2.643
105	1.675	1.764	1.853	1.942	2.030	2.119	2.208	2.297	2.385	2.474	2.563	2.651
110	1.680	1.769	1.859	1.948	2.037	2.126	2.215	2.304	2.393	2.482	2.571	2.660
115	1.685	1.775	1.864	1.953	2.043	2.132	2.221	2.310	2.400	2.489	2.578	2.667
120	1.690	1.779	1.869	1.958	2.048	2.138	2.227	2.316	2.406	2.495	2.585	2.674
125	1.694	1.784	1.874	1.963	2.053	2.143	2.233	2.322	2.412	2.502	2.591	2.681
130	1.698	1.788	1.878	1.968	2.058	2.148	2.238	2.328	2.418	2.507	2.597	2.687
135	1.702	1.792	1.882	1.972	2.063	2.153	2.243	2.333	2.423	2.513	2.603	2.693
140	1.706	1.796	1.886	1.977	2.067	2.157	2.247	2.338	2.428	2.518	2.608	2.699
145	1.709	1.800	1.890	1.981	2.071	2.161	2.252	2.342	2.433	2.523	2.614	2.704
150	1.712	1.803	1.894	1.984	2.075	2.166	2.256	2.347	2.437	2.528	2.618	2.709
155	1.715	1.806	1.897	1.988	2.079	2.169	2.260	2.351	2.442	2.532	2.623	2.714
160	1.718	1.809	1.900	1.991	2.082	2.173	2.264	2.355	2.446	2.537	2.627	2.718
165	1.721	1.812	1.903	1.994	2.085	2.177	2.268	2.359	2.450	2.541	2.632	2.723
170	1.724	1.815	1.906	1.997	2.089	2.180	2.271	2.362	2.453	2.545	2.636	2.727
175	1.726	1.818	1.909	2.000	2.092	2.183	2.274	2.366	2.457	2.548	2.640	2.731
180	1.729	1.820	1.912	2.003	2.095	2.186	2.278	2.369	2.460	2.552	2.643	2.735
185	1.731	1.823	1.914	2.006	2.098	2.189	2.281	2.372	2.464	2.555	2.647	2.738
190	1.733	1.825	1.917	2.009	2.100	2.192	2.284	2.375	2.467	2.558	2.650	2.742
195	1.735	1.827	1.919	2.011	2.103	2.195	2.286	2.378	2.470	2.562	2.653	2.745
200	1.738	1.830	1.921	2.013	2.105	2.197	2.289	2.381	2.473	2.565	2.657	2.748

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