

**Signature verification:** On receiving the signature  $\{z, r, g, \beta\}$  any  $t$  of  $n$  verifiers can verify the signature of message  $m$ . Let  $t$  verifiers be denoted as  $u_i, i = 1, 2, \dots, t$ , with public information  $x_i, i = 1, 2, \dots, t$ . First, they need to work together to generate the public key  $y$ , associated with the secret key  $s$  as  $y = \beta^s \text{ mod } p$ .

**Theorem:** With the knowledge of  $g$ , and  $t$  secret shadows,  $S_i, i = 1, 2, \dots, t, y$ , can be generated.

**Proof:** With the knowledge of the secret shadow  $S_1, u_1$  computes

$$SK1 = g_v \left( S_1 \left( \prod_{j=1, j \neq 1}^t \frac{-x_j}{x_1 - x_j \text{ mod } q} \right) \text{ mod } w \right) \text{ mod } p$$

$$= g_v \left( \alpha \left( f(x_1) \prod_{j=1, j \neq 1}^t \frac{-x_j}{x_1 - x_j \text{ mod } q} \right) \text{ mod } w \right) \text{ mod } p$$

SK1 is sent to  $u_2, u_2$  uses his secret shadow  $S_2$  to compute

$$SK2 = SK1 \left( S_2 \left( \prod_{j=1, j \neq 2}^t \frac{-x_j}{x_2 - x_j \text{ mod } q} \right) \text{ mod } w \right) \text{ mod } p$$

$$= g_v \left\{ \begin{array}{l} \left( \alpha \left( f(x_1) \prod_{j=1, j \neq 1}^t \frac{-x_j}{x_1 - x_j \text{ mod } q} \right) \right) \\ \times \left( \alpha \left( f(x_2) \prod_{j=1, j \neq 2}^t \frac{-x_j}{x_2 - x_j \text{ mod } q} \right) \right) \end{array} \right\} \text{ mod } p$$

Just by repeating the same procedure until the  $t$ th verifier has used his secret shadow to work on the value obtained from its predecessor, the public key  $y_v$  can be finally obtained as

$$SKt = g_v \left( \alpha \left( \sum_{i=1}^t f(x_i) \prod_{j=1, j \neq i}^t \frac{-x_j}{x_i - x_j \text{ mod } q} \right) \text{ mod } w \right) \text{ mod } p$$

$$= g_v \left( \prod_{i=1}^t S_i \left( \prod_{j=1, j \neq i}^t \frac{-x_j}{x_i - x_j \text{ mod } q} \right) \text{ mod } w \right) \text{ mod } p$$

$$= g_v^{\alpha^t} \text{ mod } p \quad (\text{from eqn. 1})$$

$$= \beta_v^s \text{ mod } p \quad (\text{from eqn. 2})$$

$$= y_v \quad \text{QED}$$

The signature of  $m$ , can then be verified by checking the following relation as:

$$\beta_v^{m \cdot v} = r_v^{z \cdot v} y_v^{r \cdot v} \text{ mod } p$$

If the above relation does hold, the signature of  $m$ , has been verified.

**Security discussion:** In this scheme, user  $A$  uses the secret key  $s$  to sign messages repeatedly; but the corresponding public key  $y$ , is different for each message. This is because  $y_v$  is revealed after verifying each message and thus it cannot be used again, otherwise it will lose the property of the shared verification signature scheme. On the other hand, even multiple public keys associated with the same secret key  $s$  have been revealed; to derive the secret key we have to solve the discrete logarithm problem. The secret shadow for each verifier is also protected by the discrete logarithm problem during the public key derivation process.

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## Modified Chang-Hwang-Wu access control scheme

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*Indexing terms:* Cryptography, Information theory

It is found that some security classes in the Chang-Hwang-Wu access control scheme can be combined to derive the secret key of their immediate ancestor in some cases. Some slight modifications to the proposed scheme to enhance the security levels are also given.

**Introduction:** In [1], the authors proposed an efficient cryptographic key assignment scheme for solving the access control problem in a partially ordered hierarchy. Basically, the scheme is based on the Newton interpolation method and a predefined one-way function. The scheme not only reduces the amount of storage required for storing public parameters, but also is simple and efficient in generating and deriving keys. However, some security classes can be combined to derive the secret key of their immediate ancestor in some cases. We also give some modifications to slightly modify that subject scheme so that the security will be greatly improved.

**Weakness of proposed scheme:** In [1], the authors assumed that there is a trusted third party in the system that is responsible for generating and distributing keys. They assigned each security class  $C_i$  an associated distinct pair  $(a_i, b_i)$  as the public parameter. Assume that the security class  $C_i$  has  $d$  immediate successors  $C_{i1}, C_{i2}, \dots, C_{id}$ . The security class  $C_i$ , using the Newton interpolation method, constructs an interpolating polynomial  $H_i(X)$  of degree  $d$  by interpolating on the points  $(0, K_i), (a_{i1}, b_{i1}), (a_{i2}, b_{i2}), \dots, (a_{id}, b_{id})$  over  $GF(P)$ . Let  $H_i(X) = (K_i + \sum c_j X^j) \text{ mod } P$ , where  $c_j$  is an integer between 0 and  $P-1$ . The secret key  $K_{ij}$  of  $C_{ij}$  is calculated by  $K_{ij} = f(c_{ij}) \text{ mod } P$ , for  $j = 1, 2, \dots, d$ , where  $c_j$  is the coefficient of the term  $X^j$  in  $H_i(X)$ .

The key derivation is quite similar to the key generation. Using the Newton interpolation method, they reconstruct the interpolating polynomial  $H_i(X) = (K_i + \sum c_j X^j) \text{ mod } P$  by interpolating on points  $(0, K_i), (a_{i1}, b_{i1}), (a_{i2}, b_{i2}), \dots, (a_{id}, b_{id})$ . The secret key of  $C_{ij}$  is thus obtained from  $K_{ij} = f(c_{ij}) \text{ mod } P$ , where  $c_j$  is the coefficient of the term  $X^j$  in  $H_i(X)$ .

In the proposed scheme, the pairs of public parameters  $(a_{ij}, b_{ij})$ s, the prime number  $P$  and the predefined one-way function  $f$  are known to all security classes in the hierarchy. The security class  $C_i$  only keeps its own secret key  $K_i$  secretly.

We now show the weakness in the security of the above scheme. Let  $C_{i1}, C_{i2}, \dots, C_{id}$  be  $d$  immediate successors of the security class  $C_i$ . Because the points  $(a_{i1}, b_{i1}), (a_{i2}, b_{i2}), \dots, (a_{id}, b_{id})$  for  $C_{i1}, C_{i2}, \dots, C_{id}$ , respectively, are known to each security class, we can construct an interpolating polynomial  $H_i(X) = (K_i + \sum c_j X^j) \text{ mod } P$  with one unknown point  $(0, K_i)$  and  $d$  known points  $(a_{i1}, b_{i1}), (a_{i2}, b_{i2}), \dots, (a_{id}, b_{id})$ , based on the Newton interpolation method [2]. The formula is as follows:

$$H_i(x) = (K_i + g_1(K_i)X + g_2(K_i)X^2 + \dots + g_d(K_i)X^d) \text{ mod } P \quad (1)$$

where  $g_j(K_i)$  can be represented as a linear polynomial with one

unknown variable  $K_j$ . (i.e.  $g_j(K_i) = g_1 K_i + g_0$ ,  $g_1, g_0$  are two integers.) Now suppose  $f$  is a one-way function of degree  $d$ . Then the coefficient  $g_j(K_i)$  in eqn. 1 is substituted for the function  $f$ . Consequently, we have

$$\begin{aligned} f(g_j(K_i)) &= K_j \\ &= (e_{jd} K_i^d + e_{j(d-1)} K_i^{d-1} + \dots \\ &\quad + e_{j1} K_i + e_{j0}) \bmod P \quad \text{for } j = 1, 2, \dots, d \end{aligned}$$

We have  $d$  equations and  $d$  variables, by the Gauss elimination method [3], thus when  $d$  immediate successors of the security class  $C_i$  collude, the secret key  $K_i$  of  $C_i$  is revealed. In fact, only one security class is needed to break the scheme when the one-way function is in quadratic residue modulo (i.e.  $f(X) = g_2 X^2 + g_1 X + g_0 \bmod P$ ) [4]. Therefore, when the one-way function is a polynomial of degree  $d$ , they need  $d - 1$  immediate successors for conspiracy attack.

**Modifications and discussions:** From the above statements and example, we see that  $d - 1$  security classes can attack the one-way function of degree  $d$ . We now give two modifications for withstanding this attack.

(i) **Modification 1:** Each security class has its own private one-way function. This means that the one-way function  $f$  has to be kept secretly by the security class  $C_i$  and the third party.

(ii) **Modification 2:** Choose a one-way function of degree  $d + 2$  where  $d$  is a maximal number of immediate successors of each security class in the whole system.

Although Modification 1 can prevent conspiracy attack, it wastes a large amount of storage space to store these one-way functions. Modification 2 not only keeps the advantage of the proposed scheme but also enhances the security.

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## Measuring technique for characterising the electrical properties of piezoelectric tubular devices

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Indexing terms: Piezoelectric devices, Polymers

The Letter describes a simple measuring technique which is useful for characterising in a nondestructive manner the electrical properties of tubular devices made from piezoelectric polymers. The proposed measuring technique allows us to estimate the so-called  $d_{33}^*$  constant, a parameter often used for evaluating piezoelectric activity.

**Introduction:** Recent substantial advances in the production of synthetic polymers have made possible the conception, fabrication and testing of a variety of medical devices to be used in basic research as well as in clinical investigations. Among them, piezoelectric materials, such as polyvinylidene fluoride (PVDF) [1], or electret materials, such as polytetrafluoroethylene (PTFE) [2], have been proposed and used as guidance channels, that is small tubular devices, in a number of applications that require some physical agents for promoting regenerative processes. The ability of piezoelectric materials to enhance tissue regeneration is believed to be an effect of their electrical properties. Hence, simple means for characterising these properties in a nondestructive manner are required. A thorough investigation of the properties of piezoelectric materials requires considerable effort [3]; in some cases, however, a simple characterisation is sufficient for the purpose of comparing different devices under identical operating conditions [4], so as to select samples with roughly equivalent properties for the intended application.

**Theory of measurement:** The aim of this Letter is to describe the simple apparatus we propose for measuring the so-called  $d_{33}^*$  constant of piezoelectric tubular devices. Refer to Fig. 1 for an explanation of the terms and notation usually adopted in dealing with piezoelectric planar films [3]. The transverse and machine directions refer to the directions along which the mechanical stretching of the film must be exerted; the poling direction relates to the direction of the electric field to be applied to the stretched material during the fabrication of the device. Both processing steps are necessary for inducing proper conformation changes in the crystalline structure of the polymer so as to make it piezoelectrically active.

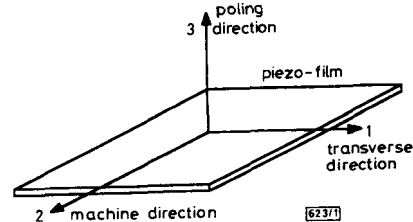


Fig. 1 Axis nomenclature for piezoelectric films

Piezoelectric measurements on polymer films are usually made by properly applying a mechanical tension to the sample and measuring the short-circuit charge or the open circuit voltage on opposing electrodes. Owing to the anisotropy of piezoelectric materials, it is important to specify the directions along which the mechanical tension is applied or the electrodes are placed. The  $d_{33}^*$  constant is defined as

$$d_{33}^* = \frac{\Delta Q}{\Delta F} \quad (1)$$

where  $\Delta Q$  is the charge developed in a double-sided metallised film of piezoelectric material with surface area  $A$ , as a consequence of a force  $\Delta F$  applied in the normal direction, that is direction 3 in Fig. 1. Let  $E$  denote the transverse electric field existing between the two electrodes. The underlying boundary condition  $E=0$  is assumed for eqn. 1, therefore the electrodes must be short-circuited: when the charge  $\Delta Q$  is measured by a charge amplifier, the short-circuit conditions are virtually met. The  $d_{33}^*$  constant is