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Thermophoretic deposition efficiency in a cylindrical tube taking into account developing flow at the entrance region

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Abstract

This study investigates the effect of developing flow in a circular tube on the thermophoretic particle deposition efficiency using the critical trajectory method numerically. When both flow and temperature are fully developed (combined fully developed), present results agree with previous theories for the long tube where the flow temperature approaches that of the wall. When the flow is fully developed and temperature is developing, it is found that only near the thermal entrance region (or temperature jump region) of the tube the deposition efficiency is slightly higher than the combined fully developed case (flow and temperature), while the deposition efficiency remains the same for the long tube. When both flow and temperature are developing (or combined developing), the deposition efficiency is about twice that of the combined fully developed case for the long tube and is much higher near the entrance of the tube. Non-dimensional equations are developed empirically to predict the thermophoretic deposition efficiency in combined developing and combined fully developed cases under laminar flow conditions.

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Keywords: Thermophoretic deposition efficiency; Developing flow effect; Particle deposition

1. Introduction

Thermophoresis is a physical phenomenon in which aerosol particles move toward the direction of decreasing temperature when subjected to a thermal gradient. Many previous investigators have studied thermophoresis-related subject such as thermophoretic coefficient (Brock, 1962; Derjaguin, Rabinovich, Storozhilova, & Shcherbina, 1976; Talbot, Cheng, Schefer, & Willis, 1980), thermophoretic deposition in tube flow (Nishio, Kitani, & Takahashi, 1974; Walker, Homsy, & Geyling, 1979; Batchelor & Shen, 1985; Montassier, Boulaud, Stratmann, & Fissan, 1990; Montassier, Boulaud,

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Nomenclature

B	dynamic mobility $B = C/3\pi\mu d_p$
C	slip correction factor
C_d	drag coefficient
C_m	momentum exchange coefficient
C_p	specific heat capacity at constant pressure
C_s	thermal slip coefficient
C_t	temperature jump coefficient
d_p	diameter of the particles
D	particle diffusivity
D_t	tube diameter
k_g	gas thermal conductivity
k_p	particle thermal conductivity
K_{th}	thermophoretic coefficient
L	tube length
m_p	particle mass
Pe_g	gas Peclet number $(u_m \cdot r_0^2)/(\alpha \cdot L)$
Pr	gas Prandtl number
r_0	tube radius
r_c	critical radial position
R	radial coordinate
R	dimensionless radial coordinate r/r_0
R_c	dimensionless critical radial position r_c/r_0
Re	Reynolds number
Re_p	particle Reynolds number
Q	volumetric flow rate
T	gas temperature
T_e	gas temperature at tube entrance
T_m	mixing-cup temperature
T_w	wall temperature
$\vec{\nabla}T$	temperature gradient
u_m	average gas velocity
U	gas velocity (z -component)
V	gas velocity (r -component)
\vec{V}_{th}	thermophoretic velocity
z_+	dimensionless axial coordinate $z/(r_0 Re Pr)$
Z	axial coordinate
Z	dimensionless distance from the entry $Z = z/(0.05 D_t Pe_g)$

Greek letters

α	thermal diffusivity $k_g/(\rho C_p)$
β_1	thermophoretic parameter $\beta_1 = Pr K_{th}/(T_e - T_w)/T_w$

η	thermophoretic deposition efficiency
θ^*	dimensionless temperature $T_w/(T_e - T_w)$
λ	mean free path of air
μ	air dynamic viscosity
ν	kinematic viscosity
ρ	gas density
ρ_p	particle density
τ	particle relation time $\tau = \rho_p d_p^2 C / 18\mu$

& Renoux, 1991; Shimada, Seto, Okuyama, 1994; Stratmann, Otto, & Fissan, 1994; Zoulalian & Albiol, 1995; Chiou, 1996; Romay, Takagaki, Pui, & Liu, 1998), thermophoretic deposition in a plate-to-plate thermal precipitator (Tsai & Lu, 1995). The experimental data of Tsai and Lu (1995) suggested that the equation of thermophoretic coefficient proposed by Talbot et al. (1980) is accurate.

The thermophoretic velocity can be calculated as

$$\vec{V}_{th} = -\frac{\nu K_{th}}{T} \vec{\nabla} T, \tag{1}$$

where K_{th} is the thermophoretic coefficient defined by Talbot et al. (1980) as

$$K_{th} = \frac{2C_s C}{(1 + 3C_m(2\lambda/d_p))} \times \left(\frac{k_g/k_p + C_t(2\lambda/d_p)}{1 + 2(k_g/k_p) + 2C_i(2\lambda/d_p)} \right). \tag{2}$$

For complete accommodation, the reasonable values of C_m , C_s and C_t are 1.14, 1.17 and 2.18, respectively (Talbot et al., 1980; Montassier et al., 1991).

Previous theories on thermophoretic deposition efficiency in laminar tube flow, listed in Table 1, are restricted to fully developed flow only. These equations are applicable for a long tube where gas temperature approaches that of the wall. Also shown in Table 1 for comparison are predicted equations of the present study. In previous theories, Walker et al. (1979) and Batchelor and Shen

Table 1

Theoretical expressions of thermophoretic deposition efficiency for a long tube where the temperature of hot gas has approached the tube wall temperature

Walker et al. (1979)	$\eta = \frac{Pr K_{th}}{T_w} (T_e - T_w)$
Batchelor and Shen (1985)	$\eta = Pr K_{th} \left(\frac{T_e - T_w}{T_e} \right) \left(1 + (1 - Pr K_{th}) \left(\frac{T_e - T_w}{T_e} \right) \right)$
Stratmann et al. (1994)	$\eta = 1 - \exp \left(-0.845 \left(\frac{Pr K_{th} + 0.025}{T_w/(T_e - T_w) + 0.28} \right)^{0.932} \right)$
This study (fully developed flow)*	$\eta_f = 0.783 \beta_1^{0.94}$
(developing flow)*	$\eta_d = 0.549 \beta_1^{0.48}$

*The thermophoretic parameter β_1 is defined as $\beta_1 = Pr K_{th} \frac{T_e - T_w}{T_w}$.

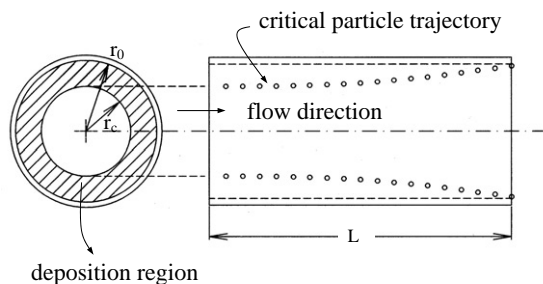


Fig. 1. Critical particle trajectory.

(1985) considered particle transport due to convection and thermophoresis only, thermal diffusivity of particle was neglected. Such an assumption is valid only when the dimensionless deposition parameter for laminar diffusion, ε ($\varepsilon = DL/Q$), is much less than 0.0001 (Hinds, 1999). Walker et al. (1979) developed two models, one for a short tube, and another for a sufficiently long tube where the gas temperature approaches that of the tube wall. For the short tube, Walker et al. solved the particle transport equation analytically and developed an equation as a function of $Pr K_{th}$, θ^* and z/Pe_g . In the case of the long tube, they solved the particle transport equation by a particle trajectory method and found that the thermophoretic deposition efficiency is a function of $Pr K_{th}$ and θ^* (see Table 1). Batchelor and Shen (1985) also found that deposition efficiency for a long tube is a function of $Pr K_{th}$ and θ^* . The predicted thermophoretic deposition efficiency of Batchelor and Shen (1985) agrees well with Walker et al. (1979) only when $Pr K_{th} = 1$. However, when $Pr K_{th}$ and T_e/T_w are small, the predicted thermophoretic deposition efficiency does not agree well with Walker et al. (1979).

Stratmann et al. (1994) utilized the extended SIMPLER algorithm developed by Patankar (1980) to calculate thermophoretic deposition efficiency and a non-dimensional thermophoretic deposition model was developed neglecting thermal diffusion as shown in Table 1.

From the above discussion, it can be seen that the entrance flow effect on particle deposition efficiency in tube flow has rarely been investigated. Recently, Fan, Cheng, and Yeh (1996) found that the gas collection efficiency of an annular diffusion denuder is higher for developing flow than fully developed flow. Since the highest temperature gradient and uniform velocity near the wall occur at the entrance of a tube where both flow and temperature are developing, thermophoretic deposition in the entrance region may be enhanced and is the objective of this study.

In this study, the critical particle trajectory method is used to obtain its thermophoretic deposition efficiency assuming that particle diffusion is negligible. The critical particle trajectory is shown in Fig. 1. A particle starting at the critical radial position, r_c , at the entrance will deposit just at the end of the tube of length L . When flow is fully developed, an analytical temperature field is available and the particle equations of motion can be solved to obtain the critical radial position and the thermophoretic deposition efficiency. However, when both flow and temperature are developing, there is no analytical equation for the temperature field. In this study, we found that developing velocity field simulated numerically by the SIMPLE algorithm (Patankar, 1980) was not accurate. Therefore, the analytical velocity distribution developed by Sparrow, Lin, and Lundgren (1964) was used to calculate the developing temperature profile numerically, and then the critical particle trajectory and deposition efficiency was obtained by solving the particle equations of motion.

2. Present method for calculating the thermophoretic deposition efficiency

2.1. Velocity and temperature profiles

In laminar flow, the entry length for the velocity profile to become fully developed is given by [Incropera and De wit \(1996\)](#) as

$$\left(\frac{z}{D_t}\right)_{\text{dep}} \cong 0.05Re \tag{3}$$

and the entry length for the temperature profile to become fully developed is ([Kays & Crawford, 1980](#))

$$\left(\frac{z}{D_t}\right)_{\text{dep}} \cong 0.05Re Pr = 0.05Pe_g. \tag{4}$$

As a result, the temperature is fully developed earlier than velocity when $Pr < 1$. If the thermal entrance length is much shorter than the total tube length, one can assume that the temperature is fully developed. In fully developed flow, the velocity profile is parabolic and can be written as

$$u(r) = 2u_m \left[1 - \left(\frac{r}{r_0}\right)^2 \right]. \tag{5}$$

For a developing flow, the two-dimensional developing velocity field in the tube was derived by [Sparrow et al. \(1964\)](#) as

$$\frac{u}{u_m} = \omega = 2(1 - R^2) + \sum_{i=1}^{\infty} \frac{4}{\sigma_i^2} \left[\frac{J_0(\sigma_i R)}{J_0(\sigma_i)} - 1 \right] e^{-\sigma_i^2 Z^*}, \tag{6}$$

where

$$\sigma_1 = 5.13562, \quad \sigma_{i+1} = \sigma_i + \pi, \quad i = 1, 2, 3 \dots,$$

$$Z^* = \frac{z^* v}{u_m r_0^2},$$

$$z = \int_0^{z^*} \varepsilon \, dz^*,$$

$$\varepsilon = \frac{\int_0^1 (2\omega - 1.5\omega^2)(\partial\omega/\partial X^*)R \, dR}{(\partial\omega/\partial R)_1 + \int_0^1 (\partial\omega/\partial R)^2 R \, dR}.$$

Eq. (6) was used to calculate the developing temperature field for developing flow numerically by solving the following 2-D cylindrical energy equation:

$$u \frac{\partial T}{\partial z} + v \frac{\partial T}{\partial r} = \alpha \left(\frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} \right) \tag{7}$$

with boundary conditions

$$T(r, 0) = T_e, \quad T(r_0, z) = T_w, \quad \frac{\partial T}{\partial r}(0, z) = 0. \quad (8)$$

For a fully developed flow and assuming all relevant physical properties are constant, Eq. (7) can be non-dimensionalized as

$$(1 - r_+^2) \frac{\partial \theta}{\partial z_+} = \frac{\partial^2 \theta}{\partial r_+^2} + \frac{1}{r_+} \frac{\partial \theta}{\partial r_+}, \quad (9)$$

where

$$\theta = \frac{T_w - T}{T_w - T_e}, \quad r_+ = \frac{r}{r_0}, \quad z_+ = \frac{z/r_0}{Re Pr}.$$

When the temperature is fully developed, or when the dimensionless distance Z is greater than the dimensionless thermal entry length, the dimensionless temperature profile $(T_w - T)/(T_w - T_m)$ is invariant in the axial direction, and is a function of radial coordinate only. The fully developed temperature profile is obtained by solving Eq. (9) analytically and is given (Skelland, 1974) as

$$\frac{T_w - T}{T_w - T_m} = \frac{\sum_{j=1}^{j=\infty} B_j \phi_j(r_+) \exp(-\beta_j^2 z_+)}{\sum_{j=1}^{j=\infty} (-4B_j/\beta_j^2) (d\phi_j/dr_+)_{r_+=1} \exp(-\beta_j^2 z_+)}, \quad (10)$$

where

$$r_+ = \frac{r}{r_0}, \quad z_+ = \frac{z/r_0}{Re Pr},$$

$$\phi_j(r_+) = \sum_{i=0}^{i=\infty} \alpha_{ji} r_+^i, \quad \alpha_{ji} = 0, \quad \text{for } i < 0,$$

$$\alpha_{ji} = 1 \quad \text{for } i = 0, \quad \alpha_{ji} = -\beta_j^2 (\alpha_{i-2} - \alpha_{i-4}) / i^2,$$

$$\beta_j = 4(j-1) + \frac{8}{3} \quad j = 1, 2, 3, \dots,$$

$$B_j = (-1)^{j-1} \times 2.84606 \beta_j^{2/3}$$

$$\frac{-B_j}{2} \left(\frac{d\phi_j}{dr_+} \right)_{r_+=1} = 1.01276 \beta_j^{-1/3}.$$

For a developing temperature profile, the analytical solution of Eq. (9), Graetz's problem, is (Skelland, 1974)

$$\frac{T_w - T}{T_w - T_e} = \sum_{j=1}^{j=\infty} B_j \phi_j(r_+) \exp(-\beta_j^2 z_+). \quad (11)$$

We have compared the dimensionless developing temperature profile of Eq. (11) with that described in Grigull and Tratz (1965) for the fully developed flow and found both profiles agree very well.

In the numerical simulation of developing temperature for developing flow at the entrance of a tube, the finite volume method and SIMPLE algorithm were used. In the test run, the number of grid points in the computational domain was either 4000 (100 in the axial direction \times 40 in radial direction), 12,000 (200 in the axial direction \times 60 in radial direction) or 24,000 (300 in the axial direction \times 80 in radial direction). The numerical results showed that the number of grid of 12,000 is accurate enough and was adopted in the further study. The grid spacing is finer near the wall and inlet where temperature gradients in radial direction are expected to be larger. In the simulation, the influence of radial fluid velocity and temperature-dependent fluid properties on thermophoretic deposition efficiency was accounted for.

2.2. Particle trajectory and corresponding thermophoretic deposition efficiency

Particle equations of motion were solved to obtain the particle trajectory and thermophoretic deposition efficiency. In cylindrical coordinates, the particle equations of motion in the z (radial) and r (axial) directions are

$$\frac{d^2z}{dt^2} = C_d \frac{Re_p}{24} \frac{(u - dz/dt)}{\tau}, \tag{12}$$

$$\frac{d^2r}{dt^2} = C_d \frac{Re_p}{24} \frac{(v - dr/dt)}{\tau} + \frac{V_{th}}{Bm_p}, \tag{13}$$

where C_d is the drag coefficient, which was proposed by [Rader and Marple \(1985\)](#) as

$$C_d = \frac{24}{Re_p} (1 + 0.0916 Re_p), \quad Re_p < 5, \tag{14}$$

$$C_d = \frac{24}{Re_p} (1 + 0.158 Re_p^{2/3}), \quad 5 < Re_p < 1000. \tag{15}$$

In order to calculate the thermophoretic deposition efficiency, one needs to know the critical radial position of particle trajectory, r_c . For the combined fully developed case, the analytical equation, Eq. (A.6), written in the appendix is solved to obtain r_c and the corresponding efficiency. For the case of fully developed flow and developing temperature, the particle equations of motion have to be solved numerically by the fourth-order Runge–Kutta method. For the combined developing case, the particle equations of motion of Eqs. (12) and (13) were integrated numerically by means of the fourth-order Runge–Kutta method. As the particle equations of motion are integrated through the domain of interest, the initial velocity at the entrance is taken equal to the average gas flow velocity. The new particle position and velocity after a small time increment is calculated by numerical integration. The procedure is repeated until the particle hits the tube wall or leaves the calculation domain.

After obtaining the critical radial position, r_c , the efficiency of thermophoretic deposition for fully developed flow is calculated in the following equation assuming the particle concentration is uniform at the inlet:

$$\eta_f = \frac{\int_{r_c}^{r_0} 2u_m \left(1 - \frac{r^2}{r_0^2}\right) 2\pi r \, dr}{u_m \pi r_0^2} = 1 - 2 \left(\frac{r_c}{r_0}\right)^2 + \left(\frac{r_c}{r_0}\right)^4 \tag{16}$$

For the combined developing case, besides assuming uniform particle concentration, the velocity profile is known to be uniform at the entrance of the tube, and the deposition efficiency can be calculated as

$$\eta_d = 1 - \left(\frac{r_c}{r_0} \right)^2. \quad (17)$$

3. Results and discussion

3.1. Thermophoretic deposition efficiency for fully developed T - and V -field

The fluid and particle properties used in the calculation were estimated at the averaged temperature of inlet gas and tube wall. Fig. 2 compares the thermophoretic deposition efficiency of the present study and previous theories at a flow rate of 5 l/min for the pipe geometry described in the experiment of Romay et al. (1998). The tube length is 0.905 m, tube diameter is 0.0049 m and the Reynolds number of the gas flow equals to 1423 which is in the laminar flow region. The thermal conductivity is 6.0 W/(m K) for NaCl particle (Romay et al., 1998). Fig. 2 shows that the deposition efficiency of submicron particle agrees well with the prediction of Stratmann et al. (1994) and Batchelor and Shen (1985) for the long tube, the deviation is smaller than 2%. It can be seen that the thermophoretic deposition efficiency increases at first with an increasing inlet gas temperature and decreasing particle size, but when particle size is further decreased to 0.05 and 0.03 μm , the thermophoretic deposition efficiency remains almost the same (Fig. 2).

In Fig. 3, the deposition efficiency calculated by the expression of Stratmann et al. (1994) (see Table 1) is compared with the present study. It shows that the present theory is in very good agreement with the expression of Stratmann et al.

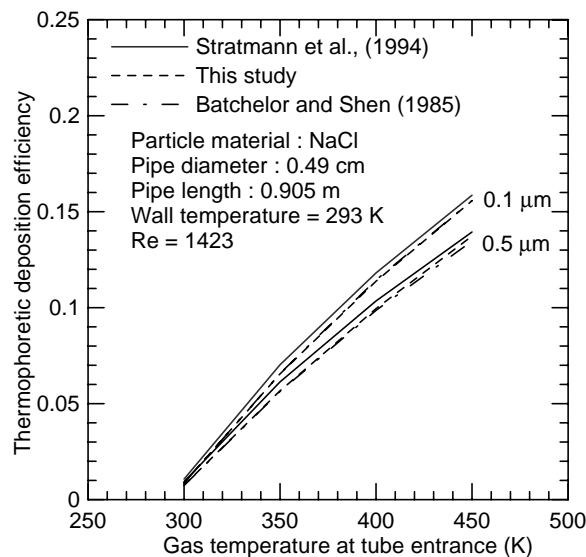


Fig. 2. Comparison of theoretical deposition efficiency with previous theories in laminar tube flow.

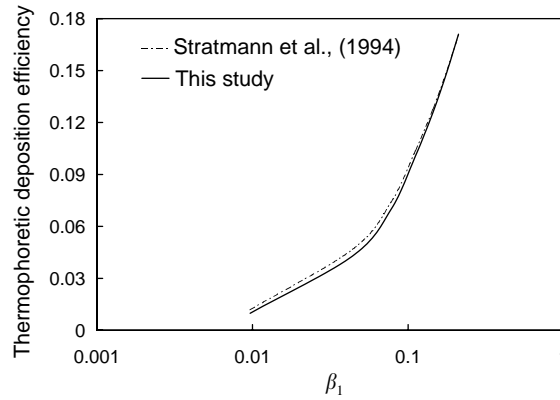


Fig. 3. Comparison of present theoretical thermophoretic deposition efficiency with numerical prediction of Stratmann et al. (1994) in laminar tube flow.

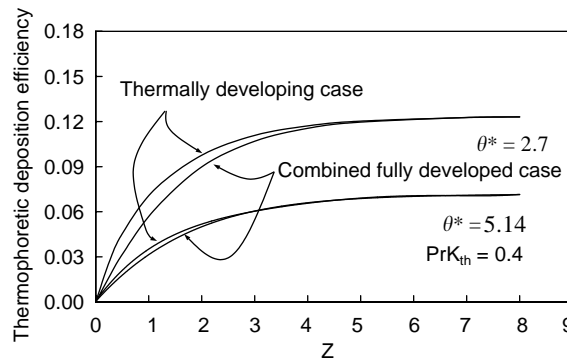


Fig. 4. Comparison of accumulated thermophoretic deposition efficiency between different θ^* 's versus Z , fully developed flow.

3.2. Thermophoretic deposition efficiency for fully developed flow but developing temperature

When the flow is fully developed, the temperature could still be developing when there is a sudden temperature jump in the tube wall. The specific problem is as follows: The gas enters with a uniform particle concentration and temperature T_e and flow through a tube with a wall temperature equal to T_e . At some distance far enough downstream such that the flow is fully developed, the wall temperature is decreased suddenly to T_w , which is different from T_e . This creates a “temperature jump” and the temperature field will start to develop from there. The developing temperature gradient in the radial direction is higher near the position of temperature jump, and the deposition efficiency is then expected to be higher than in the fully developed case.

Fig. 4 shows that accumulated thermophoretic deposition efficiency calculated for the developing temperature case is higher than for the fully developed temperature case when the dimensionless distance from the entry, Z , is less than 5.0. Z is defined as $Z = z / (0.05D_tPe_g)$, where z is the distance from the position of temperature jump, and $0.05D_tPe_g$ is the thermal entry length. It can

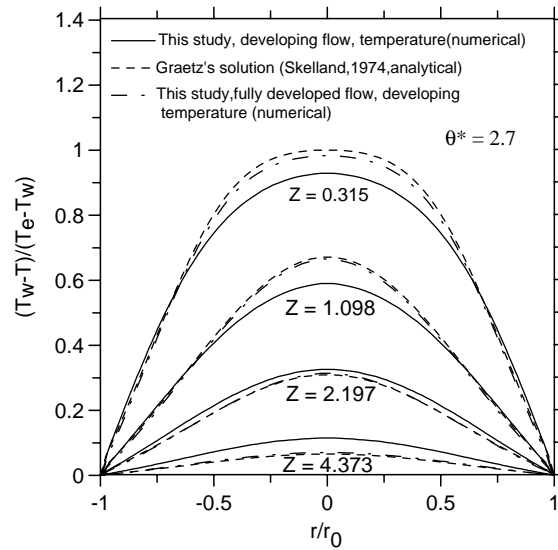


Fig. 5. Dimensionless temperature profile as a function of dimensionless radial and axial coordinates.

be seen that the deposition efficiency increases from zero at the position of temperature jump, and approaches an asymptotic limit after Z is greater than about 5.0, when the temperature of hot gas approaches that of the tube wall. After Z becomes greater than 5.0, the deposition efficiencies for both cases are the same. Fig. 4 also shows that when θ^* is higher (T_e is close to T_w), the deviation of the deposition efficiency between developing temperature case and fully developed temperature case is smaller. For example, at $Z=0.25$ and when θ^* equals 2.70, the deviation is 37%; and when θ^* equals 5.14, the deviation is 20%.

3.3. Thermophoretic deposition efficiency for developing flow and developing temperature

Fig. 5 illustrates the effect of developing flow on the temperature distribution of a tube when θ^* equals 2.7. In order to make sure the simulated temperature field is correct, a fully developed velocity profile was first used to simulate the developing temperature field numerically, which is then compared with Graetz's analytical solution, Eq. (11). Good agreement seen in Fig. 5 indicates that the present simulation is accurate. The simulated developing temperature profile based on the developing flow profile (Eq. (6)) shows that the temperature gradient close to the tube wall is higher than the case when the flow is fully developed, as depicted in Fig. 5.

Fig. 6 shows the accumulated thermophoretic deposition efficiency for a tube at different Z positions with $Pr K_{th} = 0.31$ and $\theta^* = 5.14$ based on different numbers of grid points. As the resolution of the flow improves beyond 12,000, the deposition efficiency curves do not change appreciably. Therefore, 12,000 grids were used in the subsequent simulation.

Fig. 7 shows the effect of the developing velocity on the accumulated thermophoretic deposition efficiency at different θ^* values. It can be seen that the deposition efficiency approaches an asymptotic limit when the hot gas temperature is close to the tube wall, after Z is greater than about 5.0. For

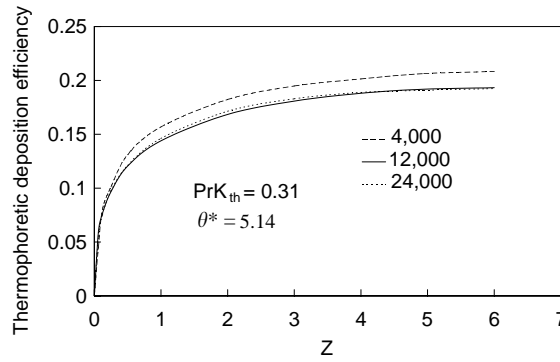


Fig. 6. Accumulated thermophoretic deposition efficiency of aerosol particles using different numbers of grids, combined developing case.

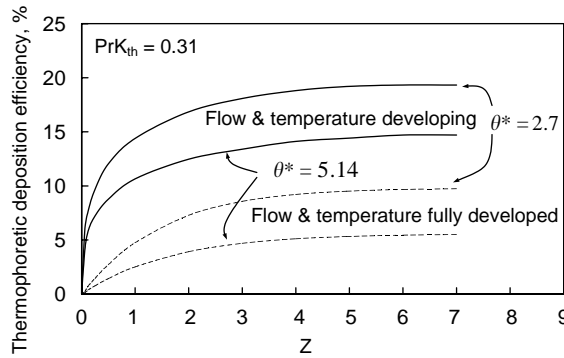


Fig. 7. Comparison of accumulated thermophoretic deposition efficiency between different θ^* 's versus Z.

the combined developing case, the limiting value for the thermophoretic deposition efficiency of an infinite long tube is higher than the combined fully developed case. For example, when PrK_{th} equals 0.31 and θ^* equals 2.7, the deposition efficiencies are 19.3% and 9.8%, respectively. Table 2 gives a list of the critical radial position and the corresponding accumulated thermophoretic deposition efficiency of the combined developing and combined fully developed cases when $PrK_{th} = 0.31$ and $\theta^* = 2.7$. It shows that the critical radial position of the combined developing case is larger than that of the combined fully developed case indicating that the inward radial velocity tends to increase r_c and reduce deposition efficiency. And the effect of inward radial velocity is larger than the increase of thermal gradient near the inlet on the position of r_c . However, since both the flow velocity and particle concentration are uniform at the tube entrance for the combined developing case, the resulting deposition efficiency is larger than the combined fully developed case in which the velocity and hence the particle flux is almost zero near the wall.

Table 2
Accumulated thermophoretic deposition efficiency of combined developing case and combined fully developed case at different positions of a tube

Z	Combined developing		Combined fully developed	
	R_c	Eff. (%)	R_c	Eff. (%)
0.06	0.9698	5.96	0.9689	0.37
0.12	0.9610	7.64	0.9561	0.74
0.25	0.9504	9.67	0.9373	1.48
0.50	0.9378	12.05	0.9132	2.76
1.00	0.9253	14.39	0.8836	4.81
2.00	0.9119	16.85	0.8534	7.38
3.00	0.9051	18.08	0.8691	8.40
4.00	0.9011	18.80	0.8333	9.34
4.95	0.8990	19.19	0.8304	9.64
6.00	0.8982	19.32	0.8288	9.80

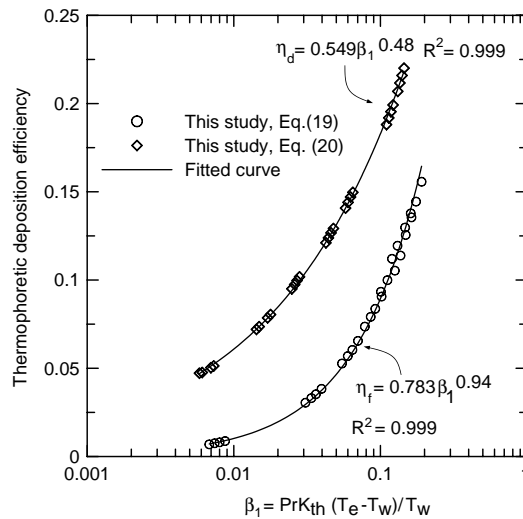


Fig. 8. Thermophoretic deposition efficiency as a function of thermophoretic parameter β_1 in laminar fully developed flow and developing flow.

3.4. Empirical equation of thermophoretic deposition efficiency for the case of a long tube

The thermophoretic deposition efficiency is a unique function of the dimensionless parameter β_1 . Fig. 8 shows this relationship; correlation equations are also indicated.

It can be seen from Eq. (A.6) in the appendix that particle transport due to combined convection and thermophoresis depends on three parameters, the product of the Prandtl number and thermophoretic coefficient, $Pr K_{th}$, the dimensionless temperature $(T_e - T_w) / T_e$ and the gas Peclet

number Pe_g . The thermophoretic deposition efficiency is shown not to depend on the gas Peclet number by Walker et al. (1979), but only on the thermophoretic parameter β_1 , which is

$$\beta_1 = Pr K_{th} \frac{T_c - T_w}{T_w}. \tag{18}$$

In this study, the empirical equation for the combined fully developed is found to be

$$\eta_f = 0.783\beta_1^{0.94}, \quad 0.007 < \beta_1 < 0.19 \tag{19}$$

and for the combined developing case, the empirical equation is found to be

$$\eta_d = 0.549\beta_1^{0.48}, \quad 0.006 < \beta_1 < 0.15. \tag{20}$$

Fig. 8 also shows that the correlation equation fits the present numerical results very well.

4. Conclusion

This study investigates the effect of developing flow and temperature of a cylinder tube on the thermophoretic deposition efficiency. It is found that by taking into account the effect of developing flow at the entrance region, a higher deposition efficiency is obtained than that of fully developed flow. Although the developing temperature gradients in the radial direction of developing temperature profiles are higher than those of fully developed temperature profiles at the entrance of a tube or the position of temperature jump, the increase of deposition efficiency is almost negligible for a long tube, if the flow is fully developed. However, when both flow and temperature are developing, the deposition efficiency is significantly higher than the case of fully developed flow in which the fluid velocity, and hence the particle flux is zero near the wall. Equations are also developed empirically to predict the thermophoretic particle deposition efficiency in both the combined developing and combined fully developed cases under laminar flow conditions.

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Appendix A. Derivation of equation for the combined fully developed case

Assuming steady, laminar fluid flow in a circular tube, thermophoretic velocity $V_{th}(r,z)$ in the radial direction is a function of r and z , and the particle equations of motion, Eqs. (10) and (11), can be simplified as

$$\frac{dr}{dt} = V_{th}(r,z), \tag{A.1}$$

$$\frac{dz}{dt} = u(r) = 2u_m \left[1 - \left(\frac{r}{r_0} \right)^2 \right]. \tag{A.2}$$

The critical particle trajectory can be calculated by

$$\int_{r_c}^{r_0} \frac{dr}{V_{th}(r, z)} = \int_0^L \frac{dz}{u(r)}. \quad (\text{A.3})$$

The temperature gradient in the radial direction can be found by the energy equation as

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) = \frac{2u_m}{\alpha} \left(\frac{dT_m}{dz} \right) \left[1 - \left(\frac{r}{r_0} \right)^2 \right] \frac{T_w - T}{T_w - T_m}. \quad (\text{A.4})$$

The mixing-cup temperature distribution is a function of z only and is given by Incropera and De wit (1996) as

$$\frac{T_m(z) - T_w}{T_{in} - T_w} = \exp\left(-\frac{\pi D_t h z}{\rho Q C_p}\right), \quad (\text{A.5})$$

where $h = (Nu_D \times k_g)/D_t$, Nu_D (Nusselt number) = 3.66 for the constant wall temperature condition.

Combining Eq. (A.5) and the invariant fully developed temperature profile, Eq. (7), Eq. (A.4) can be solved analytically to obtain the temperature gradient, and the corresponding thermophoretic velocity can be obtained as the product of two functions: $g(r)$ and $h(z)$, where $g(r)$ depends on r while $h(z)$ depends on z only. Further separation of variable of Eq. (A.3) results in the following dimensionless analytical equation which can be solved to obtain the dimensionless critical radial position, R_c :

$$\int_{R_c}^1 f(R) dR = -Pr K_{th} \ln\left(\frac{T_w}{T_e} + \left(\frac{T_e - T_w}{T_e}\right) \exp\left(\frac{-3.66\alpha L}{u_m r_0^2}\right)\right), \quad (\text{A.6})$$

where

$$f(R) = \frac{1 - R^2}{0.903R - 0.0136R^2 - 1.323R^3 + 0.355R^4 + 0.581R^5 - 0.248R^6}$$

and $R_c = r_c/r_0$ is the dimensionless critical radial position. Once R_c is obtained, the particle thermophoretic deposition efficiency can be calculated.

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