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The small-sized herringbone groove journal bearing (HGJB), i.e., so-called "magnetic bearing," filled with Newtonian ferrofluid lubrication is investigated via finite difference analysis (FDA), with consideration of cavitation zones in HGJB. The FDA starts with constructing the mass flux equations of the HGJB filled with ferrofluid. Discretization for FDA is next performed over the bearing clearance domain, from which algebraic finite difference equations based on the mass flow balance over the clearance domain are derived. Solving the equations, rotordynamic coefficients, cavitation zones, and side leakage rate are successfully predicted to show effectiveness in enhancing bearing performance by ferrofluid. © *2009 American Institute of Physics*.
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I. INTRODUCTION

Small-sized herringbone groove journal bearings (HGJBs), so-called "magnetic bearings," are popular to support motor spindles in hard disk drives. Figure 1 shows a photo of HGJB and its assembly with the rotor. Early analyses of HGJBs with gas lubricated, carried out mainly in the 1960s and 1970s, were based on the narrow groove theory.[1] Recently, Osman *et al.*[2] conducted research on ferrofluid-lubricated bearings with the focus on wire-generated magnetic fields. Chao and Huang[3] used the finite difference method to calculate rotordynamic coefficients of a HGJB with ferrofluid. However, the effects of cavitation on the performance of magnetic bearing have never been considered.

The present study pays effort to model cavitation zones in magnetic HGJB. The investigation starts with building theoretical HGJB models, and followed by finite differencing and soling for solutions.

II. FINITE DIFFERENCE ANALYSIS

For a ferrofluid under magnetic field, the induced magnetic force per unit volume can be described by[4]

$$f_m = (\text{curl } \vec{\mathbf{H}}_m) \times B + \mu_0 M^* \text{ grad } \vec{\mathbf{H}}_m, \tag{1a}$$

where h_m represents the induced free current, μ_0 is the permeability of free space, and X_m satisfies $M^*=X_m h_m$, where M^* is the magnetization vector and strength of the ferrofluid. The ferrofluid are nonconductive and no free currents are induced. The first term in Eq. (1a) can then be canceled. For linear behavior of the magnetic fluid, i.e., $M^*=X_m h_m$, Eq. (1a) can be reduced to

$$f_m = \mu_0 X_m h_m \text{ grad } h_m. \tag{1b}$$

Starting from the basic Navier–Stokes equation, using the coordinates defined in Fig. 2 (x along the circumference while z along the axis of the bearing) and seeing the magnetic force as an external body force, the momentum equations become

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \mu_0 M^* \frac{\partial h_m}{\partial x},$$

$$\frac{\partial p}{\partial y} = 0,$$

$$\frac{\partial p}{\partial z} = \frac{\partial}{\partial y}\left(\mu \frac{\partial w}{\partial y}\right) + \mu_0 M^* \frac{\partial h_m}{\partial z}. \tag{2}$$

Equation (2) is next integrated twice along the ferrofluid film. The fluid velocities can be obtained as

$$u(y) = \frac{1}{2\mu}\left[\frac{\partial p}{\partial x} - f_{mx}\right] y(y-h) + \frac{y}{h} R\omega, \tag{3}$$

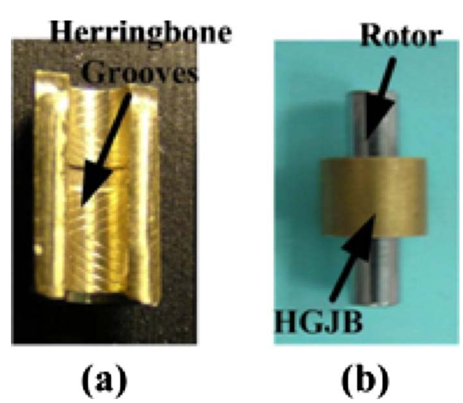

FIG. 1. (Color online) Photo of the small-sized HGJB and (b) assembly of the rotor and HGJB.

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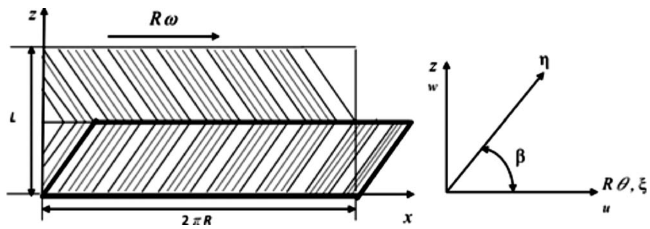

FIG. 2. Domain of herringbone grooves used in finite differences.

$$w(y) = \frac{1}{2\mu}\left[\frac{\partial p}{\partial z} - f_{mz}\right] y(y-h), \tag{4}$$

where $f_{mx}=\mu_0 X_m h_m(\partial h_m/\partial x)$ and $f_{mz}=\mu_0 X_m h_m(\partial h_m/\partial z)$ are magnetic body forces. The mass flux per unit width, that is, fluxes in the x and z directions, can be calculated by $M_x = \rho\int_0^h u(y)dy$ and $M_z=\rho\int_0^h w(y)dy$. Substituting the obtained flux into continuity equation, a modified Reynolds equation is obtained.

The analytical technique developed by Elrod[5] is employed to predict cavitation in HGJB. The cavitation index, g, is adopted, which is generally called a switch function physically while mathematically a unit step function. Furthermore, Elrod introduced a variable θ_c, which is termed the fractional film content and defined as the ratio of the density of the lubricant to the density of the lubricant at the cavitation pressure, i.e.,

$$\theta_c = \frac{\rho}{\rho_c}. \tag{5}$$

Elrod[5] related the density and the fractional film content to the film pressure through the bulk modulus as follows:

$$\beta_b = \rho\frac{dp}{d\rho}. \tag{6}$$

Integration on the above yields

$$p = p_c + g\beta_b \ln \theta_c, \tag{7}$$

where $g=1.0$ when $\theta_c \geqq 1$ (full film region) and $g=0.0$ when $\theta_c<1$ (cavitated region). Substituting Eq. (7) into Eq. (4) and nondimensionaling yield

$$\frac{1}{4\pi}\frac{\partial(\theta_c H)}{\partial\bar{x}} + \frac{\alpha(L/D)^2}{12\pi^2}\left[\frac{\partial(H^3 H_m\theta_c)}{\partial\bar{x}}\frac{\partial H_m}{\partial\bar{x}}\right] + \frac{\alpha}{12}\left[\frac{\partial(H^3 H_m\theta_c)}{\partial\bar{z}}\frac{\partial H_m}{\partial\bar{z}}\right] = \frac{\bar{\beta}_b}{48\pi^2}\left[\frac{\partial}{\partial\bar{x}}\left(gH^3\frac{\partial\theta_c}{\partial\bar{x}}\right)\right] + \frac{\bar{\beta}_b}{48(L/D)^2}\left[\frac{\partial}{\partial\bar{z}}\left(gH^3\frac{\partial\theta_c}{\partial\bar{z}}\right)\right], \tag{8}$$

where dimensionless parameters were designed as follows: $H=h/C$, $\alpha=(h_{m0}{}^2 X_m\mu_o C^2)/(\mu\omega L^2)$, $H_m=h_m/h_{m0}$, $\bar{\beta}_b(\beta_b/\mu\omega)(C/R)^2$, $\theta_c=\rho/\rho_c$, $\bar{x}=x/2\pi R$, and $\bar{z}=z/L$.

The dynamic characteristics of a HFJB are next investigated for chosen steady-state conditions; the process of which is started by perturbing the steady-state position of the journal by the amount of δX and δY, i.e., the journal position, resulting in the following first-order approximations on the film thickness H:

$$H = \gamma + \varepsilon_s\cos(\bar{\theta}-\phi_s) + \delta X\sin\bar{\theta} + \delta Y\sin\bar{\theta}, \tag{9}$$

where $\varepsilon_s=e/C$ is the eccentricity normalized by clearance C, i.e., the so-called eccentricity ratio, ψ_s is the steady-state attitude angle, and γ is defined as the circumferential film thickness normalized by clearance C at the ridge zone of the grooves while the journal and bearing sleeve are in the concentric position. Henceforth, $\gamma=1$ at the ridge zone of the groove and thus $\gamma>1$ at the groove region.

III. NUMERICAL METHOD

The governing equation is transformed from the Cartesian space (x,y) to the computational space (ξ,η), which is a rectangular grid, as shown in Fig. 2. Equation (8) can be rewritten in the computational space. The finite difference analysis based on mass flow balance is next performed to solve for pressure distribution inside the HGJB, while the finite differencing on the shear and pressure-induced flow components will be performed separately. For simplicity, the shear flow term is expressed by

$$\frac{\partial}{\partial\xi}\left(\frac{\xi_x\theta_c H}{J}\right) = \frac{\partial E}{\partial\xi}, \tag{10}$$

where J is the Jacobian of transformation as $J=\xi_x\eta_z-\xi_z\eta_x$. Based the technique of central differencing, one can arrive at for one finite difference grid

$$\frac{\partial}{\partial\xi}\left(\frac{\xi_x\theta_c H}{J}\right)_{i,j} = -\frac{1}{\Delta\xi}\left[\left(\frac{\xi_x H}{J}\right)_{i-1,j}(1-g_{i-1,j})\theta_{ci-1,j} - \left(\frac{\xi_x H}{J}\right)_{i,j}(1-g_{i,j})\theta_{ci,j} + \frac{\left(\frac{\xi_x gH}{J}\right)_{i-1,j}}{2}(2-g_{i-1,j}) + \frac{\left(\frac{\xi_x gH}{J}\right)_{i,j}}{2}(g_{i-1,j}-2+g_{i+1,j}) - \frac{g_{i+1,j}}{2}\left(\frac{\xi_x gH}{J}\right)_{i,j}\right]. \tag{11}$$

The pressure-induced flow is taken care of next for finite differencing, which is

$$\frac{\partial}{\partial\xi}\left(gH^3\frac{\xi_x^2}{J}\frac{\partial\theta_c}{\partial\xi}\right). \tag{12}$$

This pressure-induced flow exists in the full film zone ($\theta_c \geqq 1$), but vanishes in the cavitated zone ($\theta_c<1$). Equation (12) can be treated in a variety of ways, which ensure that it is central differenced in the full film and vanish in the cavitated zone. Consider

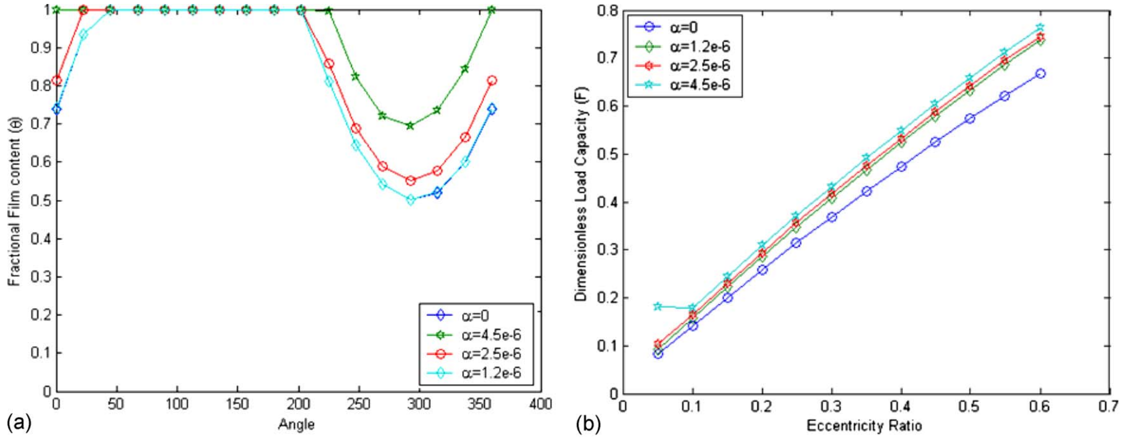

FIG. 3. (Color online) (a) Cavitation zone and (b) load capacity, for different magnetic force coefficients α using the concentric finite wire magnetic model.

$$g\frac{\partial \theta_c}{\partial \xi} = g\frac{\partial(\theta_c - 1)}{\partial \xi} = \frac{\partial g(\theta_c - 1)}{\partial \xi} - (\theta_c - 1)\frac{\partial g}{\partial \xi}, \tag{13}$$

where the last term at the RHS vanishes for all values of θ_c since the spatial derivative of g vanishes for all θ_c except at $\theta_c = 1$. Inserting Eq. (13) into Eq. (12) and central differencing yield

$$\frac{\partial}{\partial \xi}\left(gH^3\frac{\xi_x^2}{J}\frac{\partial \theta_c}{\partial \xi}\right) = \frac{1}{\Delta\xi^2}\left[\left(H^3\frac{\xi_x^2}{J}\right)_{i+1/2,j} g_{i+1,j}(\theta_{ci+1,j} - 1) - (H^3{}_{i+1/2,j} + H^3{}_{i-1/2,j})g_{i,j}(\theta_{ci,j} - 1) + H^3{}_{i-1/2,j}g_{i-1,j}(\theta_{ci-1,j} - 1)\right]. \tag{14}$$

Equations (12) and (15) are ready to be inserted into the original governing Eq. (9) in coordinates (ξ, η), then resulting in the final finite difference equations in the form of

$$A_{i,j}(\theta_c)_{i,j} = A_{i+1,j}(\theta_c)_{i+1,j} + A_{i-1,j}(\theta_c)_{i-1,j} + A_{i,j+1}(\theta_c)_{i,j+1} + A_{i,j-1}(\theta_c)_{i,j-1} + S, \tag{15}$$

where the coefficients $A_{i,j}$, $A_{i+1,j}$, $A_{i-1,j}$, $A_{i,j+1}$, and $A_{i,j-1}$ involve the contributions in ξ and η directions and S is the source term. The above system can be solved by Gaussian elimination method.

IV. RESULTS AND DISCUSSION

Efforts are paid in this section to examine the effects of choosing ferrofluid as media on various performance indices of a HGJB. The parameter values of bearings and magnetic fields considered herein, which are those for a typical small-sized fluid bearing used in precision data-storage drives with three different applied magnetic fields, are shown in Fig. 3. The magnetic field is assumed generated by a finite long wire concentric with the HGJB. The results of the proposed magnetic field models and analysis are obtained for a bearing with a length-to-diameter ratio equal to 1.5. Figure 3 shows the computation results in terms of bearing performance index for different magnetic force coefficient α. As α increases, (1) cavitation zone is moderately decreased, (2) load capacity is significantly increased, and (3) side leakage is significantly decreased. The above results all attribute a better bearing performance.

[1]J. H. Vohr and C. Y. Chow, ASME J. Basic Eng. **87**, 558 (1965).
[2]T. A. Osman, G. S. Nada, and Z. S. Safar, Tribol. Lett. **11**, 61 (2001).
[3]C. P. Chao and J. S. Huang, Tribol. Lett. **19**, 101 (2005).
[4]Q. Hang, S. Chen, S. H. Winoto, and E.-H. Ong, IEEE Trans. Magn. **37**, 2647 (2001).
[5]H. G. Elrod, ASME J. Tribol. **103**, 350 (1981).